Dense Inclined Flows: Theory and Experiments

Final Report

December 1, 1995

by

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Contract number: DE-AC22-91PC90183

Contract Period of Performance: January 1, 1991 to December 31, 1995

DOE Project Manager: Sean Plasynski

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1. Introduction

Rapid, gravity-driven flows of granular materials down inclines pose a challenge to our understanding. Even in situations in which the flow is steady and two-dimensional, the details of how momentum and energy are balanced within the flow and at the bottom boundary are not well understood. Thus we have undertaken a research program integrating theory, computer simulation, and experiment that focuses on such flows.

The effort involves the development of theory informed by the results of simultaneous computer simulations and the construction, instrumentation, and use of an experimental facility in which the variables necessary to assess the success or failure of the theory can be measured.

This work is part of DOE's Solid Transport Program. Its goal is to provide a sound theoretical and experimental base for a better understanding of the behavior and properties of multiphase flow and solid transport.

The project has involved the participation of several graduate students. The following summarizes their respective contributions. Chao Zhang developed constitutive relations for frictional, nearly elastic spheres. Eb Askari obtained solutions to boundary value for steady shearing flows.

Hongder Chang, Samuel Foerster and Mark Tuccio constructed and designed the chute facility. Samuel Foerster obtained measurements of particle impact properties. After his graduation, two undergraduate students, Adam Lorenz and Chris Tuozzolo, continued his experiments. In the last two years of the project, Steve Keast was hired as a research technician to complete construction of the chute facility and its instrumentation and to assist in the corresponding experiments.

Our theoretical approach is to derive constitutive relations and to solve boundary value problems for rapid granular assemblies moving down inclined planes. Because computer simulations can be interrogated in far greater detail than experiments, we generally employ simulations to verify the predictions of theory and to guide its development. Because the theory requires a specific model for the impacts between grains, an essential aspect of our project has been to verify experimentally the validity of such a model for the small spheres commonly used in rapid granular flows. Finally, experiments in the chute facility have identified flow regimes where rapid granular theories are likely to succeed and where they should be supplemented by other treatments based on enduring contacts among grains.

We begin the present report with a description of the collision experiments for small spheres. We follow this with an outline of the development of constitutive relations for frictional collisions in the interior of the flow. Next, we describe our efforts to verify the form of existing boundary conditions through computer simulations. We then discuss boundary value problems arising in flows down inclined planes. Finally, we outline our experimental activities with the chute facility and summarize its data pending a more complete publication of experimental results.

2. Measurement of Particle Impact Properties

Theories of rapid granular flows adopt a model for a collision between a pair of spheres, then proceed to calculate the average properties of the flow using appropriate velocity distribution functions. In order to keep the corresponding integrations tractable, such theories employ a simple model of individual collisions based on constant impact coefficients, rather than describing the evolution of each collision in detail.

In this context, Jenkins calculates the velocities of the sphere emerging from a collision by considering the balance of linear and angular momenta in the collision. The usual coefficient of restitution e characterizes the incomplete restitution of the normal component of $\mathbf{g}$, the relative velocity of the spheres at the contact point,

$$\mathbf{n} \cdot \mathbf{g}' = -e \mathbf{n} \cdot \mathbf{g}, \quad (1)$$

where $0 \leq e \leq 1$, $\mathbf{n}$ is the unit normal vector joining the centers of the spheres at contact, and primes denote conditions after the collision.

In collisions that involve sliding, the sliding is assumed to be resisted by Coulomb friction and the tangential and normal components of the impulse J are related by the coefficient of friction μ ,

$$|\mathbf{n} \times \mathbf{J}| = \mu (\mathbf{n} \cdot \mathbf{J}), \quad (2)$$

where $\mu \geq 0$. As the angle between $\mathbf{g}$ and $\mathbf{n}$ increases, sliding stops and

$$\mathbf{n} \times \mathbf{g}' = -\beta_0 \mathbf{n} \times \mathbf{g}, \quad (3)$$

where $0 \leq \beta_0 \leq 1$ is the tangential coefficient of restitution. The idea is that some of the elastic strain energy stored in the solid during impact is recoverable, so the tangential velocity of the point of contact may be reversed.

Before carrying out a meaningful test of the theory in a granular flow experiment, it is essential to verify that the impact coefficients employed in the theory provide an adequate description of the collision and, if so, to determine their values. Unfortunately, because their measurement involves the precise control of the trajectories of small spheres, these coefficients were never previously measured. Thus, comparisons with the theory generally relied on assumed values of the impact coefficients.

