Pion-Nucleon Scattering in $P_{11}$ Channel and the Roper Resonance

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Abstract

The $\pi N$ scattering in $P_{11}$ channel is investigated within the two-channel model of Pearce and Afnan$^6$. The model consists of: (1) vertex interactions $b \rightarrow \pi N, \pi \Delta$ with $b$ denoting either a bare nucleon or a bare Roper state, (2) a background potential $v_{\pi B, \pi B}$ with $B = N, \Delta$. Assuming that $v_{\pi B, \pi B}$ can be phenomenologically parameterized as a separable form and the $\pi N$ inelasticity can be accounted for by dressing the $\Delta$ in the $\pi \Delta$ channel by a $\Delta \leftrightarrow \pi N$ vertex, it is found that the fit to the $P_{11}$ phase shifts up to 1 GeV favors a large mass of the bare Roper state. Our results are consistent with the findings of Pearce and Afnan$^{12}$ that if the mass of the bare Roper state is restricted to be $\leq 1600$ MeV, then a physical Roper will have a width which is too narrow causing a rapid variation of the phase shifts at energies near the resonance energy.

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I. INTRODUCTION

An important direction in nuclear research is to investigate the propagation of the nucleon resonances ($N^*$) in nuclear medium. To proceed, it is necessary to construct a theoretical model to describe the hadronic and electromagnetic productions of $N^*$ on the nucleon. Previous studies have concentrated mostly on the $\Delta(1232)$, the one with the lowest excitation energy. Recently, more attentions have been paid to the higher mass nucleon resonances. The study of the Roper $N^*(1440)$, which has the same spin-isospin quantum numbers of the nucleon, is of particular importance since several interesting questions concerning its dynamical origin have been raised. Both the bag model and the nonrelativistic quark potential model predict that the first orbital excitation with negative parity lies below the first positive parity excitation ($0s \rightarrow 1s$)[2]. It has also been suggested by Arndt, Ford and Roper[3] that two independent resonance poles are required to fit the observed energy dependence of $\pi N$ scattering in $P_{11}$ channel. These two results can be understood more easily if we assume that the Roper could be a hybrid state rather than the ordinary $q^3$ state. One possibility is that the bare $q^3$ bag state associated with the Roper is strongly dressed by the meson cloud which can be introduced into the bag model according to chiral symmetry. Thus it is interesting to explore the extent to which this can be substantiated in a dynamical model which can describe the $\pi N$ scattering in $P_{11}$ channel. An accurate $\pi N$ model in $P_{11}$ channel is also needed for investigating the dynamics of the $\pi NN$ system. For example, Afnan and McLeod[4] demonstrated that $\pi d$ elastic scattering is sensitive to the $\pi N P_{11}$ potential and Stevenson et al[5] found that an accurate $\pi N P_{11}$ amplitude is indispensable to explain the polarization observables in $\pi^+ d$ elastic
scattering at 50 MeV.

It has been difficult[6-13] to obtain a satisfactory theoretical description of the $\pi N$ phase shifts in $P_{11}$ channel. At low energies $E_L < 400$ MeV, it requires a detailed cancellation between the repulsion originating from the nucleon pole and an attractive background. Previous investigations have shown that part of the needed attraction can be attributed to the excitation of the Roper resonance (called $R$ thereafter). The resonance excitation can be most easily described by introducing a vertex interaction $N^* \rightarrow \pi N, \pi \Delta$ in a dynamical model. One can justify this approach from the point of view of the cloudy bag model[14]. This has been the starting point of the formulation developed by Pearce and Afnan[6,12]. They have been able to give a good description of the $P_{11}$ scattering at low energies $E_L \leq 400 MeV$. At energies near the Roper resonance, they found that if the mass of the bare Roper is assumed to be less than about 1600 MeV, the predicted phase shifts vary too rapidly, implying that identifying the lowest radial bag excitation with the Roper leads to a physical Roper that is much too narrow in width. However, in the diagramatic approach of Pearce and Afnan[6,12], the unitarity condition forces them to include only the pole and the one-particle-exchange mechanisms in the resulting scattering equation. It is possible that the difficulty encountered by them could be resolved if additional mechanisms are included. From rather extensive $\pi NN$ studies[15], it is known that to account for the $NN$ data, one needs to include mechanisms other than that can be derived diagramatically within the constraint of the $\pi NN$ unitarity condition. These mechanisms can be identified primarily by phenomenological means. One example is a series of studies of $\pi NN$ processes by Lee and Matsuyama[16] within a Hamiltonian
formulation of the problem. In this work we will make a similar attempt to explore whether the difficulty encountered by Pearce and Afnan[12] can be resolved if the non-pole mechanisms (called the background interactions) are treated purely phenomenologically. Needless to say, our objective is very limited. It is mainly aimed at getting a \( P_{11} \) model which can accurately describe the data and therefore can be used to investigate the role of the Roper resonance in determining nuclear dynamics at intermediate energies. Nevertheless, our study can provide some additional information for further exploring the cloudy bag model description of the Roper resonance.

