Image Compression Using the W-Transform

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ABSTRACT

We present the W-transform for a multiresolution signal decomposition. One of the differences between the wavelet transform and W-transform is that the W-transform leads to a nonorthogonal signal decomposition. Another difference between the two is the manner in which the W-transform handles the endpoints (boundaries) of the signal. This approach does not restrict the length of the signal to be a power of two. Furthermore, it does not call for the extention of the signal thus. the W-transform is a convenient tool for image compression. We present the basic theory behind the W-transform and include experimental simulations to demonstrate its capabilities.

Keywords: W-transform, wavelet transform, multiresolution, image compression, filter banks, subbands

1 INTRODUCTION

The concept of multiresolution signal decomposition has received considerable attention in the research community over the past several years. This type of signal decomposition scheme has proved useful in a variety of applications, especially in signal compression and coding. One reason for its widespread use in signal compression is that each resulting frequency band or subband of the decomposition can be quantized and encoded independently from all the other subbands. The corresponding quantization error in each subband is then constrained to that particular band in the reconstruction of the signal.[1] Another reason is that the frequency bands can be matched to some of the properties of the human visual system. Thus, each band can be quantized based on its relative importance to the visual system. Finally, such decompositions can be implemented efficiently by using a pyramidal algorithm.

The traditional approach to multiresolution signal decomposition leads to an orthogonal wavelet representation of the signal.[2] This wavelet representation has been related to an infinitely iterated two-band filter bank, where the low-pass version of the signal at each stage is split into two bands. Combined with additional work by Daubechies[3] and Vetterli,[4] this decomposition technique has evloved into an efficient image compression scheme.[5] However, to ensure proper implementation of this scheme, certain assumptions and constraints must be enforced. First, the input image is assumed to be infinite in length, which requires some method of signal extension (e.g., symmetric or periodic). Second, the filter coefficients are assumed to form an orthonormal basis. Third, the length of the input image is assumed to be a power of two. Finally, an additional regularity or smoothness property is placed on the filter coefficients. This property ensures that in the limit, the filter coefficients lead to a continuous wavelet function. Although techniques have been developed to satisfy these assumptions,[3] such

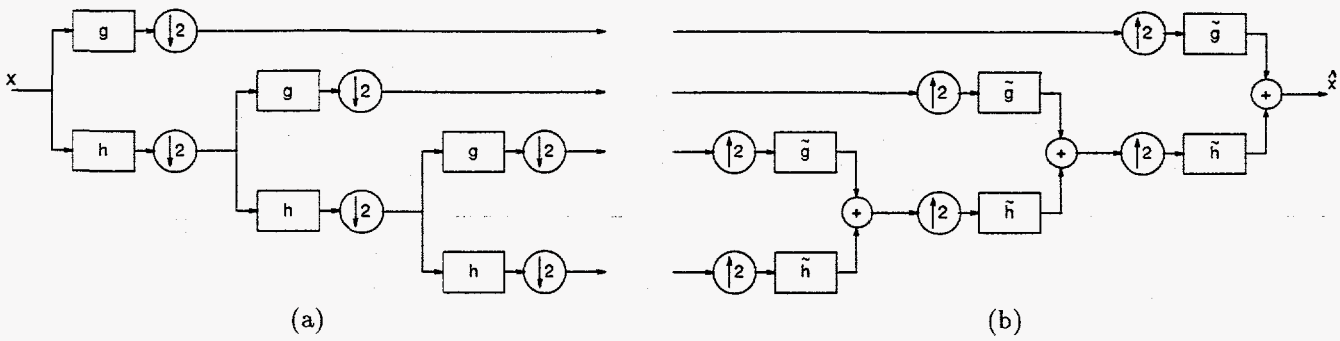


Figure 1: Irregular (octave-band) tree structure filter bank: (a) analysis section (b) synthesis section.

techniques usually impose heavy constraints on the filter design method.

In this paper we describe the W-transform to obtain a multiresolution signal decomposition.[6] This allows for a broader class of finite impulse response (FIR) filters possessing perfect reconstruction to be used in multiresolution analysis. Unlike the wavelet transform, no assumption is made on the orthogonality of the filter coefficients, and the W-transform in general leads to a nonorthogonal multiresolution analysis. Also, this approach does not restrict the length of the image to be a power of two, nor does it call for any signal extension. Thus, the W-transform becomes a convenient tool for image compression. The regularity property is maintained in the W-transform to allow for smooth approximations to the original image.

2 FILTER BANKS AND WAVELETS

This section is not intended to be an exposition on filter banks and wavelets. For our analysis we are concerned with their basic properties and structures. For a more detailed explanation, the works of other authors can be consulted.[7,8] In our analysis, the image is filtered assuming separability, so we will present the results for the 1D case only.

