Discrete Anomaly and Dynamical Mass in $2+1$ dimensional $U(1)_V \times U(1)_A$ Model

Deog Ki Hong¹

Institute for Fundamental Theory, University of Florida

Gainesville, FL 32611, U.S.A.

and

Department of Physics, Pusan National University²

Pusan 609-735, Korea

Abstract

We note that in $(2+1)$ -dimensional gauge theories with even number of massless fermions, there is anomalous Z_2 symmetry if theory is regularized in a parity-invariant way. We then consider a parity invariant $U(1)_V \times U(1)_A$ model, which induces a mutual Chern-Simons term in the effective action due to Z_2 anomaly. The effect of the discrete anomaly is studied in the induced spin and in the dynamical fermion mass.

PACS numbers: 12.50.Lr, 11.15.Pg, 11.30.Hv, 12.20.Ds

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

¹E-mail address: dkhong@hyowon.cc.pusan.ac.kr

²Permanent address

MASTER

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

The symmetry of classical lagrangians often breaks down upon quantization. A well-known example is the axial anomaly in quantum electrodynamics [1], where any gauge invariant regularization necessarily breaks the axial symmetry. On the other hand the irreducible spinor representation of Lorentz group in odd dimensions does not have γ_5 -like object. Namely, there is no matrix anti-commuting with all γ matrices in odd dimensions. For instance, in three dimensions the irreducible spinor representation is two dimensional and the product of all γ -matrices $\Gamma_0\Gamma_1\Gamma_2 = 1$. Therefore there is no axial anomaly in odd dimensions. But, a discrete symmetry might be anomalous in odd dimensional gauge theories due to the incompatibility of the gauge-invariant regulator with the discrete symmetry. The anomalous discrete symmetry is realized as an induced quantum number for the vacuum [2].

Redlich [3] has shown that parity is anomalous in $(2+1)$ -dimensional $SU(N)$ gauge theories since the parity invariant regularization results in an effective action, which is not invariant under large gauge transformations, because $\Pi_3(SU(N)) = \mathbb{Z}$ for $N \geq 2$, and thus one needs a parity-violating Chern-Simons term to recover the gauge invariance in the effective action. For the abelian case, parity is anomalous in perturbation theory [3] and for time-independent gauge fields the parity anomaly can be understood as the $(1+1)$ D axial anomaly [4].

However, when the number of fermions is even, one can find a parity-preserving Pauli-Villars regulator of four-component fermions [5, 6]. Then, parity is no longer anomalous and the Chern-Simons term is not induced in the effective Lagrangian. In this paper, we note that for even number of two-component fermions there is another anomalous discrete symmetry, which is not parity, and we study the effect of this discrete anomaly in $U(1)_V \times U(1)_A$ model. This model itself is also interesting since it might be realized in parity-invariant planar superconductivity [7].

The model is described by

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} + \sum_{i=1}^N \bar{\Psi}_i \gamma^\mu D_\mu \Psi_i, \quad (1)$$

where Ψ_i is a four-component spinor made of a pair of two-component spinors as

$$\Psi_i(x) = \begin{pmatrix} \psi_i(x) \\ \sigma^3 \psi_{N+i}(x) \end{pmatrix} \quad (2)$$

(We consider even number of two-component massless spinors.) The covariant derivative $D_\mu = \partial_\mu - ieA_\mu - ig\gamma_5 B_\mu$ and the field strength tensors $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, $B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$. The gamma matrices for the four-component spinors are defined as

$$\gamma^0 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}, \quad \gamma^i = \begin{pmatrix} 0 & -\sigma^i \\ \sigma^i & 0 \end{pmatrix}, \quad \gamma^5 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}. \quad (3)$$

