HYBRID MODELING OF THE FORMATION AND STRUCTURE OF
THIN CURRENT SHEETS IN THE MAGNETOTAIL

Author(s):
Micahel Hesse, NASA/GSFC
Dan Winske, XPA
Joachim Birn, NIS-1

Submitted to:
Proceedings of the European Conference on Substorms
Versailles, France, May 1996

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

Los Alamos National Laboratory, an affirmative action/equal opportunity employer, is operated by the University of California for the U.S. Department of Energy under contract W-7405-ENG-36. By acceptance of this article, the publisher recognizes that the U.S. Government retains a nonexclusive, royalty-free license to publish or reproduce the published form of this contribution, or to allow others to do so, for U.S. Government purposes. The Los Alamos National Laboratory requests that the publisher identify this article as work performed under the auspices of the U.S. Department of Energy.
DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.
HYBRID MODELING OF THE FORMATION AND STRUCTURE OF THIN CURRENT SHEETS IN THE MAGNETOTAIL

Michael Hesse
NASA/Goddard Space Flight Center, Greenbelt, MD
USA
email: hesse@gssfc.nasa.gov

Dan Winske, Joachim Birn
Los Alamos Nat. Lab., Los Alamos, NM
USA

ABSTRACT

Hybrid simulations are used to investigate the formation of a thin current sheet inside the plasma sheet of a magnetotail-like configuration. The initial equilibrium is subjected to a driving electric field qualitatively similar to what would be expected from solar wind driving. As a result, we find the formation of a new current sheet, with a thickness of approximately the ion inertial length. The current density inside the current sheet region is supplied largely by the electrons. Ion acceleration in the cross-tail direction is absent due since the driving electric field fails to penetrate into the equatorial region.

1. INTRODUCTION

In recent years, spacecraft observations have demonstrated the formation of thin current sheets in the Earth's magnetotail during the substorm growth phase [Refs. 6, 7, 10]. These current sheets are observed at radial distances in the range from ~7 to ~15RE, with thicknesses estimated to be of the order of a few thermal ion Larmor radii (≈ 500–2000 km). The detailed current density profile and plasma properties, however, are as yet unknown.

Because of the large current density associated with the thin current sheets, electrons and ions must have very high relative velocities, reaching values comparable to Alfvén speeds for current sheet thicknesses comparable to ion Larmor radii. Hence thin current sheets are likely sites for kinetic dissipation processes. It was realized that in a variety of cases thin current sheets must form when an initially smooth quasi-equilibrium state develops quasi-statically under ideal MHD conditions [Ref. 8].

Schindler and Birn [Ref. 11] showed by means of analytical theory that thin current sheets form if a suitable perturbation is applied at the lateral boundary of an initially one-dimensional current sheet. This boundary condition represents the fact that the tail magnetic field lines have to match the rigid near-Earth dipole field and that there may be a mismatch between the electric fields applied at an open magnetopause and at the ionosphere.

An approach which takes into account the presence of the ubiquitous normal magnetic field $B_z$, was recently undertaken by Wiegelmann and Schindler [Ref. 12]. The magnetotail in this model evolves as the consequence of flux transfer from the front side. As a result Wiegelmann and Schindler found the formation of a thin current sheet.

The formation of thin current sheets in the magnetotail was further investigated by two- and three-dimensional MHD simulations [Ref. 3] in configurations which included an inner magnetotail region with a more dipolar magnetic field. Including driving electric fields as expected from solar wind driving, these simulations provide convincing evidence that thin current sheets also form under these more general conditions by slow ideal evolution away from smooth initial states, although it is not clear whether or under what conditions these thin sheets develop into tangential singularities.

Thus MHD based approaches show clearly the formation of thin current sheets even if the initial tail model as well as the applied driving electric field are smooth. They are subject to the criticism, however, that MHD might not be a valid model if length scales become comparable to ion Larmor radii. To investigate the ion behavior during thin current sheet formation, particle or hybrid simulations are required. In contrast to the one-fluid MHD model the particle (or multi-fluid) simulations also provide information about the current carriers. It should be noted that the question whether ions or electrons are the major current carriers in this frame is quite important. For instance, a transformation into the electron rest frame would introduce very large background electric fields, which need to be taken into account in any ki-
While the electron pressure tensor $P_e$ is usually assumed to be isotropic, we here employ the electron pressure tensor model developed by Hesse et al. [Ref. 4] for a realistic electron mass $m_e = 1/1800$. For details, we refer to Hesse et al. [Ref. 5]. The time advance is provided by an explicit predictor-corrector scheme, which also includes the pressure tensor evolution.