In this context, we have developed an experimental apparatus that measures the collision properties of spheres as small as 1.5mm in diameter. The resulting impact coefficients now permit unequivocal comparisons between rapid granular theories and experiments, and they justify the form of the simple impact model that Jenkins employs. The apparatus includes a mechanism that brings two identical particles into a collision without initial spin, and a stroboscopic setup that photographs the dynamics of their flights. The setup can also produce impacts between a single sphere and a flat plate. The details of the apparatus, the experiments and the data interpretation are provided by Foerster, et al (1994). A typical photograph is shown in Fig. 1.

Fig. 1. Typical photographs with acetate spheres of 6mm diameter.

A convenient way to interpret the data is to follow Maw, Barber and Fawcett (1976, 1981) and produce a plot of $\Psi_2 \equiv -(\mathbf{g}' \cdot \mathbf{t})/(\mathbf{g} \cdot \mathbf{n})$ versus $\Psi_1 \equiv -(\mathbf{g} \cdot \mathbf{t})/(\mathbf{g} \cdot \mathbf{n})$, where $\mathbf{t}$ is a unit vector located in the collision plane $(\mathbf{g}, \mathbf{n})$ and tangent to both spheres. In collisions that involve gross sliding,

$$\Psi_2 = \Psi_1 - \frac{7}{2}(1+e)\mu \text{ sign}(\mathbf{g} \cdot \mathbf{t}); \quad (4)$$

and in collisions that do not,

$$\Psi_2 = -\beta_0 \Psi_1. \quad (5)$$

Fig. 2. Results for binary collisions of 3mm glass spheres. The dashed line is a least-squares fit of the data through (4) and (5). The solid line is the corresponding prediction of the model of Maw, Barber and Fawcett. The insert is an enlarged view of the region where sticking contacts occur.

Figure 2 shows typical experimental results for collisions of two identical glass spheres of 3mm diameter. These results clearly show that the model based on three constant coefficients (dashed lines) is an adequate representation of the individual impacts.

Table 1 summarizes impact parameters for other grains used by GFARO participants and other scientists involved in granular flows. Details of the corresponding experiments are provided by Lorenz, Tuozzolo and Louge (1995) and Louge, Tuozzolo and Lorenz (1995).

3. Constitutive Relations for Frictional Collisions

The purpose of this activity was to use methods from the kinetic theory of dense gases to formulate balance laws and constitutive relations for frictional, nearly elastic spheres. Our desire was to incorporate the collision

model discussed in the previous section in a theory that was as simple as possible in structure but that captured the essence of the physics.

To this end, we restricted our attention to nearly elastic collisions of slightly frictional spheres in order to insure that the distribution of translational velocities does not differ so much from that for smooth spheres and, in particular, to insure that the deviatoric part of the second moment of the translational velocity fluctuations is small relative to its isotropic part. In this case, the mean fields of interest are the mass density; the mean translational velocity and the mean angular velocity, about which the actual particle velocities fluctuate; and the granular temperature and the spin temperature that measure the energy per unit mass of the fluctuations in translational and angular velocity, respectively.

The balance laws for mass, linear momentum, and the translational energy have the familiar local forms. For slightly frictional spheres, the balance of angular momentum reduces to the requirement that the mean spin of the spheres be equal to half the vorticity of their mean velocity. Numerical simulations of steady shearing flows of more frictional spheres indicate that this relationship between the mean spin and the mean vorticity is satisfied everywhere but in the immediate neighborhood of the boundary. We assume that it applies everywhere in a flow and satisfy the balance of angular momentum approximately in this way. Finally, we satisfy the balance of spin energy approximately by requiring that the net rate of production of the energy of the fluctuations in angular velocity is zero. In this event, the rate at which translational energy is converted to spin energy is balanced by the rate at which spin energy is dissipated in collisions.