Lagrangian (1) has a global $SU(N) \times SU(N)$ symmetry whose Nöther currents are

$$J_a^\mu = \bar{\Psi} \gamma^\mu T_a \Psi, \quad J_{a5}^\mu = \bar{\Psi} \gamma^\mu T_a \gamma^5 \Psi \quad (4)$$

where T_a 's are the generators for $SU(N)$. Note also that we impose parity and a discrete Z_2 symmetry to forbid mass terms for fermions and gauge fields in the Lagrangian (1). By forbidding the mass terms for fermion, we have $U(1)_3 \times U(1)_{35}$ “chiral symmetry”, generated by $i\gamma^3$ and $\gamma^3\gamma^5$. This “chiral symmetry”, which mimics (3+1)-dimensional chiral symmetry, is not really chiral symmetry but a part of the flavor symmetry $U(2)$ for the two two-component spinors constituting four-component spinors. Parity P is a space-time transformation, $(t, x, y) \mapsto (t, -x, y)$, under which fermion and gauge fields transform as

$$\Psi(t, x, y) \xrightarrow{P} \Psi'(t, -x, y) = -\gamma^1 \gamma^5 \Psi(t, x, y) \quad (5)$$

$$(A_0(x), A_1(x), A_2(x)) \xrightarrow{P} (A_0(x), -A_1(x), A_2(x)) \quad (6)$$

$$(B_0(x), B_1(x), B_2(x)) \xrightarrow{P} (-B_0(x), B_1(x), -B_2(x)). \quad (7)$$

We see that A_μ and B_μ transform like an ordinary vector and an axial vector, respectively. Under Z_2 transformation,

$$A_\mu \xrightarrow{Z_2} A_\mu, \quad B_\mu \xrightarrow{Z_2} -B_\mu, \quad \Psi \xrightarrow{Z_2} i\gamma^3\Psi. \quad (8)$$

Z_2 tantamounts to the charge conjugation for $U(1)_A$, the “axial” coupling.

In the perturbation of (2+1)-dimensional gauge theories, only the vacuum polarization and the triangle graph are ultraviolet divergent. One may regularize the divergences with the Pauli-Villars regulator. One has then two choices for the regulator mass for each flavor i . One is parity-invariant but Z_2 -violating ($M\bar{\Psi}_i\gamma^3\Psi_i$) and the other is Z_2 -invariant but parity-violating ($M\bar{\Psi}_i\gamma^3\gamma_5\Psi_i$). Therefore either parity or Z_2 (but not both) is anomalous, namely PZ_2 is always anomalous.

Integrating out the fermions, one gets $-i\text{Tr}\ln i\cancel{D}$ in the effective action. In the perturbation theory, if one uses the parity-invariant Pauli-Villars regulator, one gets an effective Lagrangian given by

$$\mathcal{L}_{\text{eff}} = eg \frac{N}{2\pi} \frac{M}{|M|} \epsilon_{\mu\nu\lambda} B^\mu F^{\nu\lambda} + \dots \quad (9)$$

where $\dots$ denotes the higher order terms and M is the regulator mass signifying the Z_2 anomaly. The leading term in the effective Lagrangian (9) can be obtained from the Feynman diagram in Fig. 1. This term is similar to the Chern-Simons term but it couples two different gauge fields. We call this a mutual Chern-Simons term. It leads to mutual fractional statistics and is believed to be realized in a layered Hall system exhibiting a filling factor of even denominator [8]. One can see easily that the mutual Chern-Simons term in $U(1)_V \times U(1)_A$ theory is the only term in perturbation theory which breaks Z_2 in the effective action. Had we chosen Z_2 -invariant Pauli-Villars regulator, we would have gotten Chern-Simons terms for each gauge fields breaking parity.

The radiative generation of the mutual Chern-Simons term is also noted in references [7], where they argued that $U(1)_V \times U(1)_A$ arises in a model of dynamical holes in a planar quantum antiferromagnet in the large spin and small doping limit. But, here, we point out the origin of the mutual Chern-Simons term in (9) is Z_2 anomaly and we argue that one can not avoid it in parity-invariant theories in 2+1 dimensions because the parity-invariant regulator necessary breaks Z_2 .

