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### Title
Development of In-Structure Design Spectra for Dome Mounted Equipment on Underground Waste Storage Tanks at the Hanford Site

### Key Words
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### Abstract
In-structure response spectra for dome mounted equipment on underground waste storage tanks at the Hanford Site are developed on the basis of recent soil-structure-interaction analyses. Recommended design spectra are provided for various locations on the tank dome.
DEVELOPMENT OF IN-STRUCTURE DESIGN SPECTRA FOR DOME MOUNTED EQUIPMENT ON UNDERGROUND WASTE STORAGE TANKS AT THE HANFORD SITE

August 1995

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INDEPENDENT REVIEW

Document Reviewed Development of In-Structure Design Spectra for Dome Mounted Equipment on Underground Waste Storage Tanks at the Hanford Site

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The subject document has been reviewed by the undersigned. The reviewer reviewed and verified the following items as applicable [EP.4.1].

- Engineering Specification
- Design Input
- Basic Assumption
- Approach/Design Methodology
- Related Information
- Conclusion/Result Interpretation

E. O. Weiner 
Reviewer

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Document Reviewed: Development of In-Structure Design Spectra for Dome Mounted Equipment on Underground Waste Storage Tanks at the Hanford Site

Author: L. J. Julyk

Yes No N/A

[ ] [ ] Problem completely defined.
[ ] [ ] Necessary assumptions explicitly stated and supported.
[ ] [ ] Computer codes and data files documented.
[ ] [ ] Data used in calculations explicitly stated in document.
[ ] [ ] Data checked for consistency with original source information as applicable.
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1.0 INTRODUCTION

Generally, previous seismic analyses of equipment or components attached to the dome of an underground waste-storage tank at the Hanford Site have been conducted assuming no amplification from the dynamic motion of the dome. However, more recent analyses have indicated that significant amplification may occur in the vertical direction under vertical seismic excitation. The results from these more recent analyses are reviewed herein in an attempt to estimate appropriate in-structure response spectra that may be used in the design and analysis of dome mounted equipment on underground waste-storage tanks located at the Hanford Site. Recommendations provided herein should be considered as preliminary because of certain limitations in the analyses reviewed and because of pending implementation of the Department of Energy (DOE) Order 5480.28, Natural Phenomena Hazards Mitigation (DOE 1993). The implementation of DOE Order 5480.28 at the Hanford Site is pending the final approval and issuance of all supporting DOE Standards.
2.0 OBJECTIVE AND SCOPE

The objective of this study is to develop recommended horizontal and vertical acceleration response spectra that may be used in the seismic evaluation of equipment mounted to the dome of typical underground waste storage tanks located at the Hanford Site. The recommend in-structure design spectra are developed herein on the basis of results from recent soil-structure interaction (SSI) seismic analyses for three separate tank designs in which in-structure response spectra were calculated. The analyses were originally conducted in support of the following projects:

- Multiport Flange (MPF) installation for million-gallon double-shell tanks 241AW101 and 241SY101 (URS/Blume 1994),
- Project W320 associated with waste retrieval activities for the ¼-million-gallon single-shell tank 241C106 (Wallace 1994), and
- Project W236A associated with the design of the Multi-Function Waste Tank Facility (MWTF) (Wagenblast 1995) for potential construction of additional million-gallon double-shell tanks.

None of the above analyses were performed specifically to determine the in-structure design spectra for dome mounted equipment. Hence, the application of the results from these analyses must be tempered by consideration of their limitations. Each of the above analyses used the SASSI (Lysmer et al. 1991) computer code which employs a general 3-D finite element substructuring impedance method of analysis. This is a linear (elastic) "multi-step" analysis based on iterated strain-compatible equivalent-linear soil properties obtained from a local 1-D soil column response model for the strain level achieved with the design free-field ground motion applied.

The SSI analyses in support of the MPF (URS/Blume 1994) and the C106 retrieval (Wallace 1994) were conducted in accordance with Hanford site-specific seismic requirements given in SDC-4.1 (1993) on the basis of UCRL-15910 (UCRL 1990) and DOE Order 6430.1A (DOE 1989). Because of pending changes in seismic requirements as a result of DOE Order 5480.28, the seismic criteria applied to the design of the MWTF were modified (WHC 1995) based on methodology given in BNL 52351 (1993) with revised site-specific free-field design spectra developed by Geomatrix (1994) in accordance with DOE Order 5480.28 and draft supporting standards. The resulting "equal-hazard" free-field design spectra for the 200 West and 200 East Area differ from the corresponding Newmark-Hall design spectra of SDC 4.1 (see Figure 1 and 2). In particular, the vertical equal-hazard free-field response spectrum is taken as 2/3 of the horizontal spectrum for frequencies less than or equal 2.0 Hz and is taken equal to the horizontal spectrum for frequencies greater than 3.3 Hz. This change in vertical spectra is based on a preliminary draft revision of Standard ASCE 4-86 and is in contrast to the current version of SDC-4.1 (1993) and the current version of Standard ASCE 4-86 (1986) where the vertical free-field response spectrum is taken as 2/3 of the horizontal spectrum over the complete frequency range.
3.0 CONCLUSIONS

The following conclusions are drawn from the results of this review:

- The vertical in-structure response spectrum for dome mounted equipment is significantly amplified for vertical free-field seismic excitation. The amplification depends on the vertical frequency, damping, and attachment location of the secondary system. The greatest amplification is at the dome apex and reduces significantly as the haunch is approached. At the dome apex the zero-period acceleration (ZPA) is increased by a factor of approximately 2.3. For a secondary system with near resonant coupling with the dome (between 3 and 12 Hz) the peak amplification factor is approximately 5.5 and 9.2 at a secondary system structural damping of 5 and 2%, respectively. The vertical acceleration response is further increased by a factor of approximately 1.5 if the site-specific "equal-hazard" free-field spectrum (Geomatrix 1994) is considered relative to the current design free-field Newmark-Hall spectrum of SDC-4.1 (1993).

- The vertical in-structure response spectrum for dome mounted equipment is not significantly amplified for horizontal free-field seismic excitation.

- The horizontal in-structure response spectra for dome mounted equipment is not significantly amplified for either horizontal or vertical free-field seismic excitation.

- Because of analysis limitations discussed in Section 4.0, the resulting recommended in-structure vertical response spectra for dome mounted equipment as provided in Section 5.0 should be considered as preliminary.
4.0 LIMITATIONS

The following limitations or uncertainties exist for the analyses reviewed herein:

- The reviewed analyses were based on essentially uncracked concrete section properties. Concrete shrinkage, dead and live loads, and operating thermal induced loads can induce micro and macro cracks in the concrete structure. In addition, the response of the tank structure to the seismic induced ground motion can induce additional cracks. The effect of these cracks is to reduce the stiffness of the concrete support structure and hence reduces its response natural frequencies. Some cracking was accounted for in the C106 analysis but this did not address potential shrinkage induced cracks.

- Exposure to temperatures greater than 200 °F results in a degradation of both the concrete strength and the modulus properties. A reduction in strength properties provides a greater potential for cracking under load. A reduction in modulus reduces the effective stiffness of the structure. The net effect is a reduction in the natural frequencies of the concrete support structure. These effects were addressed to some degree in both the C106 and MWTF analyses but not in the MPF analysis.

- The backfill soil above the dome is included in the SASSI models. The stiffness of this soil layer may artificially increase the effective stiffness of the dome and hence artificially increase the natural frequency of the dome.

- Variations in local soil properties from tank-site to tank-site location need to be considered if the resulting in-structure spectra are to be applied to all of the single-shell and double-shell tank configurations at the Hanford Site. This may require additional analyses since the reviewed analyses do not consider this type of variation in soil properties.

- To properly determine the design in-structure response spectra for sub-components mounted on a supporting structure, the structural damping value to be used in the elastic response analysis of the supporting structure to define the input to the sub-component should be based on the actual response level reached throughout the "majority" of the supporting structure (BNL 1993). In the case of the analyses reviewed herein, the structural damping assumed for the supporting structure was 7% in the C106 and MWTF analyses and 5% in the MPF analysis. These damping levels correspond approximately to response levels 2 ($D_r/C_e = 0.5$ to 1.0) and 1 ($D_r/C_e \leq 0.5$), respectively, for reinforced concrete structures per Table 3.2 of BNL 1993. $D_r$ is defined as the total elastic-computed demand (seismic plus non-seismic) and $C_e$ is the code strength capacity for the supporting structure. Because the recommended in-structure response spectra curves for dome mounted equipment are based on achieving at least a level 2 response in the concrete tank structure for a 0.37 g equal-hazard earthquake, application of these curves to lower-level earthquakes may be unconservative if less than level 2 response is attained in the tank structure.
5.0 RECOMMENDATIONS

The recommended in-structure vertical response spectra for dome mounted equipment are given in Figure 3 for dome mounted secondary systems with 5% damping. The corresponding amplification factors are shown in Figure 4. Because of the pending implementation of DOE Order 5480.28 (DOE 1993) and the completeness of the MWTF analysis, the recommended design curves are based on the MWTF analysis. The in-structure design curves are given relative to the free-field vertical ground motion spectrum corresponding to the "equal-hazard" horizontal free-field design spectrum normalized to a peak ground acceleration (PGA) of 1.0 g.