With the assumptions that collisions do not dissipate much energy and that the mean spin is equal to half the mean vorticity, the influence of friction on the collisional transfer of linear momentum and translational energy is negligible and the stress and the translational energy flux are identical to those for smooth, elastic spheres. At this order of approximation, only the rates of dissipation of translational and rotational energy are influenced by friction. In order to calculate these, we assume that the fluctuations in spin obey a Gaussian distribution with the spin temperature as its standard deviation. Then the rate of decrease of translational fluctuation energy per unit volume and the rate of decrease of spin fluctuation energy per unit volume may be calculated (Zhang, 1993). When the equation of balance of rotational energy is approximately satisfied by setting the latter equal to zero, the rotational energy is determined in terms of the translational energy. When this determination is used in the expression for the rate of dissipation of translational energy, an effective coefficient of restitution may be calculated. For slightly frictional, nearly elastic grains, this is the only modification of the theory for frictionless grains that need be made. The predictions of the resulting theory are in good agreement with numerical

simulations of simple shear carried out by Lun and Bent (1993) in a periodic cell for spheres with $\mu = 0.123$, $\beta_0 = 0.40$, $e = 0.93$.

4. Boundary Conditions at a Flat, Frictional Wall

Recently, Jenkins (1992) developed the first theory for rapid granular flows interacting with a flat, frictional wall. To this end, he focused his attention on collisions and incorporated Coulomb friction into the collisional interactions with the wall. He employed a simple velocity distribution function to integrate the collisional impulse and change in fluctuation energy over all possible collisions in, and near, two limits.

In the first limit, the coefficient of friction is so large that the particles do not slide upon contact with the wall. In the second, the coefficient is so small that all collisions involve sliding. In that work, Jenkins derived boundary conditions that provide the ratio S/N of shear to normal stress exerted by the particles onto the wall and the flux Q of granular fluctuating energy transferred through the wall. The flux is normalized by the product $N\sqrt{3T}$ of the normal stress N and the square root of the translational granular temperature T at the wall. In flow regimes where particles interact with the wall through collisions, the present results constitute a set of boundary conditions that may be incorporated in models of the particle phase.

In this project our objective was to test Jenkins' theory through computer simulations. In particular, we examined the assumptions that Jenkins employed to render his calculations tractable, and we explored a wide range of collision properties for the wall and the flow particles.

Fig. 3. Sketch of the periodic Couette cell arrangement. The shaded hemispheres of the top wall thermostat move at the average velocity U shown by the arrow.

The numerical simulations are carried out using an algorithm described by Hopkins and Louge (1991). Here we consider flows of identical, homogeneous, inelastic, frictional spheres in a periodic Couette geometry bounded by a flat, frictional wall and a two-dimensional, smooth array of hemispheres colliding and moving on a plane parallel to the flat wall (Fig. 3). The hemispheres of the top wall are massive enough to be unaffected by collisions with interior particles and their mutual interactions do not dissipate energy. In this case, their two-dimensional granular temperature T_0 is conserved, so they behave like an adjustable "thermostat" that serves to energize the particles in the interior.

Fig. 4. Variations of the dynamic friction coefficient S/N with μ for $e=0.9$, $\beta_0=0$. The abscissa is the normalized slip g_0 at the point of contact $r \equiv g_0/\sqrt{3T}$. The squares, diamonds, circles and triangles are $\mu=0.1, 0.2, 0.3$ and 0.4 , respectively. The dashed lines represent Jenkins' predictions in the "Large Friction/No Sliding" and the "Small Friction/All Sliding" limits, respectively.

Typical results of the simulations are shown in Fig. 4. There the ratio S/N exhibits two asymptotic limits correctly predicted by Jenkins. Other results are published by Louge (1994). They include the behavior of the energy flux in the low and high friction regimes, the histograms of the velocity distribution functions at the wall, the behavior of the normal stress and the collision frequency, the structure of the flow near the wall, and the role of the spin and of its fluctuations.

Informed by these simulations, Jenkins and Louge (1995) recently refined the calculations of Jenkins (1992) for the flux of fluctuating energy and showed that a correlation between two orthogonal components of the fluctuating velocities of the points of contact of the grains with the wall provides a substantial correction to the flux originally predicted. The new expression for the flux agreed well with the computer simulations.

5. Boundary Value Problems

The simplest theory for dense flows of slightly frictional, nearly elastic spheres has been employed to solve two types of boundary value problems: steady, fully developed flows with a free surface, driven by gravity down a bumpy, frictional incline; and steady, fully developed shearing flows driven by the relative motion of two parallel, bumpy, frictional boundaries. The latter is meant to model a relative narrow region of intense shear with collisional interactions at the base of an otherwise passive heap. The former is fully fluidized throughout its depth. Experiments by Johnson, Nott, and Jackson (1990) indicate that the flows involving the greatest mass flux are those supported upon a basal shear layer. These evolve from the fully fluidized flows as the depth of the flow increases.