In section II, we introduce a simple extension of the formulation of Pearce and Afnan[6] to account for the inelasticity due to the decay of the \( \Delta \) in the \( \pi \Delta \) channel. The results are given in section III. Section IV is devoted to summary and discussions.

II. BASIC EQUATIONS

For practical \( \pi NN \) or nuclear calculations, the two-channel model of Pearce and Afnan[6] is more tractable than their model with three-body \( \pi \pi N \) unitarity [12]. It is therefore worthwhile to examine whether with some phenomenological procedures this model can be made to describe the \( \pi N \) data up to 1 GeV. According to the formulation of Ref.[6], the scattering amplitude in the coupled-channels space \( S = \pi N \oplus \pi \Delta \) is defined by the following operator equation

\[
T_{\pi B,\pi B'}(E) = V_{\pi B,\pi B'} + V_{\pi B,\pi B''} G_{\pi B''}(E) T_{\pi B'',\pi B'}(E),
\]

where \( B \) denotes either a \( N \) or a \( \Delta \) state. The summation over the intermediate \( \pi B'' \) states is understood in Eq. (1). We will use the same simplified
notations in all equations presented in the rest of the paper. To have a more realistic description of the \(\pi N\) inelasticity, we include the decay of the \(\Delta\) in the \(\pi\Delta\) channel and therefore define the \(\pi B\) propagator in Eq. (1) as

\[
G_{\pi B}(E) = \frac{1}{E - E_B(\vec{p}) - E_\pi(\vec{k}) - \Sigma_{\pi B}(E - E_\pi(\vec{k})) + i\varepsilon}
\]  

(2)

where \(E_B(\vec{p}) = (m_B^2 + \vec{p}^2)^{1/2}\) and \(E_\pi(\vec{k}) = (m_\pi^2 + \vec{k}^2)^{1/2}\). \(\vec{k}\) and \(\vec{p}\) denote respectively the three-momentum of the pion and the baryon \(B\), \(m_\pi\) and \(m_B\) are their masses. In the \(\pi N\) channel, \(m_N\) is taken to be the physical nucleon mass \(M_N\) and hence we set \(\Sigma_{\pi N}(E) = 0\). On the other hand, the self-energy of the \(\Delta\) is determined by a \(\pi N \leftrightarrow \Delta\) vertex

\[
\Sigma_{\pi \Delta}(\omega) = f_{\Delta,\pi N} \frac{1}{\omega - E_N(\vec{p}) - E_\pi(\vec{k}) + i\varepsilon} f_{\pi N,\Delta}
\]  

(3)

where \(f_{\pi N,\Delta}\) is the hermitian conjugate of the vertex function \(f_{\Delta,\pi N}\). By fitting the low energy \(P_{33}\) phase shifts, it is found\[16\] that the bare mass of the \(\Delta\) is \(m_\Delta = 1280\) MeV and \(f_{\Delta,\pi N}(q) = 0.98/(\sqrt{2}(m_\pi + m_N))(q/m_\pi)(\Lambda^2/(\Lambda^2 + q^2))^2\) with \(\Lambda = 358\) MeV/c.

