*A Numerical Model of Hydro-Thermo-
Mechanical Coupling in a Fractured Rock Mass*

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*A Numerical Model of Hydro-Thermo-
Mechanical Coupling in a Fractured Rock Mass*

Kathleen Marie Bower

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A NUMERICAL MODEL OF HYDRO-THERMO-MECHANICAL COUPLING IN A FRACTURED ROCK MASS

by

Kathleen Marie Bower

Abstract

Coupled hydro-thermo-mechanical codes with the ability to model fractured materials are used for predicting groundwater flow behavior in fractured aquifers containing thermal sources. The potential applications of such a code include the analysis of groundwater behavior within a geothermal reservoir. The capability of modeling hydro-thermo systems with a dual porosity, fracture flow model has been previously developed in the finite element code, FEHM. FEHM has been modified to include stress coupling with the dual porosity feature. FEHM has been further developed to implicitly couple the dependence of fracture hydraulic conductivity on effective stress within two dimensional, saturated aquifers containing fracture systems. The cubic law for flow between parallel plates was used to model fracture permeability. The Barton-Bandis relationship was used to determine the fracture aperture within the cubic law. The code used a Newton Raphson iteration to implicitly solve for six unknowns at each node.

Results from a model of heat flow from a reservoir to the moving fluid in a single fracture compared well with analytic results. Results of a model showing the increase in fracture flow due to a single fracture opening under fluid pressure compared well with analytic results. A hot dry rock, geothermal reservoir was modeled with realistic time steps indicating that the modified FEHM code does successfully model coupled flow problems with no convergence problems.

1.0 Introduction

Fluid flow in fractured reservoirs is of interest in applications where fracture permeability is influenced by temperature, fluid pressure, or changing stress conditions of the medium. One such application is the geothermal hot dry rock reservoir (Richards, *et.al.*, 1994); cool water is pumped through an injection well to saturated rock at great temperatures and great depth. The hydraulic pressure opens pre-existing fractures in the rock which increases the reservoir permeability. The water acquires thermal energy from the rock and is then extracted from a production well. The acquired heat is usable as a source of energy. As the thermal energy is extracted, the rock cools down and contracts, opening the fractures further. If the fracture permeability is large, the fluid moves through the reservoir removing thermal energy faster than it can be replenished by conduction through the rock mass. The temperature of the water at the production well decreases and extracting usable energy is more difficult. At some point the reservoir is no longer productive. Thus the transient behavior of the fluid flow and energy transfer in the fracture system needs to be quantified in order to predict long term hot dry rock reservoir performance.

Dual porosity finite element models (described in Section 3.0) have been developed to model fluid flow through fracture systems. Current models are inadequate for some applications. These involve the introduction of a change in stress or temperature to the groundwater system of interest. The current research modified a dual porosity model to describe behavior in reservoirs with coupled thermal, mechanical and hydraulic phenomena. Besides hot dry rock geothermal systems, other applications could be the burial of heat producing radioactive waste in a nuclear waste repository, oil production in fractured systems, and ground

subsidence when using traditional methods of extracting geothermal energy (Miller, *et.al.*, 1980).

Dual porosity models are based on the concept that fractures have much larger hydraulic conductivities and more rapid fluid flow than the rock matrix (Douglas and Arbogast, 1990). The fracture system and rock matrix can be considered as isolated systems that interact with each other. In a dual porosity model, all the fractures are grouped into one continuum while the rock matrix blocks are grouped into another continuum whose configuration is identical to and occupies the same physical space as that of the fracture continuum. The two groups are mathematically coupled by a fluid mass transfer function based on geometry and flow properties within the model.