Due to the mutual Chern-Simons term, fermions get a fractional spin $s = \frac{1}{N}$ by the usual Aharonov-Bohm effect [9]. At long distances a particle carrying unit (axial) charge g will look like a localized vortex of magnetic flux $\Phi = 2\pi/eN$ (modulo a sign which is not important here) for a particle of unit (vector) charge e . Therefore a fermion orbiting around another fermion will get a Aharonov-Bohm phase $e\Phi$ and thus the induced spin $s = e\Phi/2\pi = \frac{1}{N}$. The spin of the four-component spinors is invariant under parity:

$$s = \int d^2x \Psi^\dagger \frac{i}{4} [\gamma_1, \gamma_2] \Psi \xrightarrow{P} \int d^2x (\gamma_1 \gamma_5 \Psi)^\dagger \frac{i}{4} [\gamma_1, \gamma_2] (\gamma_1 \gamma_5 \Psi) = s \quad (10)$$

The induced spin for a four-component fermion therefore does not break parity. This is not the case for the two-component fermion which can have only one direction for spin, while the four-component fermion has two two-component spinors which have spins of opposite direction. The parity-violating Chern-Simons term affects the dynamical generation of parity-even mass for fermion in a rather interesting way [10, 11]. It tends to break parity maximally. Namely, it reduces both of critical flavor number for mass generation and the magnitude of mass itself. We study how the (radiatively generated) mutual Chern-Simons term affects the dynamical generation of parity-even fermion mass. According to a general theorem by Vafa and Witten [12, 13], parity-odd fermion mass cannot be generated dynamically in a parity-invariant $U(1)_V \times U(1)_A$ model. We use the $1/N$ expansion, since it not only gives a systematic way of treating nonperturbative phenomena but also softens

the IR divergences of perturbative three-dimensional gauge theories [14]. To have a well-defined field theory in large flavor (N) limit, we keep $\alpha_V \equiv e^2 N$ and $\alpha_A \equiv g^2 N$ finite as N goes to infinity.

In leading order in $1/N$ expansion, the gauge-boson propagators get contribution from the fermion loops and they get mixed. They are

$$D_{\mu\nu}^{AA}(p) = \frac{-i(g_{\mu\nu} - p_\mu p_\nu / p^2)}{p^2} \frac{1 + \frac{1}{\pi} \alpha_A F}{(1 + \frac{1}{\pi} \alpha_A F)(1 + \frac{1}{\pi} \alpha_V F) + \frac{m^2}{4\pi^2} \alpha_A \alpha_V (G/p^2)^2}, \quad (11)$$

$$D_{\mu\nu}^{AB}(p) = \frac{\epsilon_{\mu\nu\lambda} p^\lambda}{p^2} \frac{\frac{m}{2\pi} \kappa G / p^2}{(1 + \frac{1}{\pi} \alpha_A F)(1 + \frac{1}{\pi} \alpha_V F) + \frac{m^2}{4\pi^2} \kappa^2 (G/p^2)^2}, \quad (12)$$

$$D_{\mu\nu}^{BA}(p) = D_{\mu\nu}^{AB}(p), \quad (13)$$

$$D_{\mu\nu}^{BB}(p) = D_{\mu\nu}^{AA}(p)(\alpha_A \leftrightarrow \alpha_V), \quad (14)$$

where the superscript AB means gauge fields A_μ propagate to gauge fields B_μ etc. and $\kappa = \sqrt{\alpha_A \alpha_V}$. The functions F and G are

