Comment on
"Flux creep with logarithmic $U(j)$ dependence"

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Comment on "Flux creep with logarithmic $U(j)$ dependence"

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Abstract The numerical calculation by Wang and Dong [Phys. Rev. B **49**, 698 (1994)] of flux density profiles across a slab sample exposes a possible misunderstanding of the solution given earlier by Vinokur, Feigel'man and Geshkenbein, but also predicts a kink in the magnetisation relaxation curve which Schnack and Griessen have already shown to be erroneous.

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In the problem studied by Wang and Dong¹, and earlier by Vinokur, Feigel'man and Geshkenbein² the logarithmic dependence of the activation energy, U on current density, J results in a power law relation between electric field and J , and in a nonlinear diffusion law for magnetic flux. The problem is of very general interest because it touches on the notions of fractals and self organised criticality. The only characteristic length or time scales are related to the sample thickness, d and the only characteristic scales of fields or of J are related to δB . Vinokur et al² concentrated on highly nonlinear behaviour, not far removed from a stationary critical state while Wang and Dong¹ also considered more moderately nonlinear behaviour, less far removed from Ohm's law. The superconducting sample is a slab located between planes $x = 0$ and $x = d$, permeated by flux density $B_0 + \delta B$ perpendicular to Ox ("parallel to the slab"). We comment primarily on the field cooled case in which the perturbation δB , switched on at time $t = 0$ is much smaller than the prior existing flux density B_0 , and neglect a small difference between the two calculations due to having¹ $\delta B/B_0 = 0.1$ instead of² $\delta B/B_0 \rightarrow 0$.

Figure 3a of Ref.1 shows calculated profiles of the flux density at various instants before and after the flux fronts penetrating from opposite sides have met at the sample centre, at $t = t^*$. The minimum in B appears to grow sharper at $t \gg t^*$ than it was at $t = t^*$. This differs from the prediction² that the $t = t^*$ profile would decay without deformation (Eqs.(11) and (12) of Ref.2) but also from the illustrated profiles (Fig. 1a of Ref.2) which become distinctly more rounded (in relative terms) at $t \gg t^*$.

Our first, but less important comment is that the $t = t^*$ profile cannot decay entirely without deformation, and that the deformation, if at all appreciable, goes in the sense of sharpening the minimum in B , though the curve remains smooth and does not acquire a sharp kink as in Fig. 3a of Ref.1. For $t = t^*$, the exact analysis² gives the current density

$$J(x, t^*) = \frac{4}{\sqrt{\pi}} \frac{\Gamma(3/2 + 1/\sigma)}{\Gamma(1 + 1/\sigma)} \frac{\delta B}{\mu_0 d} \left(1 - \frac{4x^2}{d^2}\right)^{1/\sigma} \quad (1)$$

for $0 < x < d/2$, where σ is the exponent in $E \propto |J|^\sigma J$. If, when $t > t^*$ or when $t \gg t^*$, J is factorisable into a temporal decay function and a depth profile:

$$J(x,t) = \frac{4}{\sqrt{\pi}} \frac{\Gamma(3/2 + 1/\sigma)}{\Gamma(1 + 1/\sigma)} \frac{\delta B}{\mu_0 d} f_1(t) f_2(x) \quad (2)$$

then $f_1 = [t^*/C(t+t_0)]^{1/\sigma}$ where t_0 is a constant to be determined numerically. If it is chosen to put $C = \sigma/2(\sigma+1)(\sigma+2)$, then f_2 (while >0) is a solution to $d^2(f_2^{1+\sigma})/d(2x/d)^2 + \sigma^{-1}f_2 = 0$. For all $0 < \sigma < \infty$, the corresponding flux density profile is found to have a sharper minimum at $x = d/2$, in relative terms than the profile corresponding to Eq.(1) but the difference is only appreciable for small σ values. It is visible for $\sigma = 1$, in Fig.1. It arises from the small amount of flux which penetrates much further than the mean penetration depth, so that $t = t^*$ at an early stage in the penetration process. The effect is therefore rather artificial and trivial. A new numerical calculation shows that the transition from Eq.(1) towards Eq.(2) occurs fairly quickly after $t = t^*$. This indicates a continuity of behaviour as $\sigma \rightarrow 0$, since in the Ohm's law case, the lowest spatial higher harmonic, the 3rd, decays 9 times faster than the fundamental. On the other hand for $\sigma \geq 2$, the two curves, suitably scaled, are scarcely distinguishable, so the $t = t^*$ solution multiplied by the temporal decay function is a good approximation, though not an analytically exact one, except asymptotically as $\sigma \rightarrow \infty$ or as $t/t^* \rightarrow \infty$. This is the sense in which the results of Ref.2 are intended to be understood.⁴ By contrast, in the case of zero field cooling the $t = t^*$ curve already has a kink, as in Fig.(2) but the curves become smooth as soon as B is everywhere nonzero.