In a fractured, saturated porous medium the hydraulic conductivity of the fractures will have a strong influence on the groundwater system. The fracture hydraulic conductivity is strongly dependent on the fracture aperture which is influenced in turn by the effective stress across the fracture. It is important to model this behavior when determining the coupled effect in a fractured groundwater system. As the temperature of the fluid and rock changes, the stress across the fractures changes. This in turn causes deformation of the fractures which can affect the conductivity of the fractures and influence the fluid pressure. As the fluid pressure changes, the effective rock stress and the influence of the thermal source may change.

Codes used to model fluid behavior in fractured reservoirs where thermal changes are expected should have the thermal, hydraulic, and mechanical behavior of the fractured rock fluid mass coupled. This means that the equations of fluid mass

conservation, energy conservation, and force balance are solved simultaneously. Often codes are written with the fluid mass conservation, energy conservation, and force balance equations solved in a staggered fashion (Millard, *et.al.*, 1995); that is, the results of the fluid mass conservation equation are used as input into the energy conservation equation whose results are used in the force balance equation at every time step. This solution technique is known as an explicit solution and requires small time steps to keep error small and achieve convergence.

In the implicit method the equations for a node at any time step are solved simultaneously in terms of the unknown values at surrounding nodes at that same time step. This results in smaller error and larger time steps may be used. Thus an implicit solution method is more useful in modeling coupled situations involving long time scales.

Table 1 lists some existing numerical codes that model coupling in fractured rock. Noorishad and Tsang (1990) have developed ROCMAS II, a finite element program with hydro-thermo-mechanical coupling for fractured porous media. ROCMAS II calculates the fracture aperture as a function of stress. The fractures must be modeled explicitly as linear elements. Swenson (1994) has developed a similar code, GEOCRACK, which models implicit coupling of mechanical and hydraulic phenomena with explicit coupling of thermal phenomenon. GEOCRACK uses a discrete fracture network and ignores fluid storage in the rock matrix (Sprecker, 1994). The DECOVALEX project (Jing, *et.al.*, 1995) is an international cooperative research project for studies of coupled processes in hard rocks. Within the project, code results are compared using numerical codes that model hydro-thermo-mechanical coupled processes. None of the codes used for this project use a dual

porosity feature to model these processes and thus none capture fracture behavior accurately without the need to explicitly define fractures within the mesh. Explicitly defining fractures in the model results in cumbersome description of fracture geometry in model input and results in greater solution time requirements due to the greater number of nodes used.

Table 1: Properties of fractured reservoir coupled codes

Model	Type	Coupled Behavior	Solution Scheme	Joint Model	Comments
FEHM (old) (Zyvoloski <i>et.al.</i> , 1991)	Finite Element	Thermal Hydraulic Mechanical	Implicit	Normal -Gangi Shear - none	1. Dual porosity or dual porosity/dual permeability 2. Permeability of joints is constant through all time steps 3. 3-dimensional capabilities
UDEC (Hart, 1991)	Discrete Element	Thermal OR Hydraulic Mechanical	Explicit	Normal - Barton-Bandis Shear - Coulomb slip	1. No flow or storage in the porous medium 2. Joints modeled discretely 3. Heat flow by conduction only 4. Generally relies on very small time steps for stability but contains a limited implicit formulation to allow fluid flow time steps of up to a few days 5. 2-dimensional
ABAQUS (Gosh, <i>et.al.</i> , 1993)	Finite Element	Thermal OR Hydraulic Mechanical	Explicit	Shear - Coulomb slip	1. Equivalent continuum model
GEOCRACK (Swenson, <i>et.al.</i> , 1994)	Finite Element	Thermal Hydraulic Mechanical	Implicit Hydraulic and Mechanical, Explicit Thermal	Normal - Gangi Shear - none	1. No flow or storage in porous medium 2. Joint properties assigned randomly 3. Joints modeled as discrete fracture network 4. 2-dimensional capabilities
ROCK-MASSII (Noorishad and Tsang, 1990)	Finite Element	Hydraulic Mechanical Thermal		Normal - Goodman Shear - Lanyir and Archambault	1. Joints modeled discretely 2. Employs options of a mixed Newton-Raphson solution scheme or direct iteration