Alternate in-structure design curves are given in Figure 5 with corresponding amplification factors shown in Figure 6. The alternate design curves are given relative to the free-field vertical ground motion spectrum corresponding to the Newmark-Hall horizontal free-field design spectrum with a PGA of 1.0 g.

Numerical values for both cases are listed in Table 3. Both cases are based on an assumed 7% damped support structure which implies that the support structure (tank) reaches at least a level 2 response ($D_t/C_c = 0.5$ to 1.0). The recommended curves have been frequency broadened in accordance with NUREG 1.122 (1978) and ASCE 4-86 (1986). Additional frequency broadening down to 3 Hz was performed in an attempt to address the limitations discussed in Section 4.0. This additional frequency broadening is based on a lower-bound estimate of approximately 3.5 Hz for the fundamental natural frequency of the dome. This estimate is based on a frequency analysis of the 241SY101 tank that included the mass of the dome supported soil (excluding soil stiffness) and minimum cracked-section concrete properties (Fox et al. 1990).

Curves are given for application to equipment mounted to the dome at the apex, at 10-, 21-, and 32-ft radius, and at the dome haunch. Linear interpolation can be applied at dome locations between values given. Linear scaling can be applied to the normalized curves to obtain spectrum for the specified horizontal PGA level appropriate for the safety class designation.

For secondary system damping values other than 5%, the following approximate equation may be used to scale the peak values of the in-structure 5% damped response spectrum curves:

$$\text{Peak Response at } \%\text{-Damping} = \frac{2.1053 - 0.6631 \log_{10}(\%\text{-Damping})}{\text{Peak Response at 5\% Damping}}.$$

The resulting factors to be applied to the 5% damped in-structure peak vertical response values are summarized in Table 1 for a dome mounted secondary system with 2, 5, 7, and 10% damping.

The above recommended design spectra for dome mounted equipment are preliminary and additional analysis is recommended to properly address the limitations discussed in Section 4.0. However, the current recommended spectra may be considered adequate for very stiff (fundamental natural frequency greater than 40 Hz in vertical direction) dome mounted equipment.
6.0 DISCUSSION

The in-structure response spectra of dome mounted equipment for underground waste storage tanks at the Hanford Site were developed by the time history SSI analysis method through application of the SASSI (Lysmer et al. 1991) computer code. The results from three different recent analyses were considered. This includes the Multiport Flange (MPF) analysis for tank SY101 (URS/Blume 1994), the tank C106 SSI analysis (Wallace 1994), and the Multi-Function Waste Tank Facility (MWT) SSI analysis (Wagenblast 1995). A summary of the analysis parameters of interest is given in Table 2. Following a brief discussion on frequency broadening to account for uncertainties, the results from each analysis are discussed below. This is followed by a discussion on the effect of secondary system damping.

6.1 FREQUENCY BROADENING OF IN-STRUCTURE RESPONSE SPECTRA

Frequency broadening was applied to raw in-structure response spectra in accordance with NUREG 1.122 (1978) and ASCE 4-86 (1986). Although variations in soil properties were considered explicitly in some of the reviewed analyses, the enveloping spectrum peaks were further broadened by ±15% in the frequency domain to account for other uncertainties. No reduction in peak response was applied (as suggested in ASCE 4-86) for the individual analyses.

However, the final recommended design spectra given in Section 5.0 were further broadened down to 3 Hz to address the limitations discussed in Section 3.0 and a 15% reduction from the peak response is applied, except at the haunch. The additional frequency broadening is based on a frequency analysis of the 241SY101 tank that included the mass of the dome supported soil and cracked-section concrete properties (Fox et al. 1990). On the basis of this frequency analysis, a lower-bound estimate of the fundamental natural frequency of the dome is conservatively estimated at approximately 3.5 Hz. The frequency broadening down to 3 Hz assumes a constant amplification factor relative to the input spectrum as shown in Figures 4 and 6 for the equal-hazard and Newmark-Hall input spectra, respectively. This assumption is recognized as being approximate, as the amplification is actually frequency dependent, but is believed to be consistent with the methodology of ASCE 4-86.

6.2 IN-STRUCTURE RESPONSE SPECTRA RESULTS

In all cases considered the vertical in-structure response spectrum for dome mounted equipment is not significantly amplified for horizontal free-field seismic excitation. In addition, the horizontal in-structure response spectra is not significantly amplified for either horizontal or vertical free-field seismic excitation. The most significant amplification occurs in the vertical direction under vertical seismic excitation because of the dynamic response of the soil supporting flexible dome. Hence, only the later is emphasized below for each analysis reviewed.
6.2.1 Multiport Flange Analysis

The original analysis was conducted in support of the MPF installation for underground waste storage tanks 241AW101 and 241SY101 (URS/Blume 1994). These are million-gallon double-shell tanks. In-structure response spectra were calculated at the dome apex and at the MPF located off the center of the dome. Analysis parameters of interest are given in Table 2 for the SY-tank MPF analysis. The SASSI model employed in the SSI analysis was considered to be crude and only best-estimate soil properties were considered. Best-estimate shear moduli for the soil were obtained from URS/Blume 1974. Full section uncracked concrete stiffness properties were assumed. The free-field seismic excitation is based on the Newmark-Hall spectrum with a vertical component equal to 2/3 of the horizontal over the full frequency range (SDC-4.1). The horizontal PGA was 0.12 g with an assumed structural damping for the support structure (reinforced concrete tank) of 5% of critical which implies that the tank is limited to a level 1 response (approximately).

Figure 7 shows the resulting in-structure vertical response spectra at the dome apex for 5% damped secondary systems. Note that two distinct resonant peaks were observed with the dominant peak centered at 4.5 Hz and the other centered at about 13 Hz. This is not typical of the results obtained from either the C106 analysis or the MWTF analysis which indicated only one dominant resonant peak centered about 10-12 Hz. The amplification factor on the ZPA is about 1.7 and near resonance coupling the amplification factor is about 2.7 which are much less than the corresponding amplification factors observed in the C106 and MWTF analyses. The results from the MPF analysis appear to be inconsistent with the results obtained in the other analyses, although it is not clear why. Hence, the MPF analysis results were not considered further in establishing the recommended in-structure design spectra.

6.2.2 C106 Analysis

The seismic analysis of the ½-million-gallon single-shell tank 241C106 is reported in Wallace 1994. In-structure response spectra were calculated at the dome apex. Analysis parameters of interest are given in Table 2 for the C106 tank analysis. The SASSI model employed in the SSI analysis was considered to be more refined and best-estimate and lower-bound soil properties were considered. Best-estimate shear moduli for the soil were obtained from Dames & Moore 1988. Lower-bound shear moduli were taken as the best estimate values divided by 2.0. Concrete stiffness properties were assumed based on best-estimate concrete properties at 55 years of service which accounted for some predicted cracking.

The free-field seismic excitation is based on the Newmark-Hall spectrum with a vertical component equal to 2/3 of the horizontal over the full frequency range (SDC-4.1). The horizontal PGA was 0.20 g with an assumed structural damping of 7% of critical which implies that the tank is expected to reach a level 2 response. The assumption of 7% damping is conservative with respect to the evaluation of the structural integrity of the tank which was the primary purpose of the analysis, but is not necessarily conservative in establishing in-structure response spectra unless at least a level 2 response is attained, as it was in this case.
For best-estimate soil properties, Figure 8 shows the resulting in-structure 7% damped response spectra at the dome apex for both vertical and horizontal free-field seismic excitation. No significant amplification is indicated from the horizontal seismic excitation. Figure 9 includes the effect of lower-bound soil properties. The amplification factor on the ZPA is about 2.2 and near resonance coupling the amplification factor is about 5.0. Figure 10 includes the effect of lower-bound soil properties with a 100-ton concentrated mass at the dome center. In this case the amplification factor on the ZPA is about 3.1 and near resonance coupling the amplification factor is about 7.6. The inclusion of the 100-ton equipment mass greatly affects the results suggesting that a coupled analysis may be required in such cases.

6.2.3 MWTF Analysis

In support of Project W236A associated with the design of the Multi-Function Waste Tank Facility (MWTF) for potential construction of additional million-gallon double-shell tanks, tank seismic analyses were performed. Results from the seismic analysis are reported in Wagenblast 1995. In-structure response spectra were calculated at a number of locations which included the dome apex, at the 10-, 21-, and 32-ft radius, and at the dome haunch. Analysis parameters of interest are given in Table 2 for the MWTF tank analysis. The SASSI model employed in the SSI analysis was also considered to be more refined. Best-estimate, lower-bound and upper-bound soil properties were considered. Best-estimate shear moduli for the soil were obtained from Shannon & Wilson 1994. Upper-bound shear moduli were taken as 1.5 times the best estimate and the lower-bound shear moduli were taken as the best estimate divided by 1.5. The effect of waste volume was also considered by considering both a full and empty tank condition. Full-section uncracked concrete stiffness properties were assumed.

