Argonne National Laboratory

CONDENSATION OF METAL VAPORS: MERCURY AND THE KINETIC THEORY OF CONDENSATION

by

Donald J. Wilhelm
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CONDENSATION OF METAL VAPORS:
MERCURY AND THE KINETIC THEORY OF CONDENSATION

by

Donald J. Wilhelm

Reactor Engineering Division, ANL
and
Associated Midwest Universities

Reproduced from a Thesis Submitted in Partial Fulfillment of the Requirements for the Degree of Doctor of Philosophy in the Graduate School of The Ohio State University.

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FOREWORD

This report is one of a series that describes heat-transfer and fluid-flow studies performed at Argonne under a program sponsored jointly by the Associated Midwest Universities and the Argonne National Laboratory.

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          Flow through a Rectangular Channel
          Richard J. Thome

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<td>Description</td>
<td>Units</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
<td>-------</td>
</tr>
<tr>
<td>A</td>
<td>Area</td>
<td>m²</td>
</tr>
<tr>
<td>Ag</td>
<td>Perturbation parameter</td>
<td>N/m²</td>
</tr>
<tr>
<td>Aₖ</td>
<td>Symbol for manometer</td>
<td>-</td>
</tr>
<tr>
<td>Aₘ</td>
<td>Symbol for a mixer</td>
<td>-</td>
</tr>
<tr>
<td>c</td>
<td>Concentration of corrosion products</td>
<td>mol/l</td>
</tr>
<tr>
<td>cₐ</td>
<td>Vector velocity</td>
<td>m/s</td>
</tr>
<tr>
<td>G₀</td>
<td>Heat capacity specific heat at constant pressure</td>
<td>J/kg°C</td>
</tr>
<tr>
<td>G₁</td>
<td>Outside tube diameter</td>
<td>m</td>
</tr>
<tr>
<td>E</td>
<td>Kinetic energy flux</td>
<td>J/m²s</td>
</tr>
<tr>
<td>f</td>
<td>Velocity distribution function</td>
<td>sec⁻¹</td>
</tr>
<tr>
<td>g</td>
<td>Gravity, weight fraction vapor in wet vapor</td>
<td>-</td>
</tr>
<tr>
<td>gᵣ</td>
<td>Gravitational acceleration</td>
<td>m/s²</td>
</tr>
<tr>
<td>G</td>
<td>Mass flux</td>
<td>kg/m²s</td>
</tr>
<tr>
<td>G₁₁₀</td>
<td>Net mass flux</td>
<td>kg/m²s</td>
</tr>
<tr>
<td>G₁₂</td>
<td>&quot;Condensing,&quot; or incident mass flux</td>
<td>kg/m²s</td>
</tr>
<tr>
<td>G₂₂</td>
<td>Evaporating mass flux</td>
<td>kg/m²s</td>
</tr>
<tr>
<td>h</td>
<td>Heat transfer coefficient</td>
<td>m²/K</td>
</tr>
<tr>
<td>hₘ</td>
<td>Heat of reaction</td>
<td>J/mol</td>
</tr>
<tr>
<td>k</td>
<td>Thermal conductivity</td>
<td>W/mK</td>
</tr>
<tr>
<td>kᵣ</td>
<td>Thermodynamic equilibrium constant</td>
<td>mol⁻¹</td>
</tr>
<tr>
<td>L</td>
<td>Condenser length</td>
<td>m</td>
</tr>
<tr>
<td>Lₗ</td>
<td>Mean free path</td>
<td>m</td>
</tr>
<tr>
<td>m</td>
<td>Mass per molecule</td>
<td>g/mol</td>
</tr>
<tr>
<td>M</td>
<td>Molecular weight</td>
<td>g/mol</td>
</tr>
<tr>
<td>M₂₀</td>
<td>Molecular weight of monomer</td>
<td>g/mol</td>
</tr>
<tr>
<td>M₂₂</td>
<td>Molecular weight of dimer</td>
<td>g/mol</td>
</tr>
<tr>
<td>m</td>
<td>Number of moles</td>
<td>mol</td>
</tr>
<tr>
<td>mₙ</td>
<td>Number density</td>
<td>molecules/m³</td>
</tr>
<tr>
<td>P₀</td>
<td>Probability that molecule will condense</td>
<td>-</td>
</tr>
<tr>
<td>p</td>
<td>Partial pressure</td>
<td>Pa</td>
</tr>
<tr>
<td>P</td>
<td>Vapor pressure, total pressure</td>
<td>Pa</td>
</tr>
<tr>
<td>Pₙ</td>
<td>Vapor pressure corresponding to T₁</td>
<td>Pa</td>
</tr>
<tr>
<td>Pₙ₂</td>
<td>Vapor pressure corresponding to T₂</td>
<td>Pa</td>
</tr>
<tr>
<td>Pᵣ</td>
<td>Preheat number, cₐ/kₐ</td>
<td>-</td>
</tr>
<tr>
<td>q</td>
<td>Heat transfer rate</td>
<td>W/m²</td>
</tr>
<tr>
<td>qₜ</td>
<td>Heat flux</td>
<td>W/m²</td>
</tr>
<tr>
<td>R</td>
<td>Universal gas constant</td>
<td>J/molK</td>
</tr>
</tbody>
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<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Units</th>
</tr>
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<tbody>
<tr>
<td>R</td>
<td>Corrosion rate</td>
<td>-</td>
</tr>
<tr>
<td>Rᵥ</td>
<td>Reynolds number</td>
<td>-</td>
</tr>
<tr>
<td>t</td>
<td>Time</td>
<td>s</td>
</tr>
<tr>
<td>t₀</td>
<td>Condenser film thickness</td>
<td>m</td>
</tr>
<tr>
<td>T</td>
<td>Temperature, absolute</td>
<td>K</td>
</tr>
<tr>
<td>T₀</td>
<td>Condenser film temperature on coolant side</td>
<td>K</td>
</tr>
<tr>
<td>Tᵥ₀</td>
<td>Temperature of molecules reflected from vapor liquid interface</td>
<td>K</td>
</tr>
<tr>
<td>Tᵥ₁</td>
<td>Temperature of molecules reflected from vapor liquid interface, if reflected with maximum velocity distribution</td>
<td>K</td>
</tr>
<tr>
<td>Tₑ</td>
<td>Temperature of liquid surface</td>
<td>K</td>
</tr>
<tr>
<td>Tₑ₁</td>
<td>Evaporating temperature</td>
<td>K</td>
</tr>
<tr>
<td>Tₑ₂</td>
<td>Condenser surface temperature</td>
<td>K</td>
</tr>
<tr>
<td>U</td>
<td>Net stream velocity in x-direction</td>
<td>m/s</td>
</tr>
<tr>
<td>Uₓ</td>
<td>Molecular velocity in x-direction</td>
<td>m/s</td>
</tr>
<tr>
<td>Uᵧ</td>
<td>Molecular velocity in y-direction</td>
<td>m/s</td>
</tr>
<tr>
<td>Uₘ</td>
<td>Molecular velocity in z-direction</td>
<td>m/s</td>
</tr>
<tr>
<td>W</td>
<td>Condensate flow rate</td>
<td>m³/s</td>
</tr>
<tr>
<td>X</td>
<td>Distance coordinate normal to condenser plate</td>
<td>m</td>
</tr>
<tr>
<td>X₀</td>
<td>Male fraction</td>
<td>-</td>
</tr>
<tr>
<td>X₁</td>
<td>Male fraction monomer</td>
<td>-</td>
</tr>
<tr>
<td>X₂</td>
<td>Male fraction dimer</td>
<td>-</td>
</tr>
<tr>
<td>Y</td>
<td>Distance coordinate parallel to condenser plate</td>
<td>m</td>
</tr>
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### Greek Letters

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>α</td>
<td>Thermal diffusivity, kₒ/kₘ</td>
<td>m²/s</td>
</tr>
<tr>
<td>η</td>
<td>Thermal accommodation coefficient</td>
<td>-</td>
</tr>
<tr>
<td>B</td>
<td>(kₒ/kₘ)²</td>
<td>-</td>
</tr>
<tr>
<td>Φ</td>
<td>Condensate flow rate per unit breadth of condenser surface</td>
<td>m³/s</td>
</tr>
<tr>
<td>T₁₂</td>
<td>Correction factor, defined by Equation 121</td>
<td>-</td>
</tr>
<tr>
<td>T₁₁₂</td>
<td>Correction factor, defined by Equation 112</td>
<td>-</td>
</tr>
<tr>
<td>T₁₃</td>
<td>Correction factor, defined by Equation 123</td>
<td>-</td>
</tr>
<tr>
<td>δ</td>
<td>Condensate film thickness</td>
<td>m</td>
</tr>
<tr>
<td>Δ</td>
<td>Signifies difference between properties</td>
<td>-</td>
</tr>
<tr>
<td>εₒ</td>
<td>Eddy diffusivity of heat</td>
<td>m²/s</td>
</tr>
<tr>
<td>εᵣ</td>
<td>Eddy diffusivity of momentum</td>
<td>-</td>
</tr>
<tr>
<td>ξ</td>
<td>Molecular velocity</td>
<td>-</td>
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<tr>
<td>λ</td>
<td>Latent heat of vaporization</td>
<td>J/kg</td>
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### Greek Letters

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<th>Symbol</th>
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<tbody>
<tr>
<td>μ</td>
<td>Viscosity</td>
<td>-</td>
</tr>
<tr>
<td>υ</td>
<td>Kinematic viscosity</td>
<td>-</td>
</tr>
<tr>
<td>ν</td>
<td>Collision frequency</td>
<td>-</td>
</tr>
<tr>
<td>θ</td>
<td>Angle between inclined surface and horizontal plane</td>
<td>deg</td>
</tr>
<tr>
<td>φ</td>
<td>Molecular property, function of velocity</td>
<td>-</td>
</tr>
<tr>
<td>ρ</td>
<td>Density</td>
<td>g/cm³</td>
</tr>
<tr>
<td>rₒ</td>
<td>Condensation coefficient</td>
<td>-</td>
</tr>
<tr>
<td>r₁</td>
<td>Condensation coefficient of monomer</td>
<td>-</td>
</tr>
<tr>
<td>r₂</td>
<td>Condensation coefficient of dimer</td>
<td>-</td>
</tr>
<tr>
<td>rₑ</td>
<td>Evaporation coefficient</td>
<td>-</td>
</tr>
<tr>
<td>rₑ₁</td>
<td>Evaporation coefficient of monomer</td>
<td>-</td>
</tr>
<tr>
<td>rₑ₂</td>
<td>Evaporation coefficient of dimer</td>
<td>-</td>
</tr>
<tr>
<td>rₑₙ</td>
<td>Evaporation coefficient of dimer</td>
<td>-</td>
</tr>
<tr>
<td>rₒₑ</td>
<td>Fraction of incident energy lost to liquid</td>
<td>-</td>
</tr>
<tr>
<td>rₒₑₙ</td>
<td>Fraction of incident momentum lost to liquid</td>
<td>-</td>
</tr>
<tr>
<td>ν₀ₑ</td>
<td>Shear</td>
<td>Pa⁻¹</td>
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### Subscripts

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<th>Symbol</th>
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<th>Units</th>
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<tbody>
<tr>
<td>c</td>
<td>Condensing, or coolant side</td>
<td>-</td>
</tr>
<tr>
<td>e</td>
<td>Evaporation</td>
<td>-</td>
</tr>
<tr>
<td>i</td>
<td>Interface, Incident</td>
<td>-</td>
</tr>
<tr>
<td>k</td>
<td>Knudsen Index</td>
<td>-</td>
</tr>
<tr>
<td>L</td>
<td>Liquid phase</td>
<td>-</td>
</tr>
<tr>
<td>M</td>
<td>Maxwellian</td>
<td>-</td>
</tr>
<tr>
<td>n₀</td>
<td>Minimum value</td>
<td>-</td>
</tr>
<tr>
<td>Nₒ</td>
<td>Nusselt's theory value</td>
<td>-</td>
</tr>
<tr>
<td>o</td>
<td>Local</td>
<td>-</td>
</tr>
<tr>
<td>R</td>
<td>Reflected</td>
<td>-</td>
</tr>
<tr>
<td>s</td>
<td>Liquid surface at vapor-liquid interface</td>
<td>-</td>
</tr>
<tr>
<td>v</td>
<td>Bulb vapor</td>
<td>-</td>
</tr>
<tr>
<td>w</td>
<td>Condenser wall</td>
<td>-</td>
</tr>
</tbody>
</table>

### References

- Refers to nonmonomer
- Refers to dimer
CONDENSATION OF METAL VAPORS:
MERCUry AND THE KINETIC THEORY OF CONDENSATION

by

Donald J. Wilhelm

ABSTRACT

Condensation data and condensation theories concerning metal vapors were extensively reviewed. Nusselt's and improved theoretical film-condensation heat transfer rates for metal vapors were roughly five to 30 times as large as observed experimental rates. Because this discrepancy has been attributed by some researchers to a controlling resistance at the vapor-liquid interface, described by the kinetic theory of condensation, the latter concepts were also critically reviewed to clarify the concept and nature of this resistance.

In addition to the mass transport equation, commonly used, the kinetic theory equations of momentum and energy transport across the vapor-liquid interface in film condensation were written, and the composite velocity distribution for vapor at the interface was normalized. The simultaneous solution of these equations depends strongly upon the velocity distribution of vapor molecules reflected from the liquid surface. Solutions of these equations for six different cases, two of which permitted inelastic reflection, failed to yield physically consistent answers for heat flux and overall thermal driving force.

A parametric study of these equations yielded expected trends in the "condensation coefficient" (the fraction of molecules striking the interface that actually condenses). The available mercury film-condensing data of Misra and Bonilla and the recent data of Sukhatme and Rohsenow were analyzed in the manner of the parametric study. It was shown that heat transfer coefficients and condensation coefficients calculated from data cannot be directly compared without regard for the system conditions. Because the two series of data were obtained over significantly different pressure ranges, Misra's data remain to be checked.

The literature survey and subsequent analyses revealed serious shortcomings in the kinetic theory of condensation. Supporting reasons were not found for the common assumption that the condensation and evaporation coefficients are identical. An adequate theory of condensing vapor transport at the vapor-liquid interface remains to be developed. Sukhatme's data and the literature review of condensation coefficients indicated that collisional
energetics ought to be considered in any theory of the condensation coefficient. Moreover, published results of molecular beam determinations of condensation coefficients, particularly in condensation-crystallization studies, suggest that surface migration of adsorbed vapor is an important mechanism in dropwise condensation.

Design details and operating difficulties of mercury vapor rotating disk condenser tests (which comprised the original and a major portion of this program), aimed at evaluating the vapor-liquid interfacial resistance, have been included.

I. INTRODUCTION

Steady-state condensation, i.e., the change of phase from the vapor to the liquid state with simultaneous heat transfer and condensate removal, can be considered to occur in three basic manners: (a) bulk nucleation or droplet formation in the bulk vapor, (b) dropwise condensation on a cool surface, and (c) film condensation on a cool surface.

Nucleation occurs spontaneously in sufficiently supersaturated pure vapor, or in vapor "seeded" with foreign nuclei. Under proper conditions of heat and condensate removal and/or vapor flow, the result can be the steady formation of liquid droplets in the bulk vapor.

In dropwise condensation, the vapor impinges upon the cold wall to which it loses some of its energy and thereupon liquefies, forming droplets which continue to grow by condensation and by coalescence with other neighboring droplets until the droplets are swept along or off the cold surface by gravity, surface rotation, and/or flowing vapor. As the droplets are swept off the surface, they coalesce with droplets in their paths thus sweeping the surface clean, exposing a bare portion upon which condensation starts again. The details of the dropwise condensation mechanisms are not completely understood. Hence, no satisfactory theory exists for predicting dropwise condensation rates. Dropwise condensation of steam produces heat transfer coefficients sometimes as much as 10 to 20 times as large as film condensation,\(^1,2\) and dropwise condensation of mercury yields heat transfer coefficients only about twice as large as film condensation.\(^3\) Misra and Bonilla\(^3\) suggested that heat transfer coefficients for dropwise and film condensation of metal vapors might be expected to be roughly the same order of magnitude, if the overall thermal resistance is much higher than that of the condensate film, as for metal condensates of high thermal conductivity. Alkali metals have not been visually observed condensing, but have been assumed to condense filmwise because of their high reactivity and "wettability."
In film condensation, the liquid condensate "wets" the surface, forming a continuous film which covers the surface. This film flows over the surface under the action of gravity, surface rotation, and/or vapor flow. Nusselt\(^4\) developed a theory of film condensation which adequately predicts condensation rates and heat transfer coefficients for condensing steam and organic vapors. The theory is based upon the assumptions that the total resistance to heat transfer lies in the condensate film; i.e., the temperature at the vapor-liquid interface is the saturation temperature of the bulk vapor, and the falling film is in laminar flow with zero shear at the vapor-liquid interface.

Available data on rates of condensation for metal vapors fall far below Nusselt's theoretical predictions, and various improvements of laminar film condensation theory have not removed this discrepancy. Data of Misra and Bonilla\(^3\) on condensing sodium and mercury vapors up to atmospheric pressure indicate that film condensation heat transfer coefficients of mercury and sodium vapors are roughly 5 to 15% of the values predicted by Nusselt's theory. The data of Cohn\(^5\) on condensing mercury and cadmium vapors and the data of Engelbrecht\(^6\) on condensing potassium and rubidium vapors apparently substantiate Misra's original findings and indicate that heat transfer coefficients for condensing metal vapors are about 1 to 15% of the Nusselt theory coefficients. Similar discrepancies between classical theory and data are shown in the recent data of Sukhatme and Rohsenow\(^7\) for film condensation of mercury up to 0.33 psia. Sukhatme measured condensate film thicknesses by gamma attenuation and found that the results agreed approximately with Nusselt's Reynolds number film-thickness predictions. He also found that the condensing film heat transfer coefficient increased with increasing vapor pressure, while the addition of a noncondensible gas to the condensing system caused a decrease in heat transfer coefficient with increasing vapor pressure. Misra's mercury data show a decrease in heat transfer coefficient with increasing vapor pressure for systems supposedly free from noncondensible gases.

Based upon these findings, Sukhatme concluded that previous data for film condensation of metal vapors contain errors due to the presence of noncondensible gases, and that "during film condensation of liquid metals, a significant thermal-resistance can exist at the liquid-vapor interface." He analyzed this resistance in terms of Schrage's kinetic theory of condensation\(^8\).

The kinetic theory of condensation implicitly includes a thermal resistance at the vapor-liquid interface and treats this resistance as an abrupt thermal discontinuity or "temperature jump." The temperature of the liquid surface is lower than the saturation temperature of the bulk vapor. The magnitude of the "temperature jump" increases with a decrease in the condensation coefficient (i.e., the fraction of molecules striking the interface that actually condenses).
A. Statement of the Problem

Schrage's theory does not explain why condensation coefficients less than unity might exist. The theory has not been used to attempt to predict, quantitatively, film condensation system behavior over wide vapor pressure ranges without a priori assumptions about the condensation coefficient. Nor has the theory ever been examined to clarify the concept and nature of the interfacial resistance. Sukhatme showed that this resistance can be significant for condensing metals. However, he did not analyze Misra's mercury film condensation data according to the kinetic theory of condensation. Nor did he check Misra's data over the same vapor pressure range as the latter studied.

The objectives of this research then are critical analysis of condensation theories and data for metal vapors and experimental evaluation of the resistance to condensation for a representative metal such as mercury.

B. Summary of Literature Survey

The viewpoint was taken that the interrelationships among film condensation, dropwise condensation, and nucleation cannot be disregarded. Concepts and data for one mode of condensation might be helpful in understanding the other modes. The complete literature survey appears in Appendix A. This review is summarized now, emphasizing previous work that is pertinent to the analytical and experimental portions of this report.

Film condensation data for steam and organic vapors followed Nusselt's theory within engineering accuracy.\(^{1,9-12}\) The theory also fit the data for condensing cryogenic fluids.\(^{13-17}\) However, data for condensing metal vapors were consistently below Nusselt's theoretical predictions.

Misra and Bonilla\(^{(3)}\) condensed mercury and sodium up to atmospheric pressure. The resulting heat transfer coefficients were about 5 to 15% of Nusselt's values. Cohn's\(^{(5)}\) mercury and cadmium condensing data were about 1 to 13% of theory. Condensing rubidium and potassium heat transfer coefficients, measured by Engelbrecht,\(^{(6)}\) were about 2 to 10% of theoretical values. Similar potassium condensing data were obtained at General Electric,\(^{(18-21)}\) Sukhatme's\(^{(7)}\) data for condensing mercury up to 0.33 psia showed a substantial lowering of the temperature of the vapor-liquid interface below the saturation temperature of the bulk vapor, indicating that the interfacial thermal resistance must be included in descriptions of condensing metals.

Recent theoretical improvements of Nusselt's laminar film condensation theory included the effects of acceleration, convection, and vapor-induced drag at the vapor-liquid interface. Nusselt neglected these effects.
Chen's (22) analysis, which is representative of all these efforts, illustrates that the results of these theories are not substantially different from Nusselt's results for values of $c_p \Delta T / \lambda$ encountered in most experiments.

The temperature dependences of physical properties were not included in any of the foregoing theories, but Edwards and Tellep (23) showed that heat transfer for constant thermal conductivity is only about 40% higher than that predicted by including a temperature-dependent conductivity that varies by a factor of four across a liquid-metal boundary layer. Zozulya (24) has shown that the temperature dependency of viscosity must be considered for extremely viscous fluids, such as glycerol, but mercury and the alkali metals are fluids of relatively low viscosity.

Lee (25) showed that turbulent-film condensation theory is practically the same as Nusselt's theory for low values of $c_p \Delta T / \lambda$, in which range the available condensing data were taken. Measurements of falling-film thickness (26, 27) indicated that Nusselt's theory of laminar, falling-film flow is approximately correct. This is supported by Sukhatme's (7) measurements of condensing-film thicknesses for mercury.

Information on thermal contact resistance at a solid-liquid interface indicates that such a resistance is negligible in a pure system in which the fluid wets the surface. (28-35)

Noncondensible gases can impose a significant resistance to condensation. (1, 9, 36-40) Sparrow and Lin (41) analytically described film condensation in the presence of noncondensible gases to show the importance of gas diffusion normal to the surface and free convection parallel to the condensate surface. They only applied their theory, however, to steam condensation in the presence of air. Sukhatme (7) qualitatively demonstrated the effect of noncondensible gases in two of his condensing-mercury tests.

Baer and McKelvey (42) combined Nusselt's theory with the kinetic theory of condensation in an attempt to analyze the resistance to film condensation of methanol in the presence of air. Their results illustrate the decrease in the apparent condensation coefficient with increasing noncondensible gas concentration. Balekjian and Katz (43) and Silver and Simpson (44) combined Nusselt's theory with the kinetic theory of condensation to analyze the effect of vapor superheat. They concluded that superheat causes a lowering of the condensate surface temperature below the saturation temperature of the vapor and a corresponding lowering of the heat flux. The analysis of Sparrow and Eckert (45) based upon macroscopic boundary layer concepts, predicts an increase in heat transfer with increasing superheat.

Among recent analytical investigations of enhancing condensate removal from condenser surfaces, Sparrow and Gregg (46) analyzed laminar film condensation on a rotating disk. The results showed that condensate
film thickness is uniform over the entire disk and is inversely proportional to the square root of the speed of rotation. Nandapurkar and Beatty\(^{(47)}\) found that this theory approximately describes condensation of methanol, ethanol, and Freon-113 on a rotating disk, but no one has employed this method of controlling condensate film thickness to study the resistance to condensation of metal vapors.

The controlling resistance to film condensation of metal vapors also appears to be a large factor in dropwise condensation of metals, for Misra and Bonilla\(^{(3)}\) and later Gel'man\(^{(48)}\) observed dropwise condensation rates of roughly the same order of magnitude as film condensation rates for condensing mercury.

Hays' tests\(^{(49)}\) in which he condensed mercury vapor on a jet of cold liquid mercury, are analogous to film condensation. His data apparently indicate a vapor-liquid interfacial resistance at low pressures, for he concluded that "as the vapor becomes rarefied, the limiting resistance in the heat transfer process becomes the vapor flux attainable at the jet surface rather than the internal liquid heat transfer."

Sukhatme's conclusion\(^{(7)}\) that the interfacial resistance to condensation of metals can be important seems to be supported by the available data. Concepts of the vapor-liquid interface and the kinetic theory of condensation were therefore more closely examined.

Schrage\(^{(8)}\) and Michaels\(^{(50)}\) presented arguments supporting the idea that the thickness of the vapor-liquid interface is of the order of one molecular diameter. However, Hill\(^{(51-53)}\) discussed surface tension theories, which include a model of the interface as a transition zone, the breadth of which increases with increasing temperature. Chang et al.\(^{(54)}\) calculated surface tensions for argon, nitrogen, and methane based upon the transition zone model. Their results showed excellent agreement with data. The theory was not applied to liquid metals.

A temperature jump at the vapor-liquid interface is implicitly included in the kinetic theory of condensation. Present\(^{(55)}\) and Kaminsky\(^{(56)}\) discussed the temperature jump at a gas-solid interface, explaining that the temperature jump concept was originally introduced as a mathematical device that avoids taking account of the more rapid temperature variation within several mean free paths of the surface due to collisions with the surface.

In a related problem, Patterson\(^{(57)}\) treated the diffuse reflection of a gas from a solid surface in nonisentropic flow of the gas over the surface. He used a perturbed Maxwellian velocity distribution for nonisentropic flow in the equations of mass, momentum, and energy conservation to find an expression for the temperature jump. Although the condensation problem need
only consider flow normal to the interface, it is more complex than Patterson's reflection problem because of the condensation and evaporation processes at the vapor-liquid boundary and the possibility of inelastic reflection at that boundary.

The kinetic theory of condensation at a vapor-liquid interface was developed to its present form by Schrage. The kinetic theory of gases shows that the net flux of molecules in any one direction in a gas at equilibrium is given by

$$G = P \sqrt{\frac{M}{2\pi RT}}.$$  \hspace{1cm} (1)

Schrage assumed that Equation (1) holds for molecules approaching and leaving the liquid surface. He wrote the net rate of mass transfer in the vapor as the difference between the absolute evaporating mass flux and the absolute condensing mass flux,

$$G_{\text{net}} = \sigma_c \Gamma_1 G_v + \sigma_e G_s,$$  \hspace{1cm} (2)

where subscripts v and s indicate that $G_v$ and $G_s$ are evaluated at bulk vapor and liquid surface conditions, respectively, and $\Gamma_1$ is a negative correction factor for net condensation. The condensation coefficient, $\sigma_c$, has traditionally been considered to be the fraction of molecules striking the liquid surface that actually condenses. The evaporation coefficient, $\sigma_e$, corrects the absolute evaporating molecular flux.

Most analyses of the kinetic theory of condensation did not consider the equations of momentum and energy transport across the interface. Nor did they fully consider possible contributions of the reflection of vapor molecules from the liquid to net momentum and energy exchange. Zwick improved the theory somewhat by including the momentum and energy exchange processes, but he treated reflection as an elastic process.

Schrage and others used $\sigma_e$ and $\sigma_c$ as identical quantities. Plesset stated that such an assumption may not be justified. Paul and Courtney pointed out that the kinetics of evaporation and condensation may be quite different. Ackerman et al. showed that $\sigma_e$ and $\sigma_c$ need be equal only at thermodynamic equilibrium. They suggested that the kinetic theory of condensation is an insufficient basis for the discussion of the problem, which ought to be treated in terms of potentials at the condensing interface.

Paul's survey of evaporation coefficients indicated that pure metals exhibit the maximum evaporation coefficient of unity for evaporation into high vacua. Papers by Knacke and Stranski and Mortensen and Eyring represented treatments of the evaporation coefficient in terms of changing internal energy states of molecules during evaporation.
Molecular beam determinations of condensation coefficients for metal vapors were reviewed by Wexler.\textsuperscript{(72)} His survey indicated that the condensing coefficient increases with decreasing surface temperature and with increasing thickness of the deposited layer.

Kaminsky's discussion of thermal accommodation coefficients\textsuperscript{(56)} indicated that reflection is generally an inelastic process and stated that the accommodation coefficient "determines an equilibrium condition, not a rate." Reviews by Goodman,\textsuperscript{(73-75)} Gilbey,\textsuperscript{(76)} and Allen and Feuer\textsuperscript{(77)} showed the complexity of estimating accommodation coefficients from theory.

Sirovich\textsuperscript{(78)} discussed a kinetic model similar to the Boltzmann equation, which might prove helpful in describing vapor conditions near a condensing interface, if a more complex theory is required.

The five general theories of bulk nucleation, including the classical liquid-drop model, were reviewed by Courtney.\textsuperscript{(68)} A critical supersaturation of vapor is required so that nuclei of finite radius may form and grow. Growth is commonly described in terms of the kinetic theory of condensation. Hirth\textsuperscript{(79)} treated two-dimensional nucleation at a solid surface, but apparently no one examined nucleation at a vapor-liquid interface as a mechanism causing an interfacial resistance.
II. MODEL OF FILM CONDENSATION

Film condensation, particularly the transport of mass, momentum, and energy across the vapor-liquid interface, is re-examined here on the basis of those fundamental ideas recalled by the literature survey. Coupling the macroscopic transport equations for the vapor and the liquid phases at the vapor-liquid interface is analyzed briefly. The kinetic theory of condensation is then rewritten and expanded in detail. Its corresponding mass, momentum, and energy equations are solved simultaneously in an effort to predict quantitatively the vapor-liquid interfacial resistance and the corresponding lowering of the liquid surface temperature below the saturation temperature of the vapor for the case of a condensing metal over a wide range of conditions.

Since the breadth and structure of the vapor-liquid interfacial transition zone may have a significant influence on the interfacial transport processes, it is assumed, in the absence of a more complete understanding of the interface, that the transition zone is small enough so that an interphase boundary and a liquid surface temperature, $T_S$, may be assigned. Then the rate of evaporation of molecules from the liquid and the rate of heat conduction across the condensate film both depend directly upon $T_S$. Moreover, it is assumed that the interface is relatively undisturbed by shear between the two phases. These have been basic assumptions of the kinetic theory of condensation.$^{(8,59)}$

Experimental and theoretical evidence indicates that Nusselt's theory is an approximately valid description of heat transfer across a condensate film, provided the liquid surface temperature is accurately known and there is no solid-liquid, interfacial, thermal contact resistance. If complete wetting is achieved in a clean system, evidence again indicates that a thermal contact resistance is negligible. Nusselt's theory can be used to describe heat transfer across the condensate film and to estimate $T_S$, if the heat flux and $T_W$ (the condenser surface temperature) are known.

It is also assumed that a nearly constant pressure $P_V$ is maintained in the vapor up to the liquid surface, but the interfacial pressure must ultimately be determined from a momentum balance.

The model considered then is the film condensation of a pure vapor, free from contamination, upon a vertical cool surface. If the saturated vapor temperature, $T_V$, and pressure, $P_V$, and a condenser surface temperature are known, a unique heat flux, $q/A$, should be specified by these conditions for a particular fluid. Schrage$^{(8)}$ recognized this requirement. Figure 1 gives the schematic representation of this model. The "temperature jump" might exist as a thermal gradient, as illustrated, if the vapor
near the interface is subcooled by thermal conduction. Bulk nucleation theory, which requires a substantial degree of supersaturation to maintain net nucleation, permits subcooling.