All physical quantities are normalized to combinations of the ion skin depth $c/\omega_i$, the ion cyclotron frequency in the lobe field $\Omega_i$, a typical lobe magnetic field $B_0$, the proton mass $m_p$, and a typical density $n_e$. Using, for example, $c/\omega_i \approx 600\text{km/s}$, and $\Omega_i^{-1} \approx 0.52\text{s}$, we find, a magnetic field normalization of $B_0 = 20\text{mT}$, a thermal ion energy of $14\text{keV}$, a density normalization of $n_e = 0.14\text{cm}^{-3}$, and an electric field normalization of $23\text{mV/m}$. Although other normalizations are possible, we will in the following for the sake of demonstration adopt the above units.

The initial condition is given by a two-dimensional magnetotail-like Vlasov equilibrium provided by analytic theory [Ref. 2]. The magnetic field is derived from the flux function

$$A = -\ln \cosh(F(x)z/\alpha) + \ln(F(x))$$

where the variation in the tail direction was chosen as

$$F(x) = (1 + \delta z/(\nu \alpha))^{-\nu}$$

with $\nu = 0.6$ and a normalized magnitude of $B_z$ at $x = 0$ of $\delta = 0.05$. The initial sheet half width, again at $z = 0$ was selected as $\alpha = 8$. Equilibria of this type are usually applied to the tail region of the magnetosphere, outside of a radial distance of some $10R_E$.

The system dimensions are $0 \leq z \leq 200c/\omega_i$ and $-20 \leq z/c/\omega_i \leq 20$. For the simulation, we use $202$ and $94$ cells in the $x$ and $z$ directions, respectively. The ion initialization involves two species. One species supports the pressure and currents. The 614400 particles of this species are initialized as drifting Maxwellians. The second species, consisting initially of 307200 particles, provides a background density of $n_b = 0.2$ in order to limit the Alfvén speed in the lobe regions. It is initialized with a very low temperature of $T_b = 0.01$. More particles of species 2 are added during the driven phase in order to maintain a finite lobe density. Finally, the electrons are initialized with an isotropic pressure, corresponding to an electron-ion temperature ratio $T_e/T_i = 0.2$.

2. NUMERICAL APPROACH AND INITIAL CONDITIONS

The hybrid code provides a full treatment of the ion kinetic effects through a particle description of the ion dynamics. Electron time scales are excluded by a fluid treatment assuming negligible inertial effects. Quasi-neutrality is assumed instead of a Poisson equation for the electric field, which is here derived from the electron equation of motion neglecting electron inertia

$$\mathbf{E} + v_e \times \mathbf{B} = -\frac{1}{ne} \nabla \cdot \mathbf{P}_e + \eta \mathbf{j}$$

(1)

Here, $v_e$ denotes the electron flow velocity, $e$ the elementary charge, $\eta$ an (anomalous) resistivity, $j$ the current density, and $n$ the number density of both ions and electrons. In order to control high frequency Whistler-type fluctuations, a small nonzero resistivity corresponding to a magnetic Lundquist number

$$S = \frac{\mu v_A L}{\eta} = 10^6$$

(2)

is introduced in the code. Here $v_A$ is the Alfvén velocity, and $L$ denotes the initial current sheet thickness. Consistent with (1), the displacement current is neglected.

Particle simulations of thin current sheet formation were first undertaken by Pritchett and Coroniti [Ref. 9]. Pritchett and Coroniti, using a mass ratio of $m_i/m_e = 16$ initially imposed on a magnetotail configuration a global dawn-to-dusk electric field. As a result, they found the formation of a thin current sheet where the initially applied electric field was strongly reduced, and a newly formed electrostatic electric field provided $\mathbf{E} \times \mathbf{B}$ drifts which turned the electrons into the major current carriers.