2.1 Filter Structure

Consider the diagram in Figure 1a of an irregular tree structure filter bank. In the diagram, h and g represent low- and high-pass analysis filters, respectively. Only the low-pass output signal is further split into two bands, which is shown here for a three-level decomposition. In the frequency domain this tree structure leads to unequal-sized frequency bands and is sometimes referred to as an octave-band subband tree structure,[9] where at each level the low-pass signal represents a blurred version of the original signal and the high-pass signal represents the detail (edge) information. We also note that the filter bank is critically sampled, since the decimation factor (2) is equal to the number of subbands at each level.

This octave-band tree structure is also used to perform an orthogonal wavelet decomposition.[10] Observe that as we travel down the tree, the subband bandwidth at each stage decreases, while the corresponding time function width increases. That is, for a large number of decomposition levels we increase the frequency resolution and decrease the time resolution, and vice versa. This fundamental time-frequency trade-off is what the wavelet transform offers in a signal decomposition scheme. The property that sets the wavelet transform aside from the subband coding techniques is the manner in which the filter coefficients are selected. Apart from this difference, the two techniques are essentially equivalent. In our analysis, the W-transform also uses this type of filter bank

structure; here, the focus will be on the properties of the filter coefficients.

2.2 Filter Properties

In this section we discuss some properties of the filter coefficients used in the implementation of the wavelet transform. Note that these coefficients could also be used in subband coding systems. One property the filter coefficients can possess is that of perfect reconstruction (PR). This ensures that after the synthesis operations (assuming no quantization), we can reconstruct the original signal without any aliasing or distortion. PR is a standard property in most subband coding systems, although there are some cases for which this property does not hold.[11]

Another property is orthogonality. In this context, orthogonality means that the analysis and synthesis filters are the same. From Figure 1, this means implies that $h = \tilde{h}$ and $g = \tilde{g}$. In most subband coding systems, orthogonality is not critical to the operation of the system, although in the filter design method only one set of filters needs to be determined. This type of filter bank is sometimes referred to as a paraunitary filter bank.[10]

Linear phase is another desired property. Although both linear phase and orthogonality cannot be realized simultaneously, having linear phase filters ensures that the decomposition does not result in any nonlinear phase distortions of the signal.

Finally, we consider the property of regularity.[3] Previously, the concept of regularity was not an explicit design criterion for subband systems; however, it is an important criterion for the design of wavelet filters. Regularity has been shown to be related to the number of zeros located at $z = -1$ on the unit circle. In the next section, we show how the filter bank in Figure 1a leads to an orthogonal wavelet decomposition. This will be followed by the description of the W-transform.

3 W-TRANSFORM

3.1 Analysis of Filter Bank

Before we present the theory behind the W-transfrom, we follow the the procedure as described by Vetterli and Herley[4] for the analysis of the filter bank in Figure 1a. Assuming the input x is infinite and h and g are FIR filters, we can write the filter matrices $\mathbf{H}$ and $\mathbf{G}$ as

$$\mathbf{H} = \begin{bmatrix} \ddots & & & & & & & & & & \\ & h_{N-1} & h_{N-2} & \cdots & h_1 & h_0 & & & & & \\ & & h_{N-1} & h_{N-2} & \cdots & h_1 & h_0 & & & & \\ & & & & & \ddots & & & & & \\ & & & & & h_{N-1} & h_{N-2} & \cdots & h_1 & h_0 & \\ & & & & & & & & & & \ddots \end{bmatrix} \tag{1}$$

and

$$\mathbf{G} = \begin{bmatrix} \ddots & & & & & & & & & & \\ & g_{N-1} & g_{N-2} & \cdots & g_1 & g_0 & & & & & \\ & & & g_{N-1} & g_{N-2} & \cdots & g_1 & g_0 & & & \\ & & & & & & \ddots & & & & \\ & & & & & & g_{N-1} & g_{N-2} & \cdots & g_1 & g_0 \\ & & & & & & & & & & \ddots \end{bmatrix}, \tag{2}$$

where $\mathbf{H}$ and $\mathbf{G}$ are infinite in length. The rows of $\mathbf{H}$ and $\mathbf{G}$ are shifted over by two because of the decimation operation. If h and g are assumed to form an orthonormal set, then

$$\mathbf{H}\mathbf{H}^T = I \tag{3}$$

and

$$\mathbf{G}\mathbf{G}^T = I, \tag{4}$$

where I is the identity matrix and the superscript T represents transposition. Also, we have that

$$\mathbf{G}\mathbf{H}^T = \mathbf{0}. \tag{5}$$

Therefore, $\mathbf{H}$ and $\mathbf{G}$ are orthogonal to each other; and thus they span two disjoint signal spaces. Furthermore, the inverses of $\mathbf{H}$ and $\mathbf{G}$ are $\mathbf{H}^{-1} = \mathbf{H}^T$ and $\mathbf{G}^{-1} = \mathbf{G}^T$. The projection of x onto the subspace spanned by $\mathbf{H}$ and $\mathbf{G}$ is given by $\mathbf{H}x$ and $\mathbf{G}x$, respectively. Since the projections of x are onto two orthogonal subspaces, we have that