![Fig. 1: Model of Condensate Film and Vapor Phase Boundary Layer](image)

A. Macroscopic Transport Approach

The Bernoulli equation and the equation of conductive and convective energy transport across the vapor boundary layer were applied to condensation of superheated steam by Silver and Simpson
\(^{44}\) in an attempt to relate the "temperature jump" to the vapor superheat. Also, for condensing saturated vapors, it can be assumed that a thermal gradient in the vapor does exist, as shown in Fig. 1. In this case, the energy equation can be readily written. The momentum equation need not be considered, because it is assumed that the pressure is constant across the vapor boundary layer at \( P_V \). The solution of the differential equation of energy transport is then given by

\[
\frac{T - T_s}{T_v - T_s} \approx \exp \left\{ \frac{q}{A \lambda_v} \frac{c_p V}{k_v} x \right\},
\]

where \( T \) is the temperature at some point "\( x \)" in the vapor boundary layer.

The heat flux is given by the rate of release of the latent heat of condensation, including the effect of vapor subcooling, at the vapor-liquid interface, as follows:

\[
\frac{q/A}{A} \approx \rho_v \bar{u}_v \lambda_v \approx \rho_v \bar{u}_v \lambda_s - k_v \left( \frac{\partial T}{\partial x} \right)_s.
\]

Using the derivative of Equation (3) in Equation (4) results in

\[
\frac{q/A}{A} \approx \rho_v \bar{u}_v \lambda_v \approx \rho_v \bar{u}_v [\lambda_s - c_{pV}(T_v - T_s)].
\]

Equations (4) and (5) indicate that while the temperature jump, \( T_v - T_s \), does not markedly alter the heat released per pound of condensate, a relatively large value for \( T_v - T_s \) reduces the driving force for conduction heat
transfer across the condensate film, so that \( q/A \) and \( \rho_vu \) are reduced accordingly. The macroscopic approach does not predict the magnitude of the temperature jump, but it does suggest that an accurate theory of the temperature jump must include subcooling of the vapor boundary layer by molecular conduction.

Fixing a value of the thermal boundary layer thickness depends upon arbitrarily specifying a ratio \((T - T_s)/(T_s - T_v)\) near unity. The same arbitrary ratio should not be picked for each heat flux, but in some cases values can be specified that result in a ratio of boundary layer thickness to mean free path, \( \Delta x/L \), that is approximately unity. If \( \Delta x/L \) is indeed nearly unity, then bulk vapor conditions prevail up to the vapor-liquid interface, and the kinetic theory of condensation may be validly applied, as long as the evaporation and scattering processes can be described.

Schrage\(^{(8)}\) recognized that the "temperature jump distance," \( \Delta x \), may be greater than one mean free path and that beyond \( \Delta x \) the vapor must be described as "a uniform gas in simple mass motion," but he did not attempt to specify \( \Delta x \). Subsequent use of Schrage's theory by most authors has been based upon the assumption that bulk vapor conditions prevail up to the vapor-liquid interface, thus implying that \( \Delta x/L \approx 1 \). A notable exception is the work of Silver and Simpson,\(^{(44)}\) who took \( \Delta x \) to be one mean free path and described gradients in vapor properties beyond \( \Delta x \) in terms of the macroscopic transport equations for condensing superheated steam.

B. Approximate Kinetic Theory Approach

To describe interfacial transport phenomena in film condensation according to the kinetic theory of condensation, it is necessary to retain the assumption that interactions between condensing, evaporating, and reflecting molecules do not alter the velocity distribution and density of vapor adjacent to the condensate surface. Thus, bulk vapor conditions are assumed to prevail up to one mean free path of the liquid surface. Lack of fundamental knowledge about interfacial collisional processes is the main reason for making such an assumption. The test of this approximation must depend upon the physical consistency of the results to be obtained. It is expected that among these results reasonable predictions of the condensation and evaporation coefficients should be found, in addition to heat flux and liquid surface temperature.

In accord with Schrage,\(^{(8)}\) one would expect these results to depend uniquely upon \( T_v, P_v, T_w \), and the fluid specified. The equations that govern the process must be solved simultaneously. These equations are the conservation of mass, momentum, and energy and a normalization of the composite velocity distribution function somewhat similar to that used by Zwick.\(^{(59)}\)

(The bases for the following kinetic theory treatment of condensation can be found in References 8 and 59.)
The model for the analysis is shown in Fig. 2, where the net flux of each property is given by the sum of condensing (incident), evaporating, and reflected molecular fluxes. Whenever a group of molecules is assumed to be Maxwellian, the ideal gas law applies to those molecules.

![Kinetic Theory Model of Vapor-liquid Interface during Net Condensation](image)

The Maxwellian velocity distribution for a gas having a mass velocity, \( \bar{u} \), in direction normal to the interface is given by

\[
f_v(c) = \frac{n_v \beta_v^3}{\pi^{3/2}} \exp\left\{-\beta_v^2[(u_v - \bar{u})^2 + v^2 + w^2]\right\}.
\] (6)

The vector velocity, \( c \), is given by

\[
c^2 = u_v^2 + v^2 + w^2
\] (7)

and

\[
u = u + \bar{u},
\] (8)

where \( u \) is the velocity relative to the net stream velocity \( \bar{u} \).

The general equation for the transport of any molecular property, \( \psi \), of \( c \) in the \( x \)-direction is given by

\[
\bar{\psi} = \iiint u\psi f(c) \, du dv dw.
\] (9)

1. The Net Mass Flux and the Evaporation and Condensation Coefficients

The incident mass flux (i.e., the total number of molecules approaching the liquid from the vapor) is given by

\[
G_c = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\bar{u}} \rho_v(u + \bar{u}) \frac{\beta_v^3}{\pi^{3/2}} \exp\left\{-\beta_v^2(u^2 + v^2 + w^2)\right\} \, du dv dw,
\] (10)
which yields, upon integration and use of the ideal gas law,

\[ G_c = \Gamma_1 P_v \sqrt{\frac{M}{2\pi RT_v}} = \Gamma_1 G_v, \] (11)

where \( \Gamma_1 \) is negative and is given by

\[ \Gamma_1 = \beta_v \bar{u} \pi^{1/2} [1 + \Phi(-\beta_v \bar{u})] - e^{-\beta_v \bar{u}}. \] (12)

It was shown experimentally(8,67,71) that the absolute rate of evaporation of a liquid at temperature \( T_s \) into a high vacuum is given by the kinetic theory of gases as

\[ G_{e,\max} = \sigma_{e,\max} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{0}^{\infty} \rho_s u \frac{\beta_s^3}{\pi^{3/2}} \exp\{-\beta_s^2(u^2 + v^2 + w^2)\} \, du \, dv \, dw, \] (13)

that is, by the Maxwellian description of an ideal vapor at \( T_s \). If the ideal gas law applies to the evaporated molecules and \( P_s \) is the vapor pressure corresponding to \( T_s \), then \( G_{e,\max} \) may be written as

\[ G_{e,\max} = \sigma_{e,\max} P_s \sqrt{\frac{M}{2\pi RT_s}} = \sigma_{e,\max} G_s, \] (14)

where \( \sigma_{e,\max} \) is the maximum value of the evaporation coefficient \( \sigma_e \) achieved for evaporation of a pure liquid into a high vacuum. The evaporation coefficient, \( \sigma_e \), "corrects" the evaporating flux for steric or structural effects associated with polyatomic molecules and the equilibration of their internal degrees of freedom in passing from initial to activated states in evaporation. Knacke and Stranski(71) and Mortensen and Eyring(61) discussed these effects theoretically and showed that spherically symmetric molecules such as \( \text{CCl}_4 \) and monatomic molecules have \( \sigma_e \)'s equal to unity, while unsymmetric polyatomic molecules have values of \( \sigma_e \) between zero and unity. Paul's(67) survey of experimental evaporation coefficients indicated that measured values of \( \sigma_{e,\max} \) for liquid metals are near unity and that polyatomic molecules in general have \( \sigma_e \)'s between zero and unity as the theory predicts. However, when evaporation and condensation occur simultaneously with a relatively low net mass flux, \( \sigma_e \) may be reduced below \( \sigma_{e,\max} \) by the interaction of the evaporating flux with the condensing flux. Thus, in the following treatment, \( \sigma_e \) may be considered as the correction factor for the combined effects of steric hindrance to, and interference of, condensing molecules with the gross rate of evaporation from the liquid surface, as well as departures of \( f_s \) from the equilibrium Maxwellian form assumed. In general, then,
0 < \sigma_e \leq \sigma_{e, \text{max}}. \tag{15}

and only when evaporation occurs into a vacuum does \( \sigma_e \) approach \( \sigma_{e, \text{max}} \). For liquid metals, \( \sigma_{e, \text{max}} \) approaches unity. Thus, the flux of evaporating molecules will be written as

\[
G_e = \sigma_e G_s. \tag{16}
\]

The net flux of mass is the difference between the part of the incoming flux that actually condenses and the evaporating flux. \( G_{\text{net}} \) is negative for net condensation and positive for net evaporation. Hence Equation (2) was written as

\[
G_{\text{net}} = \sigma_C \Gamma_1 G_V + \sigma_e G_s,
\]

where \( \sigma_C \) has heretofore been considered to be the fraction of incident molecules that actually condenses; the remaining fraction, \( 1 - \sigma_C \), is reflected and contributes nothing to the net mass flux. Note that \( \sigma_C \) "corrects" the incident molecular velocity distribution for deviations from its assumed Maxwellian form, so that interpretations of this coefficient must recognize that it represents more than the singular physical significance attached to it in previous work. Thus \( \sigma_C \) may be only approximately the fraction of incoming molecules condensed, if the real \( f_V \) is not Maxwellian (i.e., if the velocity distribution of incident vapor molecules is distorted by interactions between incident molecules and molecules leaving the vapor-liquid interface).

Furthermore, there appears to be no compelling reason to assume that \( \sigma_e = \sigma_C \), except for the sake of simplifying the equations. Generally, only as \( G_{\text{net}} \) approaches zero (i.e., as thermodynamic equilibrium is approached, and \( \Gamma_1 G_V \) and \( G_s \) become identical), must \( \sigma_e \) approach \( \sigma_C \). Each coefficient is associated with a different mechanism, i.e., capture by and escape from a strong intermolecular force field. Thus, \( \sigma_e \) and \( \sigma_C \) ought to be determined by simultaneous solution of a closed set of equations describing the condensation process. (To this author's knowledge, condensation coefficients have never been measured at thermodynamic equilibrium. Hence specific equilibrium values of \( \sigma_C \) are unknown.)

One might consider \( \sigma_C \) to be related to the probability, \( p_C \), that a molecule will condense. This probability would undoubtedly depend upon the energetics of collisions between two molecules and between a molecule and clusters of molecules in the liquid. The condensing coefficient might then be given by an expression such as

\[
\sigma_C = \frac{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} p_C m u v f_V \, du \, dv \, dw}{\Gamma_1 G_V}. \tag{17}
\]
The use of $\sigma_C$ can be avoided if Equation (2) is rewritten to include the flux of reflected molecules, $G_R$, the form of which must be determined by the proper velocity distribution function for the reflected molecules. It is expected that this distribution function $f_R$ will not be Maxwellian except at thermodynamic equilibrium, because of (a) inelastic collisions at the vapor-liquid interface and (b) the possibility of condensation favoring the less energetic molecules approaching the liquid. Thus,

$$G_{\text{net}} = \Gamma_1 G_V + \sigma_e G_S + G_R, \quad (18)$$

where

$$G_{\text{net}} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \rho_v(u + \bar{u}) \frac{\beta_v^3}{\pi^{3/2}} \exp\{-\beta_v^2(u^2 + v^2 + w^2)\} \, du \, dv \, dw, \quad (19)$$

and

$$G_R = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{0}^{\infty} m u f_R \, du \, dv \, dw, \quad (20)$$

and

$$G_R = (1 - \sigma_C) \Gamma_1 G_V. \quad (21)$$

2. **Momentum Flux and the Interfacial Pressure Balance**

The net flux of the x-component of momentum across the interface is given by

$$M_{\text{net}} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \rho_v(u + \bar{u})^2 \frac{\beta_v^3}{\pi^{3/2}} \exp\{-\beta_v^2(u^2 + v^2 + w^2)\} \, du \, dv \, dw, \quad (22)$$

or

$$M_{\text{net}} = \rho_v/(2\beta_v^2) + \rho_v\bar{u}^2 = P_v + 1/(\rho_v) G^2_{\text{net}}. \quad (23)$$

This net flux or pressure on the interface is the sum of the normal momentum fluxes of molecules striking the interface, and those evaporating and reflecting from the interface, so that

$$M_{\text{net}} = M_C + M_e + M_R, \quad (24)$$
where

\[ M_c = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{-\bar{u}} \rho_v (u + \bar{u})^2 \frac{\beta_v^3}{\pi^{3/2}} \exp\left\{-\beta_v^2 (u^2 + v^2 + w^2)\right\} \ dudvdw \tag{25} \]

\[ M_c = \rho_v \Gamma_2 / (4 \beta_v^2) = P_v \Gamma_2 / 2 \tag{26} \]

where

\[ \Gamma_2 = [1 + \Phi(-\beta_v \bar{u})] (1 + 2 \beta_v^2 \bar{u}^2) - 2 \beta_v \bar{u} e^{-\beta_v^2 u^2}, \tag{27} \]

\[ M_e = \sigma_e \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{0}^{\infty} \rho_s u^2 \frac{\beta_s^3}{\pi^{3/2}} \exp\left\{-\beta_s^2 (u^2 + v^2 + w^2)\right\} \ dudvdw, \tag{28} \]

and

\[ M_e = \sigma_e \rho_s / (4 \beta_s^2) = \sigma_e P_s / 2. \tag{29} \]

It is assumed that the reflected momentum flux can be written in the form of \( P_R \) as follows:

\[ M_R = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{0}^{\infty} mu^2 f_R \ dudvdw = P_R / 2. \tag{30} \]

Thus the momentum balance can be written as

\[ P_v + G_{\text{net}}^2 / \rho_v = P_v \Gamma_2 / 2 + \sigma_e P_s / 2 + P_R / 2, \tag{31} \]

or

\[ (2 - \Gamma_2) P_v + 2 G_{\text{net}}^2 / \rho_v = \sigma_e P_s + P_R. \tag{32} \]

The pressure term \( P_R \) cannot be written in a manner analogous to the mass flux term \( G_R \). That is,

\[ P_R \neq (1 - \sigma_c) P_v \Gamma_2, \tag{33} \]

except at thermodynamic equilibrium, because some momentum may be lost to the liquid in the inelastic reflection process. Moreover, \( \sigma_c \), if related to the probability that a molecule condenses, should be evaluated inside the integral describing the transport of molecular property. The
integrations will then yield different correction factors for the condensing fluxes of mass, momentum, and energy. Thus, if \( p_C \) is the energy-dependent probability that a molecule condenses, and \( \sigma_M \) is considered to be that fraction of incident momentum flux transferred to the liquid, then

\[
G_C - G_R = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\bar{u}} p_C m(u + \bar{u}) f_v \, du \, dv \, dw = \sigma_C \Gamma_1 G_v; \tag{34}
\]

and

\[
M_C - M_R = \Delta M + \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\bar{u}} p_C m(u + \bar{u})^2 f_v \, du \, dv \, dw = \sigma_M \Gamma_2 P_v/2; \tag{35}
\]

where \( \Delta M \) is the additional amount of momentum transferred to the liquid in the inelastic reflection process at the liquid surface. Thus, by comparison, \( \sigma_C \neq \sigma_M \), except at equilibrium.

3. **Kinetic Energy Balance**

   The following treatment holds only for monatomic molecules. The potential energy or latent heat of each molecule is referred to a common temperature and hence is of no concern. The treatment may be extended to cover polyatomic molecules by including vibrational and rotational energy terms, but these effects were not considered, because metal vapors will be treated in this study as monomeric (i.e., monatomic).

   The net kinetic energy flux is given by

\[
E_{\text{net}} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\bar{u}} (u + \bar{u}) \frac{m}{2} c^2 n_v \frac{\beta_v^3}{\pi^{3/2}} \exp\{-\beta_v^2 (u^2 + v^2 + w^2)\} \, du \, dv \, dw; \tag{36}
\]

or

\[
E_{\text{net}} = \rho_v \bar{u}^3/2 + 5RT_v \rho_v \bar{u}/2M = G_{\text{net}}/(2 \rho_v \bar{u}) + 5RT_v G_{\text{net}}/2M. \tag{37}
\]

   The incoming kinetic energy flux is given by

\[
E_C = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\bar{u}} (u + \bar{u}) \frac{m}{2} c^2 n_v \frac{\beta_v^3}{\pi^{3/2}} \exp\{-\beta_v^2 (u^2 + v^2 + w^2)\} \, du \, dv \, dw; \tag{38}
\]

or

\[
E_C = (\Gamma_1 + \Gamma_3) G_v \, RT_v/M; \tag{39}
\]
where

$$\Gamma_3 = \left( \frac{3}{2} + \beta \nu \bar{u}^2 \right) \pi^{1/2} \beta \nu \bar{u} \left[ 1 + \Phi(-\beta \nu \bar{u}) \right] - e^{-\beta \nu \bar{u}^2} \left( 1 + \beta \nu \bar{u}^2 \right). \quad (40)$$

The evaporating kinetic energy flux is given by

$$E_e = \sigma_e \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{0}^{\infty} \frac{\rho_s}{2} uc^2 \frac{\beta^3_s}{\pi^{3/2}} \exp \left\{ -\beta^3_s (u^2 + v^2 + w^2) \right\} \, du \, dv \, dw; \quad (41)$$

or

$$E_e = 2\sigma_e G_s \frac{R T_s}{M}; \quad (42)$$

and the reflected kinetic energy flux is given by

$$E_R = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{0}^{\infty} \frac{m}{2} uc^2 f_R \, du \, dv \, dw. \quad (43)$$

The exact form of $E_R$ is not known unless $f_R$ is known. Hence, in the absence of information about $f_R$, the generality of the foregoing equations might be maintained by assuming that $E_R$ has a form similar to $E_C$ and $E_e$. The kinetic energy balance can then be written as

$$(\Gamma_1 + \Gamma_3) G_v T_v - 5 G_{net} T_v / 2 - MG_{net}^3 / (2 R \rho_v^2) = -2 \sigma_e G_s T_s + 2 G_R T_R). \quad (44)$$

Writing $E_R$ as $2G_R T_R (R / M)$ may be forcing $E_R$ to have a near-Maxwellian form, but this approximation remains. The interpretation of $T_R$, the temperature of molecules reflected from the liquid surface, may be questioned unless the form of $f_R$ is known.

If $\sigma_E$ is defined as the fraction of incident kinetic energy lost to the liquid by condensation and energy exchange in reflection, it is obvious that $\sigma_E$ and $\sigma_C$ are not identical. Only if molecules are reflected completely elastically from the liquid surface, and only if the condensation probability is the same for all molecules regardless of energy, are $\sigma_E$ and $\sigma_C$ the same. This is quite clear if one considers that $\sigma_E$ must be evaluated inside the energy integral in terms of $p_c$, the condensation probability, in the same manner as $\sigma_M$ was evaluated in terms of $p_c$ inside the momentum integral on page 31.

4. **Normalization of Composite Velocity Distribution.**

Because the approximate kinetic theory of condensation implies that bulk vapor conditions prevail up to the vapor-liquid interface, consistency in the treatment requires that the composite velocity distribution of molecules adjacent to the interface be normalized. Therefore,
\[
\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} m_f v \, dudvdw = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{-\bar{u}} m_f v \, dudvdw \\
+ \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{0}^{\infty} \sigma_e m_f s \, dudvdw \\
+ \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{0}^{\infty} m_f R \, dudvdw. \quad (45)
\]

If integrated, Equation (45) becomes

\[
\rho_v = \frac{\rho_v}{2} \left[ 1 + \Phi(\beta_v \bar{u}) \right] + \frac{\sigma_e \rho_s}{2} + \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{0}^{\infty} m_f R \, dudvdw. \quad (46)
\]

Thus the conservation equations, the normalized composite velocity distribution function, and the ideal gas law provide a system of five equations, which must be solved simultaneously for \( T_S \), \( \sigma_C \), and \( \sigma_e \), given \( T_v \) and \( q/A \). Obviously, the solution depends upon the form of \( f_R \) assumed. Only if \( f_R \) is Maxwellian can the ideal gas law be applied to the reflected molecules. Equations (45) and (46) may also be considered statements of constant density.

5. **Attempted Solutions**

The foregoing conservation equations and the "constant density" equation were solved numerically using a CDC-160A computer for various combinations of vapor temperature and condensing heat flux up to 1200\(^\circ\)R and 10\(^6\) Btu/hr ft\(^2\) for mercury vapor. Mercury was chosen as the representative fluid, because the data that are subsequently analyzed concern condensing mercury vapor over the pressure range of 0.02 to 14.8 psia. Within the definitions of the kinetic theory of condensation, two requirements that a solution must meet are \( 0 \leq \sigma_C \leq 1.0 \) and \( 0 \leq \sigma_e \leq 1.0 \). If the kinetic theory of condensation adequately describes condensation heat transfer, the equations outlined should provide physically consistent solutions, illustrating systematic trends at least approximately.

a. **Maxwellian \( f_R \)**

(1) **Retaining the Normalized Composite Velocity Distribution Function.** If the molecules reflected from the vapor-liquid interface have a Maxwellian velocity distribution, Equation (46) and the mass, momentum, and energy equations can be written in terms of four unknowns: \( T_S \), \( \sigma_e \), \( P_R \), and \( T_R \). Solution of this system of equations indicates that
they are satisfied simultaneously only at thermodynamic equilibrium and at values of $T_V - T_S$ less than a few degrees. Large temperature jumps are predicted spasmodically, but only at heat fluxes below a maximum lying somewhere between $q/A = 100,000$ and $500,000$ Btu/hr ft$^2$. Moreover, multiple solutions are found in some cases where solutions exist. Misra$^3$ measured fluxes as high as $700,000$ Btu/hr ft$^2$ at atmospheric pressure. Hence it appears that a Maxwellian $f_R$ improperly describes the reflection process, when combined with the requirement of constant density up to the interface.

(2) Relaxing the Requirement of Constant Density. Solution of the mass, momentum, and energy conservation equations, without considering the normalization of the composite velocity distribution, "relaxes" the restrictions of the model somewhat. Solutions so obtained will be regarded as more approximate or qualitative in nature than solutions including the normalized velocity distribution. An additional assumption is needed to permit such a solution, because dropping the "constant density" restriction has not reduced the number of unknowns. Four cases have been calculated, based upon various descriptions of $\sigma_C$.

**Case I: $\sigma_e = \sigma_{e,\text{max}} = 1.0$.** The evaporation coefficient was assumed to exhibit its maximum value of 1.0 for mercury. Double solutions occurred, one of which yielded an almost constant value of $T_V - T_S$, regardless of the heat flux, and values of $\sigma_C \leq 0.15$ for each combination of $T_V$ and $q/A$ over the range of $0.272 \leq P_v \leq 15.7$ psia. The other solutions indicated that the equations are satisfied as $\sigma_C$ approaches $\sigma_e$ (i.e., either at thermodynamic equilibrium or at relatively small temperature jumps). Neither solution may be arbitrarily discarded. Two trends which might be intuitively expected were evident in the results. (1) The reduction of vapor pressure lowered the maximum permissible heat flux in the case of the solutions yielding high values of $T_V - T_S$. (2) At $P_v = 15.7$ and $5.31$ psia, the temperature jumps corresponding to high values of $\sigma_C$ decreased with decreasing heat flux. $T_{RM}$, the temperature that the reflected stream of molecules would have, if it were reflected with a Maxwellian velocity distribution, has been calculated, except in those four instances in which the extrapolation of $T_{RM}$ was uncertain. The multiple solutions are listed in Table I as Solution No. 1 and Solution No. 2. However, the occurrence of double solutions, one of which yields a nearly constant value of $T_S$, regardless of the heat flux, nullifies this case as an accurate description of the condensing system.

**Case II: $\sigma_C = \sigma_e$.** The nature of the results of this case is similar to that of Case I. At some combinations of $T_V$ and $q/A$, triple solutions were obtained. One solution yielded large, nearly constant values of $T_V - T_S$, no matter what heat flux level was specified. For two conditions, negative condensing coefficients were obtained, representing a meaningless result. The two expected trends mentioned in Case I were also observed for $\sigma_e = \sigma_C$. $T_{RM}$ was calculated where accurately possible.
The results are given in Table II. Solutions No. 2 and No. 3, when both occur, are usually similar. However, as long as \( 0 \leq \sigma_c \leq 1.0 \), and \( T_S < T_V \), a solution cannot be arbitrarily discarded. Thus the occurrence of multiple solutions and the absence of systematic trends, as \( q/A \) is varied at a given \( T_V \), nullify Case II as an accurate description of the system.

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<th>( \sigma_c )</th>
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Table II

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Case III: Condensation Favoring Molecules of a Particular Energy Range

(a) Molecules with \( T \leq T_S \) Condense. If the fraction of molecules striking the interface that actually condenses is considered to be the fraction having x-velocity components less than or equal to the value corresponding to \( T_S \), then \( \sigma_c \) is given by
\[
\sigma_c = \frac{\Phi(-\beta_v u) + \Phi\left(\sqrt{\frac{T_s}{2T_v}}\right)}{1 + \Phi(-\beta_v u)}.
\]

(47)

At low surface temperatures, condensation coefficients near unity have been measured. But Equation (47) predicts a maximum \(\sigma_c\) of 0.5 as \(T_s\) approaches zero. Hence, this simple model of \(\sigma_c\) is incorrect.

(b) Vapor Molecules with \(T \geq T_s\) Condense. If the condensation coefficient is considered to be that fraction of incident molecules having \(x\)-velocity components equal to or greater than \(T_s\), then \(\sigma_c\) is given by

\[
\sigma_c = \frac{1 + \Phi\left(\sqrt{\frac{T_s}{2T_v}}\right)}{1 + \Phi(-\beta_v u)}.
\]

(48)

This is equivalent to saying that the molecules that condense have sufficient energy to penetrate the potential barrier to condensation and perhaps "bury themselves" in the liquid. The energy level of the barrier has been arbitrarily set as that corresponding to \(T_s\). However, Equation (48) indicates that this model predicts condensation coefficients greater than unity at thermodynamic equilibrium. Because of this impossible result, this model also must be rejected.

Case IV: Equal Probability of Condensing All Molecules Regardless of Energy; \(\sigma_c = \sigma_M = \sigma_E\). The only solutions found for this case, which also implies elastic reflection of molecules from the liquid surface, were the trivial solutions at \(T_v - T_s = 0\) and \(q/A = 0\) (i.e., at thermodynamic equilibrium). This attempt may be compared with Zwick's\(^{(59)}\) theory, which is based partly upon the same assumptions. However, including two parameters in his "stressed" velocity distributions left enough flexibility in his equations to permit solutions, if a numerical value of \(\sigma_c\) is specified.

b. Perturbed \(f_R\)

(1) Separate Number Densities. To account approximately for the effect of inelastic reflection of molecules from the liquid surface, a Maxwellian velocity distribution, including a perturbation of the component of velocity in the \(x\)-direction, was used of the form

\[
f_R = n_R \beta_v^3 \exp\left\{-\beta_v^2 [u - A_R]^2 + v^2 + w^2]\right\}/\pi^{3/2}
\]

(49)

The perturbation parameter, \(A_R\), might be considered to be related to the average momentum loss to the liquid in the reflection process. In
attempting to solve the mass, momentum, energy, and constant density equations for \( T_S \), \( \sigma_e \), \( \rho_R \), and \( AR \) over the vapor temperature range 800 to 1200\(^\circ\)R and the heat flux range \( 10^4 \) to \( 10^6 \) Btu/hr ft\(^2\), it was found that physically consistent solutions do not exist. At the lower \( T_V \)'s around 800\(^\circ\)R, there were no solutions. At 1100\(^\circ\)R and higher, there appeared to be a continuum of values \( AR \), over a narrow range, that satisfied the equations and yielded decreasing values of \( T_V - T_S \) with increasing values of \( AR \). Obviously the assumptions made about \( f_R \) in this case are unrealistic; \( f_R \) is not so simply determined.

(2) Generalized Velocity Distribution Function for Evaporating, Reflecting, and Condensing Molecules. Because the velocity distribution function gives the number density of molecules that lie in the element of velocity space dudvdw, it appeared that some flexibility could be added to the kinetic theory of condensation without loss of rigor by a slight change in the specification of vapor density. In other words, the number density of the vapor up to the interface is written as "n," and "n" is also the same in the bulk vapor. The components of the velocity distribution could then be written as

\[
f_K = n \beta_K^3 \exp\left\{-\beta_K^2[(u - AR)^2 + v^2 + w^2]\right\}/\pi^{3/2}
\]

where \( K \), a dummy index, can be \( s \), \( v \), or \( R \), depending upon the process, and \( A_S = 0 \), \( A_V = \bar{u} \), and \( A_R = A_R \). The perturbation parameter is retained. Then \( f_K/n \) is the fraction of the number density of molecules in the \( K \) group that contribute to the total number density, \( n \), and lie in the element of velocity space dudvdw. This approach eliminates the need to assume that evaporating and condensing molecules follow ideal gas behavior, as Schrage\(^8\) assumed. The resulting mass, momentum, and energy conservation equations, and the normalized composite velocity distribution contain four unknowns: \( T_S \), \( \sigma_e \), \( AR \), and \( \beta_R \). For two combinations of \( T_V \) and \( q/A \) over the range 800\(^\circ\)R \( \leq \) \( T_V \leq \) 1200\(^\circ\)R (taking 100\(^\circ\)R increments) and \( 10^4 \leq q/A \leq 10^8 \), trivial solutions were obtained. Hence this case also appears to be an inadequate description of the interphase transport processes.

6. Parametric Study

In preparation for the analysis of the available experimental data, according to the kinetic theory of condensation, a parametric study of the conservation equations was performed, rather than a continuation of attempted solutions based upon arbitrary assumptions. The requirement of constant density up to the vapor-liquid interface was relaxed, primarily because it cannot be used without assuming a form of \( f_R \). The effects of systematically varying \( T_S \) and \( \sigma_e \) upon \( \sigma_C \), \( T_R \), \( G_R \), and \( P_R \) for various combinations of \( T_V \) and \( q/A \) were examined. Only the mass, momentum, and energy conservation equations were used, i.e., Equations (2), (21), (32), and (44).
Note that this parametric study does not yield unique solutions of the condensing heat transfer problem. The available experimental data must be analyzed in the same manner. These data did influence the choice and range of variables somewhat. Heat fluxes of $10^4$, $5 \times 10^4$, $10^5$, $5 \times 10^5$, and $10^6$ Btu/hr ft$^2$ were examined. Because the maximum $T_V - T_S$ deduced from data was about 300°F, $T_S$ was varied from $T_V - 300°F$ to $T_V - 10°F$ for each combination of $T_V$, $q/A$, and $\sigma_e$. Mercury-vapor temperature was varied from 1200°F (27.45 psia) to 700°F (0.012 psia) in 50°F increments. Since the study yielded numerous data, representative results are shown and discussed.

a. **Trends in the Condensation Coefficient.** An obvious minimum $\sigma_C$ exists that is required to maintain a given $q/A$ at $T_V$ and $P_V$, if $\sigma_e G_S << \sigma_C \Gamma G_V$. This occurs either when $\sigma_e \approx 0$, or when $T_S << T_V$. These minimum $\sigma_C$'s, listed in Table III, illustrate an important result of the kinetic theory, which has been known for many years, namely, that the minimum $\sigma_C$ decreases with increasing $P_V$ because of the corresponding increase in $\Gamma G_V$, the total number of molecules striking the interface. If $\sigma_e$ increases at fixed $T_V$, $q/A$, and $T_S$, then $\sigma_C$ must increase accordingly to maintain that $q/A$.