Our second and more important comment is that Schnack and Griessen³ demonstrated both numerically and analytically the absence of a kink in $d(\ln M)/d(\ln t)$ at $t = t^*$, illustrated by magnetic relaxation curves for $\sigma = 4$ and 8. Curves for smaller σ values, obtained numerically at that time but not published, were very similar to the two shown. For $\sigma = 0$ (Ohm's law) there is certainly no kink. In that case t^* is zero or meaningless. $d(\ln M)/d(\ln t)$ decreases continuously without limit while the flux density profile evolves gradually from error functions to a sinusoid. The smaller σ , the less abruptly the perturbation is extinguished and less is the likelihood of any abrupt change at $t = t^*$. The analytic argument³ relied on approximating the flux profiles as straight lines. It might be objected that this is less accurate for small σ values where the profiles are less nearly rectilinear. In that case the following argument could be used instead. Integrating Eq.(1), for all $t \leq t^*$ and all $\sigma > 0$,

$$\frac{M(t)}{M(0)} = 1 - \frac{1}{(1 + 1/\sigma)\sqrt{\pi}} \frac{\Gamma(3/2 + 1/\sigma)}{\Gamma(1 + 1/\sigma)} \left[\frac{t}{t^*}\right]^{1/(2+\sigma)} \quad (3)$$

We note in passing that Eq.(1) of Ref.4 is derived from this in the limit $\sigma \rightarrow \infty$ and t not too near t^* . It follows that $M(t^*)/M(0) = 5/8, (1 - 4/3\pi), \approx 1/2$ respectively for $\sigma = 1, 2, \gg 1$, while as $t \rightarrow t^*$, $d(\ln M)/d(\ln t) = -1/5, -0.184, \approx -1/(2+\sigma)$ respectively. The slope at $t > t^*$ can be estimated by evaluating the constant t_0 in Eq.(2). For example the two curves in Fig.1 most nearly coincide at $t = t^*$ if $t_0/t^* \approx 5.0$. This implies $d(\ln M)/d(\ln t) \approx -t/(t+5t^*)$. Combining these, there is slight jog between $t = t^*$ and $1.2 t^*$ due to the imperfect approximation, but if $\ln M$ is plotted vs $\ln t$ over a wide range as in Ref.1, no kink is discernable. Similarly for $\sigma = 2$, where $t_0/t^* \approx 2.0$ and $d(\ln M)/d(\ln t) \approx -t/(2t+4t^*)$. We can therefore reaffirm that the logarithmic $U(J)$ law and similar approximations lead to magnetic relaxation curves with no kink at $t = t^*$. The same is true in the case of zero field cooling⁵ and true whether δB is applied with a finite ramp rate or a small initial penetration is assumed, to avoid the discontinuity at $t = 0$.

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Caption to Figure 1

Calculated profiles of flux density across a nonlinearly conducting slab in which $E \propto J^2$, at times t after switching on a magnetic field increment (field cooled case). t^* is the instant the perturbations from each side reach the median plane. The $t = t^*$ curve is the cubic obtained by integrating Eq.(1) with $\sigma = 1$, while the $t \gg t^*$ curve, obtained from Eq.(2) is the one which would decay uniformly as $(t+t_0)^{-1}$ without further deformation. We find numerically that the $t = t^*$ profile transforms rapidly towards the other as soon as $t > t^*$. t_0 is a constant to be determined numerically or estimated by making the curves approximately coincide at $t = t^*$. Its value here is circa $5.0 t^*$.

Caption to Figure 2

Numerically calculated profiles of flux density across a nonlinearly conducting slab in which $E \propto J^2$, at various times after switching on a magnetic field (zero field cooled case). The corresponding magnetisation relaxation curve has no kink at $t = t^*$.

FIG. 1