The potential \(V_{\pi B,\pi B'}\) in Eq. (1) consists of a background term and a two-pole term

\[
V_{\pi B,\pi B'} = v_{\pi B,\pi B'} + f_{\pi B,\pi B'}^{(0)} d_{b}^{(0)}(E) f_{b,\pi B'}^{(0)}
\]  

(4)

where we have defined \(f_{\pi B,\pi B'}^{(0)}\) as the complex conjugate of the vertex \(f_{b,\pi B}^{(0)}\). \(b = N\) and \(R\) denote respectively a bare nucleon state and a bare Roper state. Their masses \(m_N^0\) and \(m_R^0\) are the free parameters of the model. The vertex interaction \(f_{b,\pi B}^{(0)}\) describes the decay of the bare particle \(b\) into a \(\pi N\) or a \(\pi\Delta\) state. In the obvious matrix notation, the propagator in Eq. (4) can be written as

\[
d^{(0)-1}(E) = \begin{pmatrix}
E - m_N^0 & 0 \\
0 & E - m_R^0
\end{pmatrix}
\]  

(5)
The most important constraint on a theoretical description of the $\pi N$ scattering in $P_{11}$ channel is that the solution of Eq. (1) must have a pole at $E = M_N$ and the residue of the pole should be related to the physical $\pi NN$ coupling constant. Explicitly, it is necessary to require that

$$\lim_{E \to M_N} T_{\pi N, \pi N}(k_0, k_0, E) = \frac{f^{(phys)}_{\pi NN}(k_0)^* f^{(phys)}_{\pi NN}(k_0)}{E - M_N} + O((E - M_N)), \quad (6)$$

where $f^{(phys)}_{\pi NN}(k)$ is the physical $\pi NN$ form factor and $k_0$ is the on-shell momentum at $E = M_N$. The form of $f^{(phys)}_{\pi NN}(k)$ within our model and its relation with the physical coupling constant $f^2/(4\pi) \approx 0.08$ will be defined later.

To derive conditions under which the limit Eq. (6) is approached, it is convenient to cast the solution of Eq. (1) into the following form

$$T_{\pi B, \pi B'}(E) = T^{(np)}_{\pi B, \pi B'}(E) + \sum_{B B'} f_{\pi B, \pi B'}(E) \delta_{\pi B, \pi B'}(E)$$

where the nonpole($np$) term is determined only by the background potential $V_{\pi B, \pi B'}$

$$T^{(np)}_{\pi B, \pi B'}(E) = \sum_{\pi B, \pi B'} G_{\pi B, \pi B'}(E) T^{(np)}_{\pi B, \pi B'}(E). \quad (8)$$

The dressed vertices and propagators in the second term of Eq. (7) are of the following forms

$$f_{\pi B, \pi B'}(E) = f^{(0)}_{\pi B, \pi B'} + T^{(np)}_{\pi B, \pi B'}(E) G_{\pi B, \pi B'}(E) f^{(0)}_{\pi B, \pi B'}, \quad (9)$$

$$d^{-1}_{bb'}(E) = d^{-1}_{bb'}(E) \delta_{bb'} - \Sigma_{bb'}(E), \quad (10)$$

with

$$\Sigma_{bb'}(E) = f^{(0)}_{b, \pi B} G_{\pi B}(E) f_{\pi B, b'}(E). \quad (11)$$
The next step is to transform the dressed propagator Eq. (10) into a diagonal form. In the matrix notation, we have

$$
\tilde{d}(E) = U d(E) U^{-1} = \begin{pmatrix} D_N(E) & 0 \\ 0 & D_R(E) \end{pmatrix},
$$

where $U$ is an orthogonal matrix ($U^{-1} = U^T$). With some straightforward derivations, we find that

$$U = \begin{pmatrix} 1 & \frac{D_N^{-1}(E) - d_{RR}^{-1}(E)}{d_N^{-1}(E)} \\ \frac{D_R^{-1}(E) - d_{RR}^{-1}(E)}{d_N^{-1}(E)} & 1 \end{pmatrix},$$