The free-field seismic excitation is based on the equal-hazard spectra (Geomatrix 1994) with a vertical component equal to 2/3 of the horizontal spectrum for frequencies less than or equal to 2.0 Hz and equal to the horizontal spectrum for frequencies greater than 3.3 Hz. The free-field horizontal PGA was 0.37 g (0.35 g is conservatively used herein in normalization process) with an assumed structural damping of 7% of critical which implies that the tank is expected to reach a level 2 response. The assumption of 7% damping is conservative with respect to the evaluation of the structural integrity of the tank which was the primary purpose of the analysis, but is not necessarily conservative in establishing in-structure response spectra unless at least a level 2 response is attained, as it was in this case.

Figures 11 through 15 show the resulting in-structure 5% damped vertical response spectra at the dome apex, at the 10-, 21-, and 32-ft radius, and at the dome haunch, respectively, for vertical free-field seismic excitation. At the dome apex the ZPA amplification factor is 2.3 (see Figure 11). For a secondary system with near resonant coupling with the dome (between 9 and 12 Hz) the peak amplification factor is 5.2. These amplification factors decrease as the dome location of interest approaches the haunch.
6.3 EFFECT OF SECONDARY SYSTEM DAMPING

The level of damping in the secondary system is dependent on the type of construction, the material of construction, the amount of non-structural constituents, and the strain level attained in the secondary system during the seismic event. Recommended damping values are provided in BNL (1993) or DOE-STD-1020-94 (DOE 1994). Except for massive, low-stressed components (pumps, motors, etc.) the response level 3 \((D_s/C_c \geq 1.0)\) damping may be used in elastic response analyses independent of the state of response actually reached, because such damping is expected to be reached prior to structural failure. For massive, low-stressed components the damping is given as 2 and 3% of critical for response level 1 \((D_s/C_c \leq 0.5)\) and 2 \((D_s/C_c = 0.5 \text{ to } 1.0)\), respectively. Response level 3 is not permitted for such components. In the above, \(D_s\) is the elastically computed total demand and \(C_c\) is the code specified capacity. Response level 3 damping ranges from 5 to 10% of critical for mounted structures and components of interest (see Table 3.2 of BNL 1993).

To determine the effect of secondary system component damping on the in-structure response spectra, in-structure response spectra were calculated for secondary system damping values of 2, 5, 7, and 10% of critical. The time history response at the dome apex from the C106 analysis was used to determine the corresponding response spectrum at each damping value through the numerical procedure given in Gupta 1990 as implemented in Wallace 1994. See Appendix A for calculations. The results are summarized in Figure 16.

For secondary system damping values other than 5%, the following best-fit equation was obtained to scale the peak value of the in-structure 5% damped response spectrum curves:

\[
\frac{\text{Peak Response at } \%\text{-Damping}}{\text{Peak Response at 5\% Damping}} = 2.1053 - 0.6631 \log_{10}(\%\text{-Damping})
\]

The additional amplification factor to be applied to the in-structure peak vertical response at 5% damping are summarized in Table 1 for a dome mounted secondary system with 2, 5, 7, and 10% damping.

Note that SDC-4.1 (1993) allows the use of 5% damping in the design and analysis of attached equipment in accordance with UCRL-15910 (1990).
7.0 REFERENCES


UCRL, 1990, Design and Evaluation Guidelines for Department of Energy Facilities Subjected to Natural Phenomena Hazards, UCRL-15910, University of California Research Laboratory, Livermore, California.


Figure 1. Comparison of Free-Field Horizontal Design Spectra at 7% Damping Normalized to Horizontal PGA = 1 g.
Figure 2. Comparison of Free-Field Vertical Design Spectra at 7% Damping Normalized to Horizontal PGA = 1 g.
Figure 3. Recommended In-Structure 5% Damped Vertical Response Spectra at Dome of Tank for Vertical Earthquake Normalized to Vertical 7% Damped Equal-Hazard Spectrum Corresponding to Free-Field Horizontal Spectrum with PGA = 1 g.

- Soil overburden depth = 7 ft
- Overburden soil density = 130 lb/ft³
- Soil properties = 200 West Area test data
- $v_{ssr} = 0.27$, (Shannon & Wilson 1994)
- Concrete = uncracked condition
- Tank = full/empty, $T_{swss} = 1.6$
- Horiz. free-field spectrum = Equal-hazard spectrum
- 7% damped normalized to PGA = 1 g
- Vertical spectrum = 2/3 Horiz. below 3.3 Hz
- = Horiz. above 3.3 Hz

Legend:
- ○ Dome apex
- □ Dome at 10-ft radius
- ▲ Dome at 21-ft radius
- ○ Dome at 32-ft radius
- ● Dome at Haunch
- — Normalized free-field vertical control motion spectra
Figure 4. Vertical Amplification Factors for Recommended In-Structure 5% Damped Response Spectra at Dome of Tank for Vertical Earthquake Normalized to Vertical 7% Damped Equal-Hazard Spectrum Corresponding to Free-Field Horizontal Spectrum with PGA = 1 g.

- Soil overburden depth = 7 ft
- Overburden soil density = 130 lb/ft³
- Soil properties = 200 West Area test data
  \( \gamma_{eq} = 0.27 \) (Shannon & Wilson 1994)
- Concrete = uncracked condition
- Tank = full/empty, \( \gamma_{wate} = 1.6 \)
- Horiz. free-field spectrum = Equal-hazard spectrum
- 7% damped normalized to PGA = 1 g
- Vertical spectrum = 2/3 Horiz. below 3.3 Hz
  = Horiz. above 3.3 Hz

- --- Dome apex
- ○--- Dome at 10-ft radius
- ○--- Dome at 21-ft radius
- ○--- Dome at 32-ft radius
- --- Dome at Haunch

- Normalized free-field vertical control motion spectra
Figure 5. Alternate In-Structure 5% Damped Vertical Response Spectra at Dome of Tank for Vertical Earthquake Normalized to Vertical 7% Damped Newmark-Hall Spectrum Corresponding to Free-Field Horizontal Spectrum with PGA = 1 g.

- Soil overburden depth = 7 ft
- Overburden soil density = 130 lb/ft³
- Soil properties = 200 West Area test data
  - ν = 0.27, (Shannon & Wilson 1994)
- Concrete = uncracked condition
- Tank = full/empty, γ_m = 1.6
- Horiz. free-field spectrum = Newmark-Hall spectrum
- 7% damped normalized to PGA = 1 g
- Vertical spectrum = 2/3 Horiz.

- Dome apex
- Dome at 10-ft radius
- Dome at 21-ft radius
- Dome at 32-ft radius
- Dome at Haunch

- Normalized free-field vertical control motion spectra
Figure 6. Vertical Amplification Factor for Alternate In-Structure 5% Damped Response Spectra at Dome of Tank for Vertical Earthquake Normalized to Vertical 7% Damped Newmark-Hall Spectrum Corresponding to Free-Field Horizontal Spectrum with PGA = 1 g.

- Soil overburden depth = 7 ft
- Overburden soil density = 130 lb/ft³
- Soil properties = 200 West Area test data
- $\nu_{\text{con}} = 0.27$ (Shannon & Wilson 1994)
- Concrete = uncracked condition
- Tank = full/empty, $\gamma_{\text{con}} = 1.5$
- Horiz. free-field spectrum = Newmark-Hall spectrum
- 7% damped normalized to PGA = 1 g
- Vertical spectrum = 2/3 Horiz.

Legend:
- Dome apex
- Dome at 10-ft radius
- Dome at 21-ft radius
- Dome at 32-ft radius
- Dome at Haunch
- Normalized free-field vertical control motion spectra

Axes:
- Amplification Factor on the y-axis
- Acceleration (g) on the y-axis
- Frequency (Hz) on the x-axis
Figure 7. In-Structure 5% Damped Vertical Response Spectra for Double-Shell Tank SY at Dome Apex.

- Soil overburden = 6.5 ft
- Overburden soil density = 135 lb/ft³
- Soil properties = best-estimate (Tank SY)
  - $v_{soil} = 0.30$, (URS/Blume 1974)
- Concrete = uncracked condition
- Tank = full, $\gamma_{max} = 1.7$
- Horiz. free-field spectrum = SDC 4.1, Rev. 11
- Newmark-Hall spectrum
  - 5% damped, PGA = 0.12 g
- Vertical free-field spectrum = 2/3 Horiz.

- Broadened spectrum (+15% on freq.)
- Vertical Motion at Dome Apex (NODE 62)
- Free-Field Vertical Control Motion (Synthetic Time History)
- Vertical Newmark-Hall Design Spectra (2/3 Horiz.)
- Amplification factor

MPF SSI analysis (URS/Blume 1994)
Figure 8. In-Structure 7% Damped Response Spectra for Single-Shell Tank C106 at Dome Apex without 100-ton Concentrated Mass at Dome Center Best-Estimate Soil Properties.

