SAND 96-0081C
CONF-960682--1

RECEIVED

JAN 16 1996

OSTI

PROBABILITY-BASED STABILITY ROBUSTNESS ASSESSMENT OF CONTROLLED STRUCTURES

R.V. Field, Jr.¹, P.G. Voulgaris, and L.A. Bergman
Department of Aeronautical and Astronautical Engineering
University of Illinois, Urbana, IL 61801

Abstract. Model uncertainty, if ignored, can seriously degrade the performance of an otherwise well-designed control system. If the level of this uncertainty is extreme, the system may even be driven to instability. In the context of structural control, performance degradation and instability imply excessive vibration or even structural failure. Robust control has typically been applied to the issue of model uncertainty through worst-case analyses. These traditional methods include the use of the structured singular value, as applied to the small gain condition, to provide estimates of controller robustness. However, this emphasis on the worst-case scenario has not allowed a probabilistic understanding of robust control.

In this paper an attempt to view controller robustness as a probability measure is presented. The probability of failure due to parametric uncertainty is estimated using first-order reliability methods (FORM). It is demonstrated that this method can provide quite accurate results on the probability of failure of actively controlled structures. Moreover, a comparison of this method to a suitably modified structured singular value robustness analysis in a probabilistic framework is performed. It is shown that FORM is the superior analysis technique when applied to a controlled three degree-of-freedom structure. In addition, the robustness qualities of various active control design schemes such as LQR, H_2 , H_∞ , and μ -synthesis is discussed in order to provide some design guidelines.

Key words. Stability, robustness, control, probability, structures, reliability, failure.

1. Introduction

The distribution of eigenvalues in uncertain dynamical systems and its relationship to the robustness of structural systems have been topics of some interest in recent years, as discussed, for example, by Field *et al.* (1994, 1995a, b), Spencer *et al.* (1992, 1994a, b), and Stengel and Ray (1991, 1993). Stengel and Ray (1991) were among the first to use large-scale Monte Carlo simulation to estimate the robustness of uncertain controlled structural systems. Using this approach, one constructs a distribution of root loci simulating the stochastic behavior of the closed-loop pole locations. Because this is a graphical method, one gains an intuitive understanding of system robustness. The results reported were quite promising, but the large number of realizations required to attain a high degree of accuracy in the distribution of the tails of the closed-loop poles may render this approach computationally unattractive.

In a recent series of papers, however, Spencer *et al.* (1992, 1994a, b), and coworkers have introduced a "systematic approach for determining the probability that instability will result from the uncertainties inherently present in a controlled structure". This probability measure is a direct indication of the robustness of the closed-loop system. In addition, as opposed to the more "brute-force" Monte Carlo simulation approach, this method provides a means of evaluating the reliability of the system directly.

As described in Spencer *et al.* (1994b), this investigation into the probability of failure of controlled structures has led to a method for characterizing the stability of a system based upon an eigenvalue criterion, namely the probability that the real part of every eigenvalue will be contained strictly in the left-half plane. First and second-order reliability methods (FORM/SORM), shown to be accurate for series-type system reliability problems by Madsen *et al.* (1986), were used for estimating the probability of system instability. A series of numerical examples were constructed in which the stability of a controlled single degree-of-freedom system with four uncertain parameters was analyzed.

Traditional methods used to assess controller robustness, involving the use of the singular value of some mapping as applied to a small gain condition, are often conservative in nature. Therefore, the focus of this work involves comparing robustness estimates using the FORM methodology with those estimates obtained using more traditional techniques. In particular, a method is introduced herein to reformulate the robustness measure gained from the structured singular value analysis (*e.g.*, Doyle *et al.* (1991)) into a probabilistic framework. Upon applying these techniques, one then has two different methods to assess the robustness of the closed-loop system.

A realistic application of these methods involves the active control of a structure subject to seismic excitation. Previous work by Field *et al.* (1995b) illustrated the validity of these

1. Currently a member of the Structural Dynamics and Vibration Control Department, Sandia National Laboratories, Albuquerque, NM, 87185-0439.

This work was supported by the United
States Department of Energy under
Contract DE-AC04-94AL85000.