with

$$D_N^{-1}(E) = \frac{1}{2} \left[ d_{NN}^{-1} + d_{RR}^{-1} - \sqrt{(d_{NN}^{-1} - d_{RR}^{-1})^2 + 4d_{NR}^{-1}d_{RN}^{-1}} \right],$$

$$D_R^{-1}(E) = \frac{1}{2} \left[ d_{NN}^{-1} + d_{RR}^{-1} + \sqrt{(d_{NN}^{-1} - d_{RR}^{-1})^2 + 4d_{NR}^{-1}d_{RN}^{-1}} \right].$$

Equations (12)-(15) agree with that presented by Pearce and Afnan[6]. By using the above definitions and introducing the transformed vertices

$$\tilde{f}_{B,B}(E) = U_{bb'} f_{b',B}(E),$$

and

$$\tilde{f}_{B,B}(E) = f_{B,b'}(E) U_{bb'},$$

Eq. (7) can then be written as

$$T_{B,B}(E) = T^{np}_{B,B}(E) + \tilde{f}_{B,B}(E)d_{bb}(E)\tilde{f}_{B,B}(E)$$

With a diagonalized propagator Eq.(12), the nucleon pole condition Eq. (6) can be obtained easily by considering the $E \to M_N$ limit

$$\lim_{E \to M_N} D_N(E) = \frac{1}{D_N^{-1}(M_N) + (E - M_N)Z^{-1}_N},$$
where
\[
Z_N^{-1} = \left( \frac{dD_N^{-1}(E)}{dE} \right)_{E=M_N}.
\] (20)

The nucleon pole condition Eq. (6) follows if we require
\[
D_N^{-1}(M_N) = 0.
\] (21)

Note that Eq. (21) is highly nonlinear in relating the physical mass \( M_N \) to the bare masses \( m^0 \). By using the definitions Eqs.(10) and (14), it is straightforward to derive from the nucleon pole condition Eq. (21) the following expression relating the bare masses, \( m^0_N \) and \( m^0_R \), of the model to the physical nucleon mass \( M_N \)
\[
M_N = m^0_N + \Sigma_{NN}(M_N) + \frac{\Sigma_{NR}(M_N)\Sigma_{RN}(M_N)}{M_N - m^0_R - \Sigma_{RR}(M_N)}.
\] (22)

An explicit expression for the renormalization constant \( Z_N \), defined in Eq. (20), has also been obtained
\[
Z_N = \frac{d_{NN}^{-1} + d_{RR}^{-1}}{\Sigma_{NR}\Sigma_{RN}^{(1)} + \Sigma_{NN}\Sigma_{RR}^{(1)} + (1 + \Sigma_{NN}^{(1)}d_{RR}^{-1}) + (1 + \Sigma_{RR}^{(1)}d_{NN}^{-1})},
\] (23)
where all quantities are evaluated at \( E = M_N \) from equations (10)-(11) and we have defined
\[
\Sigma_{bb'}^{(1)} = f_{b,\pi B}(M_N)G_{\pi B}^2(M_N)f_{\pi B,b'}(M_N).
\] (24)

By using Eqs.(12), (19) and (21), it is easy to see that Eq.(18) is reduced to the form of Eq.(6) at \( E \rightarrow M_N \) and we can relate the the physical \( \pi NN \) form-factor to the dressed vertex form factors defined by Eqs. (16) and (17). Before doing so, it is necessary to relate the present formulation to the usual lagrangian field theory in which the physical coupling constant \( f^2/(4\pi) \simeq 0.08 \) is defined in the well-known pseudo-vector coupling between
the pion field and the nucleon field. In this work, we follow Mizutani et al. [8] and Afnan [17] to assume that the present \( \pi N \) scattering equation at the nucleon pole can be identified with the Blankbecker-sugar equation which is derived from the Bethe-Salpeter equation. By further taking an appropriate nonrelativistic limit of the nucleon motion, the condition Eq. (6) leads to

\[
| f_{\pi NN}(k) |^2 = \frac{\pi m^2}{3} \frac{\pi^2 k^2}{4M^2_N} E_\pi(k) E_N(k)[E_\pi(k) + M_N] \times \quad (25)
\]

\[
\frac{M_N + W(k)}{W(k)} Z_N \left| \frac{f_{\pi NN}(k, M_N)}{k} \right|^2,
\]

where \( W(k) \) is the total center of mass energy of the \( \pi N \) system. At the on-shell momentum \( k_0 \) for \( E = M_N \)

\[
| f_{\pi NN}(k_0) |^2 = 0.079. \quad (26)
\]