A finite element code (FEHM) was developed at Los Alamos National Laboratory that solves for heat and fluid mass transfer using an implicitly coupled solution algorithm as well as solving for solute transport (Zyvoloski *et.al.*,1991). FEHM also had the capabilities of modeling fracture flow using the dual porosity and the dual

porosity/dual permeability methods. The code had been further developed to provide capabilities of coupled stress with heat and fluid mass transfer though not with the dual porosity feature (Kelkar and Zyvoloski, 1990). FEHM currently does not allow the fracture hydraulic conductivity to change as the effective stress across the fracture changes.

There is a need for a numerical code for use in modeling groundwater flow in fractured systems under the influence of thermal loading. In order to model large times the code should be coupled and fully implicit. Such a code could efficiently model the heat, fluid mass, and stress coupled effects within the fractured porous medium using a dual porosity capability. This can be done by modifying the hydraulic conductivity of the fracture so that it is a function of the changing effective stress across the fracture.

In this research project, FEHM was modified to incorporate these effects. The modifications extended the stress coupling capability to the dual porosity feature of FEHM. The modified code has allowed the fracture permeability within the dual porosity system to change as the effective stress in the surrounding matrix changes. The modifications couple hydraulic, thermal, and mechanical behavior in fracture systems using a dual porosity feature. This modified FEHM can model coupled fractured systems without the difficulty of defining each fracture with distinct nodes. This saves time spent on model input, a major improvement over coupled fracture system codes that don't use the dual porosity method. It also saves on computer solution time because the number of necessary nodes is less than if each fracture must be defined in the mesh. The implicit formulation of the coupled solution in the modified FEHM is a major improvement over previous coupled codes with

explicit coupled solutions because the implicit coupling exhibits faster convergence behavior.

The modified code has many applications. It is applicable in determining the effectiveness of using the Fenton Hill hot dry rock reservoir for energy production. It may be applicable in modeling fluid flow in a high level nuclear waste repository. In petroleum production it could be applicable to fractured reservoirs as the fluid forcing a fracture open may be at a different temperature than the surrounding rock.

The modified FEHM was verified by comparing its results with known solutions. The hydro-thermal coupling with dual porosity capabilities was tested by comparison with the analytic solution of Gringarten (1975). The hydro-mechanical coupling with dual porosity capabilities was tested by comparison with the semi-analytic solution of Wijesinghe (1986). The modified code was also used to model the Fenton Hill hot dry rock reservoir in order to determine if good convergence behavior would result (see Appendix 1 for a brief description of the Fenton Hill hot dry rock project).

In this report, Section 2 describes the governing equations of conservation of fluid mass, conservation of rock and fluid energy, and the force balance equations. Section 2.2 describes the application of finite element theory to the governing equations. Section 3 describes the theory of the dual porosity method and gives the modified equation of conservation of fluid mass applicable to the dual porosity method. Section 4 is a literature survey of the hydraulic and mechanical behavior of rock joints. The details of the modifications to FEHM allowing the fracture system permeability to vary in response to fluid flow, energy transport, and changes in

stress are given in Section 5. Section 6 describes the FEHM code subroutines that have been created or modified in this research. The details and results of three verification cases are described in Section 7. A sensitivity analysis was performed on one verification test case which is described in Section 8. Concluding remarks may be found in Section 9.

2.0 Coupled Finite Element Method

The finite element method is based upon the idea that a continuum may be modeled as a configuration of discrete elements. For each element, equations may be written that describe the interaction of the element with its neighbors. These equations may describe the hydraulic, thermal, and mechanical behavior of the elements. In coupled finite element models more than one type of equation may be described at an element and all behaviors are included so that they interact. The finite element method leads to a set of nonlinear equations which are then solved simultaneously. For a detailed presentation of the finite element method, refer to Zienkiewicz (1977). The finite element method is generally used for complicated problems that cannot be solved analytically.