In this paper, we use hybrid simulations to study the response of the central current sheet region to external driving. The hybrid model is complementary to previous investigations based on MHD [Ref. 3], or full particle models [Ref. 9]. Clearly, additional ion kinetic physics is included in the hybrid model but not in the MHD approach. On the other hand, the hybrid treatment eliminates the influence of the electron mass on the evolution. Finally, the implementation of boundary conditions in the hybrid model allow us to model solar wind driving by an application of an external electric field at the boundary only. The present investigation extends previous studies involving an initially thinner current sheet [Ref. 5].

To control high frequency Whistler-type fluctuations, a small nonzero resistivity corresponding to a magnetic Lundquist number

$$S = \frac{\mu v_A L}{\eta} = 10^6$$

is introduced in the code. Here $v_A$ is the Alfvén velocity, and $L$ denotes the initial current sheet thickness. Consistent with (1), the displacement current is neglected.

While the electron pressure tensor $P_e$ is usually assumed to be isotropic, we here employ the electron pressure tensor model developed by Hesse et al. [Ref. 4] for a realistic electron mass $m_e = 1/1800$. For details, we refer to Hesse et al. [Ref. 5]. The time advance is provided by an explicit predictor-corrector scheme, which also includes the pressure tensor evolution.

All physical quantities are normalized to combinations of the ion skin depth $c/\omega_i$, the ion cyclotron frequency in the lobe field $\Omega_i$, a typical lobe magnetic field $B_0$, the proton mass $m_p$, and a typical density $n_e$. Using, for example, $c/\omega_i \approx 600\text{km/s}$, and $\Omega_i^{-1} \approx 0.52\text{s}$, we find, a magnetic field normalization of $B_0 = 20\text{mT}$, a thermal ion energy of $14\text{keV}$, a density normalization of $n_e = 0.14\text{cm}^{-3}$, and an electric field normalization of $23\text{mV/m}$. Although other normalizations are possible, we will in the following for the sake of demonstration adopt the above units.

The initial condition is given by a two-dimensional magnetotail-like Vlasov equilibrium provided by analytic theory [Ref. 2]. The magnetic field is derived from the flux function

$$A = -\ln \cosh(F(x)z/\alpha) + \ln(F(x))$$

(3)

where the variation in the tail direction was chosen as

$$F(x) = (1 + \delta z/(\nu \alpha))^{-\nu}$$

(4)

with $\nu = 0.6$ and a normalized magnitude of $B_z$ at $x = 0$ of $\delta = 0.05$. The initial sheet half width, again at $z = 0$ was selected as $\alpha = 8$. Equilibria of this type are usually applied to the tail region of the magnetosphere, outside of a radial distance of some $10R_E$.

The system dimensions are $0 \leq z \leq 200c/\omega_i$ and $-20 \leq z/c/\omega_i \leq 20$. For the simulation, we use $202$ and $94$ cells in the $x$ and $z$ directions, respectively. The ion initialization involves two species. One species supports the pressure and currents. The 614400 particles of this species are initialized as drifting Maxwellians. The second species, consisting initially of 307200 particles, provides a background density of $n_b = 0.2$ in order to limit the Alfvén speed in the lobe regions. It is initialized with a very low temperature of $T_b = 0.01$. More particles of species 2 are added during the driven phase in order to maintain a finite lobe density. Finally, the electrons are initialized with an isotropic pressure, corresponding to an electron-ion temperature ratio $T_e/T_i = 0.2$. 
larger times, the boundary electric field is set to zero. At the left, i.e., at $z = 0$, $v_x$ is prescribed as a function of $z$. This $z$ dependence is shown in the bottom panel of Fig. 1.

At the top and bottom boundaries, a linear dependence on $z$ is assumed. At the $z = z_{max} = 200$ boundary, the boundary electric field is assumed to vanish during the entire run.

