Uniform Current Density and Divergence Control in High Power Extraction Ion Diodes*


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Abstract

A theory of radial beam uniformity in extraction ion diodes is presented. The theory is based on a locally one dimensional analysis of the diamagnetic compression of magnetic streamlines and the self consistent determination of the virtual cathode location. The radial dependence of the applied magnetic field is used to determine the critical parameters of this locally one dimensional treatment. The theory has been incorporated into the ATHETA magnetic field code to allow the rapid evaluation of realistic magnetic field configurations. Comparisons between the theoretical results, simulations with the QUICKSILVER code, and experiments on the PBFA-X accelerator establish the usefulness of this tool for tuning magnetic fields to improve ion beam uniformity. The consequences of poor beam uniformity on the evolution of ion diode instabilities are discussed with supporting evidence from simulations, theory, and experiments.

Introduction

In recent years our understanding of the mechanisms which generate beam divergence in applied-\(B\) ion diodes has greatly improved. Electromagnetic instabilities in the diode gap are recognized to be a primary cause of the beam divergence. The spectrum of fluctuations in the acceleration gap is believed to be dominated by two instabilities, the diocotron instability,\(^1\) and the ion mode instability.\(^2\)-\(^7\) The diocotron instability has the higher frequency and is less damaging to the ion beam. The ion mode instability locks in on the ion transit time and causes unacceptably large divergences.\(^3,4\) As power is fed to the diode, the diocotron mode generally appears first and generates what can be an acceptably low divergence. If no attempts are made to halt or slow the expansion of the electron sheath towards the anode, the ion mode follows quickly. Two methods of limiting the expansion of electrons are to use very high magnetic fields or to use a small non-emitting, electron collecting protrusion or blade on the anode called a limiter.\(^8\) It has been shown with 3-D QUICKSILVER\(^9\) simulations that the onset of the ion mode may be delayed or prevented by these methods.\(^3,4\) Experiments at FZK in Karlsruhe, Germany and at Cornell University in the United States, have shown that the beam divergence can be reduced with the use of an electron limiter or high fields.\(^10,11\) One striking difference between the ion mode and the diocotron mode is the symmetry of the wave. The ion mode, as observed in simulations and experiments, exhibits strong variations in amplitude along the magnetic field direction and the diocotron is very uniform\(^12\) along the field. Figure 1 shows data from PBFA II shot 6711. Five sets of Faraday cups (top and bottom) were arranged in azimuth over one...
twelfth of the diode circumference. The data indicates an oscillation in the ion beam current density that breaks the symmetry between the top and bottom halves of the diode and has a low phase velocity, consistent with the ion mode. This breaking of the symmetry between the top and bottom halves of the diode has recently been observed in QUICKSILVER simulations in good agreement with the experimental results. These observations are supported by theoretical work that establish the stronger growth of ion mode perturbations with variations along the applied magnetic field. Figure 2 shows results from Ref. 7 indicating higher growth rates and lower phase velocities for ion modes with perturbations along the magnetic field. It has been observed in 3-D simulations that if the ion beam profile is not sufficiently uniform in the direction of the applied field, the ion mode is stimulated, grows rapidly, and causes large divergence under conditions that would otherwise favor the diocotron and lower divergences.

The preferred applied-$B$ ion diode configuration for the generation of light ion beams is the extraction geometry; the insulating magnetic field is predominately radial and the ion beam propagates primarily in the axial direction. A critical factor in the performance of applied-$B$ ion diodes is the limiting voltage $V_*$, which is proportional to the applied magnetic flux between the virtual cathode and the anode. The prevailing $1/r$ dependence of the applied magnetic field in the extraction configuration implies a lower $V_*$ at larger radii. As the voltage is raised, $V_*$ is approached more closely at larger radii. If the applied field is parallel to the anode when the diode voltage is zero, the diamagnetic compression of the field will be greater at larger radii as the voltage rises, leading to a skewing of the beam to larger radii. This asymmetry of the ion beam profile would favor the development of the ion mode. If progress is going to be made in reducing the divergence of ion beams in extraction diodes, it appears that control of the beam uniformity will be an essential component. The tendency for a purely radial magnetic field to skew the beam to larger radii can be compensated for...
Fig. 2. The real and imaginary frequency components for ion mode perturbations as a function of the wavenumber perpendicular to the applied field $k_yd$. The numbers above each curve correspond to the wavenumber along the magnetic field $k_zd$.

by the choice of an appropriate tilt of the applied field towards the anode at smaller radii. In essence, the greater diamagnetic compression at larger radii is compensated for by moving the electrons closer to the anode with the applied field at smaller radii.