$$V_{-1} = V_0 \oplus W_0, \tag{6}$$

where V_{-1} is the original signal space, V_0 is the subspace spanned by $\mathbf{H}$, and W_0 is the subspace spanned by $\mathbf{G}$. This is equivalent to filtering x at the first level of the filter bank. By iterating this procedure for subsequent levels, we obtain

$$V_{j-1} = V_j \oplus W_j, \quad j = 0, 1, \ldots, \tag{7}$$

$$V_j \subset V_{j-1}, \quad j = 0, 1, \ldots, \tag{8}$$

and

$$V_{-1} = W_0 \oplus W_1 \oplus \ldots, \tag{9}$$

where j is the number of decomposition levels. This particular type of decomposition of the signal space V_{-1} has been shown to be an orthogonal multiresolution signal decomposition for discrete sequences.[4] Moreover, it leads to an orthogonal wavelet decomposition.[2] Reconstruction of the signal, x, is performed by the synthesis section of the filter bank in Figure 1b.

3.2 W-matrix

In the above discussion, we made certain assumptions about the input signal and filter coefficients. Specifically, we assumed the input to be infinite in length. In practical applications this assumption results in extending the input either periodically or symmetrically. Although the assumption of periodicity for the entire image is not unreasonable, assuming that all *local* features of an image are periodic can be an unreasonable assumption.[15] Another assumption that was made concerned the orthogonality of the filter coefficients. As a result, the filter matrices were orthogonal to each other, thereby leading to an orthogonal decomposition. Also, traditional wavelet theory assumes that the input image is a power of two (i.e, $2^N \times 2^M$).

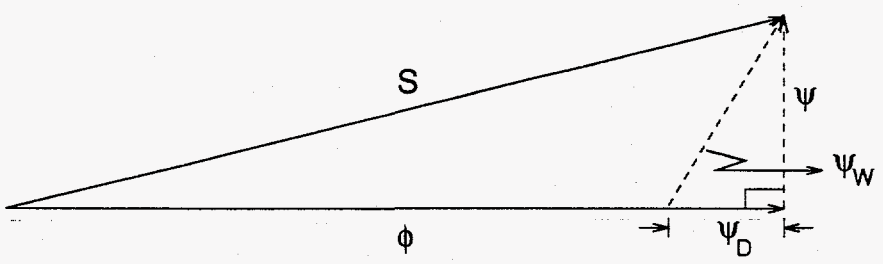


Figure 2: Vector diagram of the W-transform

In the case of the W-transform, we do not make any of the above assumptions. The W-transform treats the signals as finite and does not constrain the length to be a power of two. Note that, although we do not restrict the filter coefficients to form an orthogonal basis, we do not disregard the possibility. Hence, the W-transform leads to a nonorthogonal multiresolution signal decomposition.[6] Feauveau[12] also considers a nonorthogonal multiresolution decomposition. However, we avoid the rigorous mathematical analysis and provide a simple procedure for such a decomposition. This idea can be explained by considering the diagram in Figure 2. In the figure, S represents the signal vector, and the vector ϕ (scaling function) represents an approximation to the signal vector S. The orthogonal component to ϕ is the vector ψ, which represents the traditional wavelet decomposition. The nonorthogonal component to ϕ is the vector ψ_W, which corresponds to the wavelet used in the W-transform. The component ψ_D represents the difference or the amount of information lost in using the nonorthogonal component. Although we have not obtained a quantitative measure for the loss in information, this diagram was meant to convey the concept of the W-transform.

Now let us reconsider the filter matrix in (1) and (2). For illustration purposes, let the length of the filters h and g be $N = 4$ and let the signal, x, be an even finite length signal. Then we have that

$$\mathbf{H} = \begin{matrix} h_3 \\ \\ \\ \\ \\ \\ \\ h_0 \end{matrix} \begin{bmatrix} h_2 & h_1 & h_0 & & & & & & & \\ & & h_3 & h_2 & h_1 & h_0 & & & & \\ & & & & h_3 & h_2 & h_1 & h_0 & & \\ & & & & & & & \ddots & & \\ & & & & & & h_3 & h_2 & h_1 & h_0 \\ & & & & & & & & h_3 & h_2 & h_1 \end{bmatrix}, \tag{10}$$

where the coefficients h_0 and h_3 represent the coefficients that are excluded from the matrix because of the finite extension of the input. To include this coefficients in the matrix, we add them back to the nearest neighborhood that is retained. Thus, we obtain the following matrix

$$\mathbf{H} = \begin{bmatrix} h_3 + h_2 & h_1 & h_0 & & & & & & & \\ & & h_3 & h_2 & h_1 & h_0 & & & & \\ & & & & h_3 & h_2 & h_1 & h_0 & & \\ & & & & & & & \ddots & & \\ & & & & & & h_3 & h_2 & h_1 & h_0 \\ & & & & & & & & h_3 & h_2 + h_1 + h_0 \end{bmatrix}. \tag{11}$$