2.1 Mathematical Formulation

Details of the governing equations for fluid flow with energy transfer in a porous medium are presented in Zyvoloski (1983) and are repeated in Eqs. 1-7. For simplicity, only the two dimensional, saturated flow equations are considered. The equations shown are for an isotropic medium under plane strain, though these restrictions do not exist in the computer code, FEHM. The conservation of a compressible fluid mass within a porous medium can be described as (nomenclature is described in Section 11.0):

$$\frac{\partial A_m}{\partial t} + \nabla \cdot \vec{f}_m + q_m = 0 \quad (1)$$

where A_m is the fluid mass per unit volume given by:

$$A_m = \phi \rho \quad , \quad (2)$$

$\vec{f}_m$ is the fluid mass flux given by:

$$\vec{f}_m = \rho \vec{v} \quad , \quad (3)$$

and q_m is a fluid mass source. The fluid flow in a porous matrix can be modeled by Darcy's Law:

$$\vec{v} = -\frac{k}{\mu}(\nabla P + \rho \vec{g}) \quad (4)$$

Conservation of fluid and rock energy is given by:

$$\frac{\partial A_e}{\partial t} + \nabla \cdot \vec{f}_e + q_e = 0 \quad (5)$$

where A_e is the energy per unit volume given by:

$$A_e = (1 - \phi)\rho_r u_r + \phi \rho u \quad (6)$$

and $\vec{f}_e$ is the energy flux given by:

$$\vec{f}_e = \rho h \vec{v} - \kappa \nabla T \quad (7)$$

The density, ρ , viscosity, μ , and specific enthalpy, h are functions of pressure and temperature and are approximated by rational functions within FEHM. The porosity of the porous matrix is constant. u is the specific internal energy of the fluid and can be expressed as $u = cT$ where c is the specific heat. u_r is the specific internal energy of the rock.

In contrast to the conservation of fluid mass and energy equations, the equilibrium equations for the matrix deformation are steady-state based on the observation that stress waves travel much faster than pressure and temperature fronts in the fluid or solid. The static force balance inside a solid assuming small deformations is (Biot, 1941):

$$\frac{\partial \sigma_{ij}}{\partial x_j} + b_i = 0 \quad (8)$$

where σ_{ij} is the stress tensor in the solid and b_i is the body force (summation intended). Within a fracture potentially influential additional forces are present due to fluid pressure and thermal stresses. These additional forces are described by Kelkar and Zyvoloski (1990) where diagonal components of stress within a solid are described by (no summation intended and assuming tension is positive):

$$\epsilon_{ii} = \frac{\sigma_{ii}}{E} - \frac{\nu}{E}(\sigma_{jj} + \sigma_{kk}) + \frac{\Delta P}{3H} + \alpha \Delta T \quad (9)$$

and the off diagonal components by:

$$\epsilon_{ij} = \frac{2(1+\nu)}{E} \sigma_{ij}. \quad (10)$$

When pore pressure is present it will act to expand the porous medium if the solid is allowed to expand freely (the same effect as heating the medium). If the solid is constrained, increasing pore pressure decreases the tensile stress. Biot's constant, $\frac{1}{H}$, is the proportionality constant of the pore pressure effect. Because pore pressure is a dynamic effect in some situations, this changes the static problem of stress and strain to a time dependent problem.

In FEHM the balance of forces is actually determined in terms of the x and y -displacements at the nodes, u_1 and u_2 . The strain may be written as (no summation intended):

$$\epsilon_{ii} = \frac{\partial u_i}{\partial x_i} \quad (11)$$

$$\epsilon_{ij} = \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}, i \neq j \quad (12)$$

Eqs. 9, 10, 11 and 12 may be combined resulting in:

$$\begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \end{bmatrix} = \frac{E(1-\nu)}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1 & \frac{