The calculations have also indicated that the maximum heat flux possible decreases with decreasing $T_V$ and $P_V$. Table III also lists the cases where the flux limitation is imposed by the sonic velocity of the vapor.

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*Limited by sonic velocity (see Table VIII).

Considerations of practical condensing systems suggested that the maximum $q/A$ calculated be $10^6$ Btu/hr ft$^2$. Hence, at atmospheric and higher pressures, $G_{net}$ was always small compared with $\Gamma G_V$. When such is the case, i.e., when $G_{net} << \Gamma G_V$, the effect of $q/A$ upon $\sigma_e$ may be quite small, particularly as $T_V - T_S$ is decreased. This effect is illustrated in Figs. 3a and b, which compare $\sigma_C$ values at $q/A = 10^4$ and $10^6$ Btu/hr ft$^2$ and $T_V = 1200°F$. It can be seen that $\sigma_C$'s are numerically similar for both heat fluxes, the difference growing as
$T_s$ decreases. As vapor temperature is decreased, the differences between $\sigma_c$'s compared at different heat fluxes become more pronounced. Figures 4a and b illustrate the marked difference between $\sigma_c$'s compared for $T_v = 900^\circ\text{R}$ at $q/A = 10^4$ and $10^6$ Btu/hr ft$^2$. At this low temperature and pressure level, $G_{\text{net}}$ can be a substantial fraction of $G_G v$, whereas at atmospheric and higher pressures, any practical $G_{\text{net}}$ is but a few percent or less of $G_G v$. Figures 5a and b illustrate the same difference, exaggerated more by the decrease in $T_v$ and $P_v$ to $700^\circ\text{R}$ and 0.012 psia, in which case a fivefold increase in $q/A$ values requires roughly a $2\frac{1}{2}$-fold increase in the minimum $\sigma_c$. The estimated maximum possible heat flux at $700^\circ\text{R}$ is 80,000 Btu/hr ft$^2$; hence, Figs. 5a and b compare $q/A = 10^4$ and $5 \times 10^4$ Btu/hr ft$^2$.

![Diagram](image)

Fig. 3. Variation of Condensation Coefficient with Interface Temperature at $T_v = 1200^\circ\text{R}$ and $P_v = 27.45$ psia. (a) $q/A = 10^4$ Btu/hr ft$^2$ and (b) $q/A = 10^6$ Btu/hr ft$^2$. 
Fig. 4. Variation of Condensation Coefficient with Interface Temperature at \( T_V = 900^\circ R \) and \( P_V = 0.766 \) psia. (a) \( q/A = 10^4 \) Btu/hr ft\(^2\) and (b) \( q/A = 10^6 \) Btu/hr ft\(^2\).

Fig. 5. Variation of Condensation Coefficient with Interface Temperature at \( T_V = 700^\circ R \) and \( P_V = 0.012 \) psia. (a) \( q/A = 10^4 \) Btu/hr ft\(^2\) and (b) \( q/A = 5 \times 10^4 \) Btu/hr ft\(^2\).
The trends in $\sigma_c$ show no abrupt changes as any one parameter is varied gradually. So that these trends may be kept in mind, when the experimental data are considered, they are summarized as follows:

1) If a value of $q/A$ and $T_V$ is fixed, $\sigma_c$ must increase as $T_s$ approaches $T_V$ at a constant $\sigma_e$, to compensate for the corresponding increase in evaporation and to maintain the specified heat flux. The reverse also holds (i.e., $T_s$ must follow changes in $\sigma_c$).

2) If several $q/A$ values are compared at the same $P_V$ and $T_V$, such that $G_{net} \ll \Gamma_1G_V$, the $\sigma_c$ values will be nearly independent of $q/A$, but strongly dependent upon $\sigma_e$ and $T_s$.

3) If heat flux decreases at constant $T_V$, $T_s$, and $\sigma_e$, the condensation coefficient decreases.

4) If all other factors are constant, the condensation coefficient increases or decreases as $\sigma_e$ increases or decreases.

5) Finally, the minimum $\sigma_c$ required to maintain a given $q/A$ decreases as bulk vapor pressure increases.

b. Trends in $T_R$. The trends in $\sigma_c$ aid in understanding the trends in the estimated temperature, $T_R$, of the stream of vapor molecules reflected from the liquid surface. Temperature $T_R$ is called "estimated" because it was calculated from Equation (44) which retained the assumption that the reflected energy term could be represented by $2G_RT_R$. Generally, $T_R$ may be only a term having the units of temperature, which is related to the true temperature of the reflected molecules by the unknown velocity distribution $f_R$. The true $T_R$ is a direct measure of the mean kinetic energy of the reflected molecular flux.

The temperature, $T_{RM}$, that the reflected molecular stream would have, if its velocity distribution were Maxwellian, has also been calculated for comparison with $T_R$, $T_s$, and $T_V$. Generally, for all the condensing mercury conditions examined, $T_R$ is non-Maxwellian and $T_R \neq T_{RM}$. Approximate values of $(T_R - T_{RM})_{avg}$ and $(T_R - T_{RM})_{max}$ are listed in Table IV. The average values were taken by scanning the range of $T_s$ and $\sigma_e$ values studied and are accurate to about $\pm10\%$ above $2^\circ R$ and about $\pm30\%$ below $2^\circ R$. Absolute numerical averages are unimportant, but the approximate trends are significant.

In some cases, the maximum differences could not be accurately determined and the approximate magnitude is indicated as an inequality; e.g., at $T_V = 950^\circ R$ and $q/A = 10^6$ Btu/hr ft$^2$, $(T_R - T_{RM})_{max} > 200^\circ R$. It can be seen from Table IV that $(T_R - T_{RM})_{avg}$ increases with increasing $q/A$ and with decreasing $T_V$. Hence $(T_R - T_{RM})_{max}$ follows
the same pattern. The results suggest that the molecules are generally reflected with a Maxwellian velocity distribution only at thermodynamic equilibrium. The results also indicated that \( (T_R - T_{RM})_{\text{max}} \) always occurred at values of \( T_S \) close to \( T_V \), and hence at values of \( \sigma_e \) close to unity and \( \sigma_c \) closer to unity than to zero.

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<th>950</th>
<th>900</th>
<th>850</th>
<th>800</th>
<th>750</th>
<th>700</th>
</tr>
</thead>
<tbody>
<tr>
<td>( q/A, ) ( P_v, ) psia</td>
<td>27.45</td>
<td>17.28</td>
<td>10.42</td>
<td>5.97</td>
<td>3.23</td>
<td>1.64</td>
<td>0.766</td>
<td>0.327</td>
<td>0.125</td>
<td>0.042</td>
<td>0.012</td>
</tr>
<tr>
<td>( G_{net} ) ( \text{Btu/hr ft}^2 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
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<td></td>
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</tr>
</tbody>
</table>

Table IV

APPROXIMATE VALUES OF \( T_R - T_{RM} \) FROM PARAMETRIC STUDY

Because the effect upon \( T_R \) of changing \( T_V \) is gradual, only six illustrations are presented at the extremes of the conditions studied. Figures 6a, 6b, and 7a show typical trends in \( T_R \) when \( G_{net} \) is small compared with \( G_V \) for \( T_V = 1200 \) and \( 900\degree R \). Figures 7b, 8a, and 8b show typical trends in \( T_R \) when \( G_{net} \) is of the same order of magnitude as \( G_V \) for \( T_V = 900 \) and \( 700\degree R \).

Approaching thermodynamic equilibrium may be considered the condition of \( G_{net}/G_V \) approaching \( 0 \). Thus, either decreasing \( G_{net} \) or increasing \( T_V \) will effect an approach to equilibrium with the corresponding effect that \( T_R \) approaches \( T_V \). Otherwise, generally, \( T_R \neq T_V \) and \( T_R \neq T_S \). For \( \sigma_e = 0 \), \( T_R \) is independent of \( T_S \), and this \( T_R \) decreases with increasing \( q/A \). Thus, if \( T_R \) is representative of the mean kinetic energy of molecules reflected from the liquid surface, \( f_R \) apparently would depend strongly upon \( T_V \) and \( T_S \), or upon \( T_V \) and \( q/A \). Since \( T_R \neq T_V \), two factors appear to be possible: namely, inelastic re-}

tection with energy loss at the liquid surface \( (T_V > T_R) \), and a condensation probability that depends upon relative energy levels most likely favoring the condensation of the less energetic molecules \( (T_R > T_V) \). However, the condensation probability is not so simply depicted, if one considers that a minimum relative kinetic energy of a collision system might be required such that a potential energy barrier to condensation must be penetrated.
Fig. 6. Variation in Estimated Temperature of Molecules Reflected from Liquid Surface at $T_V = 1200^\circ R$ and $P_V = 27.45$ psia. (a) $q/A = 10^4$ Btu/hr ft$^2$ and (b) $q/A = 10^6$ Btu/hr ft$^2$.

Fig. 7. Variation in Estimated Temperature of Molecules Reflected from Liquid Surface at $T_V = 900^\circ R$ and $P_V = 0.766$ psia. (a) $q/A = 10^4$ Btu/hr ft$^2$ and (b) $q/A = 10^6$ Btu/hr ft$^2$. 
Fig. 8. Variation in Estimated Temperature of Molecules Reflected from Liquid Surface at $T_V = 700^\circ R$ and $P_V = 0.012$ psia. (a) $q/A = 10^4$ Btu/hr ft$^2$ and (b) $q/A = 5 \times 10^4$ Btu/hr ft$^2$.

Before condensing heat transfer data are analyzed on the basis of the foregoing examination of the kinetic theory, it is important to establish the effects of dimerization, noncondensible gases, and corrosion product contamination of the interface upon the value and interpretation of the condensation coefficient.

7. Condensation of Dimerized Vapor

Metal vapors tend to polymerize.\(^{(80-85)}\) Figures 9 and 10, based upon data from References 81-85 and 86, show the estimated dimeric

![Graph showing molecular weight vs. saturation temperature]

**Fig. 9. Estimated Dimeric Composition of Mercury Vapor**
compositions of saturated mercury vapor and saturated alkali metal vapors, respectively. But the mechanism of polymerization is uncertain; i.e., it is not known if polymerization occurs only in the vapor after evaporation, or if some molecules evaporate as polymers. Metal vapors are usually treated as mixtures of monomer and dimer, larger polymers being neglected. The reaction is

$$2A_1 = A_2.$$  \hspace{1cm} (51)

The net condensing flux for such a system, according to kinetic theory, would be

$$G_{\text{net}} = (\sigma_{c_1\rho_{1V}} + \sigma_{c_1\rho_{2V}}) \frac{1}{2\pi^{1/2} \beta_V} + (\sigma_{e_1\rho_{1S}} + \sigma_{e_2\rho_{2S}}) \frac{1}{2\pi^{1/2} \beta_S} ; \hspace{1cm} (52)$$

or

$$G_{\text{net}} = \left[ \sigma_{c_1M_1x_{1V}} + \sigma_{c_2M_2(1-x_{1V})} \right] \frac{\Gamma_V P_V}{2\pi^{1/2} \beta_V RT_V}$$

$$+ \left[ \sigma_{e_1M_1x_{1S}} + \sigma_{e_2M_2(1-x_{1S})} \right] \frac{P_S}{2\pi^{1/2} \beta_S RT_S} ; \hspace{1cm} (53)$$
and because $M_2 = 2M_1$, 

$$G_{\text{net}} = \left[ x_{1V} + \frac{2\sigma_{c_2}}{\sigma_{c_1}}(1 - x_{1V}) \right] \sigma_{c_1} G_{V_1} + \left[ x_{1S} + \frac{2\sigma_{e_2}}{\sigma_{e_1}}(1 - x_{1S}) \right] \sigma_{e_1} G_{S_1}. \quad (54)$$

Thus the effect of dimerization upon $G_{\text{net}}$, or upon the temperature jump, $T_\nu - T_\sigma$, can be determined if the condensing coefficients for monomer, $\sigma_{c_1}$, and dimer, $\sigma_{c_2}$, the mole fraction of monomer in the vapor, $x_{1V}$, the evaporation coefficients for monomer, $\sigma_{e_1}$, and dimer, $\sigma_{e_2}$, and the mole fraction of monomer in the evaporating flux, $x_{1S}$, are known. A complete treatment of these factors is beyond the scope of this work, but comparison of Equations (2) and (52) indicates that $\sigma_c$ and $\sigma_e$ calculated from Equation (2), which is based upon pure monomeric liquid and vapor, each include the effects of dimerization, in addition to the possible effects of non-Maxwellian velocity distributions and steric hindrance. Thus,

$$\sigma_c = \left[ x_{1V} + (2\sigma_{c_2}/\sigma_{c_1})(1 - x_{1V}) \right] \sigma_{c_1}; \quad (55)$$

and

$$\sigma_e = \left[ x_{1S} + (2\sigma_{e_2}/\sigma_{e_1})(1 - x_{1S}) \right] \sigma_{e_1}. \quad (56)$$

At ordinary pressures for metal vapors, $x_{2V} < 0.15$, so that probably, $\sigma_c \approx \sigma_{c_1}$. If $x_{2S}$ is also small, $\sigma_e \approx \sigma_{e_1}$.

It is not evident how the mechanisms or reaction kinetics of dimerization might interfere with condensation.

Hicks\(^{(87)}\) has cast doubt upon the probability of polymerization in mercury and alkali metal vapors, because these exhibit relatively continuous energy spectra with interatomic distances varying continuously.

8. Noncondensable Gases

Figure 11 and the following analysis show that values of $\sigma_c$ and $\sigma_e$, calculated for condensation in the presence of a noncondensible gas, are meaningless unless the concentration of the noncondensible gas at the interface is known and accounted for.

![Fig. 11](Model of Film Condensation in the Presence of a Noncondensible Gas)
\[ G_{\text{net}} = \sigma_{\text{ci}} \Gamma_{\text{i}i} G_{\text{vi}} + \sigma_{\text{e}} G_{\text{e}}, \]  

where the subscript, \( i \), refers to conditions in the approaching vapor at the vapor-liquid interface. If the problem were treated as if non-condensible gases were absent, a comparison of Equations (2) and (55) would show that

\[ \sigma_{\text{c}} \Gamma_{\text{v}} G_{\text{v}} = \sigma_{\text{ci}} \Gamma_{\text{li}} G_{\text{vi}}. \]  

This can be reduced to

\[ \sigma_{\text{c}} \Gamma_{\text{i}} P_{\text{v}} / \sqrt{T_{\text{v}}} = \sigma_{\text{ci}} \Gamma_{\text{li}} P_{\text{i}} / \sqrt{T_{\text{i}}}, \]  

where \( \sigma_{\text{ci}} \) is the true condensing coefficient, and \( T_{\text{i}} \) is the saturation temperature of the condensible vapor corresponding to vapor pressure \( p_{\text{i}} \) at the interface. Therefore,

\[ \sigma_{\text{c}} = \frac{P_{\text{v}}}{p_{\text{i}}} \left( \frac{T_{\text{i}}}{T_{\text{v}}} \right)^{1/2} \left( \frac{\Gamma_{\text{i}}}{\Gamma_{\text{li}}} \right) \sigma_{\text{ci}}; \]  

or

\[ \sigma_{\text{c}} = \frac{\sigma_{\text{c}}}{x_{\text{i}}} \left( \frac{\Gamma_{\text{i}}}{\Gamma_{\text{li}}} \right) \left( \frac{T_{\text{i}}}{T_{\text{v}}} \right)^{1/2} \]  

Now \( x_{\text{i}} < 1 \), depending upon the amount of noncondensible gas present in the system and upon the overall thermal driving force, \( T_{\text{v}} - T_{w} \); but \( \Gamma_{\text{i}} < \Gamma_{\text{li}} \) and \( T_{\text{i}} < T_{\text{v}} \). However, if a sizeable "temperature jump" is caused by non-condensible gases, the effect of \( x_{\text{i}} \) will be greater than the effect of \( \Gamma_{\text{i}}/\Gamma_{\text{li}} \) and \( T_{\text{i}}/T_{\text{v}} \). Thus \( \sigma_{\text{c}} \) will be greater than the apparent \( \sigma_{\text{c}} \) calculated, assuming that the vapor is free of noncondensible gases.

If noncondensible gases are present, \( x_{\text{i}} \) can be estimated from calculations based upon a theory similar to that used by Sparrow and Lin\(^{41}\) in describing the filmwise condensation of steam in the presence of a noncondensible gas. The free-convection effect must be included, for although the condensing vapor flow tends to concentrate noncondensibles at the interface, free convection tends to remove the gases from the interface. The direction of removal depends upon the relative magnitudes of the molecular weights of vapor and noncondensible gases.

An example of failure to include the diffusional resistance of noncondensible gas is the work of Baer and McKelvey\(^{42}\) on condensing methanol in the presence of known amounts of air. Their experimental condensation coefficients contain many effects, but the results do illustrate that the apparent \( \sigma_{\text{c}} \) decreases with increasing concentration of noncondensible gas (based upon \( \sigma_{\text{c}} = \sigma_{\text{e}} \)).
9. Corrosion Product Contamination of the Vapor-liquid Interface

Corrosion rates of various containment materials in liquid metals are available.\(^{(88)}\) An examination of these rates for common containment materials shows that concentration of corrosion products at the vapor-liquid interface by diffusion through the condensate film is insufficient to markedly alter the structure of the vapor-liquid interface and reduce the rate of condensation by contaminating the liquid surface.

Consider the case of perfect mixing. The film flow model is essentially the same as Nusselt's with the addition of corrosion products imposed at the solid-liquid boundary as shown in Fig. 12. If \( R = \) the rate of corrosion, \( c = \) the concentration of corrosion products at \( x \), and \( W \) is the rate of condensation, then

\[
R \approx \frac{d}{dy} \int_{0}^{\delta} cu \, dx, \quad (62)
\]

and

\[
W = \rho_L \frac{d}{dy} \int_{0}^{\delta} u \, dx. \quad (63)
\]

At any one point then,

\[
c_{\text{approx}} \approx \frac{\rho_L R}{W}. \quad (64)
\]

A corrosion rate of 1000 mils per year is considered excessive. If this were the rate for nickel, the approximate concentration of nickel in a mercury condensate film at a \( q/A \) of \( 10^5 \) Btu/hr ft\(^2\) would be

\[
c_{\text{approx}} \approx \rho_L (1.0 \text{ in./yr})(8.9)(62.4 \text{ lb/ft}^3) (1 \text{ yr}/8760 \text{ hrs}) \times (1 \text{ ft}/12 \text{ in.})(125 \text{ Btu/lb})/(10^5 \text{ Btu/hr ft}^2), \quad (65)
\]

or

\[
c_{\text{approx}} \approx 6.6 \times 10^{-6} \rho_L. \quad (66)
\]

Consequently, the small surface contamination by corrosion products would be expected to have a negligible effect upon the so-called "condensing coefficient," unless some other mechanism exists by which the contaminant concentrates excessively at the surface. The latter probability seems quite unlikely on any vertical surface.
III. DATA ANALYSIS

The only data currently available on film condensation of metal vapors on vertical surfaces are those of Misra and Bonilla (3) and those of Sukhatme and Rohsenow (7). Engelbrecht’s (6) data on condensing potassium and rubidium may be treated as film condensing data, but cannot definitely be said to be so, because the condensate was not viewed. Hence this analysis concentrates on condensing mercury data. The data of Misra (89) that were analyzed are those from tests using a nickel condenser and covering the mercury vapor pressure range from 0.5 to 14.8 psia. Sukhatme condensed mercury on a nickel tube over the vapor pressure range of 0.02 to 0.33 psia. Sukhatme did not actually check any of Misra’s data.

An examination of the authors’ methods of measuring or calculating the condenser surface temperature, $T_w$, revealed no serious errors. Thus, following the film condensation model discussed in Section II, the interface temperature was estimated, using the measured wall temperature and heat flux combined with Nusselt’s theory for each data point. Because the thermal conductivity of mercury is relatively high, as it is for all metals, the temperature drop across the condensate film is not large. Hence, for the mercury data considered in the following discussion, $T_s \approx T_w$. The pressure, temperature, heat flux, and condensation coefficient data of Misra and Sukhatme, upon which the following analyses are based, are included in Tables VIII and IX, respectively. In the analysis of several of Misra’s tests, consecutive, nearly equal data were averaged.

A. Heat Transfer Coefficients

Sukhatme implied that Misra’s data suffered errors due to the presence of noncondensible gases, primarily because his own heat transfer coefficients increase with increasing vapor pressure at constant heat flux, while Misra’s data show the opposite effect. In two runs, Sukhatme purposely added noncondensible gases, increasing $P_v$, and observed a decrease in $h$. Misra had performed some tests in which noncondensible gases were continuously removed from the condenser at various rates. He then extrapolated a plot, on Cartesian coordinates, of $h$ versus the rate of noncondensibles removed to find the $h$ corresponding to zero rate of removal of noncondensible gases (i.e., to the $h$ for pure vapor condensing). Sukhatme challenged this extrapolation, emphasizing that the concentration of noncondensibles in the system, not the leakage rate of gases from it, controls the diffusional resistance of noncondensible gases at the interface. Sukhatme supported this by replotting Misra’s heat transfer coefficients as a function of rate of removal of noncondensible gases. He showed that a straight line results on a log-log plot, which could easily be extrapolated, as noncondensible gas
concentration approaches zero, to higher heat transfer coefficients than Misra obtained. An argument in Misra's favor was his contention that the apparent reproducibility of his results would have been absent had non-condensible gases been a factor.

Gel'man's work (48) concerned dropwise condensation of mercury vapor on carbon steel, not film condensation on nickel. His data might be taken as additional evidence that Misra's data include some non-condensible gas resistance of unknown magnitude. Gel'man condensed mercury vapor flowing over a vertical tube and a horizontal tube, and found that increasing air concentration above 1% decreased his heat transfer coefficients, and that increasing $P_V$ at constant $\Delta T$ increased $h$. He noted, by comparison of his own heat transfer rates with and without air added to the system, that air concentrations less than 1% do not impair condensation.

Sukhatme, by purposely adding gas to the system following his test No. 12, conducted at 0.162 psia, demonstrated that the addition of noncondensible gases reduces the heat transfer coefficient. The subsequent total pressures at which he took data were 0.251 and 0.306 psia. Apparently he did not measure the total amount of noncondensible gases added, but the noted increases in total pressure suggest that he might have added a large percentage of gas to the system. Hence, the corresponding twofold reduction in heat transfer coefficient he observed is not surprising, and he did not prove that the addition of a relatively small percentage of noncondensible gas to the system drastically reduces heat transfer. Indeed, Gel'man implied that a minimum gas concentration exists below which the concentration of noncondensible gases is negligible. It is not impossible that Misra might have been able to maintain such a low gas concentration.

Sukhatme's results for $h$ versus $P_V$ are shown in Fig. 13. The curves at $q/A = 73,000$ Btu/hr ft$^2$ appear to "taper off" as $P_V$ approaches 0.3 psia. The 100,000 Btu/hr ft$^2$ curve even shows a

![Graph](image_url)
maximum. Whether maximum h values or asymptotic h values actually occur beyond this point is uncertain. A gap exists between 0.33 and 0.5 psia, the latter being the lowest pressure at which Misra took data that might be considered film condensation data. Figure 14 illustrates typical trends in h observed by Misra. The inverse dependence of his h values upon P_v is quite clear.

The concepts of the kinetic theory of condensation seem to support Sukhatme. One might expect h to decrease as P_v increases at constant q/A, because of the corresponding increase in the total number of molecules striking the liquid surface, and to decrease in the T_v - T_s necessary to maintain the specified net mass flux. However, the lack of fundamental knowledge of the dependence of the condensation coefficient upon T_v and T_s, and the absence of quantitative information about the percentage of noncondensible gases present in all tests but Gel'man's, postpone final judgment.

B. Condensation Coefficient

The parametric study and preceding analyses of Section II illustrated that current theoretical concepts do not permit predicting condensation coefficients. They showed that the interface temperature must be specified, and that φ_c may then be discussed in terms of T_v, q/A, T_s, and φ_e. Because T_v and T_w should supposedly specify a unique q/A for film condensation of a pure fluid, the condensation coefficients were correlated with the dimensionless quantity (T_v - T_w)/T_v, and also in one case with T_s. After T_s was estimated and φ_e was arbitrarily specified, Equation (2) was used to calculate φ_c from the data. This calculation procedure parallels the parametric study, the difference being that in the parametric study T_s was not fixed by experimental measurements. The momentum and energy equations were not used because the form of f_R is unknown. Three cases were considered, two of which are essentially Schrage's theory:(8) (1) φ_e = φ_c, which is the common usage of φ_c; (2) φ_e = φ_c,max = 1.0; (3) φ_c calculated from Zwick's theory,(59) which apparently requires no assumptions about φ_e.
Sukhatme overlooked a significant factor in analyzing his condensation coefficients. In several of his 25 tests he maintained a condenser wall temperature low enough so that the evaporating mass flux, $\sigma_e G_s$, corresponding to $T_s$ was negligibly small. Hence, the condition discussed in the parametric study, which specifies the minimum $\sigma_c$ for a given $q/A$ and $T_v$, was satisfied. Because evaporation was negligible in these tests, the corresponding $\sigma_c$ values are the actual minimum $\sigma_c$ values for the given conditions, regardless of the value of the evaporation coefficient. But each combination of $T_v$ and $T_w$ should specify a unique $q/A$ and $\sigma_c$. The $\sigma_c$ values calculated by Sukhatme for these tests varied from 0.605 to 0.368. An average $\sigma_c$ cannot describe these tests. His other $\sigma_c$ values include the effects of evaporation. Thus his use of an average $\sigma_c$ to describe all his tests over the pressure range of 0.02 to 0.33 psia is physically meaningless, as he admitted in saying that "the significant point is that in a system a reasonably constant value (of $\sigma_c$) is attainable." Such usage can only be a correlating device.

Another factor that might be important is the apparent superheat of the vapor in all of Sukhatme's tests. His maximum, observed, overall, temperature drop from superheated vapor to condenser surface was apparently 261°F, at which the apparent superheat was 34°F. His minimum $\Delta T$ was 21°F, at which the superheat was 8°F. In some tests, superheats as high as 30 to 40% of the overall $\Delta T$ values were recorded, the larger percentages occurring at the lower overall $\Delta T$ values. A superheat of 34°F or less causes a relatively small error in the calculated value of $G_v$, because $G_v$ depends upon $(T_v)^{-1/2}$. However, a possible effect of superheat upon energy exchange during interfacial collisional processes cannot be arbitrarily dismissed, even if the present status of the kinetic theory of condensation does not permit quantitative considerations of all the interfacial energy exchange processes. The presence of superheat may have contributed partly to the apparent scatter in the attempted correlations of his data in this report. Sukhatme's $\sigma_c$ values are plotted against $(T_v - T_w)/T_v$, in which term the superheat is neglected, because of its minor effect upon $G_v$, as long as the superheat is quite small compared with $T_v$. Unfortunately, in almost all his tests, changes in superheat were accompanied by changes in pressure in the same direction, so that any effect of superheat cannot be reliably evaluated.

1. **Case (i): $\sigma_e = \sigma_c$**

Figures 15a and 15b, respectively, compare Sukhatme's condensing coefficients and condensing coefficients that this author estimated from Misra's data. The discrepancies appear to be due to an effect of bulk vapor pressure not taken into account. The parametric study showed that the maintenance of a specified $q/A$ requires a higher minimum $\sigma_c$ at low $P_v$ than at high $P_v$. Sukhatme's data, taken over a much lower pressure range than Misra's, are therefore expected to yield higher condensation
coefficients than Misra's data. Hence, a comparison of numerical values of condensation coefficients without regard for the conditions under which they were obtained is meaningless.

Fig. 15. Condensation Coefficients Based upon $\sigma_c = \sigma_e$.
(a) Sukhatme's Data\(^{(7)}\) and (b) Misra's Data\(^{(89)}\)

The scatter in Sukhatme's data is evident in Fig. 15a. Heat fluxes are identified to show that $q/A$ is a dependent parameter; i.e., $\sigma_c$ shows no unique dependence upon $q/A$. Indeed, an opposite dependence seems more realistic; $q/A$ ought to depend partially upon $\sigma_c$.

Sukhatme's condensing coefficients do not appear to correlate with $(T_V - T_W)/T_V$ as well as do Misra's data. Rough grouping of Sukhatme's $\sigma_c$ values could be achieved by considering the corresponding values of $P_V$, but considerable scatter remains. The minimum values (including near-minimum values) of $\sigma_c$ are marked in Fig. 15a.

In Fig. 15b, the $\sigma_c$-versus-$\Delta T/T_V$ curve for Misra's data is analogous to the $\sigma_c$-versus-$T_s$ curves in the parametric study namely, Figs. 5, 6, and 7, up to $\Delta T/T_V \approx 0.04$. The validity of the apparent maximum of Misra's $\sigma_c$ values in the neighborhood of $\Delta T/T_V \approx 0.04$ and the decrease of $\sigma_c$ below the latter point are uncertain. It is not clear if $\sigma_c$ should decrease as equilibrium is approached, or if this trend is a peculiarity, either of Misra's data, or of the assumption that $\sigma_e = \sigma_c$. 
The data do not explain the reason for the large values of \( T_V - T_S \) observed (i.e., for the many low \( T_S \) values). Thus the possible effect of noncondensible gases may again be suggested. If the lowering of the interface temperature were due partly to the diffusional resistance of a noncondensible gas, \( T_V - T_W \) might be a measure of the amount of noncondensible gas present. The \( \sigma_c \) would be expected to decrease as \( T_V - T_W \) increases. Indeed, the data of Baer and McKelvey\(^\text{42}\) show a dependence of \( \sigma_c \) for condensing methanol upon increasing concentration of air in vapor, which might be considered analogous to the dependence which \( \sigma_c \) values from Misra's data show upon increasing \( T_V - T_W \). (Baer and McKelvey did not attempt to show their condensation coefficients as a function of \( T_V - T_W \), nor has this author examined their data in that manner.) The data that Misra did mark as having been taken during tests in which inert gases were being continuously removed from the system correlate the same as those data that supposedly were taken in gas-free systems. The possibility still remains that Misra could have maintained negligibly low gas concentrations in his system, but a steady rate of removal of gases suggests that significant amounts were present in some of his tests.

2. **Case (2):** \( \sigma_e = \sigma_{e,\text{max}} = 1.0 \)

The maximum value of the evaporation coefficient for mercury has been found experimentally to be unity.\(^\text{67}\) Hence, this case considers the values that the experimental condensation coefficients would take on if the maximum rate of evaporation occurs. In this case, as equilibrium is approached (i.e., as \( T_V - T_W \) approaches zero), \( \sigma_c \) must approach unity.