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

MASTER

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

methods for a single degree-of-freedom structural system. Here, a three-story structure, including model uncertainty and subjected to active control strategies, will be considered. Upon comparison to simulation results, one can conclude which method is more effective for this particular class of problems.

2. Problem Definition

Consider the equation of motion for an n -dimensional structure subject to controller time delay. Included is a multiplicative model of the uncertainty in the structure and delay,

$$\mathbf{M}_s(\mathbf{I} + \Delta_m)\ddot{\mathbf{q}}(t) + \mathbf{C}_s(\mathbf{I} + \Delta_c)\dot{\mathbf{q}}(t) + \mathbf{K}_s(\mathbf{I} + \Delta_k)\mathbf{q}(t) = \mathbf{B}_s\mathbf{u}(t - \tau(1 + \delta_\tau)) \quad (1)$$

Here, Δ_m , Δ_c , and Δ_k represent the uncertainty in the mass, damping, and stiffness matrices, respectively, and δ_τ represents the scalar uncertainty in the time delay of the control input. Note that, in general, the Δ_i 's are completely populated, and each element can be modeled as an independent random variable, with mean m_i and standard deviation σ_i .

When applying active control strategies to the structure described above, the robustness of individual control designs can be assessed by considering the various pole locations of the closed-loop system illustrated in Fig. 1.

Fig. 1 Closed-loop system including uncertainty.

Herein, the robustness of several control designs will be assessed using the two different methods, each to be described in the following sections.

3. The FORM Method

A method to assess the robustness of the closed-loop system using first-order reliability methods (FORM) is summarized. The dynamics of the closed-loop system shown in Fig. 1 can be represented by

$$\dot{\mathbf{x}}_{CL} = \mathbf{A}_{CL}(\Delta)\mathbf{x}_{CL}, \quad (2)$$

where Δ is used to represent the uncertainty of the entire closed-loop system and $\mathbf{x}_{CL}$ is the state vector of the closed-loop system. A stability analysis can be completed by examining the eigenvalues of

$$\det[s\mathbf{I} - \mathbf{A}_{CL}(\Delta)] = 0. \quad (3)$$

Stability requires that the real part of every eigenvalue be contained in the open left-half plane (OLHP) or, alternatively, that no one eigenvalue have real part contained in the closed right-half plane (CRHP). This notion of stability can be posed in terms of the probability of instability or probability of failure, p_f , as (Spencer *et al.* (1994b))

$$p_f = P\left[\bigcup_{j=1}^N \text{Re}[\lambda_j(\Delta)] \geq 0\right], \quad (4)$$

where λ_j is the j th eigenvalue of Eq. (3) and N denotes the order of the closed-loop system. Herein, a sufficiently accurate finite-dimensional approximation to the infinite-

dimensional time delay is used when evaluating Eq. (3) (e.g., a Pade' approximation as discussed in Spencer *et al.* (1992)).

Since this expression cannot, in general, be evaluated directly (Spencer *et al.* (1994b)), an alternative approach is to recognize that Eq. (4) is, in fact, the failure condition for a series-type system. The problem can thus be approximated by a series of "components" with limit state functions defined as

$$g_j(\Delta) = -\text{Re}[\lambda_j(\Delta)], \quad j = 1, 2, \dots, N. \quad (5)$$

If any one of the components fails (i.e., $g_j(\Delta) \leq 0$), then the entire system is considered failed. The notion of system failure is shown graphically in Fig. 2 for a three mode system with two random parameters. As illustrated, the system failure region is the union of all modal failure regions, and the number of random quantities defines the dimension of the parameter space.

Fig. 2 Stability region of three-mode system with two uncertain parameters.

The problem can be reformulated in a normalized probability space, assuming p degrees-of-randomness, through the transformation given by Spencer *et al.* (1994b)

$$z_i = T(\delta_i), \quad i = 1, 2, \dots, p, \quad (6)$$

where T is a nonlinear operator defined as

$$T: P \rightarrow N. \quad (7)$$

This transformation is always possible for continuous random variables with invertible distribution functions. Here, δ_i represents the variate in the original parameter space, P , and z_i is the variate in the normalized probability space, N , normally distributed with zero mean and unit variance. Note that the origin of N corresponds to that point where each random quantity takes on its nominal value.

