44
4-26-96 JSC

UNIVERSITY OF WISCONSIN

CENTER FOR PLASMA THEORY AND COMPUTATION

REPORT

Evolution Of Toroidal Flow During, After Mode Locking

M. Yokoyama, J.D. Callen and C.C. Hegna*

Department of Nuclear Engineering & Engineering Physics
University of Wisconsin-Madison
Madison, Wisconsin 53706-1687

November 1995

UW-CPTC 95-4

*Permanent address: Plasma Physics Laboratory, Kyoto University, Japan

MASTER

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED *BS*

MADISON, WISCONSIN 53706-1687

Evolution Of Toroidal Flow During, After Mode Locking

M.Yokoyama¹, J.D.Callen and C.C.Hegna

Nuclear Engineering & Engineering Physics Department,
University of Wisconsin, Madison, WI 53706-1687

Abstract

The response of the toroidal flow velocity to the abrupt locking of it in the vicinity of a magnetohydrodynamic (MHD) tearing-mode-induced magnetic island is examined analytically and numerically using a diffusive transport model in a cylindrical plasma model. The resultant toroidal momentum confinement is shown to degrade significantly after mode locking, as is often observed on many tokamaks and reversed field pinches (RFPs). The degraded toroidal momentum confinement time in the core and edge regions of the plasma are evaluated and compared to experimental observations. The mode locking time scale itself is also investigated by using a model toroidal torque balance equation. The decrease of mode frequency during mode locking seems to be governed mainly by the electromagnetic torque exerted on the resistive layer. For tokamak plasmas, the mode locking time scale is found to be much shorter than the diffusion time scale, which is in general agreement with experimental observations.

1 Introduction

Substantial degradation of plasma stored energy and toroidal ion momentum confinement have been observed on many tokamaks when MHD tearing-type modes are present [1-6]. A model for estimating the modest effects of these modes on energy confinement has been developed by Chang and Callen [7] assuming that the magnetic island region has much larger “perpendicular” heat diffusivity compared to the no magnetic island case. This describes the fact that the “radial” flow of plasma particles and energy are “short-circuited” through the island region by following the separatrix magnetic field lines. This enhanced diffusivity effect of the magnetic island causes a

¹Permanent address: Plasma Physics Laboratory, Kyoto University, Gokasho, Uji, 611, Japan

flattening [7] of the temperature profile in that region and a modest uniform temperature drop in the core plasma region.

In many tokamaks, the major disruption is preceded by coherent $(m, n) = (2, 1)$ magnetic fluctuations, where $m(n)$ is the poloidal (toroidal) mode number, which are apparently caused by a helical magnetic island resonant at the $q = 2$ magnetic surface in the plasma. In some cases these $(2, 1)$ modes lock to external structures such as a resistive wall or error fields, and cease oscillation and hence toroidal rotation. This process is called mode locking. In these cases, since the mode frequency is predominantly due to bulk plasma rotation, the toroidal flow velocity in the vicinity of a rational surface (especially, the $q = 2$ surface) abruptly drops to zero. Therefore, a significant degradation of toroidal momentum confinement is expected to occur. Since the momentum drops over the entire plasma volume, the momentum confinement degradation is much larger than the energy confinement degradation, in which the temperature profile is just flattened in the vicinity of the magnetic island rather than being forced to zero. Zohm et al. first analyzed the toroidal momentum loss on ASDEX [4] due to mode locking, based on a toroidal momentum balance including the force exerted by neutral beam injection (NBI) on the rotating magnetic mode structure.

In this report we evaluate the diffusive transport response of the toroidal flow velocity profile to mode locking and determine the relevant toroidal momentum confinement time for such a situation on tokamaks and reversed field pinches (RFPs). We adopt a cylindrical model which approximates a large aspect ratio, low β tokamak and RFP plasmas, although detailed geometrical factors such as toroidicity and plasma shaping (elongation, triangularity) effects have to be taken into account for a more accurate analysis [7].

Typically, the mode locking process seems to occur on a much shorter time scale than the energy diffusion time scale in tokamaks, e.g., in JET [1]. The mode locking process has been investigated theoretically by modelling the interaction of tearing modes with external structures such as a resistive wall or resonant error fields [8]; its relevant time scales have been discussed by several authors [9,10]. In Ref. [9], the plasma inertia in the island region is balanced against the decelerating electromagnetic (EM) torque taking the magnetic island evolution into account. On the other hand, in Ref. [10], the locking time scale is estimated by balancing the viscous (VC) torque against the maximum EM torque and the obtained time scale implies that it is much less than the global viscous diffusion time scale. In this report we consider

a driven reconnection case as an example of mode locking and estimate the mode locking time scale for this model.