The $\mathbf{G}$ matrix is constructed in a similar fashion. Next, we interleave the rows of the $\mathbf{H}$ and $\mathbf{G}$ matrices to obtain

the **W**-matrix[6]

$$\mathbf{W} = \begin{bmatrix}
h_3+h_2 & h_1 & h_0 & & & & & & & & \\
g_3+g_2 & g_1 & g_0 & & & & & & & & \\
 & & h_3 & h_2 & h_1 & h_0 & & & & & \\
 & & g_3 & g_2 & g_1 & g_0 & & & & & \\
 & & & & h_3 & h_2 & h_1 & h_0 & & & \\
 & & & & g_3 & g_2 & g_1 & g_0 & & & \\
 & & & & & & & \ddots & & & \\
 & & & & & & h_3 & h_2 & h_1 & h_0 & \\
 & & & & & & g_3 & g_2 & g_1 & g_0 & \\
 & & & & & & & & h_3 & h_2 & h_1+h_0 \\
 & & & & & & & & g_3 & g_2 & g_1+g_0
\end{bmatrix}. \tag{12}$$

This is considered as the even-sized **W**-matrix. For odd-length signals the odd-sized **W**-matrix is given by

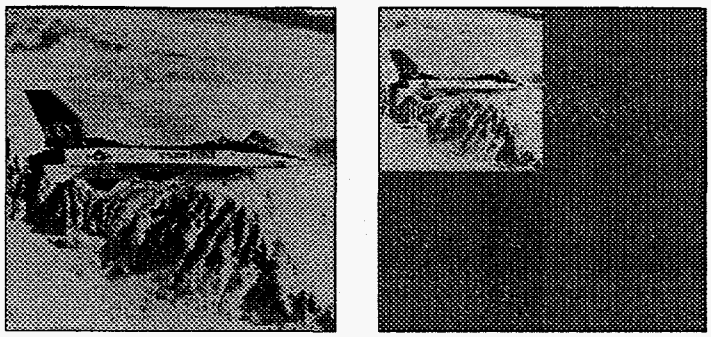
$$\mathbf{W} = \begin{bmatrix}
h_3+h_2 & h_1 & h_0 & & & & & & & & & \\
g_3+g_2 & g_1 & g_0 & & & & & & & & & \\
 & & h_3 & h_2 & h_1 & h_0 & & & & & & \\
 & & g_3 & g_2 & g_1 & g_0 & & & & & & \\
 & & & & h_3 & h_2 & h_1 & h_0 & & & & \\
 & & & & g_3 & g_2 & g_1 & g_0 & & & & \\
 & & & & & & & \ddots & & & & \\
 & & & & & & h_3 & h_2 & h_1 & h_0 & & \\
 & & & & & & g_3 & g_2 & g_1 & g_0 & & \\
 & & & & & & & & h_3 & h_2 & h_1 & h_0 \\
 & & & & & & & & g_3 & g_2 & g_1 & g_0 \\
 & & & & & & & & & & h_3 & h_2+h_1+h_0
\end{bmatrix}. \tag{13}$$

The W-transform of the signal is given by

$$y = \mathbf{W}x. \tag{14}$$

Observe that for an odd-sized **W**-matrix, the resulting low-pass signal will contain one more sample than the high-pass signal. For either case, the length of the output is always equal to the length of the input. For other techniques dealing with arbitrary-sized signals, refer to Barnard.[11] In general, $\mathbf{W}^{-1} \neq \mathbf{W}^T$ and the decomposition is not orthogonal. However, for image compression purposes, only the inverse of the transform is necessary.[13] Note that for nonorthogonal matrices, if the condition number of the matrix is large, then small data impurities may be present in the transformed signal. It turns out that the **W**-matrices have moderate condition numbers.[6] In the 2D case, we assume separability and apply the 1D W-transform to the rows and then to the columns of the image. An example of a single-level decomposed image using the W-transform is depicted in Figure 3. The decomposition results in four subbands. The subbands represent low-pass (upper-left), horizontal (upper-right), vertical (lower-left), and diagonal (lower-right) frequency information. In the analysis that follows, only the low-pass subband is decomposed further, which is equivalent to the decomposition performed by the filter bank in Figure 1. Thus, the W-transform can also be thought of as a subband coding scheme.

Kwong and Tang[6] provide a theorem for generating the coefficients of the **W**-matrix. It is shown that the coefficients exhibit compact support of length 4 and symmetry and that the associated scaling function is relatively smooth. Also the corresponding wavelet has vanishing moments up to order 2. However, Kwong and Tang make no reference to whether or not the coefficients satisfy conditions for PR. Therefore, our goal here is to show that FIR filters having PR, but not necessarily orthogonality, can be used in the **W**-matrix, which will lead to a nonorthogonal multiresolution analysis. An advantage of relaxing the orthogonality constraint is that linear phase filters can be used in the analysis, thereby ensuring no nonlinear phase distortions.