In the past, we have attempted to produce centered, uniform beams through an often tedious process of trial and error. In this paper we introduce a theoretical treatment of the beam uniformity problem for extraction diodes. Examples of this analysis as employed in the ATHETA$^{14}$ magnetic field code using actual diode fields are shown along with comparisons with experiments on the PBFA-X accelerator and simulations with the QUICKSILVER 3-D particle-in-cell simulation code.

**Extraction Diode Geometry**

A simple drawing of an extraction diode is shown in Fig. 3. The geometry is cylindrical with the anode at $z=0$. The voltage is applied by a TEM pulse in the magnetically insulated transmission line (MITL). An axial electric field develops in the diode and electrons are pulled off the cathode tip. A strong magnetic field is used to inhibit the flow of electrons to the anode. In an extraction diode the magnetic field is predominately radial and this induces the electrons to $\mathbf{E} \times \mathbf{B}$ drift in the $\theta$ direction. This rotating cloud of electrons forms a virtual cathode that defines the acceleration gap for ions drawn off the anode surface. The virtual cathode surface coincides with the magnetic streamline that passes through the cathode tip. The ions travel in the $\hat{z}$ direction. The magnetic field is provided by two cathode coils, on both sides of the extraction region, and in some diodes, another pair of coils on the anode side as well. Shown in Fig. 4 are example applied magnetic field streamlines for a diode with anode and cathode coils. In order to focus the ions on the centerline, the angular momentum of the ions must be zero. The anode coils are particularly useful in compensating for the diffusion of magnetic field into the anode surface and any charge stripping considerations in order to satisfy this constraint.
Extraction Diode Theory

A theory of applied-$B$ diode operation is detailed in Ref. 13. In that treatment, the diode parameters are assumed to be uniform over the face of the anode. Here we need to take into account the radial variations inherent in the extraction geometry. The
most significant radial variation is that of the magnetic field.

A brief summary of the results of Ref. 13 may be useful here. The virtual cathode is defined by the magnetic flux surface that passes through the cathode tip. For a given distribution of electrons in the gap and a given magnetic flux between the cathode and anode, there is a limiting voltage \( V_* \) where the self-consistent virtual cathode position approaches the anode. As \( V \to V_* \) the acceleration gap shrinks to zero and the ion current diverges. This motion of the virtual cathode can be easily understood in terms of the pressure balance in the diode. In steady-state, the total particle and electromagnetic pressure terms must be in balance:

\[
\frac{B^2}{2\mu_0} - \frac{E^2}{2} + \sum_{e,i} nMv^2 = \text{Constant.}
\]

The electron pressure is usually negligible because of the electron’s small mass. For space-charge-limited conditions on the anode and cathode, the electric field terms vanish there. Since the ions are born with zero velocity, they contribute zero pressure to the anode side of the gap. Equating the remaining pressure terms on the anode and cathode gives

\[
\frac{B_a^2}{2\mu_0} = \frac{B_c^2}{2\mu_0} + J_i \sqrt{\frac{2M}{q}} V,
\]

where \( B_a \) is the anode magnetic field, \( B_c \) is the cathode magnetic field (weakly varying), \( J_i \) is the ion current density, \( M/q \) is the ion mass to charge ratio, and \( V \) is the diode voltage. The second term on the right is just the ion stagnation pressure \( n_i Mv_i^2 \) rewritten in terms of \( J_i \). The dynamic gap is referred to here as \( g \) and the initial insulating magnetic flux as \( \psi_0 \). The term on the left will scale as \( (\psi_0/g)^2 \) because of flux conservation. The ion pressure term will scale as \( (V/g)^2 \) because of the Child-Langmuir dependence of current on voltage. Since \( \psi_0 \) is fixed, as the voltage increases the gap \( g \) must get smaller to maintain pressure balance, ultimately going to zero at the voltage where the ion pressure term approaches the anode magnetic pressure term. In this fashion the limiting voltage \( V_* \) is proportional to the applied magnetic flux in the diode. At voltages well below the limiting voltage, the diamagnetic effects scale like \( (V/V_*)^2 \). In the limit \( V \to 0 \) the ion current is proportional to the Child-Langmuir current. The proportionality constant depends on the shape of the electron density profile in the gap, 1.0 for a &delta;-function distribution at the virtual cathode and 5.5 for a flat distribution between the virtual cathode and the anode. The limiting voltages are \( 0.75c\psi_0 \) and \( 0.6c\psi_0 \) respectively. The critical voltage for electron insulation \( V_{\text{crit}} \) is approximately \( c\psi_0 - 0.5 \text{ MV} \).