This report is organized as follows. In Section 2 we discuss the diffusive transport response of the toroidal flow velocity profile to mode locking and determine the toroidal momentum confinement time, which is compared with experimental data from JET [3] and the Madison Symmetric Torus (MST) RFP [11] in a rudimentary way. Also, the momentum confinement time scale is predicted for DIII-D. In Section 3 the mode locking time scale is considered based on a driven reconnection toroidal torque balance in the vicinity of a resistive layer. The time scale obtained indicates that the mode locking occurs on a much shorter time scale compared to the diffusion process, which is similar to the results obtained by Ref. [10]. Finally, a summary of this work is given in Section 4.

2 Diffusive Transport Response of the Toroidal Flow Velocity to Mode Locking

2.1 Model description

We use a time dependent fluid momentum equation in a cylindrical plasma model. The response of the toroidal momentum in the presence of a locked mode can be described by the equation

$$\rho_m \frac{\partial V_\zeta}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} r \rho_m \chi_\zeta \frac{\partial V_\zeta}{\partial r} + F(r, t), \quad (1)$$

where ρ_m is the mass density and χ_ζ is a perpendicular viscous diffusion coefficient (e.g., for classical processes, see Ref. [12]). The force term $F(r, t)$ is the sum of two terms, $F_0(r)$ and $\delta F(r, t)$ which describe the equilibrium momentum input into plasmas, from, for example, NBI, and an EM torque due to the localized singular current in the vicinity of a resistive layer [13], which is responsible for the mode locking [8].

First, the diffusive transport response of the toroidal flow velocity profile to an abrupt mode locking event is investigated. This is modelled by setting the toroidal velocity to zero at a rational surface or within the magnetic island region associated with the mode locking. Before mode locking, the

steady state toroidal flow velocity profile is determined by integrating Eq. (1), which yields for a given momentum input

$$V_\zeta(r) = \int_r^a \frac{dr}{r \rho_m \chi_\zeta} \int_0^r F_0(r) r dr, \quad (2)$$

assuming boundary conditions $V_\zeta(a) = 0$ and $dV_\zeta/dr|_{r=0} = 0$. The mode locking is modelled by introducing $\delta F(r, t)$ in the region of the magnetic island, which is now excluded for considering the diffusive response after mode locking. After mode locking, the plasma is separated into three regions as follows:

$$\begin{aligned} \text{[A]:} & \quad \text{core} & (r < r_s - w/2), \\ \text{[B]:} & \quad \text{magnetic island} & (r_s - w/2 < r < r_s + w/2), \\ \text{[C]:} & \quad \text{edge} & (r_s + w/2 < r < a), \end{aligned}$$

where r_s is the radial position of the rational surface of the $q(r_s) = m/n$ mode, and w is the magnetic island width. Below we will write $r_s - w/2$ as r_- and $r_s + w/2$ as r_+ for simplicity. The mode locking causes $V_\zeta(r) = 0$ in region [B] and affects regions [A] and [C] diffusively through the boundary conditions

$$\begin{aligned} \text{[A]:} & \quad \frac{dV_\zeta(0)}{dr} = 0, \quad V_\zeta(r_-) = 0, \\ \text{[C]:} & \quad V_\zeta(r_+) = 0, \quad V_\zeta(a) = 0. \end{aligned}$$

The condition $V_\zeta(a) = 0$ implies that the toroidal flow velocity is clamped to zero at the plasma edge. In steady state, the toroidal velocity profile after mode locking is given by

$$\begin{aligned} \text{[A]:} & \quad V_\zeta(r) = \int_r^{r_-} \frac{1}{r \rho_m \chi_\zeta} \int_0^r F_0(r) r dr, \\ \text{[C]:} & \quad V_\zeta(r) = \{g(a)f(r) - f(a)g(r)\}/f(a), \end{aligned}$$

where

$$f(r) = \int_{r_+}^r \frac{dr}{r \rho_m \chi_\zeta}, \quad g(r) = \int_{r_+}^r \frac{dr}{r \rho_m \chi_\zeta} \int_{r_+}^r F_0(r) r dr.$$

Figure 1 shows the steady state toroidal flow velocity profile for: (a) before and (b) after mode locking for the case of $r_s/a = 0.75$ and $w/a =$

0.10. It is assumed here, for simplicity, that ρ_m, χ_ζ, F are constant in space. Figure 1 demonstrates a drastic loss of toroidal flow velocity and a significant degradation in the toroidal momentum confinement after mode locking.