$$F(m^2, p^2) = \int_0^1 dx \frac{x(1-x)}{\sqrt{m^2 - x(1-x)p^2}} \quad (15)$$

$$G(m^2, p^2) = \int_0^1 dx \frac{1}{\sqrt{m^2 - x(1-x)p^2}}, \quad (16)$$

which come from the one-loop vacuum polarization. To calculate the vacuum polarization, we need to know the exact form of the fermion self-energy, which requires full solutions to the Dyson-Schwinger equations. As an approximation, we take a constant mass for the self-energy, $\Sigma(p) = m\gamma^3$, which must be very small compared to the scale of the theory, α_A or α_V , since it is generated by a nonperturbative $1/N$ effect, and it must be also parity-even. Note also that by the $U(1)_3 \times U(1)_{35}$ “chiral symmetry” one can always rotate the fermion self energy to be $\Sigma(p) = \gamma_3 \Sigma_3(p)$, where $\Sigma_3(p)$ is a function proportional to the unit matrix.

In $1/N$ perturbation, the full vertex function can be expanded as

$$\Gamma_\mu = \gamma_\mu + O\left(\frac{1}{N}\right). \quad (17)$$

For the leading order, we take $\Gamma_\mu = \gamma_\mu$. Then, the Ward-Takahashi identity requires the wave-function renormalization constant to be 1 for a consistent $1/N$ expansion. The Dyson-Schwinger gap equation (Fig. 2) in Euclidean notation is then

$$\begin{aligned}\gamma_3 \Sigma_3(p) = & \frac{\alpha_V}{N} \int \frac{d^3 k}{(2\pi)^3} D_{\mu\nu}^{AA}(p-k) \gamma_\nu \frac{\not{k} - \gamma_3 \Sigma_3(k)}{k^2 + \Sigma_3^2(k)} \gamma_\mu \\ & + \frac{\kappa}{N} \int \frac{d^3 k}{(2\pi)^3} \left(D_{\mu\nu}^{AB} \gamma_\mu \frac{\not{k} - \gamma_3 \Sigma_3(k)}{k^2 + \Sigma_3^2(k)} \gamma_5 \gamma_\nu + D_{\mu\nu}^{BA} \gamma_5 \gamma_\mu \frac{\not{k} - \gamma_3 \Sigma_3(k)}{k^2 + \Sigma_3^2(k)} \gamma_\nu \right) \\ & + \frac{\alpha_A}{N} \int \frac{d^3 k}{(2\pi)^3} D_{\mu\nu}^{BB}(p-k) \gamma_5 \gamma_\nu \frac{\not{k} - \gamma_3 \Sigma_3(k)}{k^2 + \Sigma_3^2(k)} \gamma_5 \gamma_\mu.\end{aligned}\quad (18)$$

Since the dynamically generated mass m is exponentially small compared to the scale, α_A and α_V , and the (2+1)-dimensional gauge theories are superrenormalizable, one can think of m as an infrared cutoff and α_V (or α_A) as a ultraviolet cutoff. For the momentum p in $m < p < \alpha_V$ or α_A , one can simplify the expression for the vacuum polarization tensor. Namely, for momentum for this range,

$$F(m^2, p^2) \simeq \frac{\pi}{8|p|}, \quad G(m^2, p^2) \simeq \frac{\pi}{|p|}, \quad (19)$$

and the gauge fields propagators in Euclidean space are

$$D_{\mu\nu}^{AA}(p) \simeq \frac{g_{\mu\nu} - p_\mu p_\nu / p^2}{p^2} \frac{|p|}{8\alpha_V}, \quad (20)$$

$$D_{\mu\nu}^{BB}(p) \simeq \frac{g_{\mu\nu} - p_\mu p_\nu / p^2}{p^2} \frac{|p|}{8\alpha_A}, \quad (21)$$

$$D_{\mu\nu}^{AB}(p) \simeq 32 \frac{m}{\kappa} \frac{\epsilon_{\mu\lambda\nu} p^\lambda}{|p|^3}. \quad (22)$$

We see that the propagator $D_{\mu\nu}^{AB}$ is proportional to m/κ while the other propagators are the ones for $m \rightarrow 0$. Though by dimensional counting $D_{\mu\nu}^{AB}$ is quite suppressed compared to other propagators, it is not clear that one can neglect the second term in (18). However, if one analyzes the Dyson-Schwinger equation, keeping the second term, one finds at the end that keeping the second term is equivalent to adding a constant mass to $\Sigma_3(p)$. Therefore it is not consistent with the massless limit

approximation ($m \rightarrow 0$) taken for $D_{\mu\nu}^{AA}$ and $D_{\mu\nu}^{BB}$, if one keeps the second term in (18) which is proportional to m .