In Ref. 13, the electron distribution is modeled as a flat density profile that encompasses a fraction of the dynamic gap given by \( \rho g \), with the two limits \( \rho = 0 \) and \( \rho = 1 \) corresponding to the extremes just described. It has been shown with 3-D QUICKSILVER simulations that the electron sheath generally starts out very thin and is well approximated by the \( \rho = 0 \) or superinsulated limit, but that in response to the symmetry breaking instabilities, broadens out until electrons are collected on the anode, behaving more like the \( \rho = 1 \) or saturated limit. For the extraction diode we need to specify the corresponding parameters \( \rho(r) \) and \( \psi_0(r) \) to account for the radial variations.
As the electrons are diffusing across the magnetic flux surfaces, they are also moving in and out radially along flux surfaces, often with an energy that is a sizeable fraction of the gap energy $eV$. If electrons migrate to a flux surface that intersects the anode at some point, it is likely that those electrons will be lost to the anode. Consider the magnetic field shown in Fig. 4. As electrons diffuse across flux surfaces, they are likely to first encounter the anode at the inner radius. If we model the diode locally using the set of flat electron distributions introduced in Ref. 13, the inner radius would be modeled by $\rho = 1$ because the electrons would make it all the way to the anode at that point. Other radii would be modeled by a value of $\rho$ that is close to, but slightly less than 1. This is the basis for our extraction diode theory. At each value of $r$ the parameter $\rho(r)$ is taken to be the fraction of the initial gap flux between the virtual cathode streamline and the first streamline to intersect the anode at some point on or near the emission surface. The sheath profile function $\rho(r)$, the applied flux between the virtual cathode streamline and the anode $\psi_0(r)$, and the location of the virtual cathode streamline (at zero voltage) are all that are needed to apply the analysis from Ref. 13 for each radial position. Note that the diode voltage must be lower than the minimum value of $V_*(r)$ across the anode.

**ATHETA Modeling**

The analysis described above has been incorporated into the ATHETA magnetic field code. Diffusive calculations with the DATHEA$^{14}$ code are used to generate the fields for each individual coil as a function of time, providing a basis set of fields that can be read into ATHETA. The amplitudes of each basis field component can be specified independently. Once a complete magnetic field set is chosen, ATHETA finds the virtual cathode streamline position by following the magnetic field from the cathode tip and also finds the function $\rho(r)$ by finding the first streamline to intersect the anode as approached from the cathode side. ATHETA also finds the critical voltage for electron insulation $V_{\text{crit}}(r)$, or equivalently $\psi_0(r)$. A subroutine containing the analysis described above rapidly computes the predicted ion beam profile consistent with the total integrated current and accelerator load line. The effects of numerous combinations of anode and cathode coil currents can be quickly evaluated. Examples of predicted beam profiles from ATHETA are shown in Fig. 5. The profile indicated by the solid line corresponds to a zero trim case for a PBFA-X diode. The ions species is lithium and the diode voltage is approximately 6.5 MV. The minimum $V_{\text{crit}}(r)$ is approximately 10 MV in each case, but the location of the minimum varies with the coil currents. The cathode tip is 1.5 cm from the anode surface. A 6 MV/cm field emission threshold on the anode is used to model emission from LiF. The predicted profiles for space-charge-limited emission are generally much less peaked. The dashed line shows the effect of a 14.8% reduction in the inner coil current (negative trim). The profile indicated by the dot-dashed line shows the effect of an 11.4% reduction in the outer anode coil current (positive trim). The more highly enhanced the ion current density is above the Child-Langmuir current density, the more sensitive the profile is to slight changes in the trim. Figure 6 gives an indication of the degree of uniformity possible, at least within the context of the theory. This profile was obtained by adding an additional anode coil between the inner and outer anode coils. The extra anode coil provided a degree of tuning not possible with just the two existing anode coils. The current in the auxiliary anode coil was about 16% of the current in the outer coil.
Fig. 5.  The predicted beam profile from ATHETA for zero, -14.8 and 11.4 percent trim. The ion species is lithium and the diode voltage is about 6.5 MV. A 6 MV/cm field emission threshold is used to model LiF.