Chang and Callen proposed a model to describe energy confinement degradation due to macroscopic MHD modes [7]. Reasonable agreement with this model has been obtained for JET [3], TFTR [6] and COMPASS-C [14] discharges. Here, the same procedure is applied for the toroidal momentum balance, Eq. (1), assuming steady state. Following the formulation of Ref. [7], we obtain for the toroidal momentum confinement time τ_ζ

$$\tau_\zeta = \frac{M}{[F_{in}]_{eff}}, \quad (3)$$

where M is the stored toroidal momentum and $[F_{in}]_{eff}$ is the effective force, analogous to the effective energy input in Ref. [7], which exerts torque on the plasma. Typically mode locking occurs despite continued auxiliary heating such as NBI and ion cyclotron radiofrequency wave heating (ICRH) in JET [3]; therefore, the relative amount of stored toroidal momentum is a reasonable measure of the toroidal momentum confinement time degradation assuming $[F_{in}]_{eff}$ is approximately the same before and after mode locking. The ratio of the stored toroidal momentum before (M_0) and after (M_w) mode locking can be obtained by integrating over the toroidal flow velocity profile and is written as

$$\frac{M_w}{M_0} = \frac{\int_0^{r_-} v(r, r_-) r dr + \int_{r_+}^a \frac{1}{f(a)} \{g(a)f(r) - f(a)g(r)\} r dr}{\int_0^a v(r, a) r dr}, \quad (4)$$

where $v(r, x) = \int_r^x \frac{dr}{r \rho_m \chi_\zeta} \int_0^r F_0(r) r dr$. Figure 2 shows this ratio versus normalized rational surface position r_s/a for several island widths ($w/a = 0.0, 0.1$ and 0.2), assuming ρ_m, χ_ζ, F_0 are constant in space. The largest degradation occurs for the case of $r_s/a \sim 0.5$ and in that case $M_w/M_0 < 0.2$, even when the magnetic island width is negligible.

We have focussed on the steady state solution of Eq. (1) so far. Next, we address the dynamical evolution of the toroidal flow velocity. The effect of abrupt mode locking on the toroidal flow velocity profile can be taken into account by imposing initial conditions on Eq. (1) as follows:

$$\begin{aligned} [A]: \quad & V_\zeta(r, 0) = V_\zeta^0(r) - V_\zeta^0(r_-) \delta(r - r_-), \\ [C]: \quad & V_\zeta(r, 0) = V_\zeta^0(r) - V_\zeta^0(r_+) \delta(r - r_+), \end{aligned}$$

with the same boundary conditions as imposed for considering the steady state solutions. Here, $V_\zeta^0(r)$ is the toroidal flow velocity profile just before mode locking. Again assuming for simplicity that ρ_m, χ_ζ are constant, and solving the resultant eigenvalue problem for regions [A] and [C] yields the solution for region [A],

$$V_\zeta(r, t) = \sum_{n=1}^{\infty} \left[\int_0^t e^{-\chi_\zeta \lambda_n(t-\tau)} \bar{f}_n(\tau) d\tau \right. \\ \left. + \frac{2}{r_-^2 J_1^2(j_n)} \left\{ \int_0^{r_-} [V_\zeta(r) - V_\zeta^0(r_-) \delta(r_-)] J_0(j_n \frac{r}{r_-}) r dr \right\} e^{-\chi_\zeta \lambda_n t} \right] J_0(j_n \frac{r}{r_-}),$$

where

$$\bar{f}_n(t) \equiv \frac{2}{r_-^2 J_1^2(j_n)} \int_0^{r_-} \frac{F(r, t)}{\rho_m} J_0(j_n \frac{r}{r_-}) r dr$$

and the j_n 's are the zeros of the ordinary Bessel function of the first kind, $J_0(j_n) = 0$, and $\lambda_n = (\frac{j_n}{r_-})^2$. Therefore, the toroidal momentum confinement time in region [A] is

$$\tau_{\zeta[A]} = \frac{1}{\lambda_1 \chi_\zeta} = \left(\frac{a^2}{j_1^2 \chi_\zeta} \right) \left(\frac{r_-^2}{a^2} \right) = \tau_{\zeta 0} \left(\frac{r_-}{a} \right)^2, \quad (5)$$

where $j_1 \sim 2.405$ and $\tau_{\zeta 0}$ is the "normal" confinement time before mode locking in the cylindrical plasma model. Equation (5) implies that the momentum confinement time degradation in region [A] corresponds to a reduction in the cylindrical radius of the plasma from a to r_- — a plasma "shrinkage" effect. For region [C], not only the Bessel function of the first kind J_0 but also the second kind Y_0 could be a basis for the eigenfunction solution of Eq. (1). Thus, the first eigenvalue λ_1 determining the time scale should be obtained from the relation

$$J_0(p_1 \frac{r_+}{a}) Y_0(p_1) - J_0(p_1) Y_0(p_1 \frac{r_+}{a}) = 0, \quad (6)$$

which satisfies the boundary conditions for region [C], and here $\lambda_1 = (\frac{p_1}{a})^2$. The p_1 values for several r_+/a cases are listed in Table I. The momentum confinement time for region [C] is

$$\tau_{\zeta[C]} = \frac{a^2}{p_1^2 \chi_\zeta} = \left(\frac{a^2}{j_1^2 \chi_\zeta} \right) \left(\frac{j_1^2}{p_1^2} \right) = \tau_{\zeta 0} \left(\frac{j_1}{p_1} \right)^2. \quad (7)$$

The large degradation in momentum confinement as r_+ approaches a is expected for this region — because then the thickness of the plasma layer outside the island decreases. These analytical calculations give a basis for estimating the toroidal momentum confinement time degradation in the core and edge plasma regions.