It is straightforward to extend the previous procedures to define the Roper resonance. This amounts to requiring that the determinant of the propagator matrix \( d_{bb}^{-1} \) of Eq. (7) should vanish at a complex energy \( E_R = M_R - i \Gamma_R / 2 \), with \( M_R = 1440 \text{ MeV} \) and \( \Gamma_R \sim 450 \text{ MeV} \). However, it is more convenient to use the usual prescription to extract the resonance parameters by requiring that the \( D_R(E) \) term of the diagonalized propagator Eq. (12) has the following Breit-Wigner form as the total energy \( E \) approaches \( M_R \); namely

\[
\lim_{E \to M_R} D_R(E) \to \frac{Z_R}{E - M_R + i \Gamma_R / 2}.
\]

By expanding \( D_R^{-1}(E) \) about \( E = M_R \), it is easy to see that Eq. (27) can be obtained by requiring the following conditions

\[
Z_R^{-1} = \text{Re} \left( \frac{dD_R^{-1}(E)}{dE} \right)_{E=M_R}, \quad (28)
\]

8
The above equations allow us to express the bare mass \( m_R^0 \) in terms of the other parameters of the model and to extract the width of the Roper resonance from the numerical results.

\[ \text{Re} \left( D_R^{-1}(M_R) \right) = 0, \quad (29) \]

and

\[ \Gamma_R = 2 \frac{\text{Im} \left( D_R^{-1}(M_R) \right)}{Z_R}. \quad (30) \]

The above equations allow us to express the bare mass \( m_R^0 \) in terms of the other parameters of the model and to extract the width of the Roper resonance from the numerical results.

### III. NUMERICAL RESULTS

To proceed, we need to define the parameterizations of the interactions within the model. The vertex interactions \( \pi N \leftrightarrow b \) for \( b = N, R \) are parameterized as

\[ f_{\pi N,N}^{(0)}(k) = \frac{C_{\pi N,N}}{\sqrt{E_\pi(k)}} \left[ \frac{k}{(k^2 + \alpha_{N1}^2)^2} + \frac{a_{N}k^2}{(k^2 + \alpha_{N2}^2)^2} \right] \quad (31) \]

\[ f_{\pi N,R}^{(0)}(k) = \frac{C_{\pi N,R}}{\sqrt{E_\pi(k)}} \frac{k}{(k^2 + \alpha_{R}^2)^2} \quad (32) \]

We assume that the vertex function of \( \pi \Delta \leftrightarrow b \) have the same momentum-dependence hence set

\[ f_{\pi \Delta,\Delta}^{(0)}(k) = R_{\Delta} f_{\pi N,\Delta}^{(0)}(k), \quad (33) \]

where \( R_{\Delta} \) is an adjustable parameter in the fit. For the background potentials, we assume the following separable parameterization

\[ v_{\pi B,\pi B'}(k, k') = g_{\pi B}(k) \lambda_{BB'} g_{\pi B'}(k'), \quad (34) \]

with

\[ g_{\pi B}(k) = \frac{1}{\sqrt{E_\pi(k)}} \left[ \frac{C_{B1}k}{k^2 + \beta_{B1}^2} + \frac{C_{B2}k^2}{(k^2 + \beta_{B2}^2)^2} \right] \quad (35) \]
We set $\lambda_{NN} = -1$ for the simple reason that an attractive background potential in $\pi N \rightarrow \pi N$ is most convenient for balancing the repulsion from the nucleon pole term. Other $\lambda$'s are treated as the free parameters. Obviously, the hermiticity of the potential requires that $\lambda_{\Delta N} = \lambda_{N\Delta}$. Another parameters of the model are the bare masses $m_0^N$ and $m_0^\Delta$.