- Soil overburden = 8 ft
- Overburden soil density = 110 lb/ft³
- Soil properties = best-estimate
- \( \gamma_{\text{soil}} = 0.44 \), Grout Vault test data
- Concrete = in situ (best-estimate at 55 years)
- Tank = 1/3 full, \( \gamma_{\text{water}} = 2.0 \)
- Horiz. free-fled spectrum = SDC 4.1, Rev. 12
- Newmark-Hall spectrum
- 7% damped, PGA = 0.2 g
- Vertical free-field spectrum = 2/3 Horiz.

**Legend:**
- **-**: Vertical Motion at Dome Apex (Run ID: Q7VMAS)
- **△**: Vertical Control Motion (Synthetic Time History)
- **-**: Vertical Newmark-Hall Design Spectra (2/3 Horiz.)
- **-**: Horizontal Motion at Dome Apex (Run ID: QTTTT)
- **○**: Horizontal Control Motion (Synthetic Time History)
- **-**: Horizontal Newmark-Hall Design Spectra (0.2 g PGA)

**Graph:**
- Frequency (Hz) on the x-axis
- Acceleration (g) on the y-axis

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Figure 9. In-Structure 7% Damped Response Spectra for Single-Shell Tank C106 at Dome Apex for Vertical Earthquake Best-Estimate and Lower-Bound Soil Properties.

- Soil overburden = 6 ft
- Overburden soil density = 110 lb/ft^3
- Soil properties = best-estimate/lower bound
- $v_{soil} = 0.44$, Grout Vault test data
- Concrete = in situ (best-estimate at 55 years)
- Tank = 1/3 full, $v_{mass} = 2.0$
- Horiz. free-field spectrum = SDC 4.1, Rev. 12
- Newmark-Hall spectrum
- 7% damped, PGA = 0.2 g
- Vertical free-field spectrum = 2/3 Horiz.

- broadened spectrum (+/-15% on freq.)
- Best-estimate soil - no concentrated mass at dome center (Run ID: Q7VMAS)
- Vertical Control Motion (Run ID Q7VMAS, Synthetic Time History)
- Lower-bound soil - no concentrated mass at dome center (Run ID QLOWV)
- Vertical Control Motion * (Run ID QLOWV, Synthetic Time History)
- Vertical Newmark-Hall Design Spectrum (2/3 Horiz.)
- Amplification factor

* Control motion is applied at outcrop of first competent soil layer.
Figure 10. In-Structure 7% Damped Response Spectra for Single-Shell Tank C106 at Dome Apex for Vertical Earthquake Best-Estimate and Lower-Bound Soil Properties (with 100-ton Concentrated Mass at Dome Apex).

Soil overburden = 6 ft
Overburden soil density = 110 lb/ft³
Soil properties = best-estimate/lower bound
γnest = 0.44, Grout Vault test data
Concrete = in situ (best-estimate at 55 years)
Tank = 1/3 full, γnest = 2.0
Horiz. free-field spectrum = SDC 4.1, Rev. 12
Newmark-Hall spectrum
7% damped, PGA = 0.2 g
Vertical free-field spectrum = 2/3 Horiz.

- Broadened spectrum (+/-15% on freq.)
- Best-estimate soil - no concentrated mass at dome center (Run ID: Q7VMAS)
- Lower-bound soil with 100-ton concentrated mass at dome center (Run ID QLOWVIV)
- Vertical Control Motion * (Synthetic Time History)
- Vertical Newmark-Hall Design Spectrum (2/3 Horiz.)
- Amplification factor

* Control motion is applied at outcrop of first competent soil layer.
Figure 11. In-Structure 5% Damped Vertical Response Spectra for Double-Shell Tank MWTF at Dome Apex for Vertical Earthquake with Equal-Hazard Spectrum.

- Soil overburden depth = 7 ft
- Overburden soil density = 130 lb/ft$^3$
- Soil properties = 200 West Area test data
- $v_{soil} = 0.27$, (Shannon & Wilson 1994)
- Concrete = uncracked condition
- Tank = full/empty, $v_{tank} = 1.5$
- Horiz. free-field spectrum = Equal-hazard spectrum
- 7% damped PGA = 0.35 g (Geomatics)
- Vertical spectrum = 2/3 Horiz. below 3.3 Hz
  - Horiz. above 3.3 Hz

- Broadened spectra (+/-15% on freq.)
- ▲ Upper-bound soil - tank empty
- ▼ Upper-bound soil - tank full
- ◇ Best-estimate soil - tank empty
- ◼ Lower-bound soil - tank empty
- ■ Lower-bound soil - tank full
- Free-field vertical control motion
- Amplification factor
Figure 12. In-Structure 5% Damped Vertical Response Spectra for Double-Shell Tank MWTF at 10-ft Radius of Dome for Vertical Earthquake with Equal-Hazard Spectrum.

Soil overburden depth = 7 ft
Overburden soil density = 130 lb/ft³
Soil properties = 200 West Area test data
νₜₕ = 0.27. (Shannon & Wilson 1994)
Concrete = uncracked condition
Tank = full/empty, νₑₚₑ = 1.8
Horiz. free-field spectrum = Equal-hazard spectrum
7% damped PGA = 0.35 g (Geomatrix)
Vertical spectrum = 2/3 Horiz. below 3.3 Hz
= Horiz. above 3.3 Hz

- - Broadened spectra (+/-15% on freq.)
- - Upper-bound soil - tank empty
- - Upper-bound soil - tank full
- - - Best-estimate soil - tank empty
- - - Lower-bound soil - tank empty
- - - Lower-bound soil - tank full
- - Free field vertical control motion
- - - Amplification factor
Figure 13. In-Structure 5% Damped Vertical Response Spectra for Double-Shell Tank MWT at 21-ft Radius of Dome for Vertical Earthquake with Equal-Hazard Spectrum.

- Soil overburden depth = 7 ft
- Overburden soil density = 130 lb/ft³
- Soil properties = 200 West Area test data
  - \( v_{\text{soil}} = 0.27 \), (Shannon & Wilson 1994)
- Concrete = uncracked condition
- Tank = full/empty, \( v_{\text{tanks}} = 1.6 \)
- Horiz. free-field spectrum = Equal-hazard spectrum
  - 7% damped PGA = 0.35 g (Geomatrix)
  - Vertical spectrum = 2/3 Horiz. below 3.3 Hz
  - Horiz. above 3.3 Hz

- Broadened spectra (+/- 15% on freq.)
- Upper-bound soil - tank empty
- Upper-bound soil - tank full
- Best-estimate soil - tank empty
- Lower-bound soil - tank empty
- Lower-bound soil - tank full
- Free-field vertical control motion
- Amplification factor
Figure 14. In-Structure 5% Damped Vertical Response Spectra for Double-Shell Tank MWTF at 32-ft Radius of Dome for Vertical Earthquake with Equal-Hazard Spectrum.

Soil overburden depth = 7 ft
Overburden soil density = 130 lb/ft³
Soil properties = 200 West Area test data
\( v_{\text{ref}} = 0.27 \) (Shannon & Wilson 1994)
Concrete = uncracked condition
Tank = full/empty, \( v_{\text{meas}} = 1.6 \)
Horiz. free-field spectrum = Equal-hazard spectrum
7% damped PGA = 0.35 g (Geomatrix)
Vertical spectrum = 2/3 Horiz. below 3.3 Hz
= Horiz. above 3.3 Hz

- Broadened spectra (+/- 15% on freq.)
- Upper-bound soil - tank empty
- Upper-bound soil - tank full
- Best-estimate soil - tank empty
- Lower-bound soil - tank empty
- Lower-bound soil - tank full
- Free-field vertical control motion
- Amplification factor

Acceleration (g) vs. Frequency (Hz)
Figure 15. In-Structure 5% Damped Vertical Response Spectra for Double-Shell Tank MWTF at Dome Haunch for Vertical Earthquake with Equal-Hazard Spectrum.

- Soil overburden depth = 7 ft
- Overburden soil density = 130 lb/ft³
- Soil properties = 200 West Area test data
- γ_concrete = 0.27, (Shannon & Wilson 1994)
- Concrete = uncracked condition
- Tank = full/empty; γ_m = 1.6
- Horiz. free-field spectrum = Equal-hazard spectrum
- 7% damped PGA = 0.35 g (Geomatics)
- Vertical spectrum = 2/3 Horiz. below 3.3 Hz
- Horiz. above 3.3 Hz

- Broadened spectra (+/− 15% on freq.)
- ▲ Upper-bound soil - tank empty
- ▼ Upper-bound soil - tank full
- ○ Best-estimate soil - tank empty
- □ Lower-bound soil - tank empty
- △ Lower-bound soil - tank full
- • Free-field vertical control motion
- . Amplification factor

Acceleration (g)

Amplification Factor

Frequency (Hz)

- Soil overburden = 6 ft
- Overburden soil density = 110 lb/ft³
- Soil properties = lower bound
  \( \gamma_{soil} = 0.44 \), Grout Vault test data
- Concrete = in situ (best-estimate at 55 years)
- Tank = 1/3 full, \( \gamma_{net} = 2.0 \)
- Horiz. free-field spectrum = SDC 4.1, Rev. 12
- Newmark-Hall spectrum
  - 7% damped, PGA = 0.2 g
- Vertical free-field spectrum = 2/3 Horiz.