3. RESULTS

3.1. Current Sheet Formation

The evolution of the magnetic field in the simulation is shown in Figure 2. The figure shows clearly the enhancement of the magnetic flux in the lobes, and a compression of the plasma sheet region. The enhancement of the lobe field ceases with the driving after $t = 60$. Here, the maximum value of the lobe field base increased by about 50% compared to the initial condition. The subsequent slow evolution leads to the deformation of the magnetic field primarily in the near-Earth region which suggests that a thin current sheet may have formed.

The formation of a thin current sheet is shown in Figure 3 which depicts the cross-tail current density $j_y$ in the simulation plane for different times. The figure demonstrates clearly that a sheet of strongly enhanced current density has formed. The spatial gradient of the newly formed thin sheet are also much steeper that in the initial configuration, where they were given by the initial sheet thickness, $L = 8 c/\omega_i$. At $t = 400$, however, a typical thickness of the thin current sheet is less than $c/\omega_i$, corresponding to less than about $0.1 R_E$ in the normalisation adopted above.

Further, Fig. 3 shows a magnetic field-aligned structure of the newly formed thin current sheet. While surprising at first, the formation of this structure is easily understood since the total ion and electron pressures $p$ are almost isotropic. Under this condition, the current density is constant on lines of constant flux function $A$, i.e., on field lines. Hence, any current density enhancements forming in the equatorial ($z = 0$) plane will exhibit extensions along field lines in isotropic equilibria. The fact that this property seems to be at least approximately reproduced in Fig. 3 shows that the pressure is indeed close to isotropic, and that the current sheet is close to an equilibrium state.
3.2. Current Carriers

At a first glance, the question of which species, ions, or electrons, contribute most to the current density, might seem less important since the answer is frame dependent. Since the current density only involves relative drifts between ions and electrons, a current density entirely supported by the ions in one frame of reference can be transformed into an electron supported current density in another frame. A second inspection, however, shows that particularly in the case of thin current sheets, this transformation can introduce substantial large-scale background electric fields into the problem. The frame rooted in the Earth, however, is preferred in that in this frame the large scale background electric field is negligible. Since a possible background electric field can substantially alter growth rates and stability/instability transitions of kinetic instabilities which might operate in thin current sheets, the question of which species supports the current density in the rest frame of the Earth requires investigation.

In order to determine the current carriers in our simulation we calculated the time dependence of the maximum values of the total current density $j_T$ as well as of the maximum values of the total ion current density $j_{iy} = en_{iy}v_{iy}$. The result is shown in Figure 4. The two graphs clearly show a slight reduction in the maximum ion current density, and a substantial enhancement of the maximum value of the total current density. Since both graphs are substantially different, the conclusion remains that the major fraction of the enhanced current density in the thin current sheet is supported by the electrons. In the following, we will provide an analysis of the physical reasons for this behavior.

Equilibria are often studied in the absence of any equilibrium electric fields. A significant electron current density, however, needs to be balanced by either an electric field, or an increase in the electron pressure and its gradient. In the simplest case, the equilibrium condition for the electrons is identical to Eqn. (1) (for $\eta = 0$), with the additional requirement that the electric field be derived from a potential. The presence of a strong equilibrium electric field can significantly enhance the electron contribution to the total current density, and, at the same time reduce the ion contribution, if the induced $E \times B$ drift direction is anti-parallel to the the current density. The total current density, however, is still approximately...

Figure 3. Cross-tail current density for different times.

Figure 4. Maxima of the total and ion current densities.

The fact that the ions do not carry a significant fraction of the current might seem surprising in view of the expectation that the driving electric field might provide a viable cross-tail acceleration mechanism for the ions. Since the hybrid model calculates automatically the self-consistent electric field generated in response to the applied perturbation, we can directly compare the magnitude of the applied to the penetrated electric field. Figure 5 shows the result of such an investigation. The panels show the cross-tail electric field as a function of time during the driven phase of the simulation. From the figure it is evident that the driving electric field does not penetrate into the current sheet. In fact, the finite plasma compressibility counteracts the increased magnetic pressure due to the driving. As a result, the driving electric field is greatly reduced inside the plasma sheet, with maximum values of less than 10% of the applied electric field.
given by
\[ \mathbf{j} \times \mathbf{B} = \nabla \cdot \mathbf{P}_i + \nabla \cdot \mathbf{P}_e \] (5)

which shows that the ion pressure tensor \( \mathbf{P}_i \) can provide the major fraction of the diamagnetic even if the current is carried by the electrons. This is indeed the case here. The evolution establishes an essentially electrostatic electric field in the vicinity of the current sheet. Figure 6 displays the \( x \) and \( z \) components of this electric field. The figure demonstrates that a strong electric field has formed over the scales of the thin current sheet. This electric field is due to the Hall effect which causes separation of ion and electron scales if the gradient scale length is comparable or less than \( c/\omega_i \). In this situation, the ion current density is reduced or even inverted (as seen in the present simulation).