(a) (b)

Figure 3: Plane: (a) original; (b) decomposed at one level using the W-transform, which results in 4 subbands. The upper left subband represents a low-pass version of the original, the upper-right represents horizontal frequency components, the lower-left represents vertical frequency components, and the lower-right represents diagonal frequency components.

4 EXPERIMENTS

The W-transform was tested on several different monochrome images using different filter lengths and levels of decompositions. The subbands of the transformed image then quantized to yield a desired bit rate. Naturally, this process introduces quantization errors in each subband. However, by properly allocating the number of quantization levels (or bits), these errors can be made almost imperceivable to the human viewer. The number of bits per subband were allocated based on the bit allocation scheme given by Akansu and Liu.[14] Once the bits have been allocated, the subbands are quantized by using differential pulse code modulation (DPCM) with uniform quantization.[16] By using DPCM, the overall bit rate can be achieved while allowing the quality of the reconstructed image to be maintained.

The design of the filter coefficients was based on the spectral factorization method.[7,10] This method involves designing a half-band low-pass prototype filter and then factoring the prototype filter into two spectral factors. That is, let $T(z)$ represent a half-band low-pass filter and then factor $T(z)$ as

$$T(z) = H_0(z)H_1(-z), \tag{15}$$

where $H_0(z)$ and $H_1(z)$ represent low- and high-pass filters, respectively. If $T(z)$ has linear phase, the spectral factors $H_0(z)$ and $H_1(z)$ also have linear phase. Observe that because of the numerous ways to factor $T(z)$, $H_0(z)$ and $H_1(z)$ must be chosen with care to produce desirable frequency responses.[7] We considered two methods for the design of the prototype $T(z)$. The first method used the Lagrange interpolation formula, and the second used the Parks-McClellan algorithm.[10] Both design methods yield linear phase filters having some degree of regularity. The spectral factors $H_0(z)$ and $H_1(z)$ from the Lagrange method resulted in even-length filters, while the Parks-McClellan method resulted in odd-length filters. Also, $H_0(z)$ and $H_1(z)$ satisfy the conditions for perfect reconstruction and in general do not satisfy the orthogonality condition.[10] In order to keep the spectral factors to a reasonable amount, the length of the filters used were $N = 8, 10$ for the Lagrange method and $N = 9, 11$ for the Parks-McClellan method.

5 RESULTS

The results from the experiments are shown in Figures 4, 5, and 6. The size of each of the original images are 256×256 at 8 bits per pixel (bpp). The bit rate, number of decomposition levels, and the peak signal-to-noise

ratio (PSNR) are given along with the reconstructed image. The PSNR is determined by

$$PSNR\,(dB) = 10\log_{10}\frac{(255)^2}{\frac{1}{256^2}\sum_{i=1}^{256}\sum_{j=1}^{256}[x(i,j)-\hat{x}(i,j)]^2}, \tag{16}$$

where x is the original image and $\hat{x}$ is the reconstructed image. Figure 4 shows the reconstructed image of a two-level multiresolution decomposition. The filters used were determined from the Lagrange method and are of length $N = 8, 10$. Figure 5 shows the reconstructed image of a three-level multiresolution decomposition using the filters derived from the Parks-McClellan method. The filter lengths in this case are $N = 9, 11$. Figure 6 shows the reconstructed image for a three-level decomposition using the filters of length $N = 8$, $N = 10$, and $N = 11$. The bit rate for all three decompositions is 1.062 bpp. All simulations were implemented using MATLAB.[17]

6 DISCUSSIONS AND CONCLUSIONS

We have presented the W-transform for a multiresolution signal decomposition. The W-transform resulted in a nonorthogonal decomposition of the input as compared with the orthogonal decomposition of the wavelet transform. The W-transform was also shown to be equivalent to an irregular tree filter bank where the input image is assumed to be finite and the endpoints of the image are handled as described in Section (3.2). Even though only even-sized images where presented in the results, a method for handling odd-sized images was also described in Section (3.2). Furthermore, we demonstrated that FIR filters possessing PR, which in general do not form an orthonormal basis, can be used as filter coefficients in the W-transform. Two methods for determining the filter coefficients were also given.

The reconstructed images for bit rates in the range of $1.5 - 1.3$ bpp are visually indistinguishable from the original images. For bit rates in the range of $1.3 - 0.75$ bpp some visible distortions are noticeable. Mainly, there is the presence of granular noise, which is inherent in using DPCM. However, this type of visual distortion is much less annoying to the visual system than the blocky effects that result from conventional DCT compression schemes. For bit rates below 0.75 bpp, the edges of the reconstructed image start to become blurred. This is evident from Figure 5d. Overall, the reconstructed images using the W-transform are subjectively acceptable.