With the second term dropped, the Dyson-Schwinger equation (18) becomes exactly same as that of pure QED_3 analyzed by many other people [15, 16], except that now there are two copies of gauge fields. The analysis goes parallel to the analysis in [16]. Here we present the result in a slightly different fashion, following the analysis by Cohen and Georgi for (3+1)-dimensional gauge theories in the ladder approximation [17], where the physical meaning of constants appearing in the asymptotic behavior of the fermion self energy is identified with the operators in the operator product expansion of the fermion two-point function.

Taking the trace over γ matrices after multiplying $-\gamma_3$ and performing the angular integral in (18), we get

$$\begin{aligned}\Sigma_3(p) = & \frac{\alpha_V}{2\pi^2 N p} \int dk \frac{k \Sigma_3(k)}{k^2 + \Sigma_3^2(k)} \ln \frac{k + p + \alpha_V/8}{|k - p| + \alpha_V/8} \\ & + \frac{\alpha_A}{2\pi^2 N p} \int dk \frac{k \Sigma_3(k)}{k^2 + \Sigma_3^2(k)} \ln \frac{k + p + \alpha_A/8}{|k - p| + \alpha_A/8}.\end{aligned}\quad (23)$$

As was done in [16], we expand the logarithm in power series for $p \ll \alpha$ (here $\alpha \simeq \alpha_V$ or α_A is a typical scale of the theory) to get

$$\Sigma_3(p) = \frac{8}{\pi^2 N p} \int dk \frac{k \Sigma_3(k)}{k^2 + \Sigma_3^2(k)} (p + k - |p - k|). \quad (24)$$

Differentiating the integral equation (24), we obtain

$$\Sigma'_3(p) = -\frac{16}{\pi^2 N} \int_0^p \frac{dk}{p^2} \frac{k^2 \Sigma_3(k)}{k^2 + \Sigma_3^2(k)}, \quad (25)$$

where ' denotes differentiation with respect to p . We see from (25)

$$\lim_{p \rightarrow 0} p^2 \Sigma'_3(p) = 0 \quad (26)$$

which serves as an infrared boundary condition for $\Sigma_3(p)$. On the other hand, the

equation we get by differentiating after multiplying p

$$(p\Sigma_3)' = -\frac{16}{\pi^2 N} \int_p^\alpha dk \frac{k\Sigma_3(k)}{k^2 + \Sigma_3^2(k)} \quad (27)$$

gives an ultraviolet boundary condition

$$\lim_{p \rightarrow \alpha} (p\Sigma_3)' = 0 \quad (28)$$

Multiplying by p^2 and differentiating once again we obtain

$$p^2 \Sigma_3'' + 2p \Sigma_3' + \frac{r}{4} \frac{p^2 \Sigma_3}{p^2 + \Sigma_3^2} = 0 \quad (29)$$

where $r = N_c/N$ with $N_c = 64/\pi^2$. For small p , the solution to (29), which is consistent with the boundary condition (26) is

$$\Sigma_3(p) = m_C \quad \text{for} \quad p \ll \Sigma_3(p), \quad (30)$$

and for large p ($p \gg \Sigma_3(p)$)

$$\Sigma_3(p) = m_R \left(\frac{p}{\mu} \right)^{-\epsilon} + \frac{\kappa}{p} \left(\frac{p}{\mu} \right)^\epsilon \quad (31)$$