Fig. 6.  An example of an optimized field profile for the same diode as modeled in Fig. 5. An auxiliary anode coil carrying 16% of the outer anode current was used to help tune the beam profile. The ion emission is space-charge-limited.

for a comparable number of turns. In addition the ion emission was assumed to be space-charge-limited.
Comparisons with Experiments and Simulations

As a test of this analysis, we performed experiments on the PBFA-X accelerator. Five operating points were examined with two shots per operating point. In these experiments, each pair of anode and cathode coils were in series. The anode coil currents could be varied from one another by the use of a trim coil in parallel with either the inner or outer anode coil. This relative imbalance is referred to as the trim and is given by \( (I_{\text{inner}} - I_{\text{outer}}) \) divided by the maximum of \( I_{\text{inner}} \) and \( I_{\text{outer}} \). If the trim coil is in parallel with the inner coil, the trim is negative. Conversely for the outer coil. This convention is useful because positive trim will tend to move the beam outwards and negative trim will tend to move the beam inwards. In each case, the current in the untrimmed coil is kept at the nominal zero trim level. No attempt was made to keep \( V_{\text{crit}} \) constant.

Figure 7 shows the results of these experiments plotted as the mean beam radius relative to the mean anode radius versus the percentage of trim. The experimental radius information was obtained using an array of Faraday cups in the extraction region of the diode. With the exception of one errant data point, the experimental points are well grouped, indicating a good degree of repeatability. Also included in Fig. 7 are results from simulations with the QUICKSILVER 3-D particle-in-cell code and two sets of points from theoretical calculations. The simulations were performed for the zero trim case, the 14.8% negative trim, and the 11.4% positive trim. The simulation results are in good agreement with the experimental results. The theoretical points marked with the crosses are predicted values obtained before the experiments. The predicted points are skewed to the outer radii relative to the experimental points for the zero and negative trim cases. This suggests that the theory underestimates the electron space-charge contribution on field lines close to the anode at smaller radii. The flat electron density profile used in the theory is less weighted towards the anode than the profiles observed in simulations. The theoretical points marked with the circles were obtained after the experiments by normalizing one operating point to the upper experimental point at zero trim. The normalization consisted of a 7.5% increase in the outer anode coil current to shift the beam towards smaller radii and an adjustment to the diode operating point to account for differences between the experiments and the calculations. The rest of the adjusted theoretical points fall into place using the same normalization. This suggests a systematic shortcoming of the theory, most likely in the electron profile model, but also points out the improvement in accuracy if the theory is normalized to experimental points. In practice we have done just that, used the theory to predict initial operating points, and then normalized the theory to predict subsequent operation with very good success. A snapshot of the three simulation beam profiles around the time of peak ion current are shown in Fig. 8. The simulation profiles are rounded at the edges. This is expected from edge effects, such as magnetic tension terms, that are not included in the theory.

We have used this analysis to design a simulation of an extraction diode with a limiter. The fields used are realistic fields as opposed to the idealized fields used in earlier work. Nonuniformities in the ion beam current associated with realistic magnetic fields have made confirmation of the earlier results with idealized magnetic fields very difficult. The beam uniformity algorithm in ATHETA was used to design a magnetic field for very good uniformity. In this particular case, an additional cathode
coil was employed to improve the uniformity. (A limited set of simulations without the auxiliary coil had poorer predicted uniformity and did not show successful suppression of the ion mode.) The predicted beam current density with the extra cathode coil varied by less than ± 10% across the anode and was symmetric. The simulation showed a persistent diocotron oscillation and did not make a transition to the ion mode. The divergence was maintained at about 9 milliradians.
Conclusion

A theory of extraction diode operation has been developed that uses details of the magnetic field configuration to determine the parameters of a locally one dimensional model for each radial position in the diode. The total current is obtained by integrating over the anode surface and a load line calculation determines the voltage. This analysis provides a radial current density profile for a given magnetic field configuration, ions species, and diode voltage. By including this analysis in the ATHETA magnetic field code, we can rapidly evaluate realistic magnetic field configurations for predicted beam uniformity and tune the magnetic field for improved uniformity. The results of these calculations have been shown to be in good agreement with results from experiments on PBFA-X and simulations with the QUICKSILVER 3-D particle-in-cell code. This tool works best when it can be normalized to experimental data. We have used this analysis to design a magnetic field which allowed the suppression of the ion mode in a simulation of an extraction diode with realistic magnetic fields and an electron limiter. The ion divergence in the simulation remained below 10 milliradians.

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