Figure 16 shows \( \sigma_c \) as a function of \( (T_V - T_W)/T_V \). Misra's data fall below Sukhatme's data when \( (T_V - T_W)/T_V > 0.1 \). Below the latter point, both sets of data seem to follow the same pattern in the approach of \( \sigma_c \) to unity as \( \Delta T \) approaches zero. Again, Misra's noncondensible gas (N.C.G.) data appear to correlate with those data supposed to be free from the effects of noncondensibles. A curve drawn through most of Misra's N.C.G. data also includes the data from Sukhatme's two tests in which an inert gas was purposely added.
The scatter in Sukhatme's data was not removed by other attempted correlations based upon the concepts of the kinetic theory of condensation. Apparently this scatter is due to the strong dependence of $\sigma_C$ upon both $T_V$ and $P_V$, which is not clearly shown in a two-dimensional plot of this nature. A possible effect of superheat again might not be negligible.

Figure 17 shows the same trends in a somewhat different manner. Most of Sukhatme's $\sigma_C$ values appear to be independent of $T_S$ on this type of plot. Misra's condensation coefficients show strong dependence upon $T_S$, the curves being analogous to the $\sigma_C$-versus-$T_S$ curves of the parametric study. If $T_S$ increases, $\sigma_C$ must increase, to condense more vapor as "compensation" for the increase in $G_S$ with $T_S$ at a given $q/A$. The scatter of Misra's data about lines of constant $T_V$ is due to heat flux variations.

The minimum values (including near-minimum values) of $\sigma_C$ are marked in Figs. 16 and 17.

Misra's data have not been compared with the solutions listed in Table I for the case of $\sigma_e = \sigma_{e,\text{max}} = 1.0$, because the solutions discussed were found to be physically inconsistent. An example of this inconsistency is the occurrence of multiple solutions, neither of which could be arbitrarily discarded. Another example is the absence of expected trends evident in Solutions No. 1, in which for each vapor temperature the temperature jump appeared to be independent of the imposed heat flux. As the heat flux was decreased at constant $T_V$, the temperature jumps in Solutions No. 1 should have decreased accordingly, as intuition and the data suggest. The values in Table I are based upon a Maxwellian velocity distribution of molecules reflected from the liquid surface. No assumptions about the nature of the reflection process were made in reducing Misra's data.

A quick comparison can be made in support of the foregoing statements. Consider the solutions of Table I at $1140^\circ R$. Condensation coefficients from Solutions No. 1 would fall close to the lower end of the curve in Fig. 16, while condensation coefficients from Solutions No. 2, being near unity, would fall around the upper end of the curve in Fig. 16. However, the experimental curve in the region $(T_V - T_W)/T_V$ approaching zero was determined by data from tests in which thermodynamic equilibrium was more closely approached than in tests that yielded the data falling on the lower part of the curve at larger values of $(T_V - T_W)/T_V$. The condition $(T_V - T_W)/T_V$ approaching zero corresponds to decreasing heat flux or to decreasing values of $G_{\text{net}}/G_V$. However, the results from Solutions No. 2 yield condensation coefficients near unity at high heat fluxes, far from approaching thermodynamic equilibrium. Hence Solutions No. 2 and Misra's data at low values of $(T_V - T_W)/T_V$ do not correspond. Solutions No. 1 would scatter about the lower end of the experimental curve, but this apparent agreement can only be fortuitous,
Fig. 17. Variation of Experimental $\sigma_c$ Values with Interface Temperature, Based upon $\sigma_e = \sigma_{e,\text{max}} = 1.0$
because Solutions No. 1 yield temperature jumps independent of heat flux, showing no decrease in \((T_V - T_W)/T_V\) as \(G_{\text{net}}/G_V\) was decreased.

Thus, solutions from Table I cannot be used to evaluate data. The same holds for other solutions which have been found to be physically inconsistent.

3. Case (3): \(\sigma_C\) Based upon Zwick's Theory\(^{(59)}\)

The preceding two cases treated \(\sigma_C\) in variations of Schrage's kinetic theory of condensation\(^{(8)}\). Because Zwick's theory is considered by this author to be an improvement, condensation coefficients calculated from Zwick's results (see Fig. 37) are shown in Fig. 19. Figure 18 (with the data of Tables VIII and IX) shows that condensation coefficients based upon Zwick's model generally agree reasonably well with condensation coefficients based upon Schrage's theory, as he used it (i.e., with \(\sigma_e = \sigma_c\)). One might take this result as confirmation of the assumption that \(\sigma_e = \sigma_c\). However, such cannot be conclusively stated before an examination of the mathematical details of Zwick's model is made to ascertain that the condensation coefficient is not implicitly included in the general form of the distribution function he assumed for evaporating molecules. Indeed the latter appears to be the case.

As \(G_{\text{net}}\) approaches zero, Fig. 37 is difficult to read accurately. Zwick supplemented this figure with an equation that was used here to calculate \(\sigma_C\) values for the case in which \(\phi_V\) approaches zero. That equation is

\[
\frac{P_s}{P_V} \approx 1 + \left(\frac{1}{4 - \pi}\right)\left(\frac{2 - \sigma_C}{\sigma_c}\right) \phi_V \sqrt{\pi}.
\]  

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The discussion of Fig. 15a and 15b also applies to Fig. 18 because Zwick's theory appears to imply that \(\sigma_e = \sigma_c\).
IV. EXPERIMENTAL PROGRAM

Mercury was chosen as a representative liquid metal, primarily because it has the lowest boiling point of the metals and is relatively inert compared with alkali metals. An experimental investigation of the resistance to condensation of mercury vapor on a rotating disk was undertaken. The advantage of studying condensation on a rotating disk has been mentioned. Condensate film thickness is theoretically inversely proportional to the square root of the speed of rotation, providing a mechanical method of varying the portion of resistance to condensation due to the condensate film. Nandapurkar and Beatty\(^{47}\) demonstrated the feasibility of this experimental method in condensing organic fluids. Hence a somewhat similar 3-in.-diam., rotating-disk condenser was designed for mercury vapor condensing up to atmospheric pressure and rotational speeds up to 2500 rpm.

To ensure good data it was deemed that the system must include (1) effective sealing, to permit complete evacuation and elimination of non-condensable gases, and to exclude coolant and/or lubricant from the test fluid; (2) an efficient vacuum pumping system; (3) accurate temperature measurements in the rotating disk; (4) steady disk temperatures; and (5) steady, controllable speed of rotation.

A. Equipment

1. Safety Requirements

Because of the toxicity hazard involved in working with mercury, the first requirement was the complete enclosure of the mercury heat transfer apparatus in a walk-in type fume hood which was blower-ventilated and exhausted to the outside of the building. A mercury-vapor detector was used to continuously monitor the atmosphere in and around the hood for mercury vapor concentrations exceeding the maximum permissible level of 0.1 mg per cubic meter.\(^{90}\)

The following requirements were also set forth by the safety committee of the ANL Reactor Engineering Division: (1) The mercury heat transfer equipment could not be Type 18-8 stainless steel, nickel, or any other high-nickel alloys, even though the program was intended to be of relatively short duration. (2) A relief line (rupture disk) and containment system must be provided in case of a pressure excursion in the hot-mercury vessel due to in-leakage of cooling water. (3) In case of the "accident" possibility just mentioned, the condenser vessel must be designed to hold at the elevated pressure, before and during pressure relief.

In accordance with the foregoing requirements, mild steel (0.1 to 0.3\% carbon, plus an equal amount of silicon) was found acceptable as a
corrosion-resistant material of construction. (The corrosion resistance was supposedly enhanced during operation by the addition to the mercury of about 10 ppm titanium and 20 ppm magnesium.)

A standard, 1\(\frac{1}{2}\)-in., union-type, safety head was provided to house scored, carbon-steel, rupture disks (burst pressure: 70 to 60 psig, over the temperature range of 400 to 700°F). Difficulty in obtaining the carbon-steel rupture disks required the temporary use of Teflon-coated aluminum disks which were only satisfactory for short-term use below 500°F. The relief line was connected to a stainless steel containment tank which was essentially a spray-condenser. In the event of an excursion (which would result from a weld failure and dumping of cooling water into the hot mercury), the relief gas (mercury vapor-water vapor mixture) would empty into the containment tank and be condensed by a spray of cold water. The spray was actuated by a pressure switch in the vessel and shut off by a liquid level probe when the containment tank was filled. Another pressure switch closed the cooling water inlet solenoid valve, if the mercury vapor pressure rose above 25 psia, thus limiting the duration of any excursion. The containment vessel was made from 24-in.-OD pipe, with ellipsoidal caps welded on each end.

The mercury boiler-condenser vessel pressure and vacuum requirements were satisfied by the use of 150-lb pressure-class flanges and 1/4- to 3/8-in.-thick walls.

The condenser test section itself would be subjected to high thermal stresses. Therefore, 2\(\frac{1}{4}\)% Cr-1% Mo steel was chosen instead of mild steel. The chromium-molybdenum steel has better creep resistance than carbon steel, and the 2\(\frac{1}{4}\)% Cr-1% Mo steel has a constant k of 195 Btu in./hr ft\(^2\)-°F up to about 1100°F. It was felt that Misra's\(^3\) experience of carbon steel being wetted by mercury at the higher temperatures after several hours operation could be achieved in these tests by prolonged operation.

2. Mercury Boiler-condenser

The mercury boiler and condenser were contained in the same vessel. (This design later turned out to be a disadvantage from an equipment handling or maintenance standpoint, as well as in complicating the sealing problems for low-vacuum work.) Mild steel was the material of construction. The vessel shell consisted of an 8\(\frac{1}{2}\)-in. section of 16-in., Schedule 40 pipe, with 150-lb pressure-class flanges welded at opposite ends. The bottom or base flange was a blind flange, with a 7\(\frac{1}{4}\)-in.-diam. hole drilled through the center to provide access to the condenser section. The top flange was a standard, weld-neck flange with large tongue and groove facing. The cover was a standard blind flange with large tongue and groove facing.
The vessel interior was divided into four sections: (1) the condenser test section; (2) an annulus packed with asbestos insulation; (3) a condensate return annulus; and (4) a boiler annulus.

Figures 19 and 20 show the vessel and condenser test section design. Figure 21 shows the vessel exterior during assembly.

The condenser test section (the rotating shaft with the horizontal condenser disk mounted at the journal end of the shaft that runs in a vertical alignment bearing) was fabricated from a 2\(\frac{1}{4}\)% Cr-1% Mo steel disk, 5/16 in. thick, welded to a 2\(\frac{1}{2}\)-in.-ID-by-3-in.-OD mild steel "can," which was then tapped, over splines, onto the end of the shaft. Water ran up the shaft and cooled the bottom side of the chromium-molybdenum steel condenser disk, returning through four drain channels inside the journal to the coolant drain reservoir and drain. Six chromel-alumel thermocouples were installed in the chromium-molybdenum steel disk: three near the top surface, and three at the bottom surface. In addition, a thermocouple was placed in the journal wall, and two were placed in the coolant stream: one in the inlet, and one in the outlet before and after the condenser disk. These nine couples were brought down the rotating shaft inside 1/16-in. stainless steel tubes. At the bottom end of the shaft, the tubes were brought through slots in the shaft wall and the slots were sealed with epoxy cement. The thermocouples were then connected to the slip-ring, brush-block assembly. Figures 22-25 show typical thermocouple installations, and Figures 26-29 show the condenser test section in various stages of assembly.

The condensing surface was then the 3-in.-diam. 2\(\frac{1}{4}\)% Cr-1% Mo steel disk. To minimize heat leakage around the disk, a lava insulator was installed to surround the disk.

Heat was supplied to the boiler through three tubular, steel-sheath heaters, hairpin-bent and shaped to fit into the boiler annulus, as shown in Fig. 30. Each heater was 3.3 kW, 440 V, three-phase, giving a total power rating of 9.9 kW. Boiler heat input was controlled by a powerstat and an on-off temperature controller.

Heat losses from the boiler outer wall were matched using three 850-Watt heating tapes which were wrapped around the vessel over 1/8-in. asbestos insulation. Two 290-Watt heating tapes, wrapped around the windows and the vapor flow line to the reflux condenser, minimized heat losses through the vessel cover directly above the condenser disk. The latter was necessary to prevent condensate from dripping from above onto the condenser disk. Two on-off temperature controllers regulated the heating tapes, the controlling thermocouples being placed on the wall and cover outside the vessel.
Fig. 19. Boiler-condenser Vessel Assembly
Fig. 20. Rotary Shaft, Condenser Test Section Detail
Fig. 21. Mercury Boiler-condenser Vessel with Top Flange in Place (during Initial Stages of Assembly)

Fig. 22. Condenser Disk and Rotary Journal Cross Section Showing Location of Disk Thermocouples and Radio-active Source

Fig. 23. Typical Disk Thermocouple Installations

Fig. 24. Thermocouple Installations in Bottom Side of Disk
Fig. 25. Thermocouple Junctions at Brush-block Slip-rings Assembly, Chain Drive, and Rotary Union

Fig. 26. Rotary Shaft with Original Threaded End Coupling before Assembly

Fig. 27. Disk and Journal Exterior
Fig. 28. Rotary Shaft Components. (a) Journal Exterior and (b) Upper End, Journal Removed.

Fig. 29. Rotary Shaft Assembly, Complete. (a) Upper End and (b) Lower End.
Two thermocouples in the boiler indicated boiler temperature, while a third boiler thermocouple (all couples in deep thermowells) connected to a single-pen recorder continuously monitored the boiler temperature. A thermocouple in the thermowell in the vapor space indicated the mercury vapor temperature. A 0-30-psi pressure transducer was used to measure the mercury vapor pressure. Figure 31 indicates the locations of several additional thermocouples in the annulus of insulation between the boiler and the condenser. Thermocouples were also used to measure the inlet and outlet temperatures of the coolant streams to the condenser disk and to the reflux condenser. Pressure taps and gages at the condenser-test-section inlet and outlet provided ΔP across the test section. Rotameters were used to indicate coolant flow rates. Condensate flow-rate was measured by an electromagnetic flowmeter made from a 3/8-in., Swagelok all tube, stainless steel cross and a 5000-Gauss permanent magnet. The electrodes (which gradually dissolved in the flowing mercury and had to be replaced) were 3/8-in.-diam. copper rods. A Hewlett-Packard 425-A dc microvolt-ammeter was used to measure condensate flowmeter output (10 to 100 μV). Flowmeter output and vapor-pressure transducer output were recorded simultaneously on a Varian two-pen recorder. Thermocouples 1 through 24 were monitored continuously on a 24-point recorder.

Two methods of measuring condensate film thickness were available. Difficulties with photomultiplier tube or single-channel analyzer drift, and loss of accuracy due to critical absorption by mercury of the thulium-170, 84-keV gamma radiation above film thicknesses around 0.020 in., resulted in the elimination of the gamma attenuation method for films thicker than 0.020 in. (Because of the critical absorption problem with the 84-keV source, aqueous-mercuric acetate solutions of various concentrations equivalent to the mercury film thicknesses expected in the tests were used to attenuate the gamma radiation beam. This technique provided an accuracy check and an approximate calibration.)
For thicker films, this method would have been more accurate had another source been substituted for thulium and the single-channel analyzer difficulties been resolved. However, it appeared from tests with a mock-up of the ohmmeter-dial indicator probe that the latter would be satisfactory for the thicker condensate films encountered in the final nonrotating condensing tests. Figure 32 illustrates the design and principle of operation of this probe.

After considerable operating difficulty, as a result of liquid hold-up and carryover when excessive amounts of non-condensible gases were present in the vapor, the original jacketed-pipe, vertical reflux condenser (3/8-in. pipe) was removed from the system.

Then a reflux condenser, with separate vapor inlet and condensate return lines, was mounted above the vessel at a 45° angle. Mercury vapor condensed outside six 3/8-in.-OD x 1/16-in. wall Type 304 stainless steel tubes inside a 16-in.-long, 3-in., Schedule 40, Type 304, stainless steel shell. A vacuum line at the top of the reflux condenser permitted evacuation of the vessel and a continuous purge of noncondensible gases during operation. Figure 33 shows the reflux condenser design.

Asbestos-filled Flexitallic gaskets were used to seal the vessel at the cover and at the windows. Two, 3-in.-diam., 1/2-in. thick, Vycor-glass viewing ports were provided in the vessel cover on opposite sides of the film thickness probe. Where possible, all fittings were welded closed to prevent
leakage. Swagelok compression fittings were used with 3/8-in. x 1/32-in. wall on the condensate return lines and on the vapor and condensate lines of the reflux condenser, these lines being necessarily removable for maintenance. Swagelok fittings were also used on the 1/4-in. line to the pressure transducer and on the two 1/8-in. tubes that served as thermowells. All lines that were not subjected to heat were easily sealed with Swagelok fittings or with Teflon-taped threaded fittings. Teflon paste and Teflon tape were used to ensure a seal around the threaded end of the bellows housing of the film-thickness probe.

Figure 34 is a schematic diagram of the boiler condenser and auxiliary equipment.

The seal was a bellows-type, rotary face seal, Chicago Rawhide Company Model No. 80-2860, made of AM-355 steel, the sealing faces being lapped carbon and a chrome-plated mating ring. The seal was pressed (0.004-in. interference) into a housing below the graphite bearing. Figure 35 shows the seal, graphite bearing, and lava insulator surrounding the condenser disk.

Temperature measurements were made with chromel-alumel thermocouples located as shown in Fig. 22.

The proposed program included condensation in the 400-to-675°F temperature range (saturation pressures up to atmospheric) at speeds of rotation up to 2500 rpm. Consequently the shaft was designed to run in a radial ball bearing and a radial-sleeve alignment bearing made of carbon-graphite, after earlier problems with a Stellite alignment bearing (journal surface grinding by Stellite - almost galling). A ball thrust bearing and a bronze thrust washer supported downward and upward shaft thrust, respectively.

A 1 1/4-hp, 600- to 2000-rpm, 110-V compound motor supplied the driving power to the shaft through a miter gear box and a chain drive.

3. Operation

After passing all instrumentation and thermocouple checks, the system was evacuated to the lowest pressure attainable, usually about 0.1 to 0.5 mm Hg absolute pressure. If a leak check showed negligible or tolerably small leakage that could be removed continuously by the reflux-condenser-vacuum system, heat power was turned on. Perfect sealing for vacuum work in this apparatus appeared to be impossible, because of the number of non-welded fittings and the large sealing area between the top flange and the vessel cover. Valves and valve fittings were also leakage sources. Apiezon Q Sealing Compound was used to seal some of the troublesome vacuum joints
Fig. 34. Schematic Diagram of Mercury Vapor Condenser and Auxiliary Equipment
that did not heat up during operation. The valve on the vacuum line could be kept partially open and the vacuum pump operated continuously so that any noncondensible gases in the system could be continuously purged from the condenser during operation. The reflux condenser cooling was always maintained whenever the vessel was hot or the vacuum pump was running. After the boiler heat supply was turned on, the test section coolant flow was started. Boiler heat input and condenser coolant flow rates were adjusted until the desired level of mercury vapor pressure and temperature were reached. Beyond this point, if stable conditions could be maintained, the data were taken.

Shutdown involved turning off the heater power and leaving the coolant flows and vacuum pump turned on until the system had cooled to room temperature. At this time, the vessel was filled with dry nitrogen. To shut off the vacuum pump at the same time as the heaters, it would have been necessary to provide a nitrogen blanket for the vessel interior to prevent the in-leakage of air and the oxidation of the hot mercury and vessel interior.

4. Operating Difficulties

a. Rotary Seal.

After a few trial runs, the rotary face seal began leaking excessively. Cooling water leaked across the seal into the condensing test area during operation, adding a diffusional resistance to the condensation process and corroding the condenser surface. When reduced pressure runs were being made, the coolant leaked across the seal fast enough to prevent condensation almost entirely as the water flashed into the mercury vapor at low pressure and was swept upward and out through the vacuum pump. Attempts to operate the boiler-condenser at pressures higher than the coolant pressure were unsuccessful. Either an excessive amount of mercury leaked across the rotary seal into the coolant, or pressure and rotational speed fluctuations permitted the leakage of water into the mercury. Moreover, the removal of noncondensible gases could not be effectively achieved without vacuum pumping. Purging the gases through the reflux condenser was insufficient. It could not be determined if the rotary seal leaked at first because of damage during installation or because of pickup of lint or grit.
during installation or operation. Evidently the carbon face seal suffered some damage during subsequent disassembly-assembly operations. Repair of the seal, in place, appeared to be a costly and time-consuming operation.

b. **Bearing and Stability of Rotational Speed.**

After problems of shaft stability associated with eccentricity or "run-out" of the end coupling to the rotary union were eliminated, the shaft could be rotated at relatively high speeds for the proposed tests. However, stable rotation could not be achieved, apparently because of dust build-up into ridges on the graphite bearing surface, which then produced uneven loading or friction on the journal. Speed variations as great as ±20% were encountered during almost every revolution in the range of 360 to 1700 rpm. Higher speeds were not investigated. Apparently the shaft inertia was too small to compensate for variable journal loading. At the lower speeds, the drive chain stopped momentarily during almost every revolution as these fluctuations occurred. It was not determined if the bearing dust build-up was due to partial wetting of the bearing by coolant leaking across the seal, by improper grooving and design of the bearing for dust and particle removal during operation, or by some action of the mercury.

These difficulties were primarily responsible for the abortion of the rotating condenser tests in favor of a simpler study, which would permit the utilization of the equipment already available and yet yield some basic data on the mercury vapor condensing process in a reasonable span of time.

**B. Modified Experimental Attack**

1. **Outline**

Because of the difficulties of effectively operating the rotating condenser, it was modified somewhat to provide a positive static seal around the condenser shaft, and to permit use of the equipment already fabricated. The seal consisted of a double O-ring (silicone rubber O-rings) seat welded to the shaft to back up the rotary seal. This added seal appeared to perform satisfactorily during the remaining tests. The seals are shown in Fig. 36, a cross-sectional view of the rotary journal housing.

Analyses of condensation on a rotating disk, and then on a stationary horizontal disk, indicate that in both cases the film thickness is practically uniform over the disk, if edge effects and waves are neglected. Hence, studies of metal vapors condensing on a horizontal disk should still provide information about the condensing mechanism, if the condenser surface temperature, vapor temperature and pressure, and condensate film thickness are accurately measured. It is necessary, too, that the condensing surface be
"clean" and that noncondensible gases be absent from the system. The limitation of condensing studies on a nonrotating horizontal disk is that condensate film thickness cannot be controlled, as it could be by arbitrarily varying the speed of rotation. Films 100 or more times as thick as rotating films may result. Consequently high heat fluxes are attainable only at high $\Delta T$'s, but accurate film thicknesses can be more easily obtained.

The modified research program consisted mainly of condensing mercury vapor on the horizontal, stationary disk at a series of mercury vapor pressures, ranging from vacuum ($\sim 0.1$ psia) to atmospheric pressure, while high overall $\Delta T$'s were to be maintained by cooling with subcooled water. The resulting condensate film thicknesses were to be measured by using a dial indicator type probe and ohmmeter to locate the vapor-liquid interface. A ring was placed (Fig. 36a) over the outer edge of the condenser disk and raised 0.020 in. above the disk to ensure a more uniform distribution and flow of liquid mercury should the disk be somewhat nonlevel for some reason. This ring arrangement was checked hydrodynamically by feeding mercury onto a copper disk-ring setup with a 0.020-in. gap from a burette. It was found that film thicknesses up to about 0.15 to 0.20 in. could be expected at the higher condensing rates.

The disk was polished and plated with copper, nickel, and copper, in that order, to enhance early wetting. Subsequent operation, after completion of the assembly, was interrupted by a shutdown to repair a Vycor window that had cracked during start-up. To avoid further window cracking, the windows were replaced by stainless steel disks, eliminating observation of the mode of condensation. Henceforth, the sensitivity of the film thickness probe was relied upon to indicate the mode of condensation and the condensate surface purity. Continuation of the test resulted in gradual loss of probe sensitivity, indicating the buildup of oxides on the condenser disk. Hence the test was ended, the apparatus dismantled, and the condenser interior cleaned. Leakage of air into the vessel during this test had been sufficient to oxidize the vessel interior and form a considerable amount of oxide of mercury.
which coated the interior surfaces of the vessel, including the condenser
disk. It was concluded that the ring with a gap between itself and the disk,
while permitting the condensate to flow out, was retaining the dirt formed
or collected inside the ring. Therefore the gap was eliminated and the ring
lowered so that its top was 1/32 in. above the disk surface (see Fig. 36b).

Before starting up with the modified ring, the vessel interior
was cleaned several times with concentrated hydrochloric acid to remove
the mercury and iron oxides. Unfortunately, the vessel design is so cumber-
some, and some of the compartments somewhat inaccessible, that effective
rinsing and drying of the vessel with water and acetone could not be achieved,
even after several attempts. It was acknowledged that the oxidized vessel
walls would have to be "lived with."

The modified ring was then installed, the disk and vessel leveled,
as usual, and the disk covered with mercury (retained by the ring) so that
the initial presence of a mercury film on the disk was ensured.

The subsequent tests also produced gradual loss of probe sensi-
tivity, indicating the leakage of air into the system with its attendant oxida-
tion problems, most notably, condenser surface fouling. An attempt was
made to increase the pressure from the vacuum maintained during the initial
phases of the test (up to about 3 psia) to atmospheric pressure, with continu-
ous vacuum pumping and operation of the reflux condenser. Over a period of
24 hours, the pressure rose gradually at full boiler power input and leveled
off at 8 psia, indicating that further increases in pressure would have to be
achieved by turning off the reflux condenser or drastically reducing the re-
flux condenser flowrate. However, the film thickness probe was still com-
pletely erratic, indicating the presence of a mercury oxide coating of very
high electrical resistance on the disk. Hence the run was ended.

2. Common Operating Difficulties

During the testing under both rotating and nonrotating conditions,
the following problems were also encountered and never satisfactorily
eliminated:

a. Electromagnetic Flowmeter Zero Drift

During the last test, it was concluded that metering instru-
ment error rather than thermoelectric effects in the flowmeter was the
cause of the zero drift in the electromagnetic flowmeter. The remedy is a
simple circuit permitting bypassing of the flowmeter so that the instrument
zero may be instantaneously checked. If some thermoelectric effect in flow-
meter electrodes is evident, it could be measured by closing a valve to mo-
mentarily stop flow through the meter.
b. **Radial Temperature Gradient in Condenser Disk**

The most effective cooling apparently occurred at the stagnation point at the center of the disk, for there was typically a 100°F temperature difference between disk-center and disk-outer temperatures in the nonrotating tests, and about 50 to 75°F (the difference decreasing with increasing flowrate) in the rotating tests. The gradient problem was more pronounced in the nonrotating tests because of the thicker films and correspondingly lower heat fluxes. The major portion of heat flowed into the coolant through the bearing housing surrounding the disk. This accounts for the radial temperature gradient. These results made it obvious that the interpretation of any low heat flux data obtained in nonrotating tests with the present apparatus would be relatively useless.

c. **Fluctuating Disk Temperatures**

Fluctuations in disk temperatures may have been caused by several effects such as irregular boiling patterns on the bottom side of the disk, air in the cooling water, irregular condensate distribution and flow over the top of the disk, and shorting of the thermocouples by the cooling water. Boiling was part of the problem, because the fluctuations were minimized or eliminated by increasing the coolant flowrate. Because film condensation was never obtained, the full effect of condensate distribution upon the disk temperatures was not determined. During the last two nonrotating tests, the fluctuations in about half the thermocouples could not be eliminated entirely, and variations up to about ±0.1 mV remained. This apparently was due to shorting out of the couples by the coolant. Two disk thermocouples failed during the last tests.

d. **Air Leakage**

Air leakage occurred during tests below atmospheric pressure at undetermined locations. Helium leak tests revealed several leakage spots (primarily around fittings and the vessel cover gasket) which could only be closed definitely by welding. Moreover, some of these leaks or others may have opened up when the vessel was hot.

e. **Reflux Condenser Hold-up**

Reflux condenser hold-up was experienced in a few tests at pressures above a few psia, unless the vapor inlet line to the reflux condenser was kept hot at all times. Following the drainage of a liquid slug from the reflux condenser, the vessel pressure dropped momentarily because of the sudden exposure of the reflux condenser volume and condensation therein. This effect diminished with prolonged operation.
f. Nonwetting of Condensate Disk

Wetting of the condensate disk was never achieved, primarily because satisfactory system performance at high temperature was never achieved. However, the 2\(\frac{1}{4}\)% Cr-1% Mo steel condenser disk was an unfortunate choice, primarily because of the welding problems involved. Chromium-molybdenum steels air-harden at high temperatures. Hence, disk fabrication was a complex operation involving a controlled post-heating period after welding. The disk was actually fabricated twice. Welding difficulties with the first disk and distortion of the dimensions of the journal to which it was welded required that a new disk-journal be fabricated. Thus, to avoid another postheat operation, with the possibility of damaging the slip-fit between the journal and shaft splines, the thermocouples were installed in the disk through stainless steel tubes. These, in turn, were soft-soldered (m pt ~600°F) to the disk underside (see thermocouple installations in Figs. 22 and 23), rather than silver-soldered. The bottom temperature had to be maintained safely below 600°F to prevent loss of thermocouples, and some coolant flow had to be maintained at all times. Thus the possibility of achieving wetting by prolonged operation with no coolant flow and with the disk temperature equal to the vapor temperature was eliminated. The condenser disk should have been nickel, and the journal should have been stainless steel since wetting of the nickel could have been relatively easily achieved.

Because of these difficulties, the test program was terminated.
V. CONCLUSIONS

A. Analytical

1. A temperature jump (controlling resistance to condensation) at the vapor-liquid interface appears to be a real phenomenon, as Sukhatme and Rohsenow\(^\text{(7)}\) concluded. Temperature jumps as high as 250 to 300°F were deduced from film condensing data for mercury vapor, based upon the assumption that no diffusional resistance due to noncondensable gases existed in the corresponding experiments.

2. An interfacial resistance ought to exist for nonmetal fluids as well as metals. For a nonmetal fluid, this resistance would ordinarily be negligible because of the relatively low thermal conductivity of the condensate. For a condensing metal, the thermal resistance of the condensate film would ordinarily be quite small. (Plots of \(q/A\) versus \(\Delta T\), based upon Nusselt's theory, for mercury, cadmium, and alkali metals, compared with water and biphenyl, illustrate the differences in thermal resistance of these liquids. These plots are included in Appendix C.)

3. Coupling the macroscopic transport equations of the bulk liquid and vapor phases at the interface does not permit calculating the temperature jump, but does indicate that the phenomenon is associated with condensation subcooling of the vapor adjacent to the liquid, as well as fractional condensation of the molecular stream striking the liquid surface. (Indeed the temperature jump concept was originally devised to facilitate studies of thermal conduction at gas-solid boundaries because of inadequate understanding of the collisional processes within a few mean free paths of the solid surface.) Hence, the nature of the interfacial resistance is expected to be the same for all condensing vapors. However, the factors affecting the condensation coefficient for different materials are expected to depend strongly upon molecular structure. (Surface tensions of liquid metals are generally up to several times higher than values for nonmetals, but high surface free energy is regarded as the result, rather than the cause, of some of those same factors affecting the condensation coefficient.)