where

$$\epsilon = \frac{1 - \sqrt{1-r}}{2}. \quad (32)$$

and μ is the renormalization point. As was shown in [17], the parameters m_R and κ correspond to a renormalized mass and a fermion condensate $\langle \bar{\Psi} \gamma_3 \Psi \rangle$, respectively. If $N > N_c$, one finds that m_C has to be zero in the chiral limit ($m_R \rightarrow 0$), and thus $\Sigma_3(p) = 0$. Dynamical mass is not generated and the trivial vacuum is the only solution [17]. When $N < N_c$, the solution to (29) is

$$\Sigma_3(p) = \frac{A}{\sqrt{p}} \cos \left(\sqrt{r-1} \ln(p/\mu) + \phi \right), \quad (33)$$

where A and ϕ are arbitrary constants. We see that the operators $m_R \mathbf{1}$ and $\bar{\Psi} \gamma_3 \Psi$ are coalesced due to strong interaction when $N < N_c$ and can not be distinguished

by the operator product expansion. From the Dyson-Schwinger equation (18), we know that for high momentum $p > \alpha$

$$\Sigma_3(p) \simeq \frac{C}{p^2} \quad (34)$$

At $p \simeq \alpha$ the solution (33) for $p < \alpha$ should be smoothly connected to the solution (34) for $p > \alpha$. This condition is given by the boundary condition (28) at $p = \alpha$. Taking the renormalization point to be $\mu \simeq m_C$, we get

$$m_C = \alpha e^{-\frac{\pi-\phi}{\sqrt{N_c/N-1}}}, \quad (35)$$

where $N_c = 64/\pi^2$. We see that the dynamical mass generation in $U(1)_V \times U(1)_A$ is precisely same as pure QED_3 except that the critical flavor is now doubled.

Since Z_2 is anomalous, one may start with a bare mutual Chern-Simons term in this $U(1)_V \times U(1)_A$ model:

$$\mathcal{L}' = \mathcal{L} + \frac{\kappa_0}{2\pi} \epsilon_{\mu\nu\lambda} B^\mu F^{\nu\lambda}. \quad (36)$$

Then the Z_2 -violating (but parity-even) fermion mass will be generated radiatively in perturbation theory. However, we can still ask whether this Z_2 -violating mutual Chern-Simons term will affect the dynamical generation of parity-even (namely Z_2 violating) fermion mass. (The parity-odd mass is not generated, even in nonperturbative analysis, whether the mutual Chern-Simons term is present or not.) The analysis is again done by solving the Dyson-Schwinger equation in $1/N$ expansion. For $m \ll p \ll \alpha$ or κ_0 , the Dyson-Schwinger equation will look same as before except now the propagator $D_{\mu\nu}^{AB}$ is no longer negligible:

$$D_{\mu\nu}^{AB}(p) = \frac{\epsilon_{\mu\lambda\nu} p^\lambda}{p^2} \frac{\frac{\pi}{2\kappa}}{\left(1 + \frac{\alpha_A}{8|p|}\right) \left(1 + \frac{\alpha_V}{8|p|}\right)}. \quad (37)$$

With $D_{\mu\nu}^{AB}$ the Dyson-Schwinger equation becomes, after performing the angular integration,

$$\Sigma_3(p) = \frac{8}{\pi^2 N p} \int dk \frac{k \Sigma_3(k)}{k^2 + \Sigma_3^2(k)} (p + k - |p - k|) - \frac{1}{N\pi} \frac{\kappa}{\kappa_0} \int dk \frac{k^2}{k^2 + \Sigma_3^2(k)}, \quad (38)$$

where we keep only the leading term in p/α . We see that the mutual Chern-Simons term contributes to $\Sigma_3(p)$ by a constant, which is same as having a bare mass term in the Lagrangian. Therefore, the leading contribution of the bare mutual Chern-Simons term is radiative generation of Z_2 violating (parity-even) fermion mass. It does not affect the nonperturbative generation of fermion mass.