Assuming that the variates are mutually independent, this transformation can be performed on each independently using

$$z_i = T(\delta_i) = \Phi^{-1}[F_{\Delta_i}(\delta_i)], \quad (8)$$

where $\Phi(\cdot)$ represents the standard unit normal distribution function and $F_{\Delta_i}(\cdot)$ is the marginal cumulative distribution function of the i th random variable. Likewise,

$$G_j(\mathbf{Z}) = g_j(T(\Delta)), \quad j = 1, 2, \dots, N, \quad (9)$$

represents the set of identical limit state functions, where $\mathbf{Z}$ depicts the normalized random uncertainties, mapped to N .

Conceptually, the *reliability index*, β , is the minimum Euclidean distance from the origin to the point z^* in N , provided that the design point defines a limit state surface (see Fig. 3). This can be stated mathematically as the solution to the constrained optimization problem (Spencer *et al.* (1994b)),

$$\beta^{FORM} = \min_{G_j(Z)=0} (\min |Z|), \quad j = 1, 2, \dots, N. \quad (10)$$

This leads directly to a first-order approximation to the probability of failure, given by

$$p_f \approx p_f^{FORM} = \Phi(\mp \beta^{FORM}), \quad (11)$$

where the location of the failure surfaces in N determines the sign of β^{FORM} in Eq. (11). Note that the solution to Eq. (11) is the *global* minimum contained in N . For an N -dimensional problem, there will, in theory, be N reliability indices.

When applying Eq. (11) to determine p_f , it is imperative to consider the shape of these limit state functions in N . Highly dependent failure boundaries tend to overlap or nest, causing the failure probabilities of higher-order modes to be subsets of the lower-order modes, as shown in Fig. 3(a). In addition, and quite frequently when considering structural problems, one mode can dominate the contributions of the others (*i.e.*, $\beta_1 \ll \beta_2 < \dots < \beta_N$ as shown in Fig. 3(b)). In these situations, Eq. (11), with $\beta_1 = \beta$, provides an adequate approximation to p_f (Madsen *et al.* (1986)).

Fig. 3 Illustrations of the FORM approximation.

However, when confronted with a multiple-mode system that does not behave as illustrated in the previous figure, Eq. (11) may not provide an adequate estimate of p_f . Other FORM and SORM methods are available to combine reliability information for several modes in a system reliability analysis. See Field *et al.* (1995a, b) for further discussion.

4. The μ -Analysis Method

An alternative route in assigning a probabilistic measure of robustness utilizes some of the tools of deterministic robust control. In particular, analyses based on the notion of the structured singular value, herein termed μ -analysis, can be utilized in a probabilistic manner to provide estimates of the probability of failure.

To perform a μ -analysis, the closed-loop dynamics are represented so that the uncertainty block, Δ , that contains all of the uncertainties present, is extracted as shown in Fig. 4 (see Balas *et al.* (1991) and Doyle *et al.* (1991)).

Fig. 4 Augmented structure used in structured singular value (μ) analysis.

The Δ block is in feedback with the (stable) nominal closed-loop system H , where the latter is produced by the interconnection of the nominal plant, $P(s)$, and the controller, $K(s)$. The nominal plant, however, can be represented in an augmented form, denoted by $P_{aug}(s)$ and given by

$$\dot{x} = Ax + B_1 w_r + B_2 u \quad (12)$$

$$z_r = C_1 x + D_{11} w_r + D_{12} u \quad (13)$$

$$y = C_2 x + D_{21} w_r + D_{22} u, \quad (14)$$

where the matrices A , B_2 , C_2 , and D_{22} correspond to the state-space description of the original plant, $P(s)$, and x and u are the state and input vectors, respectively. The vectors w_r and z_r , connect the nominal closed-loop system, H , with the uncertainty block, Δ . The remaining matrices B_1 , C_1 , D_{11} , D_{12} , and D_{21} are required to augment the original plant to be compatible with these additional inputs and outputs.