In shear flows driven between parallel plates, gravity may be neglected when the weight of the material over a unit area of the bottom is a small fraction of the collisional pressure. We usually adopt this simplification; in this case, both the shear stress and the pressure are constant over the gap. Here we suppose that the top and bottom boundaries are identical flat plates to which spheres of the flow have been attached. Both the spheres of the flow and those on the boundary are nearly elastic and slightly frictional. Boundary conditions for the bumpy frictional boundary are obtained by the rather rough expedient of adding the stress and energy fluxes for a flat frictional wall and a bumpy frictionless boundary. We work in either of the two frictional limits. The balance of momentum at the wall determines the slip of the flow relative to the wall and the corresponding energy balance gives the flux of fluctuation energy as the difference between the rate of working of the slip velocity and the rate of collisional dissipation.

Fig. 5. The ratio of shear to normal stress, normalized by its value in simple shear, versus nondimensional velocity (Froude number) for a range of friction coefficients of the flow particles.

Depending on their geometry and properties, boundaries are sources or sinks of fluctuation energy. The balance of energy in the interior of the gap, used with the effective coefficient of restitution for frictional spheres, determines the fluctuation velocity of the particles; the mean velocity and volume fraction then follow from the constitutive relations for the shear stress and pressure. In order that the flow be steady, the balance of fluctuation energy and momentum at the boundaries must be consistent with that in the interior. Consequently, the boundary conditions provide relations between the shear stress, the pressure, the thickness of the gap, and the relative velocity of the plates that we must obtain if the flow is to be steady. These relations are of great interest to us because they provide the overall characteristics of the shear layer, for example, how the shear stress supported by the gap depends upon the relative velocity of the boundaries. In Fig. 5, we show such relations for a typical gap.

The analysis of a steady, fully developed flow down an incline is similar, except that gravity can not be ignored and the upper boundary is a free surface. At the free surface, the fluxes of momentum and fluctuation energy are assumed to vanish in such a way that the ratio of shear to normal stress remains constant. In Figure 6 we show the relationship between the height of the flow and tangent of the angle of inclination that results from the solution of the boundary value problem for a bottom boundary and a range of inclinations employed in the University of Florida chute.

Fig. 6 The height of a steady, fully developed flow down an incline, normalized by particle diameter, over a range of stress ratio (tangent of the angle of chute inclination) for the particles and the bottom boundary of the Florida chute.

6. Experiments in the Inclined Flow Facility

We have constructed and instrumented an inclined flow facility where grains may flow steadily at angles down to 25° from the horizontal. The inclined channel has a length of 3.6m that permits a variety of fully-developed flows. Its width of 20cm minimizes the effects of side walls. Its bottom surface is a smooth aluminum plate of 7mm thickness or a bumpy boundary of random packings of 1mm beads. The grains are collected at the bottom of the facility and continuously recycled by a corrugated conveyor belt to a hopper at the top. A small chamber located below the hopper controls the energy of the grains entering the inclined channel. Several possible entry conditions are achieved by varying the vertical drop between the chamber and the inclined plane.

Measurements of global flow parameters include the mass flow rate through the facility and the mass holdup at selected locations in the fully-developed regions of the flow. The former is recorded by weighing the solids diverted into a container during a known time. Following Johnson, Nott and Jackson (1990), the flow rate m^+ is made dimensionless with the width of the channel and the product $\rho_s \sigma \sqrt{g\sigma}$. For dilute flows, the holdup H^+ is inferred from the mass of solids trapped in a cylindrical box promptly traversed through the depth of the flow. It scales with the product of the cross section of the trapping device, the material density ρ_s of the grains and their diameter σ ,

$$H^+ \equiv \int v \, d(z/\sigma). \quad (6)$$

For denser flows, we designed a capacitance instrument producing a voltage output that is a linear function of the instantaneous mass holdup integral H^+ below the probe. This instrument revealed the presence of traveling roll waves in flows over the flat boundary (Fig. 7). We measured the speed and length of these waves for all conditions where they appeared. We are now in the process of analyzing these results. We also employed three similar capacitance probes to record the variations of mass holdup along the length of the chute. Although the measurements occasionally revealed substantial variations of H^+ along the length, the corresponding mean acceleration was always a negligible component of the overall balance of forces in any vertical section of the flow.

Fig. 7. Mass holdup waves observed by the capacitance instrument at a chute inclination of 18° and $m^+ = 10$.