If one transforms the gauge fields into new ones as

$$A_\mu = a_\mu + b_\mu, \quad B_\mu = a_\mu - b_\mu, \quad (39)$$

the mutual Chern-Simons term becomes

$$\frac{\kappa_0}{2\pi} \epsilon_{\mu\nu\lambda} B^\mu F^{\nu\lambda} = \frac{\kappa_0}{2\pi} \epsilon_{\mu\nu\lambda} a^\mu a^{\nu\lambda} - \frac{\kappa_0}{2\pi} \epsilon_{\mu\nu\lambda} b^\mu b^{\nu\lambda}, \quad (40)$$

where $a^{\nu\lambda} = \partial^\nu a^\lambda - \partial^\lambda a^\nu$ and $b^{\nu\lambda} = \partial^\nu b^\lambda - \partial^\lambda b^\nu$. And the covariant derivative becomes

$$D_\mu = \partial_\mu - \frac{e + g\gamma_5}{2} a_\mu - \frac{e - g\gamma_5}{2} b_\mu. \quad (41)$$

The gauge fields a_μ and b_μ decouple at tree level, but they get coupled through fermion loops. When $e = g$, a_μ and b_μ become the gauge fields for $U(1)_L$ and $U(1)_R$, generated by $(1 + \gamma_5)/2$ and $(1 - \gamma_5)/2$, respectively. The upper two-component spinor has $U(1)_L$ charge e but no $U(1)_R$ charge and the lower two-component spinor has $U(1)_R$ charge e but no $U(1)_L$ charge. They are completely decoupled. In this case $U(1)_V \times U(1)_A$ model is just two copies of QED_3 with a Chern-Simons term of opposite sign and N two-component spinors. They are related by parity. Under the parity, a_μ transforms to b_μ , the upper two-component spinor in a four-component spinor transforms to the lower two-component spinor, and vice versa. The symmetry is but still $U(N) \times U(N) \times P$. One interesting is that, when fermion gets dynamical mass, $U(N) \times U(N)$ breaks down to $U(N/2) \times U(N/2) \times U(N/2) \times U(N/2)$ for even N , which is shown to occur in $1/N$ expansion when $N < N_c/[1 + (16\kappa_0/\alpha)^2]$ with $\alpha = e^2 N$ [10].

In conclusion, we see that for an even number of two-component fermions in $(2+1)$ -dimensional gauge theories Z_2 is anomalous if one regularizes theory in a parity-invariant way. Due to Z_2 anomaly a parity invariant $U(1)_V \times U(1)_A$ theory induces a mutual Chern-Simons term in the effective action, which leads to fractional spin to fermions in the theory. But, the radiatively generated mutual Chern-Simons term does not affect the dynamical generation of fermion mass at least in the leading order in $1/N$ expansion. Fermions get dynamical mass when $N < 64/\pi^2$ as if we have two copies of three dimensional QED . When a bare mutual Chern-Simons term is added, Z_2 violating fermion mass is generated radiatively but the nonperturbative generation of fermion mass does not get affected.

Acknowledgments

This work was supported in part by the Korea Science and Engineering Foundation through SRC program of SNU-CTP, by NON DIRECTED RESEARCH FUND, Korea Research Foundation, and also by Basic Science Research Program, Ministry of Education, 1994 (BSRI-94-2413). The author is grateful to Prof. P. Ramond for reading the manuscript carefully and he would like to thank the high energy theory group at the Institute of Fundamental Theory, University of Florida for its support and warm hospitality.