**Peak Response**

- Peak Response at 5% Damping
  \[ \text{Response} = -0.6631 \log_{10}(-\text{Damping}) + 2.1053 \]
  \[ R^2 = 0.9909 \]

- 2% damped
- 5% damped
- 7% damped
- 10% damped

- Vertical Control Motion * (Run ID QLOWV, Synthetic Time History)
- Vertical Newmark-Hall Design Spectrum (2/3 Horiz.)
- Peak Response at 5% Damping
- 5% Damped Peak Response
- Best-Fit Curve

* Control motion is applied at outcrop of first competent soil layer.
Table 1. Additional Peak Response Factors for Secondary System Damping to be Applied to Peak of In-Structure Vertical Response Spectra Originally Based on 5% Damping for Dome Mounted Secondary System.

<table>
<thead>
<tr>
<th>Secondary system damping (%)</th>
<th>Peak response at 5% damping</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1.65</td>
</tr>
<tr>
<td>5</td>
<td>1.0</td>
</tr>
<tr>
<td>7</td>
<td>0.8</td>
</tr>
<tr>
<td>10</td>
<td>0.6</td>
</tr>
<tr>
<td>Tank</td>
<td>Type</td>
</tr>
<tr>
<td>------</td>
<td>------</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>C106</td>
<td>SST</td>
</tr>
<tr>
<td>SY2</td>
<td>DST</td>
</tr>
<tr>
<td>MUTF</td>
<td></td>
</tr>
</tbody>
</table>

2. URS/Blume 1994.
4. SST = single-shell tank, DST = double-shell tank.
5. LB = lower bound, BE = best estimate, UB = upper bound.
6. Best-estimate shear modulus of soil obtained from Dames & Moore 1988. Lower-bound shear modulus was taken as the best estimate divided by 2.0.
8. Best-estimate shear modulus of soil obtained from Shannon & Wilson 1994. Upper-bound shear modulus was taken as 1.5 times the best estimate and the lower-bound shear modulus was taken as the best estimate divided by 1.5.
10. IS = in situ (best-estimate at 55 years of service), UC = uncracked.
11. 2/3 horizontal spectrum for frequencies less than or equal to 2.0 Hz and equal to horizontal spectrum for frequencies greater than 3.3 Hz.
12. NH = Newmark-Hall design spectra (SDC-4.1 1993), EH = equal-hazard design spectra (Geomatrix 1994).
13. Actual peak ground acceleration (PGA) was 0.37 g, but conservatively used 0.35 g in normalization process to determine amplification.
Table 3: In-Structure 5% Damped Broadband Spectra for Dome Mounted Equipment Under Vertical Earthquake Normalized to Vertical 7% Damped Spectrum Corresponding to Free-Field Horizontal Spectrum with PGA = 1.9.

<table>
<thead>
<tr>
<th>Site</th>
<th>Nominal 2% Damp.</th>
<th>In-Structure 5% Damped Broadband Spectra</th>
</tr>
</thead>
<tbody>
<tr>
<td>Site A</td>
<td>10.2 m/s²</td>
<td>11.3 m/s²</td>
</tr>
<tr>
<td>Site B</td>
<td>12.5 m/s²</td>
<td>13.6 m/s²</td>
</tr>
<tr>
<td>Site C</td>
<td>15.8 m/s²</td>
<td>16.9 m/s²</td>
</tr>
</tbody>
</table>

...
APPENDIX A

CALCULATION OF THE EFFECT OF SECONDARY SYSTEM DAMPING ON THE IN-STRUCTURE VERTICAL RESPONSE SPECTRUM FOR THE C106 TANK DOME

MathCad® Calculation Sheets

\footnotetext{MathCad is a registered trademark of MathSoft, Inc.}

A-1
Computation of Nodal Acceleration Time History Given the SASSI Transfer Function and the Control Motion

Vertical Response of C106 at Dome Apex due to Vertical Excitation

2/3 Newmark-Hall (SDC 4.1, Rev 12) horizontal design spectrum (0.2 g PGA, 7% damped), lower-bound soil properties, no 100-ton live load mass, and 110-lbf/ft\(^3\) soil density.

\[ M = \text{READPRN(cmotion)} \]  
Read the control motion (associate \textit{cmotion} with \textit{qlow-v.cmo})

\[ \text{accel} = M^{\text{CD}} \]  
The vector \text{accel} is defined as the first column of matrix \( M \)

\[ \text{last} (\text{accel}) = 2303 \]  
Total number of data points

\[ \text{npts} = 4096 \]  
No. of points in SASSI control motion

\[ i3 = \text{last} (\text{accel}) + 1 \text{.. npts} \]  
\text{accel}_{\text{i3}} = 0.0  
Pad with zeros

\[ \Delta t = 0.01 \]  
Sample time increment (sec) of time history record

\[ i = 0 \text{.. npts - 1} \]  
\text{time}_i = i \cdot \Delta t  
\text{accel}_i = \text{accel}_i

\[ \text{npts} \cdot \Delta t = 40.96 \]  
Duration of strain record in seconds

\[ c = \text{fft} (\text{accel}) \]  
The complex vector, \( c \), is the FFT of the control motion

\[ N = \text{last} (c) \]  
\( N = 2048 \)

\[ j = 0 \text{.. N} \]

\[ \text{mag}_j = |c_j| \]  
SRSS real and imag components of \( c \)

\[ \text{real}_j = \Re(c_j) \]  
real component of \( c \)

\[ \text{sampf} = \frac{1}{\Delta t} \]  
Sampling frequency

\[ \text{sampf} = 100 \]

\[ \text{freq}_j = \frac{\text{sampf}}{\text{npts}} \]  
\( k = 0 \text{.. N} \)

\[ \text{freq}_N = 50 \]

Control Motion Acceleration (g) vs. Time (seconds)

![Control Motion Acceleration Graph]

\[ \text{min} (\text{accel}) = -0.15582 \quad \text{max} (\text{accel}) = 0.16763 \]
FFT of Control Motion Acceleration (g) vs. Frequency (Hz)

\[
\text{max(mag)} = 0.18948
\]
\[
\text{max(real)} = 0.17377 \\
\text{min(real)} = -0.16634
\]
\[
\text{WRITEPRN(realcm)} = \text{real} \\
\text{WRITEPRN(freqcm)} = \text{freq}
\]

Transfer Function at Dome Apex (Node 708 of SASSI Model)

\[
xfr = \text{READPRN(xfr}_\text{Node})
\]
\[
f = xfr^<0> \\
\text{amp} = xfr^<1>
\]
\[
iamp_k = \text{interp}(f, \text{amp}, \text{freq})
\]
\[
\text{SASSI}_\text{freq}_\text{cutoff} = 24
\]
\[
n_{\text{freq}_\text{cutoff}} = 982 \\
\text{freq}_\text{cutoff} = 23.97461 \\
iamp_{\text{freq}_\text{cutoff}} = 2.44092
\]
\[
k_k = 0..n_{\text{freq}_\text{cutoff}} \\
jj = n_{\text{freq}_\text{cutoff}}..2048 \\
\text{last}(iamp) = 2048
\]
\[
iamp_{jj} = 0
\]
Set amplification to zero for frequencies greater than SASSI frequency cutoff
Interpolated Transfer Function (Amplification vs. Frequency)

\[ \text{prod}_k = c_k \cdot \text{amp}_k \]  \hspace{1cm} \text{Multiply control motion by amplification (freq-by-freq basis)}

\[ \text{invprod} = \text{ifft}(\text{prod}) \]  \hspace{1cm} \text{Take inverse FFT of product to obtain nodal acceleration time history}

\[ \text{WRITEPRN}(\text{accel}_\text{Node}) = \text{invprod} \]  \text{Write to file}  \hspace{1cm} \text{(associate accel\_Node with va708.prm)}

Acceleration (g) of Dome at Apex vs. Time (seconds)

\[ \text{min(} \text{invprod} \text{)} = -0.29163 \]
\[ \text{max(} \text{invprod} \text{)} = 0.30467 \]
**Computation of Response Spectrum from Acceleration Time History**


**Input Acceleration History = Surface Free-Field Vertical Control Motion**

\( \text{M} = \text{READPRN}(\text{accel}_\text{th}) \)

\( \text{dt} = 0.01 \)

\( \text{sampr} = \frac{1}{\text{dt}} \)

\( \text{accel} = \text{M}^{<0>} \)

\( \text{last}(\text{accel}) = 2303 \)

\( \text{npts} = 2400 \)

\( \text{npts} \times \text{dt} = 24 \)

\( j = 0 \ldots \text{npts} - 1 \)