The collisional contribution to the normal stress exerted by the flow on the bottom wall was measured with a small piezo-electric load cell capable of recording small normal impact forces transmitted through a 3mm metal post mounted flush with the chute surface. Calibrations with individual beads in free-fall related the recorded peak force to the normal velocity at impact. From the individual impulses $J = -m(1+e) c \cdot n$, we calculated the contribution of individual collisions to the total normal stress N . Assuming negligible mean flow velocity in the direction normal to the chute, the weight of the column of particles is balanced by the normal stress force applied by the wall on the grain assembly,

$$N = \rho_s g \cos \alpha \int v dz = \rho_s g \cos \alpha \sigma H^+, \quad (7)$$

where α is the angle of inclination of the chute. Thus, a plot of the ratio $N / \rho_s g \cos \alpha \sigma H^+$ is a measure of the fraction of the weight of the granular assembly that is supported by collisional impulses at the base. As Fig. 8 illustrates, the fraction is often close to one, and it generally grows with mass holdup. This indicates that dense flows over a flat, frictional boundary are primarily supported by collisions with the base. In contrast, we found that flows over the 1mm bumpy boundary are dominated by interactions with the base other than collisional.

Fig. 8. Fraction of the weight of the granular assembly supported by collisional impulses at the base of the flow.

Another indication of the role of collisions is provided by our measurements of the granular temperature at the base. A robust way to determine the component of the temperature Θ_n in the normal direction is to build statistics of the collision frequency and the normal velocity at contact. Assuming negligible mean velocity in the normal direction, the balance of forces in (7) can be written in terms of the equation of state of the grain assembly at the wall,

$$N = \rho_s v g_{12}(0) \Theta_n (1+e) = \rho_s g \sigma H^\dagger \cos \alpha, \quad (8)$$

where $g_{12}(0)$ is the spatial distribution function at contact. This equation therefore predicts that the granular temperature should increase linearly with mass holdup. The excellent agreement of Fig. 8 with this prediction confirms the validity of our impact measurement at the base.

Fig. 8. Normal component of the granular temperature against mass holdup for relatively dense flows.

In our experiments, we employed monodisperse, nearly spherical glass beads of 3mm diameter (Fig. 9). Using the method summarized in section 2, we measured the impact coefficients to be $e=0.97$, $\mu=0.092$ and $\beta_0=0.44$ for binary collisions and $e=0.83$, $\mu=0.125$ and $\beta_0=0.31$ for collisions with smooth aluminum. With these beads, our facility has nearly the same relative length (L/σ) and width (W/σ) as that of Johnson, Nott and Jackson (1990), who employed smaller 1mm beads in a smaller chute. Our measurements exhibit similar trends than previously observed by these authors, although the agreement is not quantitative. In particular, we could not reproduce the two distinct branches in the $(H^\dagger, m^\dagger)$ diagram for “dense” and “energetic” entries that Johnson, Nott and Jackson had observed.

Fig. 9. Mass flow rate versus mass holdup for selected angles of inclination. The horizontal error bars represent the amplitude of the holdup waves. The vertical error bars are the corresponding statistical standard deviation of the measured mass flow rate.

7. Hydraulic Theory

We can attempt to model the narrow region of sheared, colliding particles at the base of an agitated but otherwise passive heap of grains whose free surface may be changing shape as the velocity of its base varies with space and time. We assume that the flow is traveling in the x direction down a slope with an angle of inclination $\phi(x)$, that the velocity at the top of the shear layer (the bottom of the heap) is $U(x,t)$, that the y axis is normal to the base and has its origin at the bottom of the shear layer, and that the shape of the free surface of the heap is given by $y = H(x,t)$. We ignore variations of the flow in the z direction and assume that the variations of ϕ , U , and H with x are slow compared to changes with y across the shear layer. In this case, the integrated forms of the balance of mass and momentum governing the evolution of U and H in space and time are (e.g. Whitham, 1974):

$$\frac{\partial H}{\partial t} + \frac{\partial}{\partial x}(UH) = 0, \quad (9)$$

and

$$\frac{\partial U}{\partial t} + U \frac{\partial U}{\partial x} + g \cos \phi \left(\frac{\partial H}{\partial x} - \tan \phi \right) = - \frac{S}{\bar{\rho} H}, \quad (10)$$

where $\bar{\rho}$ is the average mass density of the heap, here assumed to be constant. We assume that the thickness L of the shear layer is a small fraction of the height H of the heap. In this event, the relationship between the pressure p exerted by the heap and the height of the heap is

$$p = \bar{\rho} g H \cos \phi. \quad (11)$$

In order to close equations (9) and (10), we must relate S to U and H or, equivalently, to U and p . An example of such a relation between U , H , and ϕ can be obtained from the data relating mass flow rate to mass holdup at fixed angles of inclinations for the steady fully-developed flows of Johnson, Nott, and Jackson (1990).