4. Because the temperature jump in condensation was traditionally described by the kinetic theory of condensation, the corresponding equations of mass, momentum, and energy conservation for a monatomic gas were written. It was proposed that consistent application of the kinetic theory requires the normalization of the composite velocity distribution function for vapor next to the liquid surface. This is a required condition, not one which may be arbitrarily applied. It was shown that the solution and use of these equations depend upon the form of the velocity distribution function of molecules reflected from the liquid surface; i.e., the reflection process could not be neglected.
5. Simultaneous solutions of the four equations (mass, momentum, energy and normalized velocity distribution) for six different cases failed to yield results and trends that are considered physically consistent and systematic. In general, it was required that $0 < \sigma_e \leq 1.0$, and $0 < \sigma_c \leq 1.0$. Four cases considered a Maxwellian velocity distribution of reflected molecules. Three of these cases omitted the normalized velocity distribution function, substituting various assumptions about $\sigma_c$ (e.g., the common assumption that $\sigma_c = \sigma_e$). The other two cases considered "perturbed" Maxwellian velocity distribution functions for the reflected molecules, thereby permitting inelastic reflection. The system considered was monatomic mercury vapor condensing.

6. Although metallic vapors tend to polymerize, the extent of polymerization at ordinary pressures is not large enough to markedly alter the condensation coefficient.

7. Meaningful condensation coefficients can be calculated from condensing heat transfer data taken with noncondensable gas in the system, only if the interfacial condition of the vapor is known, and if the gas does not contaminate the liquid surface.

8. Corrosion product contamination of a liquid metal condensate film appears to be negligible, unless some unknown mechanism exists by which concentration of contaminants occurs at the vapor-liquid interface.

9. A parametric study of the kinetic theory mass, momentum, and energy equations for condensing mercury over the heat flux range of $10^4$ to $10^6$ Btu/hr ft$^2$, and the bulk vapor pressure range of 0.012 to 27.45 psia, suggests that molecules are generally reflected from the liquid surface inelastically with a non-Maxwellian velocity distribution. Apparently the ideal case is approached only as thermodynamic equilibrium is approached. This study does not yield solutions of the condensation problem. It merely shows trends and establishes minimum condensation coefficients for maintaining a given heat flux at a given pressure.

10. Condensation coefficients, calculated from data taken by Misra during condensing tests that were supposedly gas-free, and from tests during which he continuously removed noncondensible gases, correlate on the same curve. This supports Sukhatme's conclusion that Misra's data suffered some errors due to the added diffusional resistance of noncondensible gases. Gel'man's data, taken over the same pressure range as Misra's data, show an increase in the condensation heat transfer coefficient with pressure at constant overall $\Delta T$, in conformity with the criterion suggested by Sukhatme. Misra's data show the opposite effect, as Sukhatme pointed out.
However, lack of fundamental understanding of the condensation coefficient; absence of quantitative data on the concentration of noncondensible gases present in Sukhatme's and Misra's tests; and differences among Gel'man's tests (dropwise condensation on the inner tube of an annulus with a measurable effect of vapor velocity), Misra's tests (cold-finger condenser over the pressure range of 0.5 to 14.8 psia), and Sukhatme's tests (cold-finger condenser over the pressure range of 0.02 to 0.33 psia) must postpone final judgment of Misra's data until they are checked by additional experimental work.

Sukhatme's arguments for the absence of a superimposed diffusional resistance at the interface due to noncondensible gases in his own tests are convincing, but remain to be substantiated conclusively by quantitative information about the purity of the interface and the bulk vapor.

11. Individual numerical values of Sukhatme's condensation coefficients and condensation coefficients calculated from Misra's film condensation data cannot be directly compared without consideration of the different vapor pressure and condenser surface temperature conditions under which they were obtained, even if Misra's data were free from noncondensible gas effects.

12. An analysis of Sukhatme's condensation coefficients revealed that the minimum condensation coefficients necessary to maintain the measured heat fluxes were actually observed in several of his tests. These values differed. Hence an average value of the condensation coefficient used to cover a wide range of data is only a correlating device, not an accurate representation of the physical process.

13. The existence of different, measured, minimum condensation coefficients, which presently are not predictable from first principles (i.e., from $T_w$, $T_V$, and $P_V$ alone) indicates the need for a more fundamental understanding of the energetics of interfacial collisional processes which govern the condensation coefficient.

14. Trends in apparent condensation coefficients cannot be adequately evaluated from previous film condensation heat transfer data, because those primary variables, upon which $\sigma_C$ is believed to depend, were not systematically controlled in those tests analyzed. If heat flux and $\sigma_C$ vary uniquely with vapor temperature and pressure and condenser surface temperature, the latter three conditions ought to be varied systematically in evaluations of heat flux and $\sigma_C$. For example, one might vary condenser surface temperature at constant vapor pressure and temperature, observing the changes in heat flux and $\sigma_C$ with changing wall temperature. Or one might hold wall temperature and vapor pressure constant, while varying vapor superheat, observing the effect of changing high superheats upon heat flux and $\sigma_C$. However, if all these conditions
are varied simultaneously from one test to the next, it is difficult to evaluate any functional dependence of \( \sigma_c \) upon any one independent variable.

15. Analyses of Misra's and Sukhatme's data have not revealed an unquestionable functional dependence of the condensation coefficient upon vapor and condenser surface conditions. Nor have the analyses revealed any dependence of \( \sigma_c \) upon \( \sigma_e \) outside of that demanded by the kinetic theory and various assumptions that might be made about \( \sigma_e \). This illustrates a major shortcoming of the theory: \( \sigma_e \) must be arbitrarily specified before data can be analyzed.

16. Comparison of \( \sigma_c \) values calculated from Zwick's theory with corresponding values calculated from Schrage's theory, and consideration of the mathematical forms of Zwick's velocity distribution functions, indicate that Zwick's theory implicitly includes the assumption that \( \sigma_e = \sigma_c \).

17. Fundamental considerations of the evaporation and condensation coefficients, and of the approximate nature of the kinetic theory of condensation, reveal no strong reason to assume that \( \sigma_e = \sigma_c \) except to simplify the equations.

18. The approximate nature of the kinetic theory of condensation consists of its:

a. Assumption that bulk vapor conditions prevail up to the liquid surface (i.e., its neglect of the possibility that interactions between evaporating and reflecting molecules and condensing molecules might alter the velocity distribution and density of vapor near the interface).

b. Somewhat arbitrary separation of evaporation and reflection processes. Reflection of incident molecules from the liquid surface apparently cannot occur instantaneously with zero energy exchange. However, the mechanisms of energy exchange and approach of the "reflecting" molecule to equilibrium with the liquid surface during its residence time at the surface are poorly understood. Hence a velocity distribution must be assumed for the reflected molecules, in the hope that the form of this distribution will account for momentum and energy changes in the reflection process.

c. Neglect of the possible effects of collisional energetics at the liquid surface in determining the condensation coefficient as the probability that molecules of various relative energy levels will condense.

19. Condensation and evaporation coefficients calculated from film condensation heat transfer data might include more effects than those singular physical interpretations heretofore attributed to them. The coefficients include the effects of deviations from the ideal Boltzmann gas
behavior assumed in the approximate theory. For relatively large temperature jumps, as may occur for condensing metals, such errors might be quite large. Thus, the apparent condensation coefficient might not be exactly "the fraction of molecules striking the interface that actually condenses." If vapor conditions at the interface differ from the bulk vapor conditions, the apparent condensation coefficient includes the effects of this difference. The true condensation coefficient depends upon the exact vapor and liquid conditions at the interface.

20. The kinetic theory of condensation in its current form is an inadequate description of interfacial transport processes in condensation. It only provides a qualitative idea of the relative importance of various conditions governing the condensation process. The primary reasons for this conclusion are:

a. Heat flux in the case of film condensation of a pure vapor ought to be uniquely specified by the bulk vapor pressure and temperature and by the condenser surface temperature, as Schrage recognized (Reference 8, pp. 53-54). However, the theory failed to predict physically consistent, systematic trends in the behavior of the condensing mercury system. Nor have other authors' previous treatments of other condensing systems predicted system behavior, unless values of condensation coefficients were chosen a priori.

b. The approximate nature of the kinetic theory of condensation leaves fundamental questions about interfacial collision processes unanswered.

21. The foregoing conclusions suggest a need for a more exact treatment of interphase transport of mass, momentum, and energy, i.e., one that would account for the effects of interactions between condensing molecules and evaporating and reflecting molecules, and attempt to consider collisional energetics at the liquid surface in describing the condensation probability or condensation coefficient. Use of simplified forms of the Boltzmann integro-differential equation, such as the "single relaxation model" discussed by Sirovich, (78) might prove helpful in determining a more accurate velocity distribution function for vapor in the region of the vapor-liquid interface during condensation. Such an approach would imply that the temperature jump exists as a thermal gradient across a vapor phase boundary layer which is subcooled by conduction. If such is the case, greatly refined optical techniques might provide some information about the vapor-liquid interface during condensation.

22. If the temperature jump does exist as a thermal gradient rather than a sharp discontinuity, the difference between liquid surface temperature and temperature of vapor adjacent to the liquid surface would be less than the difference between liquid surface temperature and bulk vapor temperature. The true condensation coefficient depends upon the former difference.
The apparent condensation coefficient depends upon the latter difference. The true condensation coefficient would be greater than the apparent condensation coefficient calculated using bulk vapor conditions, because the true condensation coefficient is calculated from the true temperature difference that exists at the interface, not from the larger overall temperature difference.

23. Reviews of studies of interactions of vapors with solid surfaces (particularly crystallization nucleation studies) suggest that dropwise condensation is not an altogether dissimilar process. Adsorption of vapor on the solid surface, followed by surface migration of the adsorbed molecules until they evaporate or agglomerate with other molecules to form droplets, might be a contributing or even a controlling mechanism in the dropwise condensation process. In dropwise condensation of metal vapors, the vapor-liquid interfacial resistance should also be significant because of the relatively low thermal resistance of the condensate.

24. Alternative approaches to describing the interfacial resistance to condensation are briefly considered in Appendix B and are summarized as follows:

a. Vapor quality has a negligible effect upon the results of Nusselt's theory unless quality is 75% or lower. The effect of quality upon the limiting condensing vapor flow (i.e., upon the sonic vapor velocity) must be considered only at high heat flux and relatively low quality.

b. A theory of nucleation at a flat vapor-liquid interface is not evident. Such an approach, under current concepts, would reduce to the kinetic theory of condensation, which is commonly used to describe growth of droplets or nuclei.

B. Experimental

1. Continuation of the current research program (i.e., condensation on a rotating disk with the apparatus described in this report) would be more costly and time-consuming than beginning anew, primarily because of:

a. Rotary seal problems and repair costs.

b. Rotational stability problems and alignment bearing difficulties.

c. General leakage and continuing mercury oxidation problems, with resultant condenser surface fouling.
2. Stainless steel construction is superior to mild steel construction for short-term, mercury, heat transfer studies up to 700°F, primarily because of the elimination of air oxidation of the equipment. Moreover, an apparatus made of stainless steel could be tested in preliminary runs with water or some convenient organic fluid.

3. The bellows type rotary seal in these tests seemed to be unsatisfactory, chiefly from a maintenance standpoint. A spring-loaded face seal would have been much easier to repair.

4. Elimination of the seal would have been desirable.

5. The boiler and condenser sections should each have been in separate vessels for ease of maintenance.

6. With proper design and adequate safety features on a mercury loop of the type used here, the rupture disk-relief condenser system is unnecessary.

7. Condensation on a nonrotating horizontal surface merits further consideration as a technique for studying the interfacial mass transfer process, if the heat flow across the condenser surface can be well-defined and if an oxygen-free (leakage-free) system can be guaranteed, so that contamination of the vapor-liquid interface is eliminated.

8. Studies of basic condensing mechanisms on vertical surfaces appear to offer a greater net advantage than studies on rotating surfaces, because the time and cost that would otherwise be devoted to eliminating the inherent difficulties of successfully designing and operating a rotating condenser could be expended on improving the tools and techniques used to derive basic data from the simpler apparatus.
VI. RECOMMENDATIONS

A. Analytical

A more exact treatment of interfacial transport phenomena and the factors affecting the condensation coefficient is needed to eliminate the uncertainties involved in the approximate kinetic theory of condensation, and to provide an accurate description of interfacial transport rates and conditions.

B. Experimental

1. Further experimental studies of condensing metals should be performed in order to check Misra's data, to extend the range of data, and to improve the understanding of condensation mechanisms.

2. Although the kinetic theory of condensation is quantitatively inaccurate, it provides a qualitative guide for designing experiments, suggesting variables that might be systematically controlled in order to evaluate condensation mechanisms. For example, future tests could be conducted by holding a constant $T_w$ and varying $P_v$ and $T_v$ at different $T_w$ levels. The tests might be repeated, holding $T_w$ and $P_v$ constant and varying the vapor superheat. Maintaining $T_w \ll T_v$, so that the condition $\sigma_e C_s \ll \sigma_c \Gamma_1 G_v$ is satisfied, permits direct calculation of the apparent minimum condensation coefficient for the given conditions, a systematic study of which should provide important information of the dependence of $\sigma_c$ upon relative liquid surface and vapor thermal conditions for film condensation of a pure vapor.

3. Type 304 stainless steel should be used in all components of the mercury loop unless requirements dictate otherwise; e.g., in order to obtain wetting, nickel is preferred as a condenser surface material. The apparently successful use of stainless steels as construction materials in other research (References 3, 5, 7, 49, 91-94 for example) supports this recommendation. Copper might also be considered as a condenser surface material for high vacuum tests with mercury.

4. All welded construction should be used wherever possible; i.e., gaskets, compression fittings, etc., should be eliminated whenever vacuum work is to be done.

5. At least one window should be provided so that condensation mode and flow patterns may be observed. This may require more development work at high temperatures, because breakage of such acceptable window materials as Vycor may occur easily as a result of mechanical stresses.
6. Refined methods of measuring condensate film thickness need be developed to improve accuracy and to permit obtaining film thickness profiles. Gamma attenuation is more satisfactory than a micrometer type probe, if drift-free instrumentation is available.

7. Condensate flowrate should be metered with either a magnetic flowmeter or an orifice meter.

8. Facilities should be provided for baking-out the system before start-up and vacuum pumping to pressures below 1 micron. Baking-out and refilling the system with an inert gas several times would be helpful.

9. Methods for studying the effect of noncondensible gases and measuring noncondensible gas concentrations in the bulk vapor should be considered.

10. The vacuum system, which should be capable of continuous operation (i.e., continuous pumping), should be connected to the system through a reflux condenser that has separate condensate return and vapor inlet lines to prevent carryover of the condensible vapor.

11. For rotating condenser studies the following additional recommendations are given:

   a. The rotary seal must be eliminated from the design by the use of such arrangements as a magnetic clutch or an internal turbine drive.

   b. Conventional bearings should be used wherever bearings are needed.

   c. Radial temperature gradients in the condenser disk must be eliminated by effectively insulating the condenser area surrounding the disk, i.e., by effectively defining the heat transfer area (the test section) and by optimizing the disk thermal conductivity and thickness.
APPENDIX A

Literature Survey

Complete separation of the complex topics outlined in this report is physically impossible. To achieve an accurate overall view of condensation in general, and of film condensation in particular, current ideas on interfacial structure, nucleation theory, and dropwise condensation theory have been examined, in addition to the wealth of literature under the general subject of the kinetic theory of gases and the numerous theoretical and experimental investigations of film condensation. Because the extreme discrepancies between classical theory and data seem to involve only condensing metal vapors, only those experiments on condensing metals have been emphasized. Finally, because the ultimate improved understanding of the mechanisms of condensation will affect prediction of forced convection condensation and pressure drop, some recent work in this latter field is briefly mentioned.

In some cases, particularly in that of the kinetic theory of gases, excellent surveys and treatises of fundamental experiments and theories are available. These and the basic results and concepts drawn from them have been cited.

The information outlined here represents some of those basic theoretical concepts and experimental results which, when taken together and evaluated in detail, will improve the current understanding of the mechanisms contributing to the net condensation of vapors upon their own liquid species. The most notable omission is a review and discussion of potential field interactions (i.e., intermolecular forces). A thorough consideration of potentials is beyond the scope of this problem, but admittedly the ultimate solution may depend upon such considerations.

1. The Vapor-liquid Interface

Research concerning the equilibrium structure of vapor-liquid interfaces was reviewed by Schrage. With support from the results of early experiments involving reflection of light from liquid surfaces and direct microscopic observations of interfaces, Schrage concluded that the influence of one phase on the other "is restricted to a sharply defined region where the two come in contact." He also suggested that while vapor properties up to the interface should be uniform, there is a "greater possibility of surface peculiarities in the liquid." A similar review by Michaels supported the same deduction and added that the extremely high distance dependency of the potential function (of a molecule located at the surface of a condensed phase) indicated that the "second layer of molecules underlying the surface of a condensed phase is at virtually the same energy level as those deep in the bulk phase, so that the interface is of the order of a single molecule thick."
A somewhat different conclusion about the vapor-liquid interface was reached by Chang et al.\textsuperscript{(54)} following their theoretical predictions of surface tension for liquid argon, nitrogen, and methane at various temperatures. The results, which showed excellent agreement with observed values of surface tension for these fluids, were based upon a model that implies a transition zone at the interface. Their "calculations" showed that only the top two layers of a liquid contribute to the surface tension near the melting point, "but the transition zone between the gas and liquid phases becomes broader with increasing temperature." The relationship between the density transition zone (increasing in breadth with increasing temperature) and the surface tension had been calculated in somewhat similar analyses by Hill.\textsuperscript{(51-53)} Although these authors did not consider molten metals, the implications of their theory, which appears reasonably accurate, are perplexing when one ponders how to best describe an interface, between molten metal and metal vapor, across which a net transfer of mass, momentum, and energy is occurring.

In the absence of a greater understanding of vapor-liquid interfacial phenomena, the nonequilibrium (net mass transfer across the interface) interfacial structure is usually assumed to be practically the same as that at equilibrium. Furthermore, the transfer of heat, mass, and momentum through either phase is usually treated (with justification considering the foregoing assumptions) in terms of the macroscopic transport properties of the respective phases up to the interface. Then, in the absence of interfacial resistance to heat and mass transfer, the transport equations of both phases are coupled at the interface.

Schrage\textsuperscript{(8)} also discussed the phenomena of viscous slip and temperature jump, i.e., the apparent velocity and temperature discontinuities that have been deduced to occur at interfaces from observations of momentum and heat transfer between gases and solid surfaces. Present\textsuperscript{(55)} discussed "the temperature jump," giving a theoretical treatment thereof and remarking that "it is customary to use the mathematical artifice of the temperature jump" rather than take account of the more rapid temperature variation within several mean free paths of the surface due to collisions with the surface. Kaminsky\textsuperscript{(56)} did likewise and critically reviewed experimental temperature jump methods. Hurlbut\textsuperscript{(95)} reviewed the history of the temperature jump concept and the development of the understanding of interactions of rarefied gases with solid surfaces.

Net mass transfer requires some potential difference as a driving force. Thus condensation may be considered to require a lower condensate surface temperature than the bulk vapor temperature. In other words, the condensing vapor is at a higher, but not necessarily measurably higher, temperature than the evaporating molecules leaving the liquid surface. This forms the basis for the kinetic-theory-of-gases approach to evaporation and condensation, which implicitly includes a temperature jump at the vapor-liquid interface.
The foregoing suggests one of the most perplexing problems of interphase heat and mass transfer, i.e., determining accurately the liquid surface temperature. Indirect calculations of the surface temperature based upon transport equations for the bulk phase and point properties measured there are only as good as these equations and measurements. Direct measurement of liquid-surface temperatures using thermocouples at the surface was challenged by Ackerman, Thorn, and Winslow,$(69,70)$ who also questioned the use of optical pyrometers to measure liquid surface temperatures because the source of light and emitted molecules, except possibly for metals, is not the same; i.e., the light intensity is the sum of that emitted by each liquid layer multiplied by the fraction transmitted to the surface, while the evaporating molecules are emitted from the outermost molecular layer. They suggested that the most logical procedure might be to measure the total number of particles and their velocity distribution, as did McFee et al.$(96)$ for beams of potassium atoms scattered from various solid surfaces, yielding an accuracy of $\pm 25^\circ$K at $2000^\circ$K. The accuracy and the difficulty of the latter approach in a condensing heat transfer experiment present additional formidable problems.

If the interface is not sharply defined, but consists of a transition zone whose width increases with increasing temperature, as suggested by Chang et al.$(54)$ it seems that the assignment of an interface temperature loses its meaning unless this temperature is related to some position in that transition zone.

2. Kinetic Theory

Again Schrage$^{(8)}$ reviewed the attempts of various researchers to apply the kinetic theory of gases to describe condensation and evaporation, organized and clarified many of their ideas, and thereupon developed a kinetic theory of interphase mass transfer. His net rate of mass transfer is given by the difference between the rates of condensation and evaporation, so that the phenomenon of the "temperature jump" is implicitly included.

Schrage recognized that the rate of interphase mass transfer in a one-component system should be specified uniquely by the bulk vapor conditions, $P_v$ and $T_v$, and by the liquid surface conditions, principally $T_s$ (Reference 8, pp. 53-54). He assumed that at equilibrium (a) "any uncertainty in the precise location of the phase interface is small with respect to a mean free path in the gas phase," and (b) "above this interface there exists a uniform gas with the velocity and internal energy of its molecules distributed according to the same laws which are known to apply to a uniform gas in the absence of disturbing influences."

Extending these concepts to the nonequilibrium case, he reasoned that the state of the gas should have no effect upon the "absolute rate of
vaporization at the surface, which is reasonably expected to be a function only of the state of the liquid or solid surface." Thus he deduced that the rate of evaporation should have the same form in the nonequilibrium case as in the equilibrium case. The apparent validity of this deduction was demonstrated in carefully conducted experiments on evaporation into high vacua\(^{(67)}\) in which the predicted maximum rates of evaporation were achieved. To describe the rate of condensation, Schrage assumed that "the velocity distribution of a uniform gas in simple mass motion prevails very near to the interface of the two phases." This simple model of interphase mass transfer then implies that interactions between incident molecules and molecules leaving the vapor-liquid interface are negligible insofar as the bulk vapor conditions have been assumed to exist up to the interface. Subsequent applications of the theory by most researchers relied implicitly upon the latter assumption, a notable exception being Silver's and Simpson's\(^{(44)}\) treatment of condensing superheated steam in which they assumed the "temperature jump distance" to be one mean free path, beyond which gradients in the vapor were described by macroscopic transport equations.

It was shown\(^{(8,55,58)}\) that the gross flux of molecules of a uniform gas in any one direction is given by Equation (1),

\[ G = P \sqrt{\frac{M}{2\pi RT}}. \]

For the nonuniform vapor having net mass motion toward or away from the vapor-liquid interface, the bulk vapor properties are assumed to prevail up to the interface. The gross rate of condensation is given in terms of these properties as

\[ G_C = \sigma_c \Gamma_1 P_v \sqrt{\frac{M}{2\pi RT}} = \sigma_c \Gamma_1 G_v, \tag{68} \]

where \(\sigma_c\) has heretofore generally been considered the fraction of molecules striking the interface that actually condenses. The common application of Schrage's theory implies that the fraction of molecules reflected from the liquid surface, \(1 - \sigma_c\), contributes nothing to the net transfer process. Kaminsky's\(^{(56)}\) discussion of the thermal accommodation coefficient emphasized that reflection is generally an inelastic process unless \(T_v - T_s\) is smaller than a few degrees. Hence reflection in the case of larger "temperature jumps" could contribute to momentum and energy transfer to the liquid. \(\Gamma_1\) is a correction factor arising from the consideration of the vapor having net flow in one direction [see Equation (12), p. 27].

Distinguishing between reflection and evaporation is a difficult and ambiguous problem. Incident molecules reflected from the vapor-liquid interface are not reflected instantaneously, but "reside" in the interfacial force field long enough to exchange some energy with the "liquid molecules."\(^{(56,97)}\) Sufficient residence time will permit complete
equilibration of the "vapor molecule" with the liquid molecules and its "capture" thereby. Release of this same vapor molecule from the liquid after it has equilibrated, or nearly done so, with the surface might be termed evaporation rather than reflection. Evidently distinguishing between evaporation and reflection in the process just mentioned is somewhat arbitrary, but ultimately must rely upon considerations of the nature of energy exchange during the time of residence of the reflected molecule at the vapor-liquid interface. Schrage's analysis ignored the energetics of reflection, because he assumed that the reflection process is elastic. Zwick(59) did include the reflected molecules in his conservation equations in a somewhat more elaborate analysis, but he still assumed the reflection process to be elastic. Apparently no serious attempts have been made to theoretically account for the effect of inelastic collisions in the kinetic theory of condensation.

Following Schrage's assumptions, the usual rate of evaporation is derived as

\[ G_e = \sigma_e P_s \sqrt{\frac{M}{2\pi R T_s}} = \sigma_e G_s, \]  

(69)

where \( \sigma_e \) is the evaporation coefficient which has been theoretically related to internal energy changes occurring in a molecule during evaporation.\(^{(61,71)}\) Schrage found support for this vaporization equation in the work of Penner\(^{(98,99)}\) which presented an identical form of the rate of evaporation derived from the theory of absolute reaction rates.

The net rate of mass transfer is given by Equation (2) as the difference between the rates of condensation and evaporation,

\[ G_{\text{net}} = \sigma_c \Gamma_v G_v + \sigma_e G_s, \]

where the difference is due to the \( \Gamma_v \) term, which is negative.

A more extensive kinetic theory of condensation and evaporation, based upon the principles of mass, momentum, and energy conservation and normalization of an assumed perturbation of the Maxwellian velocity distribution of incident molecules, was also developed by Schrage.\(^{(8)}\) In assuming \( \sigma_e \) and \( \sigma_c \) to be unity, this analysis eliminated reflection effects at the liquid surface. Although the latter theory has not been applied by any other authors, it is important to point out an apparently impossible result of the analysis, which seems to negate its validity. That is Equation (5.6-6) of Schrage's book, which predicts imaginary values of condensation rates, or imaginary mass flow rates when the liquid surface temperature is lower than the bulk vapor temperature.

In an analysis somewhat similar to Schrage's latter theory, Zwick\(^{(59)}\) attempted to include the effects of reflection. Using a perturbed
velocity distribution for the incident vapor and reflected molecules, he assumed the reflection process to be elastic. He introduced the condensation coefficient as a parameter to be specified, upon which the solution for the "temperature jump" at a given mass flux depends. But, because Zwick did not specify any relationship between \( \sigma_e \) and \( \sigma_c \), and attempted at least nominally to account for the reflection process, his analysis must be considered an improvement of Schrage's theory. Moreover, Zwick's theory permits an estimation of \( \sigma_c \) for condensing and evaporating systems in which \( q/A, T_v, \) and \( T_s \) are measured. The ultimate interpretation of any \( \sigma_c \) thus calculated must fully consider the limits of validity of his model.

The plot of Zwick's theoretical results is reproduced in Fig. 37, showing the ratio \( P_s/P_v \) versus \( \phi_v \). The condensation coefficient, \( \sigma_c \), is the parameter. According to Zwick's model, an evaporation limit exists, such that "all molecules reaching the surface from the vapor will arrive with the same normal velocity component."

![Fig. 37. Results of Zwick's Kinetic Theory of Evaporation and Condensation (59)](image)

Kucherov and Rikenglaz\(^{100}\) used a "thirteen moment" approximation to a Maxwellian velocity distribution for a moving gas and derived an equation predicting a mass flux twice that found from the usual kinetic theory treatment. They also assumed evaporation and condensation coefficients of unity.

According to Vulliet,\(^{101, 102}\) analyses such as all the foregoing hold only for the flow of Knudsen gases; i.e., the analyses hold only when the mean free path of gas molecules is "large compared with the dimensions of the vaporizing surface." Vulliet\(^{101, 102}\) combined concepts of the theory of absolute reaction rates, a Debye frequency of the liquid, and irreversible thermodynamics and derived an equation for net mass fluxes of evaporation and condensation. He calculated theoretical values for iron, but to this author's knowledge, Vulliet's theory has not been tested experimentally.

Another theory, based upon a presumed "pseudocrystalline" liquid structure, the Polanyi-Wigner theory of escape of molecules from a solid surface, and the kinetic theory of condensation, was proposed by Lypse\(^{103}\) and supported by him with some questionable experimental verification. Apparently because of contradictions in the theory, it has not received further test (see reviewers' discussions included at end of Reference 103).
Schrage\(^{(8)}\) and others\(^{(3,7,42-44,60-65)}\) used the quantities \(\sigma_e\) and \(\sigma_c\) as being identical, using the terms condensation coefficient, evaporation coefficient, transmission coefficient, and sticking coefficient interchangeably. Some erroneously called it an accommodation coefficient.\(^{(60)}\) Used in this manner, the coefficient was considered by Schrage and his disciples as a "function only of the molecular specie under consideration and the state of the liquid or solid surface." Günther\(^{(104)}\) generated more confusion by inventing a new coefficient, which eliminates consideration of all but the incident flux.

Plesset,\(^{(66)}\) in his kinetic theory treatment of the flow of vapor between liquid surfaces, took the condensation and evaporation coefficients to be equal but admitted that such an assumption may not be justified. Paul\(^{(67)}\) and Courtney\(^{(68)}\) pointed out that the kinetics of evaporation and condensation may be quite different. Ackerman \textit{et al.}\(^{(69,70)}\) lucidly discussed that these coefficients are definitely not identical except at thermodynamic equilibrium, where it is impossible to distinguish between reflected and evaporated molecules. They emphasized the work of Langmuir\(^{(105)}\) as evidence "that condensation in general must be discussed and explained in terms of the forces or, perhaps the potentials between gaseous atoms and surface atoms," the rate of condensation and the fraction condensing depending upon "the shapes of these potentials, and the energies of the incoming particles." They suggested that the kinetic theory of gases is an insufficient basis for discussion of the problem, and briefly discussed possible particle interactions and potentials. Then examining the problem using the thermodynamics of irreversible processes, they concluded that a comparison of the latter with experimental data "is valuable in locating the rate determining boundary" and establishing explanations for nonunity evaporation coefficients. Unfortunately, the current status of irreversible thermodynamics does not permit quantitative predictions of rate processes.

Littlewood and Rideal,\(^{(106)}\) after observing that materials with high thermal conductivities, such as metals, have coefficients near unity for evaporation into vacua, suggested that thermal conditions at the interface affect the evaporation coefficient. (If the availability of energy is a limitation, a reduction in the evaporation coefficient might be expected for net condensation in which energy is being removed through the condensate by conduction.) They also recognized that the condensation coefficient is related to the "probability that an incident molecule would actually condense" and that deviations of the value of the evaporation coefficient from unity may be the result of "departure from equilibrium at the interface during evaporation" measurements. Unfortunately, they used \(\sigma_e\) and \(\sigma_c\) interchangeably.

Paul's\(^{(67)}\) survey of evaporation coefficients indicates that for most metals the maximum value of the evaporation coefficient, evaporating into a vacuum, is approximately unity. Organic and inorganic compounds show
coefficients from unity down to low fractional values, the lower values apparently being caused by marked asymmetry and/or polarity of the molecules. Knacke and Stranski(71) reviewed the theories of evaporation and discussed the effects that changing internal energy states of molecules during evaporation have upon the evaporation coefficient. Mortensen and Eyring(61) employed the theory of absolute reaction rates and considered that the evaporation coefficient ought to be given by the ratio of rotational partition functions for surface molecules to those in the vapor phase. The agreement between calculated and observed values in most cases was fair. Hirth and Pound(107) also obtained theoretical values of evaporation and condensation coefficients, but for the interaction between crystals and vapor.