\( \text{time}_j = j \times \text{dt} \)

\( \text{accel}_j = \frac{\text{accel}_j}{1.0} \)

Read in acceleration time history for point of interest (associate \( \text{accel}_\text{th} \) with input time history file)

**Time increment (sec) of input acceleration time history**

frequency of data collection (pts/second) \( \text{sampr} = 100 \)

The vector \( \text{accel} \) is defined as the first column of matrix \( \text{M} \)

Total number of data points

No. of time history data points to be considered in response spectra analysis \( i_3 = \text{last}(\text{accel}) \) \( \ldots \) \( \text{npts} \)

\( \text{accel}_{i_3} = 0.0 \)

Pad with zeros if necessary

Duration of acceleration time history in seconds

Apply scaling factor to the input acceleration history

\( \zeta = 0.07 \)

\( n = 39 \)

\( k = 0 \ldots n \)

\( f_0 = 0.4 \)

\( f_n = 40 \)

\( x = \left( \frac{f_n}{f_0} \right)^{\frac{1}{n}} \)

\( x = 1.12534 \)

\( f_{kk + 1} = f_{kk} \times x \)

\( \omega_k = 2 \times \pi \times f_k \)

Circular frequencies

\( \omega_D_k = \frac{\omega_k}{\sqrt{1 - \zeta^2}} \)

\( \exp_k = e^{-\zeta \omega_k \cdot \text{dt}} \)

**Damping ratio**

\( n \) is the no. of frequencies to define the response spectrum

\( \text{kk} = 0 \ldots n - 1 \)

\( f_{kk + 1} = f_{kk} \times x \)
\[
\begin{align*}
\forall_{i} & = \exp\left(1 - \frac{\zeta}{1 - \zeta^2}\right) - \exp\left(1 - \frac{\zeta}{1 - \zeta^2}\right) \\
\forall_{i} & = \exp\left(-\frac{\omega_c}{\omega_k} \sin(\omega_D_k \cdot dt)\right) - \exp\left(-\frac{\omega_c}{\omega_k} \sin(\omega_D_k \cdot dt)\right) \\
\forall_{i} & = \frac{\omega_c}{\omega_k} \sin(\omega_D_k \cdot dt) - \exp\left(-\frac{\omega_c}{\omega_k} \sin(\omega_D_k \cdot dt)\right) \\
\forall_{i} & = \frac{\omega_c}{\omega_k} \sin(\omega_D_k \cdot dt) - \exp\left(-\frac{\omega_c}{\omega_k} \sin(\omega_D_k \cdot dt)\right) \\
\forall_{i} & = \left[2 \cdot \omega_c^2 \sin(\omega_D_k \cdot dt) - \left(\frac{2}{\omega_k} \frac{\omega_c}{\omega_k} \sin(\omega_D_k \cdot dt)\right) + \left(\frac{2}{\omega_k^3} \frac{\omega_c}{\omega_k} \sin(\omega_D_k \cdot dt)\right) + \frac{2}{\omega_k^3} \frac{\omega_c}{\omega_k} \sin(\omega_D_k \cdot dt)\right] - \frac{2}{\omega_k^2} \frac{\omega_c}{\omega_k} \sin(\omega_D_k \cdot dt) \\
\end{align*}
\]

Step through time:

Let:
- \( d \) = relative displacement of SDOF system subject to input motion at base
- \( v \) = relative velocity of SDOF system subject to input motion at base
- \( a \) = relative acceleration of SDOF system subject to input motion at base

\[
\begin{align*}
\forall_{i} & = 0..\text{npts} - 2 \\
d_{0,k} & = 0 \quad v_{0,k} = 0 \quad \text{Initial conditions} \\
\begin{align*}
\left(d_{i+1,k}\right) & = t_{i+1,k} = \left(d_{i,k} \cdot d_{i,k} + a_{12,k} \cdot v_{i,k} + b_{11,k} \cdot \text{accel}_i + b_{12,k} \cdot \text{accel}_{i+1}\right) \\
\left(v_{i+1,k}\right) & = \left(v_{i,k} \cdot v_{i,k} + a_{22,k} \cdot v_{i,k} + b_{21,k} \cdot \text{accel}_i + b_{22,k} \cdot \text{accel}_{i+1}\right)
\end{align*}
\]

Displacement of SDOF system with frequency \( f_1 \):
\( f_1 = 0.45013 \)
\[ v_{2,0,k} = 0 \quad v_{2,1,k} = \frac{d_{1,1,k} - d_{1,0,k}}{dt} \]

Check on velocity calculation.

**Velocity of SDOF system with frequency \( f_1 \)**

\[
\begin{align*}
\frac{v_{2,1}}{v_{2,1,0.1}}
\end{align*}
\]

**Acceleration of SDOF system with frequency \( f_1 \)**

\[
\begin{align*}
a_{0,k} &= \text{accel}_0 \\
a_{1,1,k} &= \frac{v_{1,1,k} - v_{1,0,k}}{dt} + \text{accel}_{1,1}
\end{align*}
\]
Obtain maximum absolute acceleration for each frequency:

\[ \text{max}_{a_k} = \max (a^{<k>}) \]
\[ \text{min}_{a_k} = \max [-a^{<k>}] \]
\[ \text{spec}_k = \text{if}(\text{max}_{a_k} > \text{min}_{a_k}, \text{max}_{a_k}, \text{min}_{a_k}) \]

**Acceleration Response Spectrum (g)**

```
f_spec = augment(f, spec)
WRITEPRN(rspec) = f_spec
```
Computation of Response Spectrum from Acceleration Time History


Input Acceleration History = at Dome Apex for Vertical Earthquake (qlow-v.cmo)
(va708.prm)

\[
M = \text{READPRN}(\text{accel}_\text{th})
\]

\[
dt = 0.01
\]

\[
sampf = \frac{1}{dt}
\]

\[
\text{accel} = M^0
\]

\[
\text{last}(\text{accel}) = 4095
\]

\[
npts = 2400
\]

\[
npts \cdot dt = 24
\]

\[
j = 0 \ldots npts - 1 \quad \text{time}_j = j \cdot dt
\]

\[
\text{paccel}_j = \text{accel}_j \cdot 1.0
\]

Read in acceleration time history for point of interest (associate \text{accel}_\text{th} with input time history file)

Time increment (sec) of input acceleration time history

frequency of data collection (pts/second) \quad sampf = 100

The vector \text{accel} is defined as the first column of matrix \text{M}

Total number of data points

No. of time history data points to be considered in response spectra analysis \quad n = \text{last}(\text{accel}) \cdot npts \quad \text{accel}_i = 0.0 \quad \text{Pad with zeros if necessary}

Duration of acceleration time history in seconds

Apply scaling factor to the input acceleration history

\[
\zeta = 0.07
\]

\[
n = 39 \quad k = 0 \ldots n
\]

\[
f_0 = 0.4 \quad f_n = 40 \quad x = \left(\begin{array}{c}
\frac{1}{f_n} \\
\frac{1}{f_0}
\end{array}\right) \quad x = 1.12534
\]

\[
f_{kk} + 1 = f_{kk} \cdot x
\]

\[
f_0 \approx 2 \cdot \pi \cdot f_k
\]

Circular frequencies

Damping ratio \quad n is the no. of frequencies to define the response spectrum

\[
\omega_k = \sqrt{\omega_k^2 - \zeta^2}
\]

\[
\exp_k = e^{-\zeta \omega_k \cdot dt}
\]

\[
\omega D_k = \omega_k \sqrt{1 - \zeta^2}
\]

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RSPEC.MCD
\[ a_{11,k} = \exp_k \left( \cos(\omega D_k \cdot dt) + \frac{\zeta}{\sqrt{1 - \zeta^2}} \sin(\omega D_k \cdot dt) \right) \]
\[ a_{12,k} = \frac{\exp_k}{\omega D_k} \sin(\omega D_k \cdot dt) \]
\[ a_{22,k} = \exp_k \left( \cos(\omega D_k \cdot dt) - \frac{\zeta}{\sqrt{1 - \zeta^2}} \sin(\omega D_k \cdot dt) \right) \]
\[ a_{21,k} = -\frac{\omega_k^2}{\omega D_k} \exp_k \sin(\omega D_k \cdot dt) \]
\[ b_{11,k} = \exp_k \left[ \left( \frac{2\zeta - 1}{\omega_k^2} \right) \frac{\sin(\omega D_k \cdot dt)}{\omega D_k} + \left( \frac{2\zeta}{\omega_k^2} \right) \frac{\cos(\omega D_k \cdot dt)}{\omega_k^2} \right] - \frac{2\zeta}{\omega_k^2} \]
\[ b_{12,k} = \left( -\exp_k \right) \left[ \left( \frac{2\zeta - 1}{\omega_k^2} \right) \frac{\sin(\omega D_k \cdot dt)}{\omega D_k} + \left( \frac{2\zeta}{\omega_k^2} \right) \frac{\cos(\omega D_k \cdot dt)}{\omega_k^2} \right] + \frac{2\zeta}{\omega_k^2} - \frac{1}{\omega_k^2} \]
\[ b_{21,k} = \frac{1 - a_{11,k}}{\omega_k^2} - a_{12,k} \]
\[ b_{22,k} = -b_{21,k} - a_{12,k} \]