Finally, we are now working on developing a simple hydraulic model to capture the global behavior of the flow shown in Fig. 9 and the formation of waves described in section 6. Preliminary calculations indicate that the flow over a flat, frictional plane is governed by the combination of collisional interactions and more static interparticle contacts. It is this balance that permits the establishment of flows at angles of inclination greater than $\arctg(\mu)$ that do not keep on accelerating. The formation of waves appears to be the instantaneous manifestation of this balance.

8. Conclusions

At Cornell, funding from the Granular Flow Advanced Research Objective has resulted in a number of accomplishments in theory, computer simulations and experiments on the subject of dense, inclined flows. These include

- The measurement of the impact properties of small spheres.
- The derivation of constitutive relations for frictional collisions.
- The verification of theoretical predictions for the interaction with a flat, frictional wall through computer simulations.
- The solution of boundary value problems for shear flows and fluidized inclined flows.
- The construction and instrumentation of an inclined flow facility.
- Measurements of mass holdup, mass flow rate and basal granular temperature in the facility over flat and bumpy boundaries; characterization of roll waves and interpretation of their presence in the flow.

The following publications have so far resulted from this project; at the request of the Department of Energy, they are appended to the present report:

- Louge, M.Y., Jenkins, J.T. & Hopkins, M.A.: "The relaxation of the second moments in rapid shear flows of smooth disks" *Mechanics of Materials*, **16** (1993) 199-203. © Elsevier Science, Inc.; reproduced by permission.
- Hopkins, M.A., Jenkins, J.T. & Louge, M.Y.: "On the structure of 3D shear flows" *Mechanics of Materials*, **16** (1993) 179-187. © Elsevier Science, Inc.; reproduced by permission.
- Jenkins, J.T. & Hanes, D.M.: "The balance of momentum and energy at an interface between colliding and freely flying grains in a rapid granular flow" *Phys. Fluids A* **5**, 781-783 (1993). © American Institute of Physics; reproduced by permission.
- Jenkins, J.T. & Askari, E.: "Rapid granular shear flows driven by identical, bumpy, frictionless boundaries" Reprinted from: Thornton, C. (ed.), *Powder & Grains 93* -Proceedings of the second international conference on micro-mechanics of granular media, Aston University, Birmingham, UK, 12-16 July 1993. 1993. 485 pp., Hfl.230/US\$130.00. © A.A. Balkema, Old Post Road, Brookfield, Vermont 05036 (telephone: 802-276-3162; telefax: 802-276-3837).
- Zhang, C.: "Kinetic Theory for Rapid Granular Flow" Ph.D. Dissertation, Cornell University, August, 1993.
- Foerster, S. F.: "Measurements of Particle Collision Properties," M.S. Thesis, Cornell University, May 1993.
- Jenkins, J.T.: "Hydraulic Theory for a Debris Flow Supported on a Collisional Shear Layer" in *Proc. Int. Cong. Hyd.. Res.*, IAHR:Tokyo (1993).
- Jenkins, J.T. and Louge M.Y.: "Boundary Conditions for Rapid Flows of Inelastic Frictional Spheres In Sliding Collisions with a Flat Frictional Wall", *Fluidization and Fluid-Particle Systems*, Special Supplement to the Fall Annual Meeting, AIChE, NY (1994), p. 7. © AIChE; reproduced by permission.
- Jenkins, J.T.: "Rapid Granular Flow down Inclines", in *Mechanics USA* 1994, A.S. Kobayashi, ed., *Appl. Mech. Rev.* **47** (6), part 2, June 1994. © ASME; reproduced by permission.
- Jenkins, J.T.: "Rapid Granular Flow" in *Research Directions in Fluid Mechanics* (Lumley, J.L., et al., Eds.) National Academy of Engineering: Washington, D.C., in press. © American Institute of Physics; reproduced by permission.
- Foerster, S. F., Louge, M. Y., Chang, H. & Allia, K.: "Measurements of the collision properties of small spheres" *Phys. Fluids* **6**(3), 1108-1115 (1994). © American Institute of Physics; reproduced by permission.