The parameters defined above are determined from a $\chi^2$-fit to the $\pi N$ $P_{11}$ phase shift data up to 1 GeV subject to the pole conditions Eqs.(22), (25), (26) and (29). Our strategy is to use these three constraints to calculate the bare masses $m_0^N$ and $m_0^\Delta$ and the bare $\pi N N$ coupling constant $C_{\pi N N}$ of eq.(31), while all other parameters are free to vary in the $\chi^2$-fit.

We first consider the case that no bare Roper state is included in the model. This amounts to assuming that the dynamics of the Roper is purely due to the hadronic coupling between $\pi N$ and $\pi \Delta$ channels. No coupling to a Roper bag state is assumed. In Fig.1 we show that the phase shifts of Arndt et.al.[18] can be fitted (solid curves) very well with this model. The resulting parameters are listed in the set 1 of Tables 1 and 2. If we further neglect the decay of the $\Delta$ in the $\pi \Delta$ channel, i.e. setting $\Sigma_{\pi \Delta} = 0$ in Eq.(3), the best fits to the data are the dashed curves in Fig.1. The resulting parameters are in the set 2 of tables 1 and 2. Clearly, the simple procedures defined by Eqs (2) and (3) for including the $\pi \pi N$ channel through the dressing of the $\Delta$ in the $\pi \Delta$ channel is essential in getting an accurate description of $\pi N$ inelasticity near the threshold.

When the bare Roper state is included, it becomes difficult to fit the data if we assume that the bare Roper mass is less than about 1600 MeV. All of the fits yield a rapid variation in phase shifts at energies near the Roper resonance. This is similar to the result of Pearce and Afnan[12]. We have
found that this difficulty is caused by the rather severe constraints on the parameters due to the pole conditions Eqs. (22), (25), (26) and (29). It can not be removed by simply using different parameterizations of the vertex functions and background potentials. The only way to get a reasonable fit is to allow a rather large value of the bare Roper mass. It is found that no acceptable fit to the data in the considered energy region can be obtained with a bare Roper mass $m_R^0 \leq 4000$ MeV. As an example we show in Fig. 2 the results with $m_R^0 \approx 3000$ MeV and $\Gamma = 499.3$ MeV. There is a structure in the phase at energy near $E_L = 700$ MeV, and in the inelasticity at energy near $E_L = 820$ MeV. The resulting parameters are given in set 3 of tables 1 and 2. We therefore conclude that if the phase shifts of Arndt et al. [18] are correct, the bare mass of the Roper has to be unrealistically large within our model. It is difficult, if not impossible, to relate this model to the cloudy bag model description of the Roper.

There exists a different phase shift analysis by Karlsruhe-Helsinki group [19]. Its $P_{11}$ phase shifts have a more complicated structure. We have also made an attempt to fit this data. Again, we are not able to get a good fit unless a large bare Roper mass is allowed. One example is shown in Fig. 3. The fit can reproduce qualitatively the shoulder in the phase ($\delta$) and the irregular structure in inelasticity. However, the resulting mass of the bare Roper is also rather large $M_R \approx 2000$ MeV. The resulting parameters of this fit are listed in the set 4 of tables 1 and 2.

IV. SUMMARY and DISCUSSIONS

In summary, we have presented a simple extension of the two-channel
model of Pearce and Afnan [6] to investigate the $\pi N$ scattering in $P_{11}$ channel. The essential steps are to use Eqs.(2) and (3) to account for the inelasticity due to the decay of the $\Delta$ in the $\pi \Delta$ channel and to allow a pure phenomenological treatment of the non-pole mechanisms. The parameters of the vertex functions $f_{b,\pi B}^{(0)}$ and background interactions $u_{\pi B,\pi B'}$ are severely constrained by the pole conditions Eqs.(22), (25), (26) and (29). It is found that no good fit to the $P_{11}$ phase shifts can be obtained unless a very large value of the bare Roper mass is allowed.