An effect of polarity or molecular structure upon the condensation coefficient was perhaps indicated qualitatively by the work of Velkoff and Miller(108) on condensing Freon-113 in a transverse electrostatic field. They observed significant increases in condensation rates which suggest that the liquid surface and incident vapor molecules for a polar fluid orient themselves in a proper electrostatic field to effect a net increase in condensation.

Condensation or sticking coefficients, measured primarily for metal vapors condensing and crystallizing on solid surfaces, were reviewed by Wexler.(72) The survey (values for various metals tabulated) indicates that the condensing coefficients increase with decreasing surface temperature and with increasing thickness of the deposited layer. Moreover, these coefficients were determined primarily in molecular beam experiments under high vacua, conditions markedly different from typical engineering film condensation studies. Surface nucleation (crystallization) theories and the extremely high, critical supersaturation ratios required to initiate surface nucleation were discussed as well as "dwelling time" concepts and surface migration of adsorbed molecules. The latter might be important in dropwise condensation.

Mayer(97) also discussed the above ideas in his comprehensive texts on the physics and formation of thin films. Both these treatments recognized the importance of surface purity (freedom from contamination and adsorbed extraneous gases) in obtaining reliable data.

De Boer(109) discussed migration of adsorbed molecules as a "two-dimensional gas" and examined the spreading of liquids over surfaces and the wetting thereof as a phenomenon caused by surface wandering.

The accommodation coefficient was discussed with varying clarity by many authors. Schrage(8) briefly discussed the most commonly used definition of the thermal accommodation coefficient, as introduced by Knudsen,(110)
\[ \alpha_T = \frac{(T_i - T_R)}{(T_i - T_S)} \]  

where \( T_i \) is the average temperature of the incident molecule stream, and \( T_R \) is the average temperature of the reflected molecule stream, if a kinetic temperature can be assigned thereto. Thus \( \alpha_T \) "measures" the efficiency of energy transfer between a gas and a surface.

Wise\(^{(111)}\) showed theoretically how isothermal and nonisothermal adsorption of gases on solids, the adsorbate being mobile, might affect thermal accommodation and be used to calculate energy of activation for adsorption, if the accommodation and transmission coefficients are known.

Rogers\(^{(112)}\) defined normal and tangential momentum accommodation coefficients and compared their nature with that of thermal accommodation coefficients. He also analyzed the Baule model of thermal accommodation to illustrate its shortcomings.

Hartnett's\(^{(113)}\) survey of thermal accommodation coefficients indicated that all available data on thermal accommodation were inaccurate for one of the following reasons: (a) vaguely defined surface conditions, (b) "departure from free molecule flow," and (c) "excessive radiation."

Mayer\(^{(97)}\) reviewed experimental accommodation coefficients and theories of the accommodation coefficient in his treatise on particle and surface interactions. He defined the accommodation coefficient as the following limit:

\[ \alpha_T = \lim_{E_s \to E_i} \left[ \frac{(E_i - E_R)}{(E_i - E_S)} \right] \]  

The same definition was given by Kaminsky,\(^{(56)}\) who admitted, however, that this definition in terms of temperatures held only if the reflected particles had an undistorted Maxwellian velocity distribution (i.e., only when \( T_S \approx T_i \), or when the temperature difference between surface and gas was of the order of a few degrees or less). He added that the accommodation coefficient depended upon the absolute temperatures involved and the magnitude of the temperature difference between the gas and the surface, and emphasized that \( \alpha_T \) "determines an equilibrium condition, not a rate." He also presented a comprehensive critical review of accommodation coefficient theories, methods for measuring the accommodation coefficient, and experimentally-determined accommodation coefficients.

Feuer\(^{(114,115)}\) developed an improved quantum mechanical theory of the thermal accommodation coefficient for diatomic molecules, and Allen and Feuer\(^{(77)}\) reviewed the theory for monatomic molecules. Other comprehensive reviews of gas-solid (lattice) interactions influencing the accommodation coefficient were presented by Gilbey,\(^{(76)}\) by Goodman,\(^{(73-75)}\) and by Chambers and Kinzer.\(^{(116)}\) The complexities of these quantum
mechanical theories indicate the difficulties involved in estimating, from first principles, thermal accommodation coefficients for gas-solid interactions. The calculation of accommodation coefficients, condensation coefficients, and evaporation coefficients for vapor-liquid interactions would probably involve similar complexities. After reviewing published data on these coefficients, Paul (67) concluded that apparently nobody has attempted to show the interrelationships among these three coefficients, but Ackerman et al. (69,70) stated that Prüger (117) attempted to suggest how the accommodation and evaporation coefficients might be related.

It seems clear, in retrospect, that many attempts to treat film condensation of vapors upon the same liquid specie, using the kinetic theory of condensation suffered from one or more of the following shortcomings:

(a) Failure to recognize that the theory is only approximate.

(b) Because of (a), failure to recognize that the condensation and evaporation coefficients defined by the theory, if measurable or calculable from the theory and data, include not only the singular interpretations heretofore attributed to them, but the additional effects of deviations of the actual system behavior from the simple kinetic theory.

(c) Assuming the condensation and evaporation coefficients to be at all times identical.

(d) Misuse of the thermal accommodation coefficient.

(e) Neglect of the processes of momentum and energy exchange; i.e., describing the process purely in terms of mass transfer.

(f) Failure to recognize the problems of separating the reflection and evaporation processes.

(g) Neglect of the probability that reflection of incident molecules from the liquid surface is an inelastic process.

If the foregoing difficulties cannot be surmounted using the approximate kinetic theory of gases, then a more rigorous approach will be needed. This approach might be the consideration of the Boltzmann integro-differential equation for the vapor adjacent to the vapor-liquid interface. The corresponding consideration of interfacial collision processes might ultimately involve the quantum mechanical estimation of the coefficients of condensation, evaporation, and thermal accommodation. The complexity of determining such coefficients was implied by Allen and Feuer (77) among others.

As an approach to the more accurate estimation of the proper velocity-distribution function to use in describing the transport of properties
across the vapor adjacent to the interface, the kinetic model reviewed by Sirovich\(^{(78)}\) might prove helpful. The equation is

\[
\left( \frac{\partial}{\partial t} + \xi \frac{\partial}{\partial x} \right) f = \nu (f_0 - f), \tag{72}
\]

in which "\(f\) is the molecular distribution function, \(\xi\) is the molecular velocity, \(f_0\) is the local Maxwellian distribution function," and \(\nu\) is the "collision frequency," apparently determined by the collisional model of the system considered.

This equation, according to Sirovich, "mimics the Boltzmann equation"; i.e., "moments of the collision term vanish when weighted by the collisional invariants, . . . the H-theorem holds," and the local Maxwellian distribution is "its solution in limit of infinite collision frequency," the same as the full Boltzmann equation. However, in the space homogeneous case, "all moments of the distribution function other than density, temperature, and velocity have the same decay time." Hence, Equation (72) is called the "single relaxation model."

3. Nucleation

Condensation in supersaturated bulk vapor occurs by nucleation, i.e., the formation of "droplets" containing less than about 100 molecules (or atoms) which grow by molecular deposition of fresh vapor thereon or by coalescences with other nuclei or growing droplets. Summarizing Courtney's\(^{(68)}\) survey of "recent advances in condensation and evaporation," the condensation process can be described in terms of the simultaneous kinetics of nucleation, growth, and agglomeration of liquid particles. The five general theories of nucleation are comprised of: the classical liquid drop model of steady-state nucleation developed chiefly by Volmer,\(^{(118)}\) Volmer and Weber,\(^{(119)}\) Farkas,\(^{(120)}\) Becker and Döring,\(^{(121)}\) and Frenkel;\(^{(122)}\) the excess energy model; the statistical mechanical and the quantum mechanical theories of predicting cluster concentrations; and the recombination model. Nucleation may be homogeneous (pure system), or heterogeneous with impurities and foreign particles acting as nuclei. The growth of a condensate particle by molecular deposition depends upon bulk diffusion of vapor molecules to the surface of the particle, incorporation into the liquid lattice of the particle, and diffusion of the heat of condensation away from the particle. Accurate predictions of growth rates are difficult to make because of lack of understanding about the so-called accommodation, condensation, and evaporation coefficients used in describing mass transport across a vapor-liquid interface. Surface-controlled growth rates are usually described in terms of kinetic theory. Moreover, growth kinetics varies with supersaturation, temperature, and particle size. "Agglomeration can be considered to be an ordinary bi-molecular chemical reaction," and can be treated as such analytically. Particle size and turbulence are important factors in agglomeration. The
kinetics of the reverse process of growth (i.e., evaporation) may be different because of different rate-controlling mechanisms in the two processes.

The growth of water, methanol, and ethanol droplets from supersaturated vapors was investigated by Wakeshima and Takata. They compared observed growth rates with theoretical growth rates in order to calculate condensation coefficients for these fluids. Their results showed approximate agreement with published coefficients calculated by other methods.

The general theory of nucleation was also reviewed by LaMer and by Pound. Reiss reviewed the classical liquid drop model in its latest form and presented another theory based on statistical mechanics. In explaining the process of spontaneous nucleation, Rodebush considered the equilibrium stability of nuclei, assuming equilibrium concentrations were always present. Friedlander analyzed the factors affecting the particle size spectrum of a condensing vapor to show that particular size can be controlled by varying certain parameters.

Courtney conducted an extensive analytical program to solve the simultaneous equations describing nucleation kinetics using the classical, liquid-drop, homogeneous, nucleation theory and the growth law of classical theory. The ultimate applications were more accurate understanding and prediction of cloud chamber behavior and of condensation in rocket nozzles.

In an analysis somewhat similar to Courtney's, Heiskala treated homogeneous nucleation as an "irreversible process for which the mechanism of cluster formation is the stepwise addition of molecules through collision." His results compared favorably with the few available data points he chose. He concluded that transient rates of cluster formation may be adequately described by their steady-state values, and that bulk fluid properties, such as surface tension, "can be used for the properties of the nuclei initiating the condensation."

Hirth reviewed the theory of homogeneous nucleation from the vapor phase and extended the ideas therein to develop a theory of heterogeneous nucleation on a substrate. Whereas previous investigators used an analogue of the Volmer theory to treat the kinetics of two-dimensional nucleation, Hirth used a modified form of the Lothe-Pound theory. He attributes disagreements between theory and data to uncertainties in surface properties, particularly surface entropy terms, and to uncertainties in the relaxation time required to establish steady-state conditions. A two-dimensional model of nucleation on or near a liquid surface or film is not evident.
The supersaturation of vapor needed to produce spontaneous nucleation can be produced by rapid expansion of the vapor in both non-flow and flow systems. In the simplest flow system, the saturated vapor expands adiabatically until the critical supersaturation ratio is reached, near which point nucleation occurs. Near this point, pressure rises abruptly, accompanied by considerable condensation. Condensation and droplet growth continue downstream beyond this "condensation shock."

Stodola and Lowenstein\(^{(140)}\) reviewed early work on steam condensation in flow through nozzles. Stever\(^{(141)}\) reviewed nucleation kinetics and flow of condensing vapor through nozzles, considering separate conservation equations for the gas and liquid phases, i.e., treating the condensed vapor as a two-phase mixture. Condensation in wind tunnels was reviewed by Wegener and Mack.\(^{(142)}\) These five authors also reviewed existing data and experimental techniques.

Hill \textit{et al.}\(^{(143)}\) utilized the foregoing concepts to theoretically describe the condensation of metal vapors during rapid expansion through nozzles. Apparently, data for condensing flow of supersaturated metal vapors are not yet available.

4. \textbf{Dropwise Condensation}

Although dropwise condensation has been studied by many during the past 30 years,\(2,3,9,48,89,144-168\) current understanding of the mechanisms of dropwise condensation is not complete. Drew and his co-workers\(^{(144)}\) have been credited with clarifying some of the early confusion about dropwise condensation of steam.\(^{(1,P.347)}\) They concluded that film condensation is always obtained, except when the condensing surface is contaminated with a "promoter" which prevents the condensate from wetting the surface. Many organic compounds are dropwise condensation promoters for steam, whereas surface oxides or refractories and adsorbed gases are generally believed to prevent film condensation of mercury.

Jakob\(^{(145)}\) suggested that dropwise condensation results from the fracturing of thin condensate films into droplets after the film has grown to some critical thickness, and the process repeating itself over the newly exposed "bare" surface. Baer and McKelvey,\(^{(146)}\) and later Welch and Westwater,\(^{(147)}\) supported the film-fracturing theory. Welch and Westwater observed dropwise condensation of steam on copper with \(\Delta T\)'s varying from 0.4 to 47°F and took high-speed motion pictures of the process through a microscope. Their results indicated that drops large enough to be visible (0.01 mm) grew mainly by coalescences with other drops, exposing a "lustrous bare area" the lustre of which quickly faded, indicating that condensate was building up on the surface. They concluded that heat transfer occurs mainly between the droplets and that the droplets are formed by fracturing of the liquid film between the droplets when the film grows to
about 0.5 to 1 micron thick, but they could not support their conclusions with film-thickness measurements or photographs of the condensation process at higher magnification. Westwater's work is currently being continued with emphasis upon dropwise condensation of organic vapors.\(^{(148)}\)

Fatica and Katz\(^{(2)}\) had stated their fondness for the film-fracturing theory, but gave no evidence to support it. They, and later Sugawara and Michiyoshi,\(^{(149)}\) derived equations for heat transfer coefficients based upon the questionable assumptions of uniform drop-size distribution and constant fraction of condensing surface covered by droplets.

Eucken\(^{(150)}\) had suggested a mechanism of diffusion towards the droplets from supersaturated adsorbed surface layers, while Emmons\(^{(151)}\) proposed a mechanism based upon the re-evaporation of condensed molecules on the "bare" area at the temperature of the condensing surface, followed by recondensation onto the droplets. A major driving force in his mechanism was the assumed pressure fluctuations and violent local eddy currents set up in the vapor between the drops as a result of rapid condensation.\(^{(9,151)}\)

Although the film-fracturing theory seems to explain the relatively high heat transfer rates of drop condensation, several researchers supported the idea that droplets are formed at active sites or "condensation centers" on the surface. Tamman and Boehme\(^{(152)}\) and Fuks\(^{(153)}\) reported condensation centers in the condensation of moisture from wet air at low \(\Delta T\)'s. Gel'man's\(^{(48)}\) experiments on dropwise condensation of mercury vapor indicated the presence of such condensation centers.

Kutaseladze et al.\(^{(154)}\) presented a simple equation for the heat transfer coefficient in dropwise condensation, based upon the Clapeyron equation and the simple equation for mass flux under the influence of \(\Delta P\). His result qualitatively describes dropwise condensation, but predicts much higher heat transfer coefficients than those observed experimentally. He attributed this lack of agreement to the presence of non-condensable gases. This is a simple explanation for a phenomenon that may be complicated by several rate-controlling mechanisms, not necessarily the diffusional resistance of noncondensible gases.

McCormick and Baer\(^{(155)}\) proposed that numerous submicroscopic drops grow from randomly distributed sites, which are possibly faults in the condensing surface. Umur and Griffith\(^{(156)}\) supported the theory of condensation centers with the results of their studies of dropwise condensation of steam on a gold-plated surface at low \(\Delta T\)'s (less than \(1^\circ F\)). Using an optical method, and measuring the intensity and polarization of incidentally plane, polarized light reflected from the condensing surface, they were able to calculate the thickness of condensate layers between droplets. They concluded that
1. The area between the drops does not have a liquid film greater than a monolayer in thickness.

2. No net condensation takes place on the area between the drops. Therefore, nearly all the energy transferred to the cooling surface is transferred through the drops.

3. The most probable drop nucleation sites are wetted pits and grooves in the surface.

4. The growth rate of small drops is significantly dependent on the vapor pressure.

They derived equations, based upon the kinetic theory of gases and liquids and conduction of heat through the droplets, which appear to predict and support their conclusions.

The mechanism of dropwise condensation at higher $\Delta T$'s (greater than about 10°F), particularly on surfaces other than gold, still seems open to question to this author. A mechanism of surface layer diffusion somewhat similar to that suggested by Eucken does not seem unreasonable, and may merit further consideration. Rapid transport of surface layers was observed by many researchers in the field of adsorption and diffusion; e.g., Gilliland et al.,(158) Mayer,(97) and Wexler(72) discussed the migration of adsorbed vapors in their reviews of condensation and crystallization. It seems that adsorbed layer flow might support higher mass and heat transfer rates than falling film flow, depending upon the droplet distribution and fluid properties. De Boer(109) discussed wetting and spreading of liquids in terms of adsorbed layer migration. Certainly the "bare" surface areas observed between droplets are not totally inactive.

It is interesting to compare the results of Gel'man(48,154,159) from dropwise condensation of mercury on carbon steel, with the results of Cohn(5) for condensation of mercury and cadmium on Type 304 stainless steel (not visually observed, but assumed to be dropwise). Gel'man and Cohn both observed that the heat transfer coefficient is proportional to the reciprocal of $\Delta T$, i.e., that heat flux (the product h$\Delta T$) is a constant. Apparently the condensers from which these data were taken were operating at their limiting heat fluxes for the cooling methods used. Cohn's condenser was air-cooled by natural convection, and Gel'man's was water-cooled. A deviation from Nusselt's theory is expected, but it is expected to result in higher values than Nusselt's theory predicts, rather than the much lower values of heat fluxes actually observed.
5. **Film Condensation**

a. **Classical Theory**

Nusselt\(^{(4)}\) fathered the development of steady-state condensation heat transfer theory with his 1916 publication of the theory of laminar film condensation of saturated vapors on vertical and inclined surfaces. He used a highly idealized model which assumes that the saturated vapor condenses on a cold wall of uniform temperature so that the condensate completely wets the wall, forming a liquid film which flows downward due to the force of gravity. Neglecting acceleration of the film and shear at the vapor-liquid interface, but considering viscous shear at the wall, and assuming a linear \(\Delta T\) across the film, with saturation conditions at the vapor liquid interface, he arrived at the following equations for average heat flux and average heat transfer coefficient for the condensing surface:

\[
\left( \frac{q}{A} \right)_{\text{avg}} = \frac{4}{3} \left[ \frac{\rho^2 g \lambda k^3 \sin \theta}{4 \mu L} \right]^{1/4} (\Delta T)^{3/4}; \tag{73}
\]

\[
h_{\text{avg}} = \frac{4}{3} \left[ \frac{\rho^2 g \lambda k^3 \sin \theta}{4 \mu L \Delta T} \right]^{1/4}. \tag{74}
\]

Nusselt's analysis assumed also that only latent heat of condensation is conducted through the film and that convective heat transfer, axial conduction, and viscous dissipation are negligible. Figure 38 illustrates the physical model used by Nusselt.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{film-condensation-model.png}
\caption{Physical Model of Film Condensation According to Nusselt}
\end{figure}

Defining the Reynolds number of the film as

\[
\text{Re}_L = 4 \delta (L) v_L \text{avg} \frac{\rho}{\mu}, \tag{75}
\]
Nusselt obtained the following equation:

\[
\left[ h_{\text{avg}}^3 \frac{g_\mu^2}{(\rho^2 k^3)} \right]^{1/3} = 1.47 (\text{Re}_l)^{-1/3} .
\]  
(76)

The corresponding equations for condensation outside a horizontal cylinder are

\[
h_{\text{avg cyl}} = 0.725 \left[ \frac{\rho^2 g_\lambda k^3}{(\mu_D \Delta T)} \right]^{1/4} ,
\]  
(77)

and

\[
\left[ g_\mu^2 h_{\text{avg}}^3 \frac{1}{(\rho^2 k^3)} \right]^{1/3} = \frac{(\text{Re})^{-1/3}}{2},
\]  
(78)

where

\[
\text{Re} = 4\Gamma/\mu.
\]  
(79)

Jakob\(^9\) reviewed each of the cases Nusselt considered (i.e., cocurrent flow and counter-current flow of a condensing saturated vapor in a vertical tube), using the established frictional pressure-drop correlation for single-phase flow in tubes to account for shear at the vapor-liquid interface, and assuming the film so thin that the diameter for vapor flow is very nearly the tube diameter. He and co-workers\(^9, 169\) also extended Nusselt's cocurrent vapor flow case to cover nearly total condensation of the flowing vapor.

Hartmann\(^170\) examined the latter problem using a variable friction factor, dependent upon gas-phase Reynolds number, thus allowing for decreasing vapor velocity through the tube (i.e., depletion of the vapor supply). The resulting Nusselt number is roughly half that given by previous authors who assumed a constant friction factor through the tube. However, it seems to this author that using a friction factor to describe shear at the vapor liquid interface is an erroneous simplification of a process involving momentum exchange by shear and by mass transfer.

Broglio et al.\(^171\) treated the same problem as Jakob and Hartmann, but only for laminar, downward flow of vapor and condensate in a vertical tube. Their analysis was restricted by their assumption that the condensing heat flux is constant over the tube.

b. Steam and Organic Vapors

Nusselt's theory adequately described film condensation of steam and organic vapors within engineering accuracy.\(^1, 9-12\) Small
deviations of experimental heat transfer rates and heat transfer coefficients from theory were usually attributed to the presence of noncondensible gases,\(^{(1,9,36-40)}\) variations in fluid properties or condenser temperature,\(^{(1,9,24,172,173)}\) turbulence, waves, and ripples on the condensate film, and shear at the vapor-liquid interface.\(^{(1,9,174-176)}\) However, the advent of research on condensing metal vapors added more confusion to this field of simultaneous heat and mass transfer, which apparently had been developed to a point of adequate understanding for many purposes.

c. **Metal Vapors**

Brooks and Bonilla\(^{(177)}\) reviewed the status of condensing-metal heat transfer with the emphasis upon space systems, but their survey is quite brief.

The earliest available data on condensing-metal heat transfer came from the General Electric Company's mercury power plants,\(^{(178)}\) but were insufficient for comparison with Nusselt's theory. Misra and Bonilla\(^{(3)}\) later obtained enough information from GE to estimate that the heat transfer coefficients used by GE were roughly 3 to 6% of theoretical values.

Misra and Bonilla\(^{(3)}\) then undertook a program aimed at producing heat transfer coefficients for condensing metal vapors with the following results:

Heat transfer coefficients were determined for mercury vapor condensing on both water and air-cooled vertical steel, copper plated steel, nickel, and stainless steel condensers 0.5-in.-OD, and from 0.5 to 3-in. in length. Experiments were also carried out on a 4.5-in. long, 0.5-in.-OD nickel-stainless-steel-composite tube condenser, in both horizontal and vertical positions. Visual observations, still photographs, and Fastax moving pictures showed filmwise condensation on copper plated steel and nickel surfaces and dropwise condensation on stainless steel surfaces. On steel, condensation was usually dropwise near the top and filmwise near the bottom. The heat velocity varied from about 25,000 Btu/hr ft\(^{2}\) at 0.5 psia with air cooling, to about 750,000 at 15 psia with water cooling, and the heat transfer coefficient ranged from about 3,000 to about 10,000 Btu/hr ft\(^{2}\) °F for film-type condensation, and from about 4,000 to over 50,000 for dropwise condensation.

Condensing heat transfer studies were also carried out on a bimetallic nickel-stainless-steel condenser. The heat velocity varied from about 60,000 Btu/hr ft\(^{2}\) at 650°C to about about 100,000 Btu/hr ft\(^{2}\) at 870°C, giving a heat transfer
coefficient ranging from 11,000 to 13,000 Btu/hr ft² °F. The condensing heat transfer coefficients obtained for mercury and sodium are only about 5 to 15% of the Nusselt equation values for filmwise condensation.

Misra obtained the condensing heat load from a coolant heat balance, and he used four different methods (in different apparatus) to measure the temperature drop across the condensate film. Assuming a condensation coefficient of unity, he corrected the ΔT's for "temperature jump" at the vapor-liquid interface based upon kinetic theory of gases, but the correction was too small to bring the heat transfer coefficients within agreement with Nusselt's theory. Misra concluded that his high ΔT's for mercury could not have contained errors large enough to produce the disagreement observed between data and theory. He apparently avoided a discussion of errors in measuring ΔT for condensing sodium. The ΔT's for condensing sodium ranged from about 4 to 9°F, so that considerable error could result in these small ΔT's. Hence large errors could result in observed heat transfer coefficients, particularly at the lower ΔT's. He also concluded that reproducibility of results ruled out the presence of non-condensible gases, that there is no evident reason for a possible thickening of the condensate film, and that a solid-liquid interface contact resistance did not seem possible at the high temperatures studied. An effect of vapor velocity on mercury condensation was not observed at atmospheric pressure, but upon decreasing the pressure (and thus increasing the vapor velocity) he observed that the condensate became agitated, and in some cases the film or droplets were blown off the surface. But vapor shear and turbulence should increase the heat transfer coefficient. However, while he saw no theoretical explanation for such low heat transfer coefficients, he stated that they are "sufficiently high to cause relatively little additional thermal resistance in most applications."

In an effort to expand the available information on condensing metals, Cohn(5) studied the condensation of mercury, mercury amalgams (0.3% sodium and 1.0% sodium), and cadmium 1/2-in.-OD x 12-in.-long, 16 BWG Type 304 stainless steel reflux tube, air-cooled by natural convection. He measured heat transfer coefficients from 200 to 70,000 Btu/hr ft² °F at ΔT's from 0.5 to 70°F and temperatures up to 725°F for mercury and 1425°F for cadmium. The heat transfer coefficients fell within 1 to 13% of Nusselt's theoretical values,(179) but the correlation of h versus ΔT indicated that h is proportional to the reciprocal of ΔT; i.e., heat flux, the product h ΔT, is independent of ΔT and pressure or temperature. This result was apparently dictated by the limitations of natural convection cooling. Dropwise condensation probably occurred on the 304 stainless steel. Cohn could not explain the observed discrepancy between theory and data, but a glance at his apparatus indicates that non-condensible gases and possibly liquid hold-up might have caused some difficulty.
Misra's sodium condensing data were somewhat fortified by Engelbrecht's(6) studies of potassium and rubidium condensing inside 1/2-in.-OD and 1 1/4-in.-OD stainless steel-nickel composite tubes cooled by air or by natural convection and radiation. His observed heat transfer coefficients ranged from 50 to 35,000 Btu/hr ft² °F for potassium up to 1300°F with ΔT's as high as 105°F, and 1,900 to 5,700 Btu/hr ft² °F for rubidium up to 877°F and ΔT's from 2 to 4°F. These coefficients are roughly 2 to 10% of Nusselt's theoretical values. Engelbrecht attributed fluctuations in the coefficients to "unobservable variations in the mode of condensation and/or condensing surface conditions" as well as experimental error, and blamed noncondensible gases for the lower coefficients, but could not give any quantitative idea of the amount of noncondensible gas that might have been present in his systems.

Preliminary results of the General Electric Company's Space Power Program(18-21) may be taken as confirmation of earlier data on condensing metals. They obtained average and local film-condensing coefficients for potassium inside a 0.87-in.-ID x 66.5-in.-long, stainless steel condenser. While the comparison of the GE data with that of Bonilla's students seems favorable (GE data about 2 to 10% of Nusselt's theory), the meaning of such a comparison is not clear; the comparison of condensation inside horizontal tubes with Nusselt's theory is definitely stretching the applicability of theory. However, the General Electric plot of h versus ΔT correlates in approximately the same manner as much of the earlier data; i.e., h is nearly proportional to the reciprocal of ΔT. No significant variation in tube wall temperatures was observed around the tube OD.

The GE researchers tried to correlate their data(21) using Dukler's(180) turbulent condensation theory. Unfortunately, Dukler's theory incorrectly describes turbulent filmwise condensation of metals because he neglected k in favor of ε_H. Estimation of ε_H from Dukler's results indicates that his ε_H ≪ k for film flow of metals.

Noyes(181) and Reed (and Noyes)(182) presented the results of sodium condensing studies at Atomics International. They studied sodium boiling and condensing in the presence of known amounts of inert gases. The sodium vapor was condensed on the underside of a horizontal plate after passing through several perforated plates. Consequently their condensing work is of little value except in its verification of the simple diffusional theory used to describe their system.

Although the University of Michigan's liquid metal research program is apparently not intended to yield basic condensing heat transfer data, they apparently have obtained some unpublished data for sodium condensing on the bottom side of a horizontal plate.(183) Considering the general unstable nature of any films which might be formed on the bottom side of a horizontal plate, such data must be regarded as relatively useless in exposing condensation mechanisms, as are the data of Noyes and Reed.
Roth's\(^{184,185}\) condensing sodium and rubidium results, while providing more information on materials and operating experience, offer no solution to the problem of explaining observed low heat transfer coefficients. Sodium and rubidium were condensed on a Type 304 stainless steel, air-cooled U-tube up to 44 psia for sodium, and up to 63 psia for rubidium. Overall heat transfer coefficients up to 24 Btu/hr ft\(^2\) °F, and individual heat transfer coefficients up to 71 Btu/hr ft\(^2\) °F for rubidium, were obtained, all at low heat flux. No theoretical evaluation was attempted. Roth found it impossible to view the condensate through a sapphire window in the condenser because of fogging and, later, cracking of the window to thermal stresses. Mecklenburg's\(^{186}\) work is of the same nature as Roth's, with similar or identical equipment being used. However, Sukhatme\(^{7}\) estimated condensate-side film heat transfer coefficients for Roth's data and compared the results with Nusselt's theory. The comparison showed Roth's rubidium heat transfer coefficients to be less than 1% of Nusselt's theoretical predictions. Although the data are rough, the same puzzling, low heat transfer rates are evident.

Broglio et al.\(^{171}\) condensed sodium vapor flowing downward inside water-cooled tubes and channels in a natural circulation loop. The maximum working conditions achieved in their tests were a vapor temperature of 1300°F, a mass flow rate of 0.35 grams/sec, and a vapor velocity of 150 ft/sec. Their heat transfer coefficients were determined through measurements of coolant heat rise in segmented condensers, thus permitting the approximate measurement of local condensing heat transfer coefficients. Their wall temperature measurements, which they admitted were inaccurate, were not presented. They correlated their condensing data from 33 tests, according to a modification of Nusselt's theory which supposedly accounts for shear at the vapor-liquid interface. However, their theory is based upon the assumption that heat flux is constant throughout the tube, and the data they presented contain insufficient basic information which might be used to evaluate condensing mechanisms. The correlation of their data shows that vapor velocity is a parameter and that the heat transfer coefficient increases with increasing vapor velocity. Such a correlation is unenlightening, however, when one recalls that their flow was natural circulation and that the vapor velocities were calculated from the measured heat load.