Step through time:

- \( d \) = relative displacement of SDOF system subject to input motion at base
- \( v \) = relative velocity of SDOF system subject to input motion at base
- \( a \) = relative acceleration of SDOF system subject to input motion at base

\[
\begin{align*}
\text{Initial conditions:} \\
&d_{0,k} = 0, \quad v_{0,k} = 0 \\
&i := 0..npts - 2
\end{align*}
\]

\[
\begin{align*}
\begin{pmatrix} d_{i+1,k} \\ v_{i+1,k} \end{pmatrix} &= \begin{pmatrix} a_{11,k} & a_{12,k} \\ a_{21,k} & a_{22,k} \end{pmatrix} \begin{pmatrix} d_{i,k} \\ v_{i,k} \end{pmatrix} + \begin{pmatrix} b_{11,k} \cdot paccel_i \\ b_{12,k} \cdot paccel_i \end{pmatrix} + \begin{pmatrix} b_{21,k} \cdot paccel_i \\ b_{22,k} \cdot paccel_i \end{pmatrix}
\end{align*}
\]

Displacement of SDOF system with frequency \( f_1 \)

\[
f_1 = 0.45013
\]
\[ v_{2i,k} = 0 \quad v_{2i+1,k} = \frac{d_{1i+1,k} - d_{i,k}}{dt} \]

Check on velocity calculation.

**Velocity of SDOF system with frequency \( f_1 \)**

\[ \frac{v_{1i}}{v_{2i,1} + 0.01} \]

**Acceleration of SDOF system with frequency \( f_1 \)**

\[ a_{0,k} = \text{accel}_0 \quad a_{1i+1,k} = \frac{v_{1i+1,k} - v_{1i,k}}{dt} + \text{accel}_{i+1} \]
Obtain maximum absolute acceleration for each frequency:

\[ \text{maxa}_k = \max\left( a_k^{<k>} \right) \]

\[ \text{mina}_k = \max\left( -a_k^{<k>} \right) \]

\[ \text{spec}_k = \text{if} (\text{maxa}_k > \text{mina}_k, \text{maxa}_k, \text{mina}_k) \]

Acceleration Response Spectrum (g)

\[ f_{\text{spec}} = \text{augment}(f, \text{spec}) \]

\[ \text{WRITELPRN}(\text{rspec}) = f_{\text{spec}} \]
Computation of Response Spectrum from Acceleration Time History


Input Acceleration History = at Dome Apex for Vertical Earthquake (qlow-v.como)
(va708.pm)

PRNPRECISION = 6
PRNCOLWIDTH = 12

M := READPRN(accel_th)
dt = 0.01
sampf := dt
accel := M<
>
last(accel) = 4095
npts = 2400
npts*dt = 24
j = 0..npts - 1
time_j := j*dt
accel_j := accel.j 1.0

Read in acceleration time history for point of interest
(associate accel_th with input time history file)

Time increment (sec) of input acceleration time history

frequency of data collection (pts/second) sampf = 100

The vector accel is defined as the first column of matrix M

Total number of data points

No. of time history data points to be considered in response spectra analysis
i3 := last(accel) .. npts

x = 1.12534

Duration of acceleration time history in seconds

Apply scaling factor to the input acceleration history

ζ := .05

Damping ratio

n = 39

k = 0..n

f_0 := .4

f_n := 40

x = \frac{\frac{1}{n}}{f_0}

k = 0..n - 1

f_{kk} + 1 := f_k x

Circular frequencies

ω_k := 2·π f_k

ωD_k := \frac{ω_k}{\sqrt{1 - ζ^2}}

exp_k := e^{-ζω_k dt}
\[
\begin{align*}
a_{11k} &= \exp_k \left( \cos (\omega_D \cdot \text{dt}) + \frac{\zeta}{\sqrt{1 - \zeta^2}} \sin (\omega_D \cdot \text{dt}) \right) \\
a_{22k} &= \exp_k \left( \cos (\omega_D \cdot \text{dt}) - \frac{\zeta}{\sqrt{1 - \zeta^2}} \sin (\omega_D \cdot \text{dt}) \right) \\
a_{12k} &= \frac{\exp_k}{\omega_D} \sin (\omega_D \cdot \text{dt}) \\
a_{21k} &= -\frac{\omega_D}{\sqrt{1 - \zeta^2}} \exp_k \sin (\omega_D \cdot \text{dt}) \\
b_{11k} &= \exp_k \left[ \frac{2 \cdot \zeta^2 - 1}{\omega_k^2} \frac{\sin (\omega_D \cdot \text{dt})}{\omega_D} + \frac{2 \cdot \zeta}{\omega_k^3} \frac{\cos (\omega_D \cdot \text{dt})}{\omega_k^2} - \frac{2 \cdot \zeta}{\omega_k^3} \right] \\
b_{12k} &= \exp_k \left[ \frac{2 \cdot \zeta^2 - 1}{\omega_k^2} \frac{\sin (\omega_D \cdot \text{dt})}{\omega_D} + \frac{2 \cdot \zeta}{\omega_k^3} \frac{\cos (\omega_D \cdot \text{dt})}{\omega_k^2} + \frac{2 \cdot \zeta}{\omega_k^3} \right] \\
b_{21k} &= \frac{1 - a_{11k}}{\omega_k^2} - a_{12k} \\
b_{22k} &= -b_{21k} - a_{12k}
\end{align*}
\]

Step through time:

\[d = \text{relative displacement of SDOF system subject to input motion at base}\]
\[v = \text{relative velocity of SDOF system subject to input motion at base}\]
\[a = \text{relative acceleration of SDOF system subject to input motion at base}\]

\[i = 0 \to \text{npts} - 2\]

\[d_{0,k} = 0 \quad v_{0,k} = 0\] Initial conditions

\[
\begin{pmatrix}
d_{i+1,k} \\
v_{i+1,k}
\end{pmatrix} = \begin{pmatrix}
a_{11k} \cdot d_{i,k} + a_{12k} \cdot v_{i,k} + b_{11k} \cdot \text{accel}_{i} + b_{12k} \cdot \text{paccel}_{i+1} \\
a_{21k} \cdot d_{i,k} + a_{22k} \cdot v_{i,k} + b_{21k} \cdot \text{paccel}_{i} + b_{22k} \cdot \text{paccel}_{i+1}
\end{pmatrix}
\]

Displacement of SDOF system with frequency \(f_1\)

\(f_1 = 0.45013\)
\[ v_{2,0,k} = 0 \]

\[ v_{2,1+k} = \frac{d_{1+k,1} - d_{1,k}}{dt} \]

Check on velocity calculation.

**Velocity of SDOF system with frequency \( f_1 \)**

\[ v_{1,1} \]

\[ v_{2,1.1 + .01} \]

\[ \ldots \]

\[ a_{0,k} = paccel_0 \]

\[ a_{1+k,1} = \frac{v_{1+k,1} - v_{1,k}}{dt} + \text{paccel}_{k+1} \]

**Acceleration of SDOF system with frequency \( f_1 \)**

\[ a_{1,1} \]

\[ \ldots \]
Obtain maximum absolute acceleration for each frequency:

\[
\begin{align*}
\text{maxa}_k &= \max(a^{<k>}) \\
\text{mina}_k &= \max(-a^{<k>}) \\
\text{spec}_k &= \text{if}(\text{maxa}_k > \text{mina}_k, \text{maxa}_k, \text{mina}_k)
\end{align*}
\]

**Acceleration Response Spectrum (g)**

![Graph of Acceleration Response Spectrum](image)

\[f_{\text{spec}} = \text{augment}(f, \text{spec})\]

\[\text{WRITEPRN}(\text{rspec}) = f_{\text{spec}}\]
**Computation of Response Spectrum from Acceleration Time History**


**Input Acceleration History** = at Dome Apex for Vertical Earthquake (qlow-v.cmo) (va708.pm)

---

M = READPRN(accel_th)

\[ \text{dt} = 0.01 \]

\[ \text{sampf} = \frac{1}{\text{dt}} \]

\[ \text{accel} = M^{<0>} \]

\[ \text{last(accel)} = 4095 \]

\[ \text{npts} = 2400 \]

\[ \text{npts-dt} = 24 \]

\[ j = 0 \ldots \text{npts-1} \]

\[ \text{time}_j = j \cdot \text{dt} \]

\[ \text{accel}_j = \text{accel}_j \cdot 1.0 \]

Read in acceleration time history for point of interest (associate accel_th with input time history file)

Time increment (sec) of input acceleration time history

frequency of data collection (pts/second) \( \text{sampf} = 100 \)

The vector accel is defined as the first column of matrix M

Total number of data points

No. of time history data points to be considered in response spectra analysis

\( \text{i3} = \text{last(accel)} \ldots \text{npts} \)