7 ACKNOWLEDGMENTS

The author thanks Man Kam Kwong for his helpful discussions on wavelet theory, multiresolution analysis and for the diagram in Figure 2. This work was supported by the Mathematical, Information, and Computational Sciences Division subprogram of the Office of Computational and Technology Research, U.S. Department of Energy, under Contract W-31-109-Eng-38.

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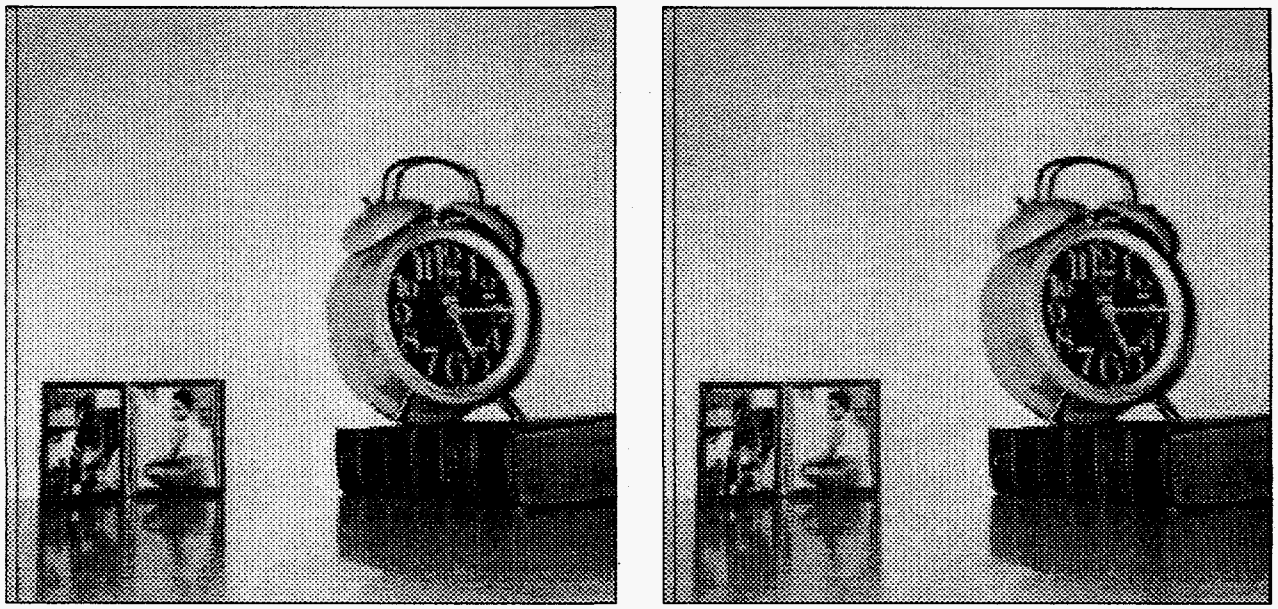
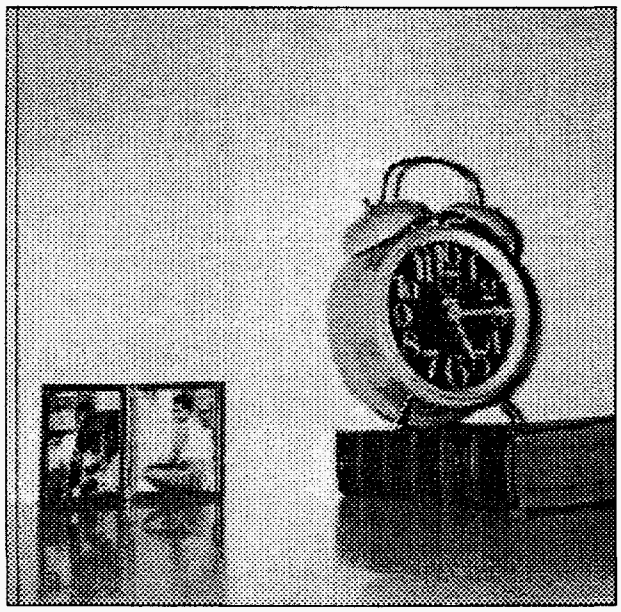
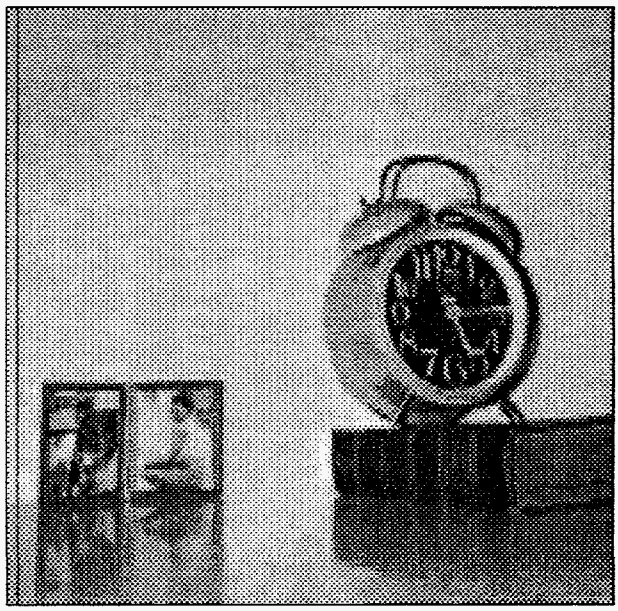