Robust stability of the closed-loop system depends on the size of $H(j\omega)$. For the structural system given by Eq. (1), the uncertainty block can be represented as

$$\Delta = \begin{bmatrix} \Delta_m & 0 & 0 & 0 \\ 0 & \Delta_c & 0 & 0 \\ 0 & 0 & \Delta_k & 0 \\ 0 & 0 & 0 & \Delta_\tau \end{bmatrix}, \quad (15)$$

where Δ_c in Eq. (15) is a (stable) error due to the uncertainty in the time delay. For the framework presented herein, this delay uncertainty can be represented as

$$\Delta_\tau(j\omega) = e^{-j\omega|\delta_d|} - 1. \quad (16)$$

This allows the uncertainty in controller time delay to be bounded by a rational transfer function (see Field *et al.* (1995b)). The structured singular value of a complex matrix

M , associated with the structure of Δ , is defined as (Doyle *et al.* (1991))

$$\mu[M] = \begin{cases} 0 & \inf\{\bar{\sigma}[Q]: \det(I-QM)=0\} = 0 \\ \frac{1}{\inf\{\bar{\sigma}[Q]: \det(I-QM)=0\}} & \text{else} \end{cases}, \quad (17)$$

where Q is any complex matrix with structure identical to that of Δ (i.e., block diagonal of the same dimensions), and $\bar{\sigma}[Q]$ is the maximum singular value of Q . Assuming that the nominal closed-loop system, H , is stable, a condition for robust stability can be given as

$$\sup_{\omega} \{\mu[H(j\omega)]\bar{\sigma}[\Delta(j\omega)]\} < 1, \quad (18)$$

where μ is the structured singular value of $H(j\omega)$.

This structured singular value analysis can be reformulated in a probabilistic framework. A probability estimate of the stability of the closed-loop system is simply the probability that Eq. (18) is satisfied. Hence, the probability of instability, or failure, in this μ -analysis framework, can be given as

$$P_f^{\mu} = 1 - P(\sup_{\omega} \{\mu[H(j\omega)]\bar{\sigma}[\Delta(j\omega)]\} < 1). \quad (19)$$

This method may provide conservative estimates of controller robustness for several reasons (Balas *et al.* (1991)). If Eq. (18) is satisfied, the closed-loop system is stable for all admissible uncertainty. Violation of Eq. (18), however, does not imply that the system is unstable since it is only a *sufficient* condition for stability. It can be shown that violation of Eq. (18) *does imply* that there exists a perturbation system, $\Delta(s)$, possibly dynamic, with size $\|\Delta\|_{\infty}$ at most equal to $(\mu[H(j\omega^*)])^{-1}$, where ω^* is that frequency where violation of Eq. (18) occurs. Nonetheless, it is not specified how *probable* this destabilizing $\Delta(s)$ may be in the operating space of the system, leading to a "worst-case" assessment of the robustness of the closed-loop system. In addition, such an uncertainty may be totally dynamic, which is immediately excluded here since mixed (i.e., real parametric and dynamic) uncertainty is considered. Finally, the computation of μ , as shown in Eq. (17), can be done, in general, only approximately by computing an upper bound (Balas *et al.* (1991) and Doyle *et al.* (1991)),

$$\mu[M] \leq \inf_D \bar{\sigma}[DMD^{-1}], \quad (20)$$

where D is any invertible matrix that commutes with the uncertainty block, Δ . The presence of these features will contribute to the conservativeness of the μ -analysis method.

5. Applications

As discussed in Field *et al.* (1995b), the FORM and μ -analysis methods have been successfully applied to assess the robustness characteristics of a controlled single degree-of-freedom structure. Consider now a three-story, single-bay building, as described by Chung *et al.* (1989). The structure is constrained to a single degree-of-freedom at each story, and active tendon control is applied at the first floor. Figure 5 illustrates this model.