![Figure 6](image)

Figure 6. \( E_x \) (top panel) and \( E_z \) (bottom panel) for \( t = 400 \)

![Figure 7](image)

Figure 7. \( B_y \) magnetic field component for \( t = 400 \)

4. SUMMARY AND DISCUSSION

In this paper we used hybrid simulations to study current sheet formation in a magnetotail-like configuration when boundary conditions similar to the driving effect of the solar wind electric field for southward IMF are applied. Hybrid simulations are able to extend previous MHD-based investigations [Ref. 3] by the inclusion of ion kinetic effects, and avoid the complications of particle simulations [Ref. 9] which typically require a relatively large electron mass to operate. In contrast to the one-fluid MHD model, the particle or hybrid simulations also provide information about the current carriers.

As in MHD simulations, a thin current sheet forms as a consequence of applying an electric field perturbation at the boundary of the modeling region only. The region of enhanced current density extends earthward from the thin current sheet in the equatorial plane, approximately along field lines, as would be required by isotropic equilibrium conditions. The current density in the enhancement regions is largely carried by the electrons, when the speed is measured in the frame where the background electric field vanishes. Ion acceleration due to the driving electric fields was not found, due to the fact that only a few percent of the applied electric field penetrate to the
neutral sheet due to inductive effects. Instead, we found the current distribution between ions and electrons to be due to the formation of strong localized electrostatic electric fields in the $x-z$ plane which reduce, through $E \times B$ drift, the ion and enhance the electron current in the rest frame of the Earth. It should be noted that the question whether ions or electrons are the major current carriers in this frame is quite important. For instance, a transformation into the electron rest frame would introduce very large background electric fields, which need to be taken into account in any kinetic analysis pertaining to possible instabilities in the thin current sheet.

The electrostatic nature of these new localized fields together with the small electron temperature in the plasma sheet implies that the associated electrostatic potential is almost constant along field lines. Therefore, it should be possible to remotely detect thin current sheets in the magnetotail in satellite observations even if the orbit is much closer to the Earth than the region investigated here. The latitudinal convergence of the magnetic field closer to the Earth implies that the gradient of the electrostatic potential increases (thus rendering the electric field easier to detect). The longitudinal convergence of the magnetic field, on the other hand, implies that the current density decreases as one approaches the Earth (thus rendering the currents more difficult to detect). Therefore, electric field measurements in the inner magnetotail may provide a new tool to diagnose remotely the presence of thin current sheets in the equatorial plane at larger radial distances.

Despite the difference in methodology, we thus found qualitatively very similar results to what was found by Pritchett and Coroniti [Ref. 9] using a full particle model. Both investigations demonstrated that a thin current sheet forms in initially smooth magnetotail models, whether a driving electric field is applied as a boundary condition, similar to the MHD models [Ref. 3], or globally as an initial condition [Ref. 9]. Therefore, we conclude that all theoretical and modeling efforts to-date appear to support thin current sheet formation in an initially smooth magnetospheric equilibrium if solar wind driving is turned on. Furthermore, driving electric fields typically do not penetrate in full magnitude into the current sheet region — in fact, the applied electric field can be substantially reduced due to inductive effects. Therefore, it is not clear at all presently to which degree solar wind driving, such as expected for southward IMF conditions, leads to the occurrence of earthward convection. Instead, earthward convection probably occurs primarily due to sporadic episodes of fast earthward convection associated with substorms or bursty bulk flows [Ref. 1].

ACKNOWLEDGMENTS

This work was supported by the NASA Supporting Research and Technology and Space Physics Theory Program.

REFERENCES