(a)

(b)

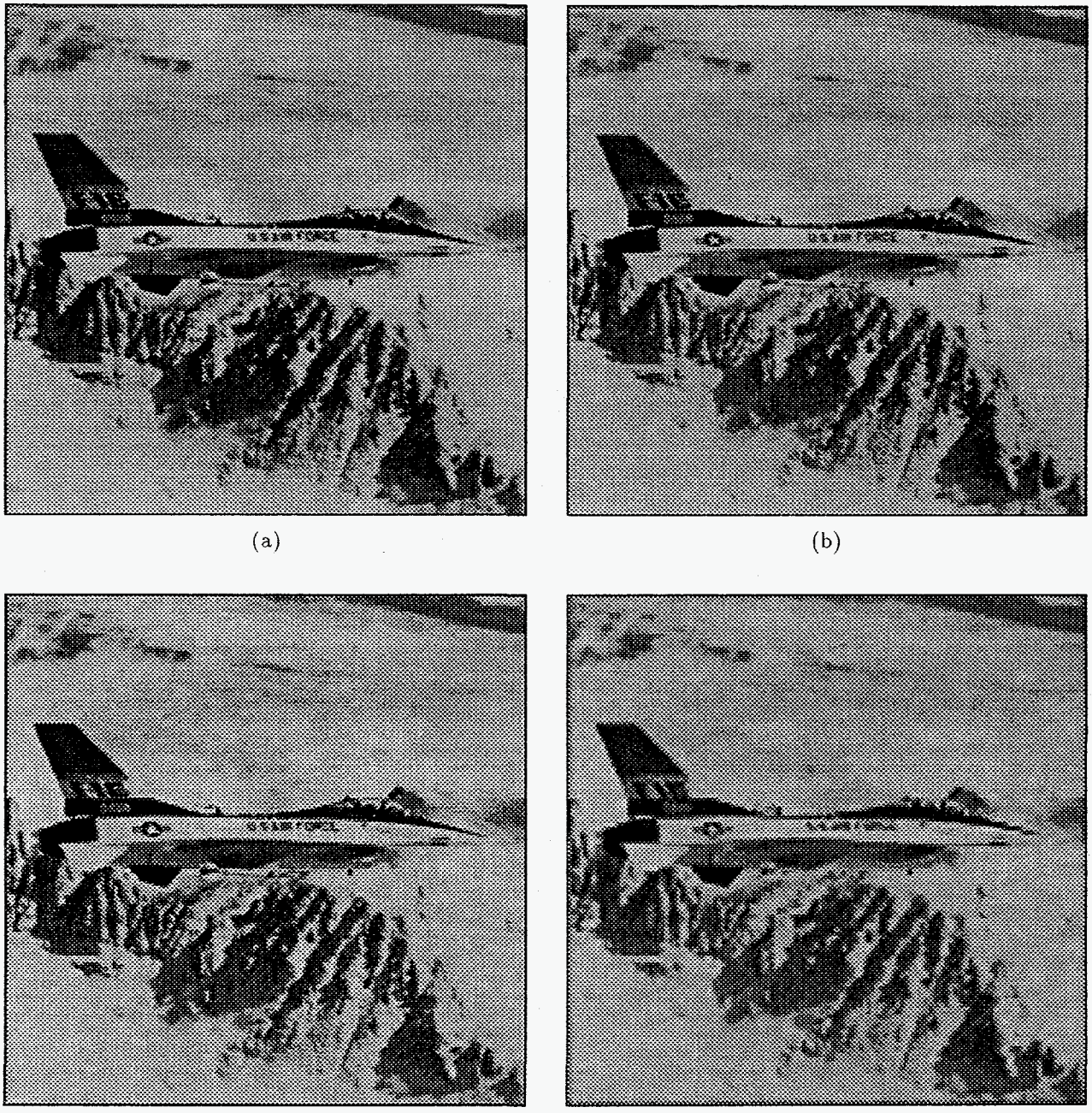


(c)



(d)