Mercury-film condensation heat transfer data for the vapor pressure range of 0.02 to 0.33 psia were published recently by Sukhatme and Rohsenow.\(^{7}\) Their results also show a marked discrepancy between theory and data. Hence they used Schrage's theory\(^{8}\) to calculate condensing coefficients (assuming \(\sigma_e\) equals \(\sigma_c\)) from their data. Although a considerable amount of scatter with no apparent regularity attended their \(\sigma\)'s, they found that an average "condensing coefficient" of 0.45 correlated their data well. They also measured condensate film thicknesses by gamma attenuation at seven different heat flux levels from 35,000 to 143,000 Btu/hr ft\(^2\). The results plotted against film Reynolds
number indicated order-of-magnitude agreement with the hydrodynamic predictions of Nusselt's theory, showing that large deviations from Nusselt's theory apparently cannot be attributed to thicker than "normal" condensate films. Finally, because their heat transfer coefficients increased with increasing vapor pressure at constant heat flux, while Misra's results showed the opposite effect (i.e., a decrease in $h$ similar to that observed by Sukhatme when he purposely added noncondensible gases in two tests), they suggested that Misra's data suffered inaccuracies due to the added diffusional resistance of noncondensible gases. The average condensing coefficient they used, 0.45, is simply a correlating factor. By coincidence, it may equal the actual condensing coefficient of some of their tests. An important sideline of their tests is that in some of their runs the condenser wall temperature, and hence the liquid surface temperature, were low enough so that the theoretical rates of evaporation from the vapor-liquid interface were negligible compared with the rates of condensation. In such runs, the coefficients measured were actually the condensing coefficients as defined by Schrage's theory, regardless of the value of the evaporation coefficient.

Although the results of Gel'man\textsuperscript{48} (already mentioned) concerned dropwise rather than film condensation, it is interesting to note that in condensing mercury vapor on carbon steel tubes, he obtained heat transfer coefficients that increased with increasing vapor pressure over nearly the same pressure range as Misra's tests. Apparently $q/A$ was nearly constant for all these runs, for he found that $h$ is proportional to $(\Delta T)^{-1}$. He also added air to the vapor in bulk concentrations up to 12% and found that below 1% air in the mercury vapor the heat transfer performance is practically unaffected, while between 1 and 12 w/o air in the vapor the heat flux varied inversely as the air concentration raised to the 0.2 power. With and without air in the vapor, he found that $q/A$ is directly proportional to pressure raised to the $1/3$ power, and that the heat transfer coefficient $h$ is inversely proportional to $\Delta T$. Vapor velocities ranged from 3.3 to 32 m/sec. The coolant was water. Gel'man developed empirical equations for $h$ with and without noncondensible gases.

Condensing data from the latest General Electric\textsuperscript{187}, NASA\textsuperscript{188} and General Motors\textsuperscript{189} research programs are expected to be published in the near future.

References 3, 5, 6, 7, and 89 graphically compare condensing metals heat transfer data with Nusselt's theory. The data are scattered well below Nusselt's predictions.

d. **Cryogenic Fluids**

At low temperatures, it is interesting to examine some condensation data published recently on cryogenic fluids. Golovinski\textsuperscript{13} studied condensing air at pressures up to about 6 atm, and observed
dropwise condensation at the lower pressure and film condensation above about 4 atm. In all tests, the heat transfer coefficients and rates were higher than Nusselt's theory predicts, particularly for dropwise condensation as would have been expected from information on condensing steam and organic fluids. However, Timmerhaus et al.\(^{(14,15)}\) published data for condensing hydrogen at about 20°K (assumed to be filmwise) which gave heat transfer coefficients as much as five times lower than Nusselt's theory predicts - the greatest deviations being observed at the lower ΔT's. The range of ΔT's was about 0.3 to 4.7°F. They attributed the discrepancy between theory and data to rough solid-liquid and vapor-liquid interfaces and convection, as well as to possible errors in the lower ΔT's.

Although the observed hydrogen condensing heat transfer coefficients appear to be up to five times lower than the corresponding Nusselt theory values, they still follow Nusselt's theory qualitatively. The Prandtl number for liquid hydrogen at 20°K is about 0.7, so that a reason for the observed low coefficient is not evident. In earlier work Haselden and Prosad\(^{(16)}\) found good agreement between Nusselt's theory and condensation results for oxygen and nitrogen, and Schmidt\(^{(17)}\) obtained experimental heat transfer coefficients about twice as high as Nusselt's theoretical values for condensing carbon dioxide, depending upon the vapor velocity.

6. Theoretical Improvements

Within the past 15 years, improvements in Nusselt's film condensation theory have been made, utilizing in most cases the developments in boundary layer theory to remove the restrictive assumptions of Nusselt's theory.

a. Laminar Smooth Film Flow

(1) Plates and Outside Tubes, Steady State. Chu et al.\(^{(190)}\) analyzed the data from tests of several organic vapors condensing on a single horizontal tube by plotting overall thermal resistance of the tube versus resistance on the cooling water side, and by evaluating the condensing film resistance, and subsequently the film heat transfer coefficient, from the value of the ordinate intercept. If a value or function independent of ΔT can be assigned to the coolant side resistance, the need for measuring wall temperatures is eliminated. Film heat transfer coefficients calculated by this method agreed well with values predicted by Nusselt's theory. The authors attributed disagreements at higher heat fluxes to turbulence in the condensate film.

To account for heat rise in condenser coolant when the tube is long and the coolant is not boiling, Chari and Kulkarni\(^{(191)}\)
assumed that $\Delta T$ varies with tube length and so modified Nusselt's derivation. The resulting condensate film thickness varied linearly with tube length, but their analysis was incorrectly derived. Hassan(173) later performed a similar analysis for inclined flat plates and cylinders assuming various surface temperature distributions.

Peck and Reddie(192) studied condensation outside a horizontal tube, using a semiempirical equation to correlate their data and other available data, and concluded that heat transfer around the tube cannot account for deviations from Nusselt's theory. Their work was substantiated by Bromley et al.,(193) who used a different theoretical approach to the same problem, but obtained their data on a slowly rotating condenser tube with a thermocouple on the tube OD. Although Nusselt's equation was shown to be adequate, if the correct average $\Delta T$ is used for calculations, the tube wall temperature was observed to vary considerably from top to bottom. Thus film heat transfer coefficients based upon $\Delta T$'s measured at the top and bottom of a horizontal condensing tube could differ markedly. Jacowitz's(194) subsequent treatment of the foregoing data showed that Nusselt's theory of condensation on horizontal tubes is adequate if the correct integrated average $\Delta T$ is used.

Bromley(195) also derived a correction for Nusselt's theory which accounts for the contribution of sensible heat to the heat flow. His equation, satisfactory for values of $\Delta Tc_p/\lambda$ up to about 3.0, predicts a slight increase in heat transfer coefficient for fluids with finite heat capacity. The effect of a nonlinear temperature distribution upon the condensation heat transfer coefficient was also studied by Rohsenow(196) who performed an analysis similar to Bromley's but included the effect of cross-flow within the film. His result, in the first approximation, is a correction the same as Bromley derived.

Yang(197) reformulated Nusselt's model of condensation to include the effects of convection and acceleration. He employed the familiar similarity variables of boundary layer analysis to transform the conservation equations to ordinary differential equations, which he then solved by assuming series expansion of the similarity variables and comparing coefficients. The results indicated that Nusselt's theory is reasonably accurate for fluids having Prandtl number around unity or larger, but that heat transfer coefficients for fluids of low Prandtl number (liquid metals) could be one-half the Nusselt theory coefficients or less, the disagreement increasing with decreasing values of the parameter $\lambda/(\Delta Tc_p)$. This result, contrary to the analyses of Bromley and Rohsenow, indicates that convection in or subcooling of low Prandtl number condensate causes an increase in condensate film thickness with corresponding reduction in heat transfer coefficient. Yang(198) repeated his analysis using another method; he solved the integral equations of motion and energy for filmwise condensation with similar results, and later(199) extended the analysis to the turbulent flow region.
Sparrow and Gregg, not familiar with Yang's work in Communist China, presented "A Boundary Layer Treatment of Laminar Film Condensation," on a vertical surface, using similarity transformations to reduce the same differential equations for conservation of mass, momentum, and energy to ordinary differential equations, which were solved numerically. The results are essentially the same as Yang's, though in different form, showing up to 50% reduction in Nusselt number for fluids of low Prandtl number and large values of the parameter \( c_p \Delta T/\lambda \). Sparrow and Gregg repeated the analysis for a horizontal cylinder with similar results. Mabuchi obtained nearly the same results, using an approximate integral method based upon truncated series expansions of the temperature and velocity profiles.

Koh et al. generalized the condensation problem as far as current similarity techniques permitted and obtained numerical solutions of the transformed conservation equations for film condensation with a drag-induced vapor boundary layer. With constant wall temperature and saturated vapor assumed, the theory accounted for convection and acceleration in the liquid film and acceleration or drag in the vapor boundary layer due to interfacial shear. Although interfacial shear noticeably reduces the Nusselt's number, the effect for liquid metals is not significantly different from the case of zero interfacial shear to fit the available data.

Koh approached the same problem differently, using an integral method. He assumed polynomial expressions for velocity and temperature profiles to approximate algebraically the heat transfer results of laminar film condensation with a drag-induced, vapor-phase, boundary layer. The results compare favorably with the numerical results of Ref. 203.

Extending the condensation model further, Sparrow and Eckert applied similarity techniques to the condensation of superheated vapor with a drag-induced vapor boundary layer and obtained numerical results for Nusselt number as a function of superheat and temperature drop across the condensate film. The results again were similar to those of Sparrow and Gregg and of Koh et al., with a slight increase in heat flux predicted due to superheat for a given \( \Delta T \) across the film. Sparrow and Eckert also qualitatively examined the effect of noncondensable gases upon condensation, concluding that an accurate analysis of the effect of noncondensibles must include the effects of free convection. Then Sparrow and Lin analyzed these effects quantitatively for condensation of fluids of moderate to high Prandtl number with a known bulk concentration of noncondensible gas in the vapor to show that the presence of noncondensibles markedly reduce the condensation rate due to the diffusional resistance of the gas. However, they only applied their theory to a few tests on the steam-air system.
Chen\textsuperscript{(22)} also examined condensation on vertical flat plates, horizontal tubes, and horizontal tube bundles, using integral momentum and energy equations for the two-phase boundary layers. His results, obtained numerically with perturbation techniques, seem less restrictive than those using similarity techniques but compare favorably with the results of Sparrow and Gregg\textsuperscript{(200)} and of Koh.\textsuperscript{(203,204)} Yet none of these similar theories accurately described the available condensation data for metal vapors as Chen illustrated in a plot of Sparrow and Gregg's theory,\textsuperscript{(200)} his theory, and Misra's data\textsuperscript{(3)} (something the other theoreticians avoid doing).

The same results for the film condensation problem were found by Lee\textsuperscript{(205)} using analytical iteration to solve the set of three simultaneous integro-differential equations describing the velocity and temperature distributions in the condensate film and the condensate film profile. He concluded that the techniques used to solve the problem might be useful when similarity does not exist, such as in turbulent condensation.

\textbf{(2) Transient Condensation.} Transient laminar film condensation on a vertical, flat surface was studied theoretically by Sparrow and Siegel\textsuperscript{(206)} and by Chung.\textsuperscript{(207)} The former authors used the method of characteristics to obtain equations for film thickness and heat transfer coefficient as functions of their respective steady-state values and time, as well as the time required to reach steady state, for the case in which the plate temperature is suddenly dropped below the saturation temperature of the vapor. In their analysis, acceleration of the condensate film and a drag-induced vapor boundary layer are neglected.

Numerical solutions of the perturbation equations yielded the ratio of the instantaneous local heat transfer rate to the hypothetical steady-state rate. "The effect of unsteady wall temperature on heat transfer was found to increase as the Prandtl number, \(1/\Delta T\), and \(c_p/\lambda\) are increased," but in the liquid metal region (i.e., \(Pr \ll 1\)), the effect of unsteady wall temperature is negligible. The effect of unsteady gravity on heat transfer increased with Prandtl number at high Prandtl numbers, and with decreasing values of \(c_p\Delta T/\lambda\) and \(g\). While the effect of \(g\) upon condensing metals is significant, it is independent of \(Pr\) at low Prandtl numbers.

Knuth\textsuperscript{(208)} analyzed a problem more closely related to the preparation of fresh water from sea water than transient film condensation. He found temperature profiles for the case of a saturated vapor and a liquid phase, each at different uniform temperatures initially, brought into contact so that net condensation (or evaporation) occurs, the latent heat being absorbed (or supplied) by the condensed phase. His analysis is too idealized to be of any use.
b. **Forced Convection**

Cess\(^{209}\) analyzed laminar film condensation on a flat surface in the absence of body forces. The motion of the condensate film was caused solely by the shearing action of the saturated vapor flowing over the plate and the film. When the usual similarity transformations and polynomial approximations of the transformed variables are used, heat transfer results (Nusselt number) are found in terms of local vapor Reynolds number, Prandtl number, and \(c_p \Delta T/\lambda\), that are valid for values of \(c_p \Delta T/Pr\lambda\) less than 3 and \(c_p \Delta T/\lambda\) less than 12. This analysis in effect neglects acceleration and convection in the film, as well as interfacial disturbances such as waves, ripples, etc. No comparison with data is given, but it can also be seen from the results that this analysis, if applied to condensing metal vapors, is valid only for values of \(c_p \Delta T/\lambda\) less than three times the Prandtl number, i.e., for low values of \(c_p \Delta T/\lambda\). This requirement can probably be met in condenser-radiators.

Chung\(^{210}\) performed an analysis similar to that of Cess but included the effects of gravity, acceleration, and convection, utilizing perturbation techniques to obtain a solution in which the important parameters are \(\mu/\mu_0\), \(c_p \Delta T/Pr\lambda\), \(c_p \Delta T/\lambda\), and Froude number. He concluded that while the interaction of all the parameters must be considered to find the net effect upon heat transfer, the condensation process appears to enhance itself as a result of condensation-induced suction at the vapor-liquid interface which increases shear and, correspondingly, the liquid velocity, decreasing the film thickness at the same time.

Koh\(^{211}\) also analyzed forced-convection film condensation on a flat plate, neglecting body forces and thus eliminating the Froude number, but exactly considering the vapor tangential velocity at the interface. Chung\(^{210}\) only approximated the latter effect. A numerical solution was achieved after appropriate similarity transformations were made. Koh's results for liquid metals compare favorably with the results of Cess, but Koh concluded, contrary to Chung, that heat transfer does depend upon Prandtl number for high Pr fluids.

Two somewhat different means of condensate removal during film condensation were studied analytically.

Singer\(^{212}\) analyzed laminar film condensation in a magnetic field, using the previously mentioned boundary layer similarity techniques. His results predict several-fold increases in Nusselt number when the condensate flow is accelerated by a magnetic field.

Jain and Bankoff\(^{213}\) used the perturbation method employed by Chen\(^{22}\) to analyze laminar film condensation on a porous vertical wall with uniform suction velocity. As expected, the results indicated
an increase in heat transfer with an increase in suction velocity, particularly for fluids of higher Prandtl number. Frankel and Bankoff\(^{214}\) repeated the analysis for horizontal tubes.

While both of these methods of condensate removal might seem impractical at a first glance, they do merit consideration for zero-gravity field applications or possibly as research tools for studying mechanisms.

c. Combined Theories

The simplest form of Nusselt's theory was combined with Schrage's kinetic theory of condensation by various authors in attempts to correlate their condensing data. Misra and Bonilla\(^{3}\) could not explain the low-condensation heat transfer coefficients for mercury and sodium on the basis of a temperature jump alone.

Balekjian and Katz\(^{43}\) considered the case of superheated vapors, experimentally condensing Freon-114 and steam on horizontal tubes, with as much as 180°F of superheat. However, they measured tube wall temperatures at only a few points on a 3/4-in.-diam. tube, so that in the light of the work of Bromley et al.\(^{193}\) it is doubtful that they used the correct integrated average tube wall temperature in their calculations. Hence, their conclusion that "the effect of superheat in the absence of excessive splashing of the condensate is a lowering of the heat flux and condensing load and a depression of the condensate surface temperature below that of the saturated vapor" remains to be verified. Sparrow and Eckert's\(^{45}\) analysis predicted a slight increase in heat flux due to superheat, while the data reviewed and theory presented in the proceedings of a United Kingdom Conference\(^{44,215}\) on the condensation of superheated steam supported the idea that increasing steam superheat caused a reduction in condensate surface temperature and a corresponding reduction in heat transfer, in concord with Balekjian and Katz. The theory used in both references was based upon Schrage's kinetic theory of mass transfer, but Silver and Simpson\(^{44}\) pointed out an error in Balekjian and Katz's use of Schrage's equation for net condensation. The most comprehensive bibliography on condensing superheated vapors is that given by Chisholm.\(^{215}\)

Baer and McKelvey\(^{42}\) analyzed the overall heat transfer coefficient in terms of the separate resistances of the condensing wall, the condensate film, and the vapor-liquid interface. Using the Clausius-Clapeyron equation to eliminate pressures from the kinetic theory expression for mass flux, they obtained expressions in terms of measured \(\Delta T\)'s from which the various resistances were calculated. Thus correlating their condensing methanol data, they concluded that under conditions of low heat flux, low condensing coefficient, or low total pressure, the interfacial resistance must be considered. Their results and their conclusions that noncondensible gases greatly reduce the condensing
coefficient were challenged by Harriot(216) who proposed that the resistance offered by noncondensible gases could be explained entirely on the basis of diffusion. Harriot's argument is valid, because the diffusional resistance of a noncondensible gas is real, as Sparrow and Lin showed,(41) Baer and McKelvey, and again McKelvey,(217) used an approach which avoids diffusional considerations, and yet yields variable values of the "condensation coefficient" which can be shown to be partly dependent upon diffusion through the noncondensible gas (see Section II-B-8). Their results illustrate the decrease of the apparent condensation coefficient (based upon $\sigma_e = \sigma_C$) with increasing concentration of noncondensible gases. However, Baer and McKelvey's data have not been re-evaluated in terms of the diffusional theory of Sparrow and Lin, in order to permit estimation of the interfacial conditions and possibly re-evaluation of the condensing coefficients.

d. Rotating Condensation

(1) Rotating Cylinders. Data and a theory for condensation on a rotating cylinder were practically nonexistent until Singer(218,219) studied the problem and developed Nusselt-type theories which describe the two film-condensing regimes observed from steam, i.e., "a laminar, gravitational flow of the condensate at low rotational speeds, with decreasing heat transfer coefficients with increasing speeds"; and "an unstable, film-like condensation at moderate speeds, with increasing coefficients with increasing speed due to thinning of the condensate film by centrifugal force." A third regime of decreasing heat transfer coefficients with increasing speed was correlated with a plot of Nusselt group versus Weber number for Weber numbers greater than about $10^3$. Singer's theories included convection in the low-speed regime, and acceleration as well as convection in the moderate-speed regime. He attributed the decreasing heat transfer coefficient at high speeds to interfacial drag.

An experimental study of condensation of steam on a rotating cylinder is being continued by Hoyle(220) at the Imperial College, London.

(2) Rotating Disk. Sparrow and Gregg(46) used the available similarity transformations from the boundary layer theory of flow over rotating disks to extend Nusselt's theory, including the added effects of acceleration and convection to condensation of saturated vapor on a rotating, horizontal disk. The interesting and useful result was that condensate film thickness is uniform over the entire disk and inversely proportional to the square root of the speed of rotation. (This result is not surprising, however, when one notes that these transformations, used by the authors in a paper(221) on heat transfer from a rotating disk, yielded for fluids of low Prandtl number, heat transfer coefficients that are directly proportional to the square root of the speed of rotation.)
Hence the possibility exists of mechanically controlling the condensate film thickness for experimental purposes. When they extended their theory to include interfacial shear,\(^{222}\) they found the effects of shear to be negligible for condensates of moderate to high Prandtl number, but significantly decreasing (about 12\% decrease) the Nusselt number for a Prandtl number of 0.008. Unfortunately, they only calculated the reduction due to drag for the one low Prandtl number. In a similar manner, Sparrow and Hartnett\(^ {223}\) analyzed condensation on a rotating cone and found that the ratio of heat transfer coefficient for condensation on a cone to that on a disk equals the quantity \(\sin \frac{1}{2} \alpha\), where \(\alpha\) is the angle between the side of the cone and the axis of rotation.

Beretsky,\(^ {224,225}\) in analyses similar to that of Sparrow and Hartnett, considered disk-type thrust bearings, lubricated by condensible vapor which is fed to the bearings at their respective centers of rotation. His analysis, valid only for laminar flow, where viscous heat generation is negligible (i.e., in the region of the center of rotation), included the effect of a radial pressure gradient which is accountable when the Clausius-Clapeyron equation is used. Although Beretsky's results are useful criteria for designing externally pressurized thrust bearings, they are not as readily adaptable to studying condensation mechanisms as are those of Sparrow and Gregg for the simpler rotating system.

Nandapurkar and Beatty\(^ {47}\) were the first to take advantage of the control of condensate film thickness offered by rotating condensation theory in an experimental study of the condensation of methanol, ethanol, and Freon-113 vapors on a 5-in.-diam. rotating disk at speeds up to about 2400 rpm. Their results fell about 25\% below predicted Nusselt number groups. Vapor drag was suggested as a possible cause of the observed lower heat transfer coefficients, which were also observed to be proportional to the 0.45 power of rotational speed, rather than the 0.50 power predicted by the theory of Sparrow and Gregg. (Their calculated film thicknesses for all three fluids, over a speed range of 400 to 2400 rpm and a \(\Delta T\) range of about 20 to 80°F, varied only from about 0.75 to 3.4 mils.) They observed flow patterns by injecting dye onto the condensate surface. The flow patterns observed by Seely et al.,\(^ {226}\) after dye was injected onto the condensate, seemed to support the assumptions, which Sparrow and Gregg used in their theory, that radial flow dominates and that axial symmetry exists in condensate flow on a rotating disk. Ripples and waves that were visible on the condensate film could have increased the vapor drag. The theory did not account for the effect of waves.

Continued studies under the direction of Beatty\(^ {227}\) indicated "increased deviation between measured and theoretical heat transfer performance" as rotational speeds were increased from 4000 to 5000 rpm. He is also investigating the effects of noncondensible gases
and elevated pressures, and the effects of disk orientation with respect to
the gravity field upon condensate flow patterns.

e. **Turbulent Condensation**

Colburn\(^{(228)}\) apparently was the first to attempt a theoretical
analysis of turbulent condensation assuming a transition from laminar to
turbulent film flow at a Reynolds number of 1600. Using the simple
analogy between heat and momentum flow developed for conduits, and
assuming the main resistance to heat transfer to be in the viscous sub-
layer near the surface, he derived an expression for heat transfer co-
efficient at high Reynolds numbers, which yielded lower heat transfer
coefficients than observed experimentally for condensing steam. Steam
condensate has a Prandtl number near unity, whereas Colburn's curve
for Prandtl number 5 fit the steam data more closely. He concluded that
the transition Reynolds number is uncertain and may be much less than
the assumed value due to shear at the vapor-liquid interface. Carpenter
and Colburn\(^{(229)}\) used the equation for a gas flowing in pipes to calculate
shear stress at the vapor-liquid interface in terms of a friction factor
and to show that increasing shear increases the heat transfer coefficient
for cocurrent flow. Specification of this friction factor, however, is
difficult.

The later papers on turbulent condensation were written
primarily on the concept that turbulent momentum and heat transfer may
be represented as diffusion processes by the familiar equations,

\[
\frac{\tau}{\rho} = -(\nu + \epsilon_M) \frac{\partial u}{\partial x} \tag{80}
\]

and

\[
\frac{q}{A\rho c_p} = -(\alpha + \epsilon_H) \frac{\partial T}{\partial x} \tag{81}
\]

where \(\epsilon_M\) and \(\epsilon_H\) are the eddy diffusivities of momentum and heat, re-
spectively. Moreover, \(\epsilon_M\) and \(\epsilon_H\) were usually taken in these papers to
be identical, but this assumption is not necessarily valid; the ratio
\(\epsilon_M/\epsilon_H\) is a position-dependent variable. Finally, all these analyses
neglected convection and acceleration in the condensate film and amounted
to Nusselt's analysis extended to turbulent flow.

Seban\(^{(230)}\) used the Prandtl-Nikuradse velocity distribution
for flow in smooth pipes to evaluate the eddy diffusivities and integrate
the momentum and energy equations. He also used the same transition
Reynolds number as did Colburn, with results similar to Colburn's.
Seban's Nusselt numbers of low Prandtl number are not drastically
different from Nusselt's theory, because molecular heat diffusivity of liquid metals is much larger than the eddy diffusivity, except at very high Reynolds numbers.

Rohsenow et al.\(^{(231)}\) extended the turbulent analysis to include the effects of vapor drag. They derived a Reynolds number for transition from laminar to turbulent film flow which included the effect of interfacial shear. In the turbulent region, they used three generalized velocity distributions for the laminar sublayer, buffer zone, and turbulent outer layer to describe the eddy diffusivities, and subsequently to integrate the momentum and energy equations. Their results indicated that interfacial shear increases heat transfer, the effect being more pronounced at higher Prandtl numbers. However, use of the results is limited by inadequate knowledge of interfacial shear; i.e., the shear must be specified before the heat transfer coefficient can be calculated.

Kutateladze\(^{(159, \text{pp.} 23-31)}\) performed an analysis similar to that of Seban with the same results. However, his analysis is only for fluids of moderate to high Prandtl number since it neglects molecular diffusivity of heat in the turbulent outer layer. Kutateladze et al.\(^{(232)}\) later reworked the analysis including molecular heat conductivity in the turbulent region and obtained results similar to Seban's for fluids of low Prandtl number. Both these analyses predicted a reduction of up to 50% in heat transfer coefficients for condensate of low Prandtl number. The explanation was that the turbulent contribution to heat transport in liquid metals is practically negligible, while the turbulent contribution to momentum transport is large, causing an increased film thickness due to increased shear at the wall with a corresponding reduction in heat transfer.

Dukler\(^{(180)}\) essentially duplicated Seban's analysis with the exceptions that he used Deissler's expression for eddy diffusivity in the laminar sublayer,\(^{(233)}\) did not use the restrictive assumption of a particular transition Reynolds number, and neglected the molecular heat conductivity in the turbulent region. While his theory permits a gradual transition to turbulence, it cannot be applied to fluids of low Prandtl number because the neglect of molecular heat conductivity in the turbulent region is fundamentally incorrect. Calculations based upon Dukler's results indicate that the turbulent contribution to heat transport is but a few per cent of molecular heat transport for metals. Moreover, Dukler's results for finite interfacial shear are difficult to use, not only because of lack of knowledge about interfacial shear, but also because of the unusual parametric groupings he employed.

Lee\(^{(234)}\) reworked Dukler's turbulent condensation theory, showing that it gives results practically identical to Seban's if molecular heat conductivity is included throughout the condensate film. A subsequent
paper of Lee's(25) considered turbulent condensation, including the effects of convection and acceleration, with results similar to his earlier paper. A qualitative analysis of vapor shear led Lee to conclude that the observed discrepancies in condensing metal vapor data may be due in part to the effect of vapor velocity, causing film thickening. Sukhatme and Rohsenow's(7) results indicate that the latter conclusion is incorrect in the 0.02 to 0.33-psia pressure range in which he measured film thicknesses that agreed approximately with Nusselt's predictions of \( \delta \) versus Re.

f. **Turbulent Flow of Liquid Metals and Eddy Diffusivities**

It is not the purpose here to cover turbulent flow of liquid metals in detail, but to represent briefly the current status of usable turbulent flow theory to illustrate that turbulent condensation theories such as Lee's(234) seem adequate insofar as the physical model of the theory applies.

Empirical correlations are available for turbulent heat transfer(28,235-239) in various types of channels. Theoretical predictions of heat transfer have been based primarily upon various analogies that assume the equivalence of turbulent momentum and heat transport and use the generalized velocity distributions developed for "two-layer" turbulent flow,(29,235,240,241) or for the "three-layer" turbulent flow(242-244) (i.e., including a buffer layer as well as the laminar sublayer and the turbulent core).

Loitsiansky(244) reviewed the development of turbulent heat transfer theory, then assumed "three-layer" flow with logarithmic velocity profiles and \( \varepsilon_H/\varepsilon_M \) equal to unity, to derive a general expression for turbulent heat transfer in liquid metals, which appears less useful than other specific expressions. Other authors tried to remove the physical inconsistencies produced by describing turbulent flow in terms of logarithmic velocity distributions of the Prandtl-Nikuradse type. Gill and Scher(245) arbitrarily modified Prandtl's mixing-length expression to derive a velocity distribution that fits the data well throughout the tube and thus describes the transition from laminar flow as well as turbulent flow. Gill and Lee(246) illustrated the use of this velocity distribution in calculating turbulent heat transfer after \( \varepsilon_H/\varepsilon_M \) was assumed to be unity. Lee(247) solved the momentum equation, including both molecular viscosity and von Karman's eddy viscosity, to show that when \( \mu \) is retained, the velocity derivatives can be matched at the turbulent boundaries, and that the velocity gradient drops to zero at the center of the pipe. He applied this solution to turbulent film flow with various values of positive and negative interfacial shear,(248) and then to turbulent condensation with zero interfacial shear and \( \varepsilon_H/\varepsilon_M \) equal to unity.(234)
The results of several experiments indicated that the ratio $\varepsilon_H/\varepsilon_M$ is not unity but a function of distance from the wall, as well as Reynolds number ($29,249-253$) and Prandtl number. A few semitheoretical attempts were made to predict the eddy diffusivity ratio ($251,252,254$) with varied success. The methods given by Kutateladze et al. ($255$) and Dwyer ($249,250$) appear to be the most useful for the average diffusivity ratio. A recent review ($257$) of methods for calculating eddy diffusivities showed good agreement between correlations for eddy diffusivities of mercury and air. Average values of $\varepsilon_H/\varepsilon_M$ published in the literature varied from as low as 0.2 at low Reynolds numbers to as high as about 1.6 at higher Reynolds numbers for Prandtl numbers around 0.02 ($251,258-260$) and ratios of less than 0.5 at Re around $10^4$, to about 1.2 at Re around $2.5 \times 10^5$ for Prandtl numbers from 0.02 to 10 ($29,253$). The lower Prandtl numbers appear to yield lower values of $\varepsilon_H/\varepsilon_M$ at low Reynolds numbers.

The effect of the ratio $\varepsilon_H/\varepsilon_M$ upon turbulent heat transfer in liquid metals was illustrated in several papers ($250,261-265$). By its very nature, decreasing $\varepsilon_H/\varepsilon_M$ decreases heat transfer, the effect becoming more pronounced as Peclet number increases. But in thin condensate films, because $k \gg \varepsilon_H$, an approximate average value of $\varepsilon_H/\varepsilon_M$ is adequate for calculations, as discussed in Section e above.

g. Remarks on Turbulent Theory

When the various problems in calculating turbulent heat transfer in liquid metals are considered, it seems evident that the development of more of the same turbulent condensation theories is fruitless and will not provide an explanation of the low-condensing heat transfer coefficients for liquid metals. Experimental heat transfer coefficients were obtained at relatively low Reynolds numbers, at which the ratio $\varepsilon_H/\varepsilon_M$ is expected to be less than unity. In the first approximation, interfacial shear may be neglected at these low Reynolds numbers so that the theories of Lee ($234$) and Seban ($230$) seem adequate. At higher Reynolds numbers, at which vapor drag would be significant, convection and acceleration could not be neglected in any analysis. If axial convection is important, then, it seems that axial conduction cannot be arbitrarily neglected.

Moreover, if vapor drag is important in turbulent flow, an accurate description of interfacial shear demands accurate knowledge of the factors determining the liquid surface structure (i.e., the formation and motion of waves and ripples, etc.).