The governing equations of motion are

Fig. 5 Three degree-of-freedom structure with active tendon control.

$$\begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix} \ddot{q}(t) + \begin{bmatrix} c_1 + c_2 & -c_2 & 0 \\ -c_2 & c_2 + c_3 & -c_3 \\ 0 & -c_3 & c_3 \end{bmatrix} \dot{q}(t) + \begin{bmatrix} k_1 + k_2 & -k_2 & 0 \\ -k_2 & k_2 + k_3 & -k_3 \\ 0 & -k_3 & k_3 \end{bmatrix} q(t) = \begin{bmatrix} -4k_c \cos \alpha \\ 0 \\ 0 \end{bmatrix} u(t - \tau) + \begin{bmatrix} m_1 \\ m_2 \\ m_3 \end{bmatrix} \ddot{d}(t), \quad (21)$$

where m_i , c_i , k_i , and q_i are the mass, damping, stiffness, and displacement associated with the i th floor of the building, and u is the position of the controlling actuator, subjected to controller time delay. Table 1 lists the nominal, or mean, values for this structure.

Table 1: Model parameters of the 3DOF structure.

Parameter	Mean, m
m_1 (lb-s ² /in)	5.6
m_2 (lb-s ² /in)	5.6
m_3 (lb-s ² /in)	5.6
c_1 (lb-s/in)	2.6
c_2 (lb-s/in)	6.3
c_3 (lb-s/in)	0.35
k_1 (lb/in)	5,034
k_2 (lb/in)	10,965
k_3 (lb/in)	6,135
τ (ms)	36
k_c (lb/in)	2,124
α (degrees)	36

To assess the robustness of the closed-loop system, consider the original equation of motion, rewritten to include the model uncertainty

$$\begin{aligned}
& \begin{bmatrix} m_{m_1} & 0 & 0 \\ 0 & m_{m_2} & 0 \\ 0 & 0 & m_{m_3} \end{bmatrix} + \begin{bmatrix} \delta_{m_1} m_{m_1} & 0 & 0 \\ 0 & \delta_{m_2} m_{m_2} & 0 \\ 0 & 0 & \delta_{m_3} m_{m_3} \end{bmatrix} q(t) \\
& + \begin{bmatrix} m_{c_1} + m_{c_2} & -m_{c_2} & 0 \\ -m_{c_2} & m_{c_2} + m_{c_3} & -m_{c_3} \\ 0 & -m_{c_3} & m_{c_3} \end{bmatrix} + \begin{bmatrix} \delta_{c_1} m_{c_1} + \delta_{c_2} m_{c_2} & -\delta_{c_2} m_{c_2} & 0 \\ -\delta_{c_2} m_{c_2} & \delta_{c_2} m_{c_2} + \delta_{c_3} m_{c_3} & -\delta_{c_3} m_{c_3} \\ 0 & -\delta_{c_3} m_{c_3} & \delta_{c_3} m_{c_3} \end{bmatrix} q(t) \\
& + \begin{bmatrix} m_{k_1} + m_{k_2} & -m_{k_2} & 0 \\ -m_{k_2} & m_{k_2} + m_{k_3} & -m_{k_3} \\ 0 & -m_{k_3} & m_{k_3} \end{bmatrix} + \begin{bmatrix} \delta_{k_1} m_{k_1} + \delta_{k_2} m_{k_2} & -\delta_{k_2} m_{k_2} & 0 \\ -\delta_{k_2} m_{k_2} & \delta_{k_2} m_{k_2} + \delta_{k_3} m_{k_3} & -\delta_{k_3} m_{k_3} \\ 0 & -\delta_{k_3} m_{k_3} & \delta_{k_3} m_{k_3} \end{bmatrix} q(t) \\
& = \begin{bmatrix} -4k_c \cos \alpha \\ 0 \\ 0 \end{bmatrix} u(t - \tau(1 + \delta_\tau)).
\end{aligned} \tag{22}$$

Utilizing a similar procedure as for the single degree-of-freedom example discussed in Field *et al.* (1995b), it can be shown that the constrained optimization problem used in the FORM method is given by

$$\begin{aligned}
\beta^{FORM} &= \min \sqrt{z_{m_1}^2 + z_{m_2}^2 + \dots + z_{\tau}^2} \\
\text{Re}(\lambda_1) &= 0 \\
\text{Re}(\lambda_j) &\leq 0, j = 2, 3, \dots, N
\end{aligned} \tag{23}$$

To assess controller robustness using the μ -analysis method, the distributions of the 10 random quantities (the m_i 's, c_i 's, k_i 's, and τ) must be specified. While any combination of distribution functions may be used here, only *identical uniform* distributions will be considered. Therefore, when formulating the Δ block, as illustrated in Fig. 6, the actual parameter uncertainty is weighted so that each $|\delta_i|$ is independent and uniformly distributed on $[-1, 1]$.