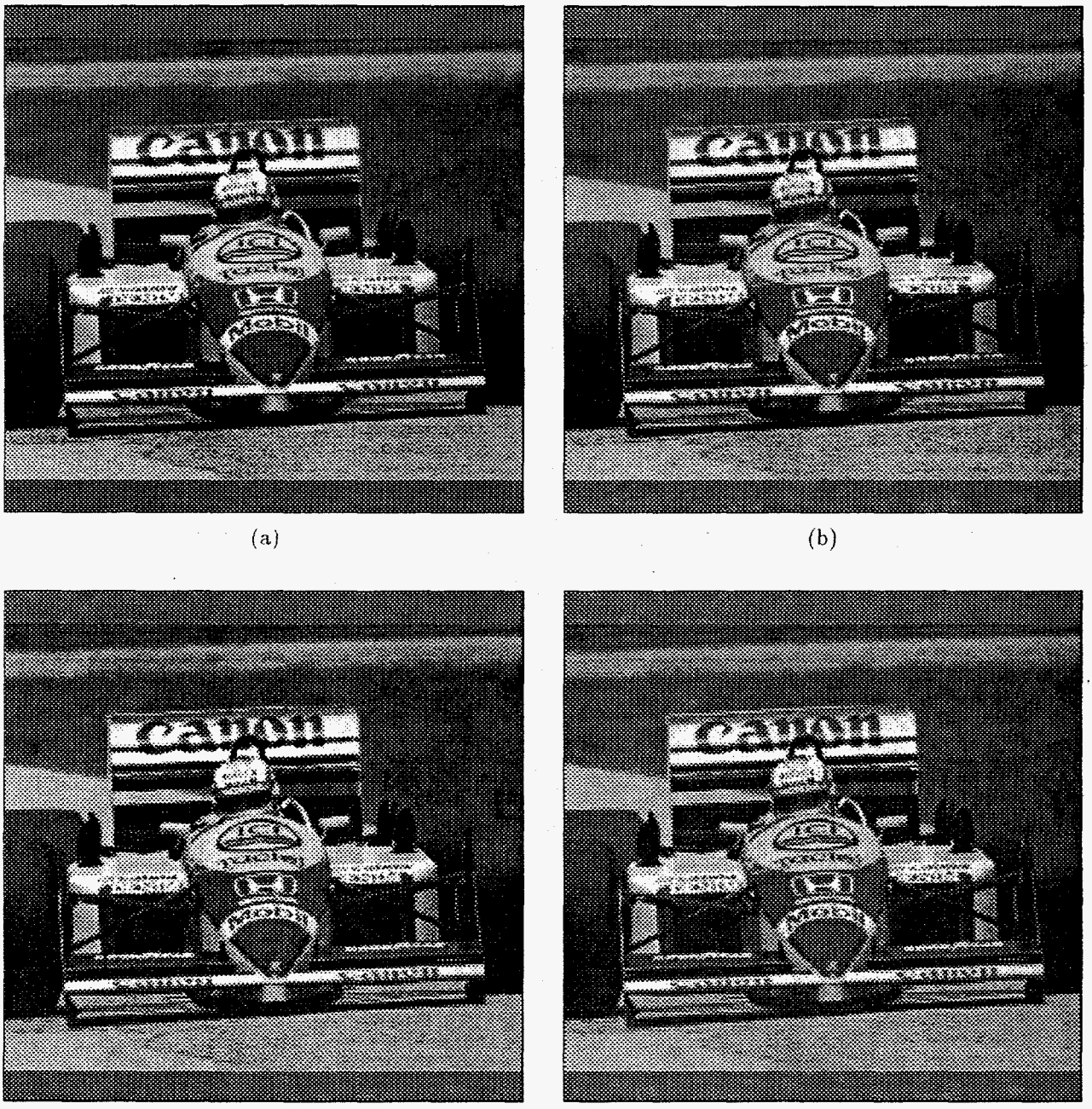
Figure 4: Clock: (a) original 8-bit image; (b) 1.5 bpp, $PSNR = 38.66$; (c) 1.125 bpp, $PSNR = 37.77$; and (d) 0.75 bpp, $PSNR = 32.76$. The filters used where designed using the Lagrange formula with (b), (c) of length $N = 8$, and (d) of length $N = 10$. The number of decomposition levels is $L = 2$.



(a) (b)

(c) (d)

Figure 5: Plane: (a) original 8-bit image; (b) 1.312 bpp, $PSNR = 36.68$; (c) 1.078 bpp, $PSNR = 35.18$; and (d) 0.5625 bpp, $PSNR = 26.73$. The filters where designed using the Parks-McClellan method with (b), (c) of length $N = 9$ and (d) of length $N = 11$. The number of decomposition levels is $L = 3$.



(a) (b)

(c) (d)

Figure 6: Car: (a) original 8-bit image; (b) $N = 8$; (c) $N = 10$; (d) $N = 11$. The bit rate, PSNR, and number of decomposition levels for (b), (c), and (d) are 1.062 bpp, $PSNR = 33.03$, and $L = 3$.