Although some work has been done on wave flow, interfacial shear in wave flow is poorly understood today.
7. Falling Films

Kapitza\(^{266}\) found an approximate solution for the flow of thin layers of a viscous liquid under the influence of a constant bulk force, taking surface tension into account. He showed that the wave regime studied by a number of authors\(^{267-269}\) is more stable than the laminar state and predicted the form of the wave profile, the phase velocity, and the amplitude. Thus the average film thickness for water in wave flow is about 20% less than that for smooth laminar flow, making the wave flow heat transfer coefficient about 25% greater than that for smooth laminar flow.

Bushmanov\(^{270}\) extended Kapitza's analysis, introducing small perturbations in the flow, to investigate the stability of laminar flow preceding wave flow.

Dukler proposed a turbulent-falling-film theory\(^{180}\) to describe some available film thickness data of nonmetal fluids reasonably well.\(^{271}\) Wrobel and McManus\(^{272}\) analytically studied the case of a turbulent gas flowing cocurrently with a liquid film, likening the film wave profile to pipe roughness in single-phase flow, in order to arrive at a two-phase friction factor. The film thicknesses predicted by their theory "agree with the available experimental data only in a limited range." Chien and Ibele\(^{273}\) used an electrical micrometer probe to study liquid film thickness and presented correlations for pressure drop, mean film thickness, and flow pattern, when the two-phase flow rates were known. Dukler and Wicks\(^{274}\) made an extensive survey of research on film thickness and interfacial structure giving the experimental conditions and film measurement techniques used.

Binnie\(^{275}\) found experimentally that the onset of wave formation on a vertically flowing water film occurred at a film Reynolds number of about 4.4. This result was fortified by a theoretical analysis of Benjamin\(^{276}\) who showed that stable wave flow can occur at low Reynolds numbers.

Wilkes and Nedderman\(^{26}\) determined the velocity profiles in falling-liquid films by stereoscopic photography of small air bubbles moving with the fluid and concluded that Nusselt's laminar velocity profile for film flow is reasonably accurate (i.e., that the velocity profile is nearly parabolic at low Reynolds numbers). The same conclusion was not reached by Portalski,\(^{27}\) who, after critically reviewing previous film thickness theories and methods, performed numerous hold-up measurements using 13 different liquids flowing down a smooth vertical plate. He found that the film thickness theory based upon an assumed universal velocity profile fit the data more accurately than Nusselt's theory. He observed that Kapitza's theory was somewhat more accurate than Nusselt's theory, and suggested
how Kapitza's theory can explain the generation of circulating eddies in vertical falling films.(277)

Sukhatme and Rohsenow(7) measured average condensate film thickness for mercury vapor condensing on a vertical nickel tube over the heat flux range of 35,000 to 143,000 Btu/hr ft², using a gamma attenuation method and a Co⁵⁷ source. Although the accuracy of the results cannot be checked, because an error analysis was not given, the measured film thicknesses were within order-of-magnitude agreement (a few per cent to about 100% higher than Nusselt's values) with Nusselt's hydrodynamic predictions. This indicated that Nusselt's theory approximately describes vertically falling mercury condensate film flow under the studied test conditions, and that the deviations of heat transfer rates from classical theory cannot be ascribed to unexpectedly thick condensate films. More such measurements are needed for condensing metals over a wide range of conditions.

Gido and Koestel(278) reviewed wetting film stability in an attempt to predict film break-up or slugging in condensation of such fluids as mercury in tubes, as an alternate attack on the problem of condensation pressure drop and flow patterns. Conclusive results of their work are not yet available.

8. Contact Resistance

Another problem which has arisen from liquid metal forced convection heat transfer studies is the "thermal contact resistance" at the solid-liquid interface. This resistance is generally believed to be caused by solid-surface contamination and/or by entrained gases in the liquid.(28-33) Some work indicates that contact resistance decreases with increasing Reynolds number(28,29) and also with time(30) until it disappears after prolonged operation, apparently as the liquid "cleans" the walls and/or is purified by trapping and separation. Other work indicates that precautions taken to ensure surface "cleanliness" and the absence of entrained gases will eliminate the contact resistance(32-35) or reduce it to an insignificant value whether or not wetting is achieved. Quittenton(33) reviewed and analyzed earlier work on liquid metal heat transfer and concluded that "nonwetting of liquid metals on metal transfer surfaces imposes a significant electrical resistance but an insignificant thermal resistance under nonboiling conditions." He suggested that electrical resistance or electromagnetic flowmeter output be used to check for wetting or nonwetting conditions. Addition of small amounts of sodium or magnesium, and titanium to mercury enhances wetting and oxide removal,(98) but these components are not sufficiently volatile to be present in condensing vapor in significant amounts. However, the foregoing evidence indicates that under prolonged operation of liquid metal systems there should be no significant thermal contact resistance at the solid-liquid interface after
the liquid has "cleaned" the surface. If the condensing metal wets the surface, nearly complete desorption of surface gases, and thereby excellent thermal contact, would apparently be guaranteed. This means then that short-term condensing tests in which the system is frequently opened for cleaning, inspection, maintenance, etc., might be initially subject to some contamination by gas, cleaning fluids, or other impurities. This threatens the possibility of a contact resistance, which is extremely difficult to evaluate unless the experiments being conducted have been specifically designed for such a purpose. Usually the latter means obtaining the apparent temperature discontinuity at the solid-liquid contact surface by measuring solid and fluid temperature profiles. This method appears wholly unsuitable to condensation studies, but the design of a magnetic flowmeter type probe, as Quittenton suggested, might be feasible.

9. **Inside Horizontal Tubes**

Condensation inside horizontal and slightly inclined tubes is somewhat more complicated than condensation outside tubes or inside vertical tubes because of the tendency of condensate to fill the bottom of the tube at low vapor flow rates. According to Jakob,\(^{9}\) however, if this effect is neglected, Nusselt's theory of condensation on horizontal tubes should apply.

Chato,\(^{279}\) in a theoretical analysis employing the integral momentum and energy equations and perturbation techniques developed by Chen,\(^{22}\) included the resistance to heat transfer through the bottom condensate layer to show that less than 2.5% of the heat is transferred through this layer, so that it may be neglected. The results predicted a significant decrease in heat transfer for metal condensates. The results of his Refrigerant-113 condensing studies indicated that Nusselt's theory does adequately describe laminar film condensation of vapors whose condensates have Prandtl numbers of the order of one or greater, that a slight downward slope of the tube significantly increases the overall heat transfer, and that for a given set of conditions "there is an optimum downward slope for which the heat transfer coefficient is a maximum."

A number of experimental and semitheoretical investigations of condensation inside horizontal tubes have provided data and correlations for the various flow regimes encountered. Akers and Rosson\(^{280}\) condensed methanol and Freon and found that in turbulent annular flow of the condensate "the condensing coefficient increases with increased liquid loading and is independent of the temperature difference." Similar results were found by Akers et al.\(^{281}\) who condensed Freon-12 and propane and used an equivalent liquid mass velocity to correlate the data on a single-phase basis. Altman et al.\(^{282}\) condensed Refrigerant-22, correlating nonsuperheated data by the method of Carpenter and Colburn\(^{229}\) and combining the analyses of Seban\(^{230}\) and Rohsenow\(^{231}\)
with the two-phase pressure drop method of Martinelli and Nelson\textsuperscript{(283)} to correlate their data in the high-vapor-shear cases. Myers and Rosson\textsuperscript{(284)} found the heat transfer coefficients for condensing methanol to be independent of vapor velocity and dependent on condensate flow rate. They developed correlations for condensation with the condensate running in the bottom of the tube, and also with the condensate flowing in an annular ring along the tube wall. Gorodinskaya\textsuperscript{(285)} published an empirical equation for Nusselt number, correlating the data of many authors for condensation inside horizontal tubes. Konsetov\textsuperscript{(286)} later published a correlation for condensing steam inside horizontal tubes, and concluded that angles of inclination up to 15 to 20 degrees can be neglected for high vapor velocities. Ananiev \textit{et al.}\textsuperscript{(287)} extended Reynolds analogy to condensate flow and found an expression relating local heat transfer coefficient to the heat transfer coefficient at the tube outlet given by single-phase flow correlations. This method seemed to correlate the data fairly well.

Work on mercury condensing inside tubes in space power cycles\textsuperscript{(91-93)} has renewed interest in the Lockhart and Martinelli\textsuperscript{(288)} two-phase pressure-drop methods applied to predicting wetting and non-wetting condensing pressure drops. Kutateladze's\textsuperscript{(159)} method was used to predict pressure drop in the SNAP 8 radiator-condenser.\textsuperscript{(93)} Hays\textsuperscript{(93)} and his co-workers compared the correlation methods of Lockhart and Martinelli,\textsuperscript{(288)} Baroczy and Sanders,\textsuperscript{(289)} and Kutateladze\textsuperscript{(159)} and found Lockhart and Martinelli's correlation offered the best agreement with test data when vapor inlet quality was greater than 60% and fractional condensation was greater than 30%. Marked disagreement between data and predictions was observed for lower inlet qualities, and motion pictures revealed flow reversals and interface disturbances due to impinging liquid droplets during lower-quality tests. A fog-flow model, including the effects of entrainment of condensate droplets in the vapor and drag on the droplets, was attempted by Gido and Koestel,\textsuperscript{(92)} who were also concerned with wetting film stability and break-up.

Recent results of the latter program\textsuperscript{(94)} yielded condensation pressure drop data "at flow rates of 1.05 to 3 lb/min at inlet pressures of 8 to 30 psia, and at condensing lengths of 4 to 8 feet" in both constant-diameter and tapered tubes. Wetting and nonwetting condensing flow patterns inside the stainless steel and Haynes 25 tubes were observed through a fluoroscope. The Lockhart-Martinelli two-phase pressure drop correlation appeared to be inadequate, especially at low qualities. The data given are insufficient to permit condensing heat transfer coefficients to be estimated.

Interest in space power systems has also produced research on mercury jet condensers.\textsuperscript{(93)} The most significant result was the "demonstration of the high values of pressure rise possible in convergent-divergent jet condensers . . . through the conversion of vapor thermal energy to mechanical energy in the outlet liquid," so that the condensers
also performed as vapor-driven circulation pumps. These researchers did not expect wetting to be an important stability or pressure rise factor, but their tests were not broad enough in scope to permit application of the results to alkali metal systems with confidence. Mercury condensing data were presented\(^{(49)}\) graphically, illustrating the jet condenser principles and performance, but the data apparently did not permit interfacial mass transfer mechanisms to be evaluated. The author did deduce from the data that turbulent mixing in the jet is the main mode of heat transfer and that "as the vapor becomes rarefied the limiting resistance in the heat transfer process becomes the vapor flux attainable at the jet surface rather than the internal liquid heat transfer."

The foregoing results suggest that jet condensers, if properly designed, might provide some basic data for the evaluation of condensation mechanisms.

Lackey\(^{(290)}\) discussed the design and operation of a Venturi condenser for immiscible fluids.

A somewhat different approach to condensation in tubes, taken by Chiarulli and Dressler,\(^{(291)}\) assumed a stationary vapor liquid interface and developed Hugoniot curves for final water conditions when the inlet steam conditions and rate of heat removal were known. However, this analysis does not predict the heat transfer coefficients or the tube lengths required for condensation.

10. Conclusions

The discrepancies observed between condensing metals heat transfer data and the predictions of Nusselt's classical theory of film condensation appear to be valid. The various mathematical "improvements" of film condensation theory within the realm of boundary layer techniques and turbulent film flow theories have not significantly reduced the discrepancies. These improved theories and falling-film studies do indicate that Nusselt's theory adequately describes heat transfer across the film, if the liquid surface temperature, \(T_S\), is known. For film condensation of a pure fluid, a contact resistance at the condenser wall should be negligible. Thus there appears to be a vapor-liquid interfacial resistance to condensation, as Sukhatme concluded, which lowers \(T_S\) and reduces the thermal driving force for conducting the heat of condensation across the film. This resistance has been discussed in terms of a temperature jump and the kinetic theory of condensation and condensation coefficients, but the theory does not permit predicting the resistance from first principles. Moreover, the interpretation of \(\sigma_c\)'s calculated from condensing heat transfer data may be questioned. A two-dimensional theory of nucleation at a vapor-liquid interface is not evident, nor is the structure of the vapor-liquid interface completely understood. Dropwise condensation remains unsolved too, but considerations of adsorption phenomena might improve the understanding of this more complex mode of condensation.
Finally, condensing pressure-drop studies have not yielded information from which condensation mechanisms might be evaluated, but properly designed jet-condenser studies might provide a tool for studying interphase mass transfer.

Because vapor quality and nucleation at a vapor-liquid interface may affect the interphase transport rates, these are discussed briefly in Appendix B. The effect of variable surface temperature upon Nusselt's theory is also discussed. Finally, calculations are shown for the approximate concentration of mercury dimer in the equilibrium vapor, but the effect of dimerization upon the interfacial resistance is discussed only in terms of the kinetic theory of condensation in Section II-B-7.
APPENDIX B
Notes on Theory

1. Effect of Vapor Quality on Nusselt's Theory

If \( f \) is the vapor quality (i.e., the weight fraction of vapor in the vapor-liquid mixture), the amount of heat released per pound of condensate is \( \lambda f \), and Nusselt's theoretical heat flux becomes

\[
q_{\text{avg}} = \frac{4}{3} \left[ \frac{\lambda f \rho^2 g k^3 (\Delta T)^3}{4 \mu L} \right]^{1/4};
\]

or in terms of the heat transfer coefficient,

\[
h_{\text{avg}} = \frac{4}{3} \left[ \frac{\lambda f \rho^2 g k^3}{4 \mu L \Delta T} \right]^{1/4}
\]

The reduction in \( h \) and \( q \) with quality is insignificant except at relatively low qualities as can be seen in Table V.

| \( f \) | \( 1 \) | 0.9 | 0.75 | 0.5 | 0.25 | 0.10 | 0.05 | 0.02 | 0.01 |
| \( q/q_{\text{Nu}} \) | \( 1 \) | 0.97 | 0.93 | 0.84 | 0.71 | 0.56 | 0.47 | 0.38 | 0.32 |
| \( h/h_{\text{Nu}} \) | \( 1 \) | 0.97 | 0.93 | 0.84 | 0.71 | 0.56 | 0.47 | 0.38 | 0.32 |

A 90% reduction in \( h \) and \( q \) is reached only when \( f = 0.0001 \). When \( \delta \) begins to thicken appreciably, inertia and convection must also be considered.

The presence of quality tends to reduce the sonic velocity\(^{292,293}\) and hence can impose flow limitations. However, condensation vapor qualities in Misra's, Cohn's, Engelbrecht's, and Sukhatme's tests presumably were close to unity so that the condensation velocities were well below sonic velocity in these tests. The problem may be raised, however, that a large "temperature jump" or degree of vapor subcooling adjacent to the vapor-liquid interface may initiate fog formation and introduce substantial qualities and the sonic-flow or droplet-diffusion limitation. The latter complex problem has not been investigated analytically.

2. Variable Condenser Surface Temperature

Nusselt's assumption that \( T_w \) is constant over the length of the condenser may not be accurate, as has been shown, but improperly treated, by Chari and Kulkarni.\(^{191}\) If the condenser surface has a well-defined beginning, at this point \( (y = 0) \), the \( \Delta T \) between vapor and wall in the absence of a "temperature jump" should be zero. In other
words, \( T_W \) is a function of condenser length. It may then be more realistic to rework Nusselt's theory with the cold side of the condenser taken to have a constant temperature \( T_C \) while \( T_C \leq T_W \leq T_V \). Then,

\[
k_W(T_W - T_C)/t = k_L(T_V - T_W)/\delta,
\]

where \( k_W \) and \( t \) are wall conductivity and thickness, respectively. Applying the remaining conditions of Nusselt's theory and integrating, we find that

\[
\frac{\mu k_L k_W(T_V - T_C)y}{(\lambda \rho^2 g)} = \left(\frac{k_W \delta}{4} + \frac{k_L t}{3}\right)\delta^3.
\]

For liquid metals, we might assume that the term \( k_W \delta^4/4 \ll k_L t/3 \), so that the heat flux is given approximately by

\[
q/A \approx k_W(T_V - T_C)/t.
\]

This result is valid insofar as the Nusselt theory assumptions are valid. It does not include the effect of a "temperature jump" but includes a more reasonable dependence of \( T_W \) upon condenser length than Nusselt's original theory presumes. The value of this analysis would lie in relating point measurements of condenser surface temperature to other local temperatures or an average surface temperature.

3. Nucleation at a Flat Vapor-liquid Surface

The possibility of extending Frenkel's\(^{(122)}\) liquid-drop nucleation model was considered in an effort to develop a mechanism for nucleation at a flat vapor-liquid interface. It is known that net nucleation in bulk vapor requires a considerable subcooling below the initial saturation temperature (i.e., a supersaturation of the vapor). This might be considered analogous to a "temperature jump" at the vapor-liquid interface. However, nucleation theory is based upon the assumed presence of nuclei or small droplets which are in equilibrium or near-equilibrium with the supersaturated vapor and exert their relatively high vapor pressure as a result of the curvature of their surfaces. Considerations of equilibrium between a droplet and vapor show that the supersaturation ratio approaches unity as the droplet shape approaches a flat film (i.e., as the droplet radius becomes large). Thus a surface nucleation theory would require the reasonable description of nuclei at or near the vapor-liquid interface. Such a description remains to be developed, if it is at all realistic. Moreover, growth of nuclei is usually described in terms of the kinetic theory of condensation, the shortcomings of which are listed in Section V-A above.
4. **Estimation of Composition of Mercury Vapor**

There is evidence\(^{(82-84)}\) that mercury vapor dimerizes to some extent as follows:

\[
2 \text{Hg} \rightleftharpoons \text{Hg}_2 \text{ (exothermic);} \tag{87}
\]

\[
P = P_{\text{Hg}} + P_{\text{Hg}_2}. \tag{88}
\]

If \(n\) is the total number of moles of \(\text{Hg}_2\) formed from one mole of mercury, the mole fractions are given by

\[
x_{\text{Hg}_2} = \frac{n}{1 - n}; \tag{89}
\]

\[
x_{\text{Hg}} = \frac{1 - 2n}{1 - n}; \tag{90}
\]

and

\[
K_p = \frac{P_{\text{Hg}_2}}{\frac{P_{\text{Hg}}^2}{P^2}} = \frac{xP}{(1 - x)^2P^2} = \frac{x}{(1 - x)^2P}. \tag{91}
\]

According to Kelley,\(^{(84)}\) mercury vapor is approximately 7\% \(\text{Hg}_2\) molecules at the normal boiling point, i.e., at 675°F (1135°R) and 1 atm. Therefore,

\[
K_{p_{675^\circ F}} \approx \frac{0.07}{(1 - 0.07)^2 (1 \text{ atm})} = 0.081 \text{ atm}^{-1}. \tag{92}
\]

The effect of \(T\) upon \(K_p\) is given by

\[
\frac{\text{d} \ln K_p}{\text{d} T} = \frac{\Delta H}{RT^2}. \tag{93}
\]

If the effect of \(T\) upon \(\Delta H\) is neglected, \(K_p\) is given approximately by

\[
K_p \approx K_{p_{675^\circ F}} \exp \left\{ \frac{\Delta H}{R} \left( \frac{1}{T} - \frac{1}{1135} \right) \right\}. \tag{94}
\]

From measured values of the heat of dimerization given in Reference 294, an average value of \(\Delta H \approx -1.5 \text{ kcal/gmole} \text{ Hg}_2\) was taken.

Solving the foregoing equations for \(x\) results in

\[
x = \pm \frac{1}{2} \left( \frac{1 + 2PK_p}{PK_p} \right)^{\frac{1}{2}} \sqrt{\left( \frac{1 + 2PK_p}{PK_p} \right)^2 - 4}, \tag{95}
\]
where

\[
K_p = 5.5 \times 10^{-3} \text{ psia}^{-1} \exp \left\{ +1.20 \frac{1135 - T}{T} \right\}.
\]  \hspace{1cm} (96)

Parameters for dimeric mercury vapor, estimated from Equations (95) and (96), are listed in Table VI.

**Table VI**

**EQUILIBRIUM PARAMETERS FOR DIMERIZED MERCURY VAPOR**

<table>
<thead>
<tr>
<th>T, °F</th>
<th>T, °R</th>
<th>P, psia</th>
<th>(KP), psia(^{-1}) (\times 10^3)</th>
<th>PK(P), (\times 10^3)</th>
<th>2PK(P), (\times 10^3)</th>
<th>x</th>
</tr>
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<tr>
<td>440</td>
<td>900</td>
<td>0.766</td>
<td>7.5</td>
<td>5.8</td>
<td>11.6</td>
<td>0.0057</td>
</tr>
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<td>540</td>
<td>1000</td>
<td>3.23</td>
<td>6.5</td>
<td>21.0</td>
<td>42.0</td>
<td>0.020</td>
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<td>640</td>
<td>1100</td>
<td>10.4</td>
<td>5.5</td>
<td>57.2</td>
<td>114.4</td>
<td>0.070</td>
</tr>
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<td>740</td>
<td>1200</td>
<td>27.4</td>
<td>5.0</td>
<td>137</td>
<td>274</td>
<td>0.11</td>
</tr>
<tr>
<td>840</td>
<td>1300</td>
<td>62.0</td>
<td>4.6</td>
<td>285</td>
<td>590</td>
<td>0.19</td>
</tr>
<tr>
<td>940</td>
<td>1400</td>
<td>124</td>
<td>4.4</td>
<td>546</td>
<td>1092</td>
<td>0.28</td>
</tr>
<tr>
<td>1040</td>
<td>1500</td>
<td>226</td>
<td>4.1</td>
<td>927</td>
<td>1854</td>
<td>0.37</td>
</tr>
<tr>
<td>1140</td>
<td>1600</td>
<td>379</td>
<td>3.8</td>
<td>1440</td>
<td>2880</td>
<td>0.45</td>
</tr>
<tr>
<td>1240</td>
<td>1700</td>
<td>598</td>
<td>3.6</td>
<td>2150</td>
<td>4300</td>
<td>0.51</td>
</tr>
<tr>
<td>1340</td>
<td>1800</td>
<td>895</td>
<td>3.5</td>
<td>3130</td>
<td>6260</td>
<td>0.57</td>
</tr>
<tr>
<td>1440</td>
<td>1900</td>
<td>1280</td>
<td>3.4</td>
<td>4350</td>
<td>8700</td>
<td>0.62</td>
</tr>
<tr>
<td>1540</td>
<td>2000</td>
<td>1760</td>
<td>3.3</td>
<td>5810</td>
<td>11620</td>
<td>0.66</td>
</tr>
</tbody>
</table>

Gaydon\(^{294}\) discussed the aspects of atomic and molecular spectra from which molecular structure of the vapor can be deduced. Hicks\(^{87}\) questioned that polymerization occurs in mercury and alkali metal vapors, because these exhibit relatively continuous energy spectra with inter-atomic distances varying continuously.
APPENDIX C

Additional Figures and Tables

To facilitate simple comparisons of the vapor properties of the liquid metals discussed in this report and to permit rapid estimations of various quantities according to the kinetic theory of gases, the following are shown graphically: vapor pressure data (Figs. 39 and 40); equilibrium mass fluxes versus temperature, as predicted by the kinetic theory of gases (Figs. 41-44); the \( \Gamma \) "correction" factors used in predicting the incident fluxes of mass, momentum, and energy from the kinetic theory of condensation (Fig. 45); and Nusselt's theory predictions of average heat fluxes, water, and diphenyl included to show that the major thermal resistance for condensing nonmetals lies in the condensate film, \( 1/k_L \) (Figs. 46 and 47).

Fig. 39. Comparison of Vapor Pressures of Alkali Metals, Mercury, \(^{86}\) and Cadmium \(^{90}\)

Fig. 40. Saturated Vapor Pressure of Mercury at Moderate Temperatures (Ref. 295, pp. 696-697)
Fig. 41. Absolute Mass Flux of Saturated, "Monomeric," Alkali Metal Vapors, Based on Equation (1)

Fig. 42

Absolute Mass Flux of Saturated, "Monomeric," Lithium Vapor, Based on Equation (1)
Fig. 43. Absolute Mass Flux of Saturated, "Monomeric," Mercury Vapor at Moderate Temperatures, Based on Equation (1)

Fig. 44. Absolute Mass Flux of Saturated "Monomeric" Vapors, Based on Equation (1). (a) Mercury and (b) Cadmium

Fig. 45. Γ Correction Factors for Kinetic Theory of Condensation Incident Mass, Momentum, and Energy Fluxes during Net Condensation
Fig. 46. Nusselt's Theory Predictions of Average Heat Flux for Film Condensation of Alkali Metal Vapors on a Vertical Surface 1 ft Long: (a) Lithium and Potassium and (b) Sodium and Rubidium

Fig. 47. Nusselt's Theory Predictions of Average Heat Flux for Film Condensation of Vapors on a Vertical Surface 1 ft Long: (a) Mercury and Cadmium and (b) Water and Diphenyl
Table VII lists sonic velocities of saturated mercury vapor, taken to be an ideal gas. Tables VIII and IX contain the reduced data, temperatures, and condensation coefficients, from the tests of Misra\(^{(89)}\) and Sukhatme and Rohsenow,\(^{(7)}\) respectively. The calculation of these coefficients from the data, using Equation (2), is straightforward.

**Table VII**

<table>
<thead>
<tr>
<th>(T_V, ^\circ R)</th>
<th>(T_V, ^\circ F)</th>
<th>(\bar{u}, \text{ft/sec})</th>
<th>(\bar{u} \times 10^{-6}, \text{ft/hr})</th>
<th>(\rho_V, \text{lb/ft}^3)</th>
<th>(G_{\text{max}}, \text{lb/hr ft}^2)</th>
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<tr>
<td>1200</td>
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<td>695</td>
<td>2.50</td>
<td>0.431</td>
<td>1,080,000</td>
</tr>
<tr>
<td>1150</td>
<td>690</td>
<td>680</td>
<td>2.45</td>
<td>0.282</td>
<td>690,000</td>
</tr>
<tr>
<td>1100</td>
<td>640</td>
<td>665</td>
<td>2.39</td>
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</tr>
<tr>
<td>1050</td>
<td>590</td>
<td>650</td>
<td>2.34</td>
<td>0.110</td>
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</tr>
<tr>
<td>1000</td>
<td>540</td>
<td>634</td>
<td>2.28</td>
<td>0.0614</td>
<td>140,000</td>
</tr>
<tr>
<td>950</td>
<td>490</td>
<td>618</td>
<td>2.22</td>
<td>0.0327</td>
<td>72,600</td>
</tr>
<tr>
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<td>602</td>
<td>2.17</td>
<td>0.0163</td>
<td>35,400</td>
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<td>585</td>
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<td>0.0071</td>
<td>15,000</td>
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<td>1.98</td>
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</tr>
<tr>
<td>700</td>
<td>240</td>
<td>531</td>
<td>1.91</td>
<td>0.00328</td>
<td>627</td>
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### Table VIII

**MISRA'S(89) MERCURY FILM CONDENSATION DATA, REDUCED**

<table>
<thead>
<tr>
<th>DATE OF</th>
<th>$P_v$</th>
<th>$T_v$</th>
<th>$T_w$</th>
<th>$\Delta T$</th>
<th>$q/A$</th>
<th>$T_s$</th>
<th>$\frac{T_v-T_w}{T_v}$</th>
<th>$\sigma_c$</th>
<th>$\sigma_c = \sigma_e$</th>
<th>$\sigma_c^a$</th>
<th>$\sigma_c$</th>
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<tr>
<td>RUN</td>
<td>psia</td>
<td>°F</td>
<td>°F</td>
<td>°F</td>
<td>Btu/hr ft²</td>
<td>°F</td>
<td>(α_e = 1)</td>
<td>MINIMUM</td>
<td>ZWICK</td>
<td>0.020</td>
<td>0.018</td>
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<td>12-1-52</td>
<td>14.7</td>
<td>675.5</td>
<td>415</td>
<td>260.5</td>
<td>404,700</td>
<td>425</td>
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<td>0.061</td>
<td>0.017</td>
<td>0.0165</td>
<td>0.020</td>
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<tr>
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<td>375</td>
<td>299.8</td>
<td>482,100</td>
<td>388</td>
<td>0.264</td>
<td>0.044</td>
<td>0.020</td>
<td>0.0197</td>
<td>0.023</td>
</tr>
<tr>
<td>1-7-53</td>
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<td>399.4</td>
<td>275.6</td>
<td>620,800</td>
<td>415</td>
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<td>0.063</td>
<td>0.026</td>
<td>0.0252</td>
<td>0.030</td>
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<tr>
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<td>14.7</td>
<td>675.0</td>
<td>382.5</td>
<td>292.5</td>
<td>508,000</td>
<td>396</td>
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<td>0.021</td>
<td>0.0207</td>
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<td>434</td>
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<td>0.025</td>
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<tr>
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<td>675</td>
<td>404.4</td>
<td>270.6</td>
<td>648,900</td>
<td>422</td>
<td>0.238</td>
<td>0.069</td>
<td>0.028</td>
<td>0.0264</td>
<td>0.032</td>
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<td>5.3</td>
<td>580</td>
<td>365</td>
<td>215</td>
<td>412,000</td>
<td>375</td>
<td>0.207</td>
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<td>0.049</td>
<td>0.0462</td>
<td>0.056</td>
</tr>
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<td>551.8</td>
<td>350.3</td>
<td>201.5</td>
<td>336,700</td>
<td>357</td>
<td>0.199</td>
<td>0.11</td>
<td>0.055</td>
<td>0.0516</td>
<td>0.063</td>
</tr>
<tr>
<td>c</td>
<td>14.7</td>
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a BASED UPON $\sigma_e \cdot G_s = 0$

b NON-CONDENSIBLE GASES CONTINUOUSLY REMOVED

c AVERAGE VALUES FOR DATES (3-24, 27, & 30-53)
Table VIII (Contd.)

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<th>T&lt;sub&gt;w&lt;/sub&gt; F</th>
<th>ΔT °F</th>
<th>q/A Btu/hr ft&lt;sup&gt;2&lt;/sup&gt;</th>
<th>T&lt;sub&gt;s&lt;/sub&gt; °F</th>
<th>T&lt;sub&gt;v&lt;/sub&gt;-T&lt;sub&gt;v&lt;/sub&gt;</th>
<th>(σ_c)</th>
<th>(σ_c = σ_e)</th>
<th>(σ_c^a) MINIMUM</th>
<th>(σ_c) ZWICK&lt;sup&gt;11&lt;/sup&gt;</th>
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<sup>a</sup> BASED UPON \(σ_e G_s = 0\)

<sup>b</sup> NON-CONDENSIBLE GASES CONTINUOUSLY REMOVED
Table IX

SUKHATME'S(7) MERCURY FILM CONDENSATION DATA, REDUCED

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<th>$T_w$ °F</th>
<th>$T_a$ °F</th>
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<th>$b$</th>
<th>$\sigma_C$ (MIN)</th>
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<th>$\frac{c/A}{T_v}$</th>
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a TOTAL SUPERHEATED VAPOR TEMPERATURE
b SUPERHEAT NEGLECTED IN CALCULATING $G_v$
c BASED UPON $\phi_v G_a = 0$
d NON-CONDENSABLE GASES PURPOSELY ADDED
ACKNOWLEDGMENTS

The author especially thanks his coadvisers, Professor Charles Dryden, who introduced him to the condensing metals problem and gave his every assistance to seeing the program underway, and Professor Robert Brodkey, who entered the program in Dr. Dryden's absence and gave freely of his time and knowledge in all phases of the study.

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