2.2 Tokamak mode locking case

The simple analytic estimate of momentum confinement time degradation described in subsection 2.1 is now applied to explain the ion momentum loss behavior in tokamak experiments. Toroidal flow velocity or rotation frequency measurements have been made in JET [3] and DIII-D [5] in order to study the effects of mode locking. Ion toroidal flow rotation is usually only observed for at most a few hundred milliseconds after mode locking in JET, which implies that the ion momentum is lost on such a time scale and continues to be rapidly lost in the presence of a locked mode. Figure 6 in Ref. [3] shows the time evolution of the radial profile of the ion (C^{6+}) toroidal rotation frequency during mode locking measured in discharge, JET#16066. In this discharge, a large sawtooth collapse at about 13.5 s drives a large oscillating (2,1) mode, which slows down and locks at about 13.79 s. The ion rotation frequency profile nearly vanishes at about 200 ms after mode locking. It is also noted that an independent measurement of the Ni^{26+} ion toroidal rotation frequency near the plasma center (20 ms sampling time) comes to rest in about 120 ms after mode locking. The $q = 2$ surface was calculated to be located at about 3.9 m ($r_s/a \sim 0.75$). Also the toroidal flow drops somewhat more rapidly in the outer edge of the plasma.

The “normal” toroidal momentum confinement time $\tau_{\zeta 0}$ is given by

$$\tau_{\zeta 0} = \frac{a^2}{j_1^2 \chi_\zeta} \sim \frac{a^2}{6 \chi_\zeta},$$

which is about 240 ms for JET ($a \sim 1.2$ m), where χ_ζ is assumed to be 1 m²/s. From Eqs. (5) and (7), the momentum confinement time for regions [A] and [C] are predicted to be $\tau_{\zeta[A]} \sim 135$ ms and $\tau_{\zeta[C]} \sim 9$ ms. The e-folding time scale of the C^{6+} ion toroidal rotation frequency can be estimated to be less than 160 ms (13.79 s $\sim$ 13.95 s) from Ref. [3] and this is reasonably close to $\tau_{\zeta[A]}$, which describes the momentum confinement degradation for

the bulk of the plasma. The ratio of the stored toroidal momentum, M_w/M_0 , is about $0.3 \sim 0.4$ from Fig. 2 for $r_s/a \sim 0.75$ and $w/a \leq 0.1$ in this case; this implies that the global momentum confinement time would be degraded to $240 \times (0.3 \sim 0.4) \sim (70 \sim 100)$ ms, which is also comparable to the JET experimental result [3]. It should be noted that these time scales are to be multiplied by an elongation factor κ when we consider the elongation of the plasma cross section.

To exhibit the dynamical evolution of the toroidal flow velocity clearly, the full time-dependent Eq. (1) has been solved numerically. The mode locking phenomenon is modelled by the abrupt setting of the toroidal flow velocity to zero inside the magnetic island region. Here, the magnetic island is assumed to have a width of $w/a = 0.1$ centered at $r_s/a = 0.75$ (as it is in the JET case [3]). Figure 3 shows the time evolution of the calculated radial profile of the toroidal flow velocity. Here, the initial profile is assumed to be parabolic in radius, as in Fig. 1(a), and $F(r, t)$ is taken to be the equilibrium force just before mode locking. Again, ρ_m, χ_ζ are taken to be constant to make the comparison with analytical results simpler. Comparing the profile at $t = 600$ ms and Fig. 1(b), it can be said that the toroidal flow velocity profile has almost reached a new equilibrium with the locked island by a few hundred milliseconds after mode locking. This is seen much more clearly in Fig. 4, where the time evolution of the toroidal flow velocity at the magnetic axis for region [A] and at $r = (a + r_+)/2$ for region [C] are shown versus time after mode locking. From these figures, it is reasonable to consider that the toroidal momentum confinement time after mode locking is of order 100 ms, which agrees well with the time scale obtained by the eigenvalue analysis. It should be noted that the toroidal momentum confinement degrades very rapidly (~ 10 ms) and significantly in the edge region, which again agrees with the analytical results both qualitatively and quantitatively.

We briefly consider a DIII-D case. From both analytic and numerical calculations, we predict that the toroidal momentum confinement time degrades from $\tau_{\zeta 0} \sim 70$ ms to about 50 ms when we assume a minor radius of 0.65 m and that the $q = 2$ magnetic island is located at $r_s/a \sim 0.78$ [5] with an island width of about $w/a \sim 0.1$.

2.3 RFP mode locking case

In the MST RFP [15], mode locking occurs in response to error fields [11,16] in a similar manner to mode locking in tokamaks. In the MST, field errors develop dynamically in a localized gap of the conducting shell as a result of sawteeth. The phase velocities of a number of pre-existing tearing modes are observed to drop to zero at the sawtooth crash due to the mode locking with the dynamically induced field errors. Additionally, spectroscopic measurements indicate that a dramatic drop in the bulk plasma toroidal flow velocity coincides with the mode locking event. Although the field error development is novel in the MST plasma, the mode locking and toroidal flow properties can also be modelled in the same manner as the tokamak case.