The present work is just a step toward a detailed understanding of the Roper resonance. It only provides some qualitative information about the relative importance between the pole mechanisms and the background mechanisms. It appears to be difficult to accomodate the cloudy-bag model description of the Roper resonance within a two-channel ($\pi N \oplus \pi \Delta$) formulation of the $\pi N$ scattering up to about 1 GeV, even if additional "phenomenological" mechanisms are introduced.

The weak point of the present work is an incomplete treatment of the $\pi \pi N$ channel. In a recent amplitude analysis of $\pi N \to \pi \pi N$ reaction by Manley and Saleski[20], the partial width of $R \to \epsilon N$ ( $\epsilon$ represent a broad $\pi \pi$ continuum) is about 33 MeV which is about half of the 88 MeV of the $R \to \pi \Delta$ considered in this work. The decay of the Roper into a $\rho N$ channel, as considered by Pearce and Afnan[12], appears to be much less important in the considered energy region. Clearly an extension of either the present simplified model or the fully unitary model of Pearce and Afnan[12] to include this $\pi \pi$ continuum will be the next step to explore the dynamical origin of the Roper resonance. This of course poses a difficult challenge which can be met only by carrying out a much more extensive numerical work than we
have undertaken so far.

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Table 1. Parameters of the vertex $f^{(0)}_{\pi B, B}$ (Eqs. (31)-(33)) and $v_{\pi B, \pi B}$ (Eqs. (34)-(35))

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<td>$\lambda_{\pi\pi}$ (unitless)</td>
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<td>1.244</td>
<td>1.715</td>
<td>1.182</td>
</tr>
<tr>
<td>$\lambda_{\Delta\Delta}$ (unitless)</td>
<td>3.486</td>
<td>2.472</td>
<td>12.359</td>
<td>2.2999</td>
</tr>
</tbody>
</table>
Table 2. Solutions of the pole conditions defined by Eqs. (22), (25), (26), and (29).

<table>
<thead>
<tr>
<th></th>
<th>Set 1</th>
<th>Set 2</th>
<th>Set 3</th>
<th>Set 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_{\pi N, N} \text{ (fm}^{-3}\text{)}$</td>
<td>8.129</td>
<td>7.679</td>
<td>0.589</td>
<td>0.692</td>
</tr>
<tr>
<td>$M_{N}^{0} \text{ (fm}^{-1}\text{)}$</td>
<td>5.219</td>
<td>7.679</td>
<td>0.589</td>
<td>0.692</td>
</tr>
<tr>
<td>$M_{R}^{0} \text{ (fm}^{-1}\text{)}$</td>
<td>$\infty$</td>
<td>$\infty$</td>
<td>15.285</td>
<td>10.173</td>
</tr>
<tr>
<td>$Z_{N} \text{ (unitless)}$</td>
<td>0.965</td>
<td>0.952</td>
<td>0.777</td>
<td>0.094</td>
</tr>
<tr>
<td>$Z_{R} \text{ (unitless)}$</td>
<td>-</td>
<td>-</td>
<td>0.900</td>
<td>0.927</td>
</tr>
<tr>
<td>$\Gamma_{R} \text{ (MeV)}$</td>
<td>-</td>
<td>-</td>
<td>499.3</td>
<td>242.4</td>
</tr>
</tbody>
</table>
FIGURE CAPTIONS

Fig. 1 The fits (solid curves) to the πN phase shifts in P_{11} channel using the model with no bare Roper state. The dashed curves are the fits when the decay of the Δ in the πΔ channel is also neglected (i.e. setting Σ_{πΔ} = 0 in Eq. (3)). The data are from Ref. 18.

Fig. 2 The fits to the πN phase shifts in P_{11} channel using the full model defined in Section II. The data are from Ref. 18.

Fig. 3 The fits to the πN phase shifts in P_{11} channel using the full model defined in Section II. The data are from Ref. 19.

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Fig. 1
Fig. 2
Fig. 3