- Louge, M. Y.: "Computer simulations of rapid granular flows of spheres interacting with a flat, frictional boundary" *Phys. Fluids* 6 (7), 2253-2269 (1994). © American Institute of Physics; reproduced by permission.
- Louge, M., Tuccio, M. and Lander, E.: "Capacitance measurements of the volume fraction and velocity of dielectric solids near a grounded wall", *Rev. Sci. Instrum.* (1995), in press.
- Louge, M.Y.: US Patent 5,459,406 "Guarded Capacitance Probes for Measuring Particle Concentration and Flow," October 17, 1995.

In addition, the following papers are currently in preparation or under review:

- Lorenz, A., Tuozzolo, C. and Louge, M.: "Measurements of impact properties of small, nearly spherical particles", *Experimental Mechanics*(1995), under review.
- Jenkins, J.T. and Zhang C.: "Kinetic Theory for Identical, Slightly Frictional, Nearly Elastic Spheres," *Phys. Fluids* (1996), in press. © American Institute of Physics; reproduced by permission.
- Jenkins, J.T. and Louge M.Y.: "Boundary conditions for rapid flows of inelastic frictional spheres interacting with a flat frictional wall", *Phys. Fluids* (1995), in preparation.
- Louge, M., Tuozzolo, C. and Lorenz, A.: "On the impact of two small liquid-filled shells", *Physics of Fluids* (1995), in preparation.
- Louge, M. and Keast S.: "Inclined flows of grains: waves, collisional stresses and granular temperature at the base", *Phys. Fluids* (1996), in preparation.

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Table 1

Particle Material	Collisions of	Particle Diameter (mm)	Impact Parameters			Relative contact velocities (m/sec)			
			e	μ	β_0	lg.nl		lg.tl	
Glass	Two Spheres	3.18	0.97 ± 0.01	0.092 ± 0.006	0.44 ± 0.07	0.64	1.2	0.06	0.41
Glass	Sphere on Smooth Aluminum Plate	3.18	0.831 ± 0.009	0.125 ± 0.007	0.31 ± 0.06	1.0	1.7	0.24	0.81
Cellulose Acetate	Two Spheres	5.99	0.87 ± 0.02	0.25 ± 0.02	0.43 ± 0.06	0.29	1.2	0.14	0.86
Cellulose Acetate	Sphere on Smooth Aluminum Plate	5.99	0.891 ± 0.003	0.208 ± 0.007	0.39 ± 0.07	0.67	1.7	0.06	1.2
Polystyrene	Two Spheres	4.00	0.952 ± 0.009	0.189 ± 0.009	0.46 ± 0.05	0.34	1.23	0.11	1.1
Acrylic	Two Spheres	4.00	0.934 ± 0.009	0.096 ± 0.006	0.22 ± 0.07	0.35	1.25	0.07	1.14
Stainless Steel	Two Spheres	5.00	0.95 ± 0.03	0.099 ± 0.008	0.32 ± 0.08	0.17	1.2	0.15	1.2
Fresh Glass*	Two Beads	2.97 ± 0.02	0.922 ± 0.021	0.048 ± 0.006	0.37 ± 0.07	0.43	1.14	0.07	1.08
Spent Glass*	Two Beads	2.97 ± 0.02	0.972 ± 0.015	0.177 ± 0.020	0.25 ± 0.08	0.24	1.11	0.14	1.1
Fresh Glass*	Bead on Smooth Aluminum Plate	2.97 ± 0.02	0.816 ± 0.013	0.131 ± 0.007	0.46 ± 0.24	0.80	1.92	0.07	1.64
Spent Glass*	Bead on Spent Aluminum Plate	2.97 ± 0.02	0.800 ± 0.010	0.141 ± 0.009	0.35 ± 0.31	0.91	1.94	0.07	1.53
Spent Glass*	Bead on Spent Bumpy Boundary	2.97 ± 0.02	0.865 ± 0.011	0.126 ± 0.014	0.34 ± 0.16	0.73	1.67	0.29	1.51
Vitamin-E pill†	Two beads	3.95 ± 0.20	0.89 ± 0.07	not applicable	0	0.37	1.16	0.09	1.02

* Used in the Cornell and Florida chutes.

† Used in the rotating drum experiments at Lovelace.