Here we apply the diffusive transport model to the MST mode locking case to clarify the dynamics of the toroidal flow velocity profile during mode locking in the presence of several tearing modes. The time evolution of the calculated radial profile of the toroidal flow velocity is shown in Fig. 5. The initial profile is rather flat in the region of $r/a \leq 0.7$ due to the mode-mode phase locking of $m = 1, n = 5 - 10$ tearing modes [11]. Here we assume $\chi_\zeta = 1 \text{ m}^2/\text{s}$ for illustration purposes. It is also assumed that each tearing-mode-induced magnetic island has a negligibly small width compared to the plasma minor radius to show the dynamical behavior of the toroidal flow velocity clearly in the presence of several tearing modes. It should be noted that neither of these assumptions is valid for the MST, where it is believed that the tearing mode induced islands are overlapped and correspondingly the anomalous transport is larger than for the tokamak case. However, we will use these assumptions as an example for multiple-island locking effects, and note that the anticipated toroidal momentum confinement degradation for the MST occurs in a similar fashion.

It can be clearly seen from Fig. 5 that the toroidal flow velocity decays very rapidly (on the time scale of less than 1 ms) in a rather wide plasma region around $r/a \sim 0.5$, where the $m = 1, n = 6 - 9$ tearing-mode-induced magnetic islands are overlapping [11]. It is noted again that we assume that the magnetic island width is negligibly small in this numerical calculation. Therefore, this result implies that the plasma region where the islands are overlapping experimentally is subject to significant degradation of the toroidal momentum confinement even if their island widths are negligibly small. It may be equivalent to consider the mode locking of one magnetic

island with a large width ($w/a \sim 0.5$). Thus, the toroidal momentum confinement property after mode locking in the MST is mainly governed by the behavior of the toroidal flow velocity in the core and edge plasma regions, as in tokamak case.

According to the eigenvalue analysis [Eqs. (5) and (7)] and the estimated radial location of the (1,5) and (1,10) magnetic islands in the MST [11], $\tau_{\zeta[A]}$ and $\tau_{\zeta[C]}$ are about 1.8 ms and 3.6 ms, respectively, which are much shorter than “normal” toroidal momentum confinement time $\tau_{\zeta 0}$ of about 45 ms. Here, [A] refers to the region inside the $q = 1/5$ surface and [C] to outside the $q = 1/10$ surface. It should be noted that the degraded toroidal momentum confinement times for the core and edge regions are almost the same due to the existence of a (1,5) island far inside the plasma ($r_-/a \sim 0.2$). In this numerical calculation, $F(r, t)$ in Eq. (1) is taken to be the equilibrium force just before mode locking. The toroidal flow velocity profile in the core region reaches zero at about 30 ms after mode locking because the assumed equilibrium force is zero in this region. On the other hand, it reaches a new equilibrium by 20 ms after mode locking in the edge region with a level of $V_{\zeta} \sim 0.4$ in this figure. Therefore, 2 ms and 4 ms are reasonable time scales for the e -folding of the toroidal flow velocity profile for the core and edge regions as expected from the eigenvalue analysis, for an assumed $\chi_{\zeta} = 1 \text{ m}^2/\text{s}$.

Both the mode frequency and the toroidal flow velocity have been observed to decelerate in about 100 μs during mode locking in the MST [11]. This deceleration of the mode can be expected from a calculation of the EM torque on the magnetic islands; however, a simple calculation of the Braginskii viscous diffusion [12] of the plasma predicts a much longer ($\sim 130 \text{ ms}$) deceleration time scale for the toroidal flow velocity [11]. We can resolve this discrepancy between the naive prediction and the observed deceleration time scale of the toroidal flow velocity, by accounting for the presence of a number of $m = 1$ locked islands in the diffusive toroidal momentum transport model calculation. In particular, our numerical and analytical calculation demonstrate a much shorter deceleration time scale than the one predicted in Ref. [11]. Therefore, it can be said that the mode locking of $m = 1$ islands with large extent in the plasma cross section causes a deteriorious effect on the toroidal momentum confinement in the MST.

Finally, it should be noted that the assumed toroidal momentum diffusivity of $1 \text{ m}^2/\text{s}$ is probably an underestimate, since the momentum diffusivity

is usually of the order of the ion diffusivity, which is believed to be an order of magnitude larger in the MST. Much faster decay of the toroidal flow velocity profile can be seen when we consider $\chi_\zeta \sim$ several tens of m^2/s in our calculation. However, in that case, our model of the mode locking event breaks down. We have assumed that the mode locking occurs spontaneously and that the toroidal momentum responds to it diffusively, on a much longer time scale. For the experimental values of the MST, this assumption is not valid; the mode locking time scale and the degraded momentum confinement time scale are comparable. Therefore, the results described in this subsection should be considered only as an indication of the deteriorious effects on the toroidal momentum confinement in the presence of several tearing modes, which is the case in the MST.