The robustness of several control schemes, including LQR, H_2 , H_∞ , and μ -synthesis will be assessed. The H_2 , H_∞ , and μ -synthesis designs were performed using a third-order Pade' approximation to the nominal time delay, and the LQR controller was designed based upon a nominal system with zero delay. For a detailed discussion of these designs, refer to Field *et al.* (1995b).

Robustness estimates of the controlled 3DOF structure are presented in Table 2. To facilitate the discussion, define the maximum structured singular value as

$$\bar{\mu} = \sup_{\omega} \{\mu[H(j\omega)]\}. \tag{24}$$

The large values (greater than 1) of $\bar{\mu}$ for the first three designs render the μ -analysis method virtually useless for probabilistic robustness assessment. This was not the case when considering a similar problem with only four random variables (see Field *et al.* (1995b)). Here, however, note that

Fig. 6 Augmented structure used in μ -analysis of three degree-of-freedom structure.

this is the case even with a relatively small level of uncertainty (*i.e.*, 5% in each of the ten random variables) since it can be shown that the probability of stability is proportional to $\bar{\mu}^{-9}$. Hence, it is possible to conclude that the μ -analysis method becomes unsuitable when studying systems exhibiting a high degree of randomness (*i.e.*, many random quantities). In contrast, FORM predictions are much more accurate, despite the relatively large order of the closed-loop system. Note that the order of the closed-loop system is 9 for LQR, 18 for both H_2 and H_∞ , and 96 for μ -synthesis designs.

When considering the robustness of the individual controllers, similar results hold as in the SDOF problem. The robustness properties of the LQR control are superior to both H_2 and H_∞ designs. In addition, the H_2 and H_∞ controllers exhibit fairly unrobust behavior. The introduction of the μ -synthesis design, however, significantly improves the robustness of the closed-loop system.

6. Conclusions

Traditional methods used to assess controller robustness may sometimes have precluded a probabilistic understanding of robust control. In this paper, methods to assess robustness as a probability measure have been presented. In addition to the valuable insight gained concerning robust stability of the closed-loop system, with the use of these methods one can

Table 2: Robustness estimates using FORM and μ , COV = 5%.

Controller	$\bar{\mu}$	β^{FORM}	P_f^μ	P_f^{FORM}	P_f^{MCS}
LQR	3.701	∞	1.0	0.0	0.0
H_2	8.626	0.0716	1.0	0.3991	0.2572
H_∞	8.501	0.1145	1.0	0.5456	0.5420
μ -synthesis	0.9065	∞	0.0	0.0	0.0

classify the robustness characteristics of various control designs for the specific problem at hand.

The first-order reliability method (FORM), as used herein to estimate robustness, is an approximation that relies upon two assumptions. First, the failure surface defining the onset of instability must be fairly linear. If it is not, the first-order curve fit to the failure surface may become inadequate, requiring a second-order fit (SORM). Second, the most probable failure condition must be sufficiently governed by a single mode. Highly uncorrelated failure modes may lead to inaccuracies of the FORM approximation, and more sophisticated FORM/SORM methods may be needed. However, for the problems considered herein, robustness estimates using the FORM method were quite adequate. This may be attributed to the high level of correlation between failure modes, typical of a structural system.

The use of the structured singular value to assess controller robustness proved to be conservative when applied in a probabilistic framework. The main premise of this method operates only upon the maximum structured singular value and, therefore, evaluates the "worst-case" scenario. In addition, the fundamental robust stability criterion used is valid for a broad class of uncertainties. The examples presented herein assumed only real parametric uncertainty, a small subclass. As a result, although μ -analysis can typically predict which controller is most robust, the predicted probability of failure can be quite conservative. Therefore, it is possible to conclude that the FORM method provides superior robustness estimates when applied to structural control problems.