\( \text{accel}_j = 0.0 \)

Pad with zeros if necessary

Duration of acceleration time history in seconds

Apply scaling factor to the input acceleration history

---

\[ \zeta = 0.02 \]

\( n = 39 \quad k = 0 \ldots n \)

\[ f_0 = 0.4 \quad f_n = 40 \]

\[ x = \left( \frac{f_n}{f_0} \right)^{1/n} \]

\[ x = 1.12534 \]

\[ f_{kk+1} = f_{kk} \cdot x \]

\[ \omega_k = 2 \pi f_k \]

Circular frequencies

\[ \omega D_k = \frac{\omega_k}{\sqrt{1 - \zeta^2}} \]

\[ \exp(-t) = e^{-\zeta \omega_k \cdot \text{dt}} \]

---

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\[
\begin{align*}
\alpha_{11,k} &= \exp_k \left( \cos \left( \omega_k \cdot dt \right) + \frac{\zeta}{\sqrt{1 - \zeta^2}} \sin \left( \omega_k \cdot dt \right) \right) \\
\alpha_{12,k} &= \frac{\exp_k}{\omega_k} \sin \left( \omega_k \cdot dt \right) \\
\alpha_{21,k} &= -\frac{\alpha_k}{\omega_k} \exp_k \sin \left( \omega_k \cdot dt \right) \\
\alpha_{22,k} &= \exp_k \left( \cos \left( \omega_k \cdot dt \right) - \frac{\zeta}{\sqrt{1 - \zeta^2}} \sin \left( \omega_k \cdot dt \right) \right) \\
\beta_{11,k} &= \exp_k \left[ \frac{2 \zeta^2 - 1}{\alpha_k^2} \frac{\sin \left( \omega_k \cdot dt \right)}{\omega_k} + \frac{2 \zeta}{\alpha_k^3} \frac{\cos \left( \omega_k \cdot dt \right)}{\omega_k^3} \right] \\
\beta_{12,k} &= -\exp_k \left[ \frac{2 \zeta^2 - 1}{\alpha_k^2} \frac{\sin \left( \omega_k \cdot dt \right)}{\omega_k} + \frac{2 \zeta}{\alpha_k^3} \frac{\cos \left( \omega_k \cdot dt \right)}{\omega_k^3} \right] + \frac{2 \zeta}{\alpha_k^3} \frac{1}{\omega_k^2} \\
\beta_{21,k} &= \frac{1 - \alpha_{11,k}}{\alpha_k^2} - \alpha_{12,k} \\
\beta_{22,k} &= -\beta_{21,k} - \alpha_{12,k}
\end{align*}
\]

Step through time:

\[d = \text{relative displacement of SDOF system subject to input motion at base}\]
\[v = \text{relative velocity of SDOF system subject to input motion at base}\]
\[a = \text{relative acceleration of SDOF system subject to input motion at base}\]

\[i = 0 \ldots \text{npts} - 2\]

\[d_{0,k} = 0 \quad v_{0,k} = 0\]

Initial conditions

\[
\begin{align*}
\begin{pmatrix}
d_{i+1,k} \\
v_{i+1,k}
\end{pmatrix} &=
\begin{pmatrix}
\alpha_{11,k} & \alpha_{12,k} & v_{i,k} & b_{11,k} \cdot \text{paccel}_{i} + b_{12,k} \cdot \text{paccel}_{i+1} \\
\alpha_{21,k} & \alpha_{22,k} & v_{i,k} & b_{21,k} \cdot \text{paccel}_{i} + b_{22,k} \cdot \text{paccel}_{i+1}
\end{pmatrix}
\end{align*}
\]

Displacement of SDOF system with frequency \(f_1 = 0.45013\)
\[ v_{2_{0}, k} = 0 \quad v_{2_{i+1}, k} = \frac{d_{i+1, k} - d_{i, k}}{dt} \]

Check on velocity calculation.

**Velocity of SDOF system with frequency \( f_1 \)**

\[ \begin{align*}
  v_{1_{1}, 1} \\
  v_{2_{1}, 1} + 0.01
\end{align*} \]

\[ \text{time}_1 \]

\[ a_{0, k} = \text{paccel}_0 \quad a_{i+1, k} = \frac{v_{i+1, k} - v_{i, k}}{dt} + \text{paccel}_{i+1} \]

**Acceleration of SDOF system with frequency \( f_1 \)**

\[ \begin{align*}
  a_{1_{1}, 1} \\
\end{align*} \]

\[ \text{time}_1 \]
Obtain maximum absolute acceleration for each frequency:

\[
\begin{align*}
\maxa_k &= \max(a^{<k>}) \\
\mina_k &= \max([-a^{<k>}] & \quad \text{spec}_k = \text{if}(\maxa_k > \mina_k, \maxa_k, \mina_k)
\end{align*}
\]

Acceleration Response Spectrum (g)

\[f_{\text{spec}} = \text{augment}(f, \text{spec})\]

\[\text{WRITEPRN}(\text{rspec}) = f_{\text{spec}}\]
Computation of Response Spectrum from Acceleration Time History


Input Acceleration History = at Dome Apex for Vertical Earthquake (qlow-v.cmo) (va708.pm)

\[ M = \text{READPRN}(\text{accel}_\text{th}) \]
\[ dt = 0.01 \]
\[ \text{sampf} = \frac{1}{\text{dt}} \]
\[ \text{accel} = M^{>20>} \]
\[ \text{last}(\text{accel}) = 4095 \]
\[ \text{npts} = 2400 \]
\[ \text{npts} \cdot \text{dt} = 24 \]
\[ j = 0..\text{npts} - 1 \quad \text{time}_j = j \cdot \text{dt} \]
\[ \text{accel}_j = \text{accel}_j \cdot 1.0 \]

Read in acceleration time history for point of interest
(associate \text{accel}_\text{th} with input time history file)

Time increment (sec) of input acceleration time history

The vector \text{accel} is defined as the first column of matrix \text{M}

Total number of data points

No. of time history data points to be considered in response spectra analysis
\[ i3 = \text{last}(\text{accel}) - \text{npts} \quad \text{accel}_{i3} = 0.0 \quad \text{Pad with zeros if necessary} \]

Duration of acceleration time history in seconds

Apply scaling factor to the input acceleration history

\[ \zeta = 1 \]
\[ n = 39 \quad k = 0..n \]
\[ f_0 = 0.4 \quad f_n = 40 \quad x = \left(\frac{f_n}{f_0}\right)^{1/n} = 1.12534 \]
\[ f_{kk+1} = f_{kk} \cdot x \]
\[ \omega_k = 2 \pi f_k \quad \text{Circular frequencies} \]

Damping ratio

\[ n \text{ is the no. of frequencies to define the response spectrum} \]

\[ \omega_{D_k} = \frac{\omega_k}{\sqrt{1 - \zeta^2}} \]
\[ \exp_k = e^{-\zeta \omega_k \cdot \text{dt}} \]
Step through time:

- \( d = \) relative displacement of SDOF system subject to input motion at base
- \( v = \) relative velocity of SDOF system subject to input motion at base
- \( a = \) relative acceleration of SDOF system subject to input motion at base

\[
\begin{align*}
i & := 0 \ldots npts - 2 \\
d_{0,k} & = 0 \quad v_{0,k} = 0 & \text{Initial conditions}
\end{align*}
\]

\[
\begin{align*}
\left\{ \begin{array}{c}
\left(d_{i+1,k}\right) = a_{11,k}d_{i,k} + a_{12,k}v_{i,k} + b_{11,k}\text{accel}_i + b_{12,k}\text{accel}_{i+1} \\
\left(v_{i+1,k}\right) = a_{21,k}d_{i,k} + a_{22,k}v_{i,k} + b_{21,k}\text{accel}_i + b_{22,k}\text{accel}_{i+1}
\end{array} \right.
\end{align*}
\]

Displacement of SDOF system with frequency \( f_1 \)  
\( f_1 = 0.45013 \)
\[ v_{2i,k} = 0 \quad v_{2i+1,k} = \frac{d_{i+1,k} - d_{i,k}}{dt} \]

Check on velocity calculation.

**Velocity of SDOF system with frequency \( f_1 \)**

\[ \frac{v_{i+1,k} - v_{i,k}}{dt} = \text{accel} \]

**Acceleration of SDOF system with frequency \( f_1 \)**
Obtain maximum absolute acceleration for each frequency:

\[
\max a_k = \max (a_k^<\theta>)
\]

\[
\min a_k = \max [(-a_k^<\theta>)]
\]

\[
\text{spec}_k = \text{if}(\max a_k > \min a_k, \max a_k, \min a_k)
\]

**Acceleration Response Spectrum (g)**  \(\zeta = 0.1\)

\[
f_n = 40, \quad \text{spec}_n = 0.31743
\]

\[
f_{\text{spec}} = \text{augment}(f, \text{spec})
\]

\[
\text{WRITEPRN}(\text{rspec}) = f_{\text{spec}}
\]