As described in this section, we have succeeded in obtaining reasonable estimates for the toroidal momentum confinement time degradation in the presence of mode locking from simple analytical and numerical calculations. The radial dependences of ρ_m, χ_ζ, F and their changes after mode locking need to be considered for more detailed comparisons with experimental data.

3 Mode Locking Time Scale

Several types of interactions of tearing modes with external structures have been investigated theoretically in Ref. [8]. Here, as an example of mode locking, we consider the driven reconnection case, in which it is assumed that the tearing stability index Δ' [13] is negative, so that the plasma is intrinsically tearing-mode stable. In this case, any magnetic reconnection is only due to the externally applied error fields, as has been investigated extensively on the DIII-D [5], COMPASS-C [14] and JET [17] experiments. The analysis described below could also be applied to other interactions, although only the driven reconnection case is considered here. The equation for the toroidal torque balance exerted in the resistive layer in the vicinity of rational surface can be expressed as [8]

$$\alpha \frac{d\omega}{dt} = -\beta \frac{1}{\tau_{A\theta}^2} \frac{\omega \tau_S}{(-\Delta')^2 + (\omega \tau_S)^2} \left(\frac{w}{a}\right)^4 - \gamma \omega_0 (\omega - \omega_0), \quad (8)$$

where the term on the left side is due to the plasma inertia, the first and second terms on the right are the EM and VC torques, respectively. Here, α, β, γ

are parameters that depend on the geometrical and plasma parameters, and $\tau_{A\theta}$ is the poloidal Alfvén time, τ_S the time constant of the resistive layer, ω the mode frequency and ω_0 the “natural” frequency of a naturally unstable (i.e., $\Delta' > 0$) tearing mode. It is to be noted that we have assumed only the plasma inertia involved in the resistive layer plays an important role in this equation; this assumption is based on experimental results from the DITE tokamak [18]. Although Eq. (8) is taken to describe the driven reconnection case, nearly all theories of tearing modes interacting with external structures can be written in the form of Eq. (8) [8,9].

For ω_0 greater than a critical frequency, which is typically the case for discharges with mode locking, there is either one or three real steady state solutions of Eq. (8) depending on the magnitude of the EM torque (i.e., mode amplitude), which has a non-monotonic dependence on mode frequency (see Fig. 6). When the peak of the EM torque becomes comparable to the VC torque, balancing the EM and VC torques occurs at three points corresponding to the three real steady state solutions (say, ω_1, ω_2 and ω_3 satisfying $\omega_1 < \omega_2 < \omega_3$). As the EM torque increases further, ω_2 and ω_3 then approach each other and eventually become degenerate. It is at this point that the sudden drop of mode frequency to ω_1 , that is a characteristic of mode locking, occurs. This situation is illustrated in Fig. 6, where the EM torque is shown by dotted lines as the mode amplitude is increased and the VC torque by a solid line for the case where the natural frequency is greater than a critical frequency. More detailed situations including RMPs (Resonant Magnetic Perturbations) on COMPASS-D are described similarly in Ref. [19].

At this level of EM torque, we determine the mode locking time scale by integrating the equation obtained from Eq. (8):

$$dt = \left(\frac{\alpha}{-\gamma\omega_0} \right) \frac{\omega^2 + ((-\Delta')/\tau_S)^2}{(\omega - \omega_1)(\omega - \omega_2)^2} d\omega,$$

which gives

$$\tau_l = \left(\frac{\alpha}{-\gamma\omega_0} \right) \left[\frac{\omega_2^2 + ((-\Delta')/\tau_S)^2}{(\omega_1 - \omega_2)(\omega - \omega_2)} + \left(\frac{-\Delta'}{\tau_S} \right)^2 \frac{1}{(\omega_1 - \omega_2)^2} \ln \left| \frac{\omega - \omega_1}{\omega - \omega_2} \right| \right]_{\omega_I}^{\omega_F}, \quad (9)$$

where τ_l is the required mode locking time scale and ω_I, ω_F are the mode frequency just before and after the mode locking process. Here it is assumed that the island width is unchanged during the mode locking process. If we

choose $\omega_I = \omega_2$ and $\omega_F = \omega_1$, τ_l is divergent. Physically, this divergence can be resolved by fluctuations in the mode frequency around ω_2 and ω_1 . Fine time scale evolution measurements of the mode frequency during the mode locking in JET [1] and COMPASS-D [19] show that the mode frequency drops almost monotonically during mode locking. Therefore, we evaluate the “reference” mode locking time scale by choosing $\omega_I = g\omega_2$ and $\omega_F = f\omega_2$ to avoid the singular problem mathematically or exclude the fluctuations in the mode frequency around ω_2 and ω_1 physically, where $g \leq 1$ and $f \geq 0.0$. Assuming the parameters $\omega_0 \sim 10^4 \text{ s}^{-1}$, $\tau_S \sim 3.6 \times 10^{-2} \text{ s}$, $(w/a) \sim 0.1$, $\omega_2 \sim \omega_0/2$ (obtained by solving Eq. (8) for steady state) and using JET geometrical parameters [3] with ω_I and ω_F as defined above yields:

$$\tau_l \sim \left(\frac{\alpha}{b}\right) \frac{\omega_0^2 (\omega_2 \tau_S)^2 + (-\Delta')^2}{4 \omega_2 \tau_S} \frac{g - f}{\omega_2 (g - 1)(f - 1)}, \quad (10)$$

where $b \equiv \beta \frac{1}{\tau_{A\theta}^2} \left(\frac{w}{a}\right)^4$. It should be noted that we drop the second term in Eq. (9) because it is generally two or three orders of magnitude smaller than the first one for these parameters. This is equivalent to neglecting the VC torque in Eq. (8) because the formulation of Eq. (9) strongly reflects the EM torque term in Eq. (8). This implies that the mode locking process is governed mainly by the EM torque and this implication seems to be similar to Ref. [9], in which the inertia term is balanced with only the EM torque for determining the mode locking time scale. If we take $g = 0.98$ and $f = 0.02$ ($\omega_F \sim 10 \omega_1$) for reference, Eq. (10) gives τ_l of order 40 ms; this time scale seems to be comparable to the time scale of the measured MHD frequency drop during mode locking in JET [1]. It is to be noted that the τ_l becomes the order of 15 ms if we choose $g = 0.95$ and $f = 0.02$ and this implies that the mode locking process is slow in its first phase and becomes faster during the significant drop of the mode frequency, which agrees qualitatively well with the mode locking time scale obtained in Ref. [9].

We avoided the divergences in τ_l by excluding the mode frequency regions where there are fluctuations around ω_2 and ω_1 in the above considerations. This difficulty does not occur if we determine the mode locking time scale from a different consideration — the average “decelerating” torque during mode locking. Equation (8) has a form:

$$\frac{d\omega}{dt} = T(\omega),$$

where $T(\omega)$ represents the sum of the EM and VC torques divided by α . If we define the average “decelerating” torque $\bar{T}(\omega_2, \omega_1)$ (although it does not have the dimensions of a torque because it is divided by α) by

$$\bar{T}(\omega_2, \omega_1) = \frac{\int_{\omega_2}^{\omega_1} T(\omega) d\omega}{\omega_2 - \omega_1},$$

then we can obtain a time scale from the relation

$$\Delta t \sim \left| \frac{\Delta\omega}{\bar{T}(\omega_2, \omega_1)} \right|, \quad (11)$$

where $\Delta\omega = \omega_F - \omega_I$ is the variation of the mode frequency during mode locking and Δt is the time evolution for that process. Evaluating this formula using the same parameters as before, we find $\tau_l \sim 1$ ms, which is much less than the order of 10 ms observed experimentally [1]. It should be noted that the contribution from the EM torque to $\bar{T}(\omega_2, \omega_1)$ is larger than that from the VC torque.

We have considered the mode locking time scales by two approaches which are both based on a model toroidal torque balance equation. It has been shown that the mode locking process is mainly governed by the EM torque for both approaches. The obtained mode locking time scales are both much shorter than the time scale for the degraded toroidal momentum confinement time after mode locking, which is governed by the viscous diffusion process. Thus, it is appropriate to neglect the mode locking dynamics in solving the transport evolution of the toroidal flow velocity profile, $V_\zeta(r, t)$, as we did in the preceding section of this report.

4 Summary

The diffusive transport response of the toroidal flow velocity to mode locking has been investigated by adopting a cylindrical plasma model. The mode locking is interpreted as forcing the toroidal flow velocity to zero in a region encompassing the magnetic island. Because diffusive momentum transport is assumed, the rest of the toroidal flow profile also slows down. There is a significant degradation of the toroidal momentum confinement since the stored toroidal momentum drops significantly from before to after mode locking. The degradation is estimated by solving eigenvalue problems for the diffusion

equation in the core and edge plasma regions in cylindrical geometry. The toroidal momentum confinement time after mode locking is mainly governed by that of the core plasma region, Eq. (5), and the degradation corresponds to a reduction in the effective cylinder radius (shrinking effect) of the plasma. The dynamical evolution of the toroidal flow velocity has also been examined numerically assuming that the equilibrium force is the same as just before mode locking. The same level of toroidal momentum confinement time is obtained as was deduced from the preceding analytic calculations. The toroidal momentum confinement time obtained analytically and numerically from our simple diffusive model seems to agree well with the measured value on JET [3]. It has also been predicted for DIII-D [5]. The more deteriorious effects on the toroidal momentum confinement in the presence of several tearing modes in the MST RFP mode locking case [11,16] has also been described.