When considering the various control schemes, the obvious choice is one that utilizes state feedback. In all cases, the LQR design possessed very good robustness qualities. However, this ideal situation rarely occurs in structural applications because the state vector may be difficult or impossible to measure. When output feedback is required by implementability, the robustness of the closed-loop system generated by the nominally optimal H_∞ and H_2 controllers may be poor, particularly if there exists any uncertainty in controller time delay. Nonetheless, the situation can be rectified with minimal nominal performance degradation by utilizing μ -synthesis design techniques. These methods allow the inclusion of model uncertainty into the design process. As a result, although the μ -analysis presented herein gave conservative quantitative results, μ -synthesis controllers can be used to dramatically improve the robustness of nominally optimal output feedback designs. Controller reduction techniques may be required since the resulting controller may be of unacceptably high order. This remains the subject of future research.

References

1. Balas, G., J. Doyle, K. Glover, A. Packard and R. Smith (1991). μ -Analysis and Synthesis Toolbox, MUSYN Inc. and The MathWorks, Inc.
2. Chung, L., R. Lin and T. Soong (1989). "Experimental Study of Active Control for MDOF Seismic Structures," *Journal of Engineering Mechanics*, Vol. 115, No. 8, pp. 1609-1627.
3. Doyle, J., A. Packard and K. Zhou (1991). "Review of LFT's, LMI's, and μ ," *Proceedings of the 30th IEEE Conference on Decision and Control*, England, pp. 1227-1232.

4. Field, R.V. Jr., W. Hall and L. Bergman (1994). "A MATLAB[®]-Based Approach to the Computation of Probabilistic Stability Measures for Controlled Systems," *Proceedings of the First World Conference on Structural Control*, Los Angeles, CA, Vol. 2, pp. 13-22.
5. Field, R.V. Jr., L. Bergman and W. Hall (1995a). "Computation of Probabilistic Stability Measures for a Controlled Distributed Parameter System," *Journal of Probabilistic Engineering Mechanics*, Vol. 10, No. 3, pp. 181-189.
6. Field, R.V. Jr., P.G. Voulgaris and L.A. Bergman (1995b). "Probabilistic Stability Robustness of Structural Systems," *ASCE Journal of Dynamics and Control*, in press.
7. Maciejowski, J. (1989). *Multivariable Feedback Design*, Addison-Wesley Publishing, New York, NY.
8. Madsen, H., S. Krenk and N. Lind (1986). *Methods of Structural Safety*, Prentice Hall, Englewood Cliffs, NJ.
9. Ray, L. and R. Stengel (1993). "A Monte Carlo Approach to the Analysis of Control System Robustness," *Automatica*, Vol. 29, No. 1, pp. 229-236.
10. Sain, P., B. Spencer, M. Sain and J. Suwardjo (1992). "Structural Control Design in the Presence of Time Delay," *Proceedings of the 9th ASCE Engineering Mechanics Conference*, College Station, TX, May 25-27.
11. Spencer, B., M. Sain, J. Kantor and C. Montemagno (1992). "Probabilistic Stability Measures for Controlled Structures Subject to Real Parameter Uncertainties," *Smart Materials and Structures*, Vol. 1, pp. 294-305.
12. Spencer, B., M. Sain and J. Kantor (1994a). "Reliability-Based Measures of Stability for Actively Controlled Structures," *Structural Safety and Reliability*, Schueller, Shinozuka and Yao (eds.), Balkema, Rotterdam, pp. 1591-1598.
13. Spencer, B., M. Sain, C-H. Won, D. Kaspari and D. Sain (1994b). "Reliability-Based Measures of Structural Control Robustness," *Structural Safety*, Vol. 15.
14. Stengel, R. and L. Ray (1991). "Stochastic Robustness of Linear Time-Invariant Control Systems," *IEEE Transactions on Automatic Control*, Vol. 36, No. 1, pp. 82-87.

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.