References

- [1] S. Adler, Phys. Rev. **177**, 2426 (1969); J. S. Bell and R. Jackiw, N. Cimento **60A**, 47 (1969).
- [2] J. Goldstone and F. Wilczek, Phys. Rev. Lett. **47**, 986 (1981); R. Jackiw and C. Rebbi, Phys. Rev. **D13**, 3398 (1976).
- [3] A. N. Redlich, Phys. Rev. Lett. **52**, 18 (1984); Phys. Rev. **D29**, 2366 (1984).
- [4] A. J. Niemi and G. W. Semenoff, Phys. Rev. Lett. **51**, 2077 (1983).
- [5] R. Jackiw and S. Templeton, Phys. Rev. D **23**, 2291 (1981); S. Deser, R. Jackiw, and S. Templeton, Ann. Phys. (NY) **140**, 372 (1982); (**E**) **185**, 406 (1988); Phys. Rev. Lett. **48**, 975 (1982); (**C**) **59**, 1981 (1987); G.W. Semenoff and L.C.R. Wijewardhana, *ibid.* **63**, 2633 (1989).
- [6] D. K. Hong and S. H. Park, Phys. Rev. **D49**, 5507 (1994).
- [7] N. Dorey and N. Mavromatos, Nucl. Phys. **B386**, 614, (1992); Phys. Lett. **B250**, 107 (1990); G. W. Semenoff and N. Weiss, Phys. Lett. **B250** 117 (1990); T. Banks and J. D. Lykken, Nucl. Phys. **B336**, 500 (1990).
- [8] F. Wilczek, Phys. Rev. Lett. **69**, 132 (1992).
- [9] F. Wilczek, in *Fractional Statistics and Anyon Superconductivity*, edited by F. Wilczek (World Scientific, Singapore 1990).
- [10] D. K. Hong and S. H. Park, Phys. Rev. D **47**, 3651 (1993).
- [11] T. Ebihara, T. Iizuka, K.-I. Kondo and E. Tanaka, Chiba University Preprint CHIBA-EP-77 (unpublished), hep-ph/9404361; K.-I. Kondo and P. Maris, Chiba/Nagoya Univ. Preprint, CHIBA-EP-84/DPNU-94-33, hep-ph/9408210;

K.-I. Kondo and P. Maris, Chiba/Nagoya Univ. Preprint, CHIBA-EP-85/DPNU-94-51, hep-ph/9501280.

- [12] C. Vafa and E. Witten *Nucl. Phys.* **B234**, 173 (1984).
- [13] S. H. Park and Y. Shamir, *Phys. Rev.* **D48**, 3352 (1991).
- [14] T. Appelquist and R. D. Pisarski, *Phys. Rev.* **23**, 2305 (1981); R. Jackiw and S. Templeton, *Phys. Rev.* **23**, 2291 (1981).
- [15] T. Appelquist, M. Bowick, E. Cohler, and L. C. R. Wijewardhana, *Phys. Rev. Lett.* **55**, 1715 (1985); T. Appelquist, M. Bowick, D. Karabali, and L. C. R. Wijewardhana, *Phys. Rev. D* **33**, 3704 (1986); R. Pisarski, *ibid.* **D 29**, 2423 (1984); T. Appelquist, M. Bowick, D. Karabali, and L. Wijewardhana, *ibid.* **D 33**, 3774 (1986); D. Boyanovsky, R. Blankenbecler, and R. Yahalom, *Nucl. Phys.* **B270**, 483 (1986); S. Rao and R. Yahalom, *Phys. Rev.* **D34**, 1194 (1986); K. Stam, *ibid.* **34**, 2517 (1986).
- [16] T. Appelquist, D. Nash and L.C.R. Wijewardhana, *Phys. Rev. Lett.* **60**, 2575 (1988); D. Nash, *ibid.* **62**, 3024 (1989); T. Appelquist and D. Nash, *ibid.* **64**, 721 (1990).
- [17] A. Cohen and H. Georgi, *Nucl. Phys.* **B314**, 7 (1989).

Figure Captions

Figure 1: Z_2 anomaly. The solid lines denote fermions, the wavy lines gauge fields.

Figure 2: Dyson-Schwinger gap equation. The (bold) solid lines denote (full) fermion propagator, the wavy lines gauge fields.