The mode locking time scale itself has also been investigated from a model toroidal torque balance equation in the vicinity of a rational surface, which describes the driven reconnection. The nonmonotonic dependence of the EM torque on the mode frequency makes the sudden drop of mode frequency (mode locking) possible. The mode locking time scales have been discussed in previous works [8,9]. We obtained a relation between time and mode frequency from the torque balance equation. Avoiding the mathematical difficulties by physical considerations implies that the mode frequency drop during mode locking is mainly governed by the EM torque, which is similar to Ref. [9]. We have obtained a mode locking time scale much shorter than that of the diffusive response of toroidal momentum; it is almost comparable to the measured value, for example, on JET [1].

Acknowledgements

One of the authors (MY) gratefully acknowledges useful discussions with Prof. M. Wakatani, Dr. Y. Nakamura and Dr. R. Fitzpatrick. This work was carried out during the visit of MY to the Center for Plasma Theory & Computation at the University of Wisconsin-Madison as an exchange scientist of the Joint Institute for Fusion Theory.

The work was partially supported by the Japan Society for the Promotion of Science Research Fellowships for Young Scientists and by U.S. Department of Energy grants DE-FG02-86ER53218 and DE-FG02-92ER54139.

References

- [1] J.A.Snipes et al., Nucl. Fusion 28, 1085(1988).
- [2] J.A.Wesson et al., Nucl. Fusion 29, 641(1989).
- [3] J.A.Snipes et al., Nucl. Fusion 30, 205(1990).
- [4] H.Zohm et al., Europhys. Lett. 11, 745(1990).
- [5] J.T.Scoville et al., Nucl. Fusion 31, 875(1991).
- [6] Z.Chang, E.D.Fredrickson and J.D.Callen et al., Nucl. Fusion 34, 1309(1994).
- [7] Z.Chang and J.D.Callen, Nucl. Fusion 30, 219(1990).
- [8] R.Fitzpatrick, Nucl. Fusion 33, 1049(1993).
- [9] M.F.F.Nave and J.A.Wesson, Nucl. Fusion 30, 2575(1990).
- [10] R.Fitzpatrick et al., Nucl. Fusion 33, 1533(1993).
- [11] D.J.Den Hartog et al., Phys. Plasmas 2, 2281(1995).
- [12] S.I.Braginskii, "Transport Properties in a Plasma,"
in Reviews of Plasma Physics (Consultants Bureau, NY, 1965), Vol.1 p.205
- [13] H.P.Furth, J.Killeen and M.N.Rosenbluth, Phys. Fluids 6, 459(1963).
- [14] T.C.Hender et al., Nucl. Fusion 32, 2091(1992).
- [15] R.N.Dexter et al., Fusion Technol. 19, 131(1991).
- [16] A.F.Almagri et al., Phys. Fluids B4, 4080(1992).
- [17] G.M.Fishpool and P.S.Haynes, Nucl. Fusion 34, 109(1994).
- [18] T.C.Hender et al., in Proceedings of the Seventh International Conference
on Plasma Physics and Controlled Nuclear Fusion Research,
Nice, 1988(IAEA, Vienna 1989) Vol.1, p.445
- [19] D.A.Gates and T.C.Hender, Resistive Wall Induced "Forbidden" Bands of
Mode Rotation Frequency on the Compass-D Tokamak, UKAEA FUS 284,
UKAEA/EURATOM Fusion Association(1995).

Table.I Values of p_1 deduced from solving Eq. (6) for several r_+/a cases.

r_+/a	0.05	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
p_1	3.07	3.32	3.82	4.40	5.18	6.24	7.83	10.5	15.7	31.4

Fig.1: Steady state toroidal flow velocity profile for: (a) before and (b) after mode locking for the case of $r_s/a = 0.75$ and $w/a = 0.1$.

Fig.2: Ratio of the stored toroidal momentum before (M_0) and after (M_w) mode locking versus r_s/a for several w/a cases.

Fig.3: Time evolution of the calculated radial profile of the toroidal flow velocity for $r_s/a = 0.75$ and $w/a = 0.1$ (JET mode locking case).

Fig.4: Time evolution of the toroidal flow velocity at the magnetic axis and in the edge region versus time after mode locking (JET mode locking case).

Fig.5: Time evolution of the calculated radial profile of the toroidal flow velocity for the MST RFP mode locking case.

Fig.6: Graphical description of the interaction of the EM (dotted lines) and the VC (solid line) torques when ω_0 (natural frequency) is greater than the critical frequency. The behavior of ω_1, ω_2 and ω_3 are shown as the EM torque is increased.

This report has been reproduced directly from the best available copy.

Available to DOE and DOE contractors from the Office of Scientific and Technical Information, P.O. Box 62, Oak Ridge, TN 37831; prices available from (615) 576-8401, FTS 626-8401.

Available to the public from the National Technical Information Service, U.S. Department of Commerce, 5285 Port Royal Rd., Springfield, VA 22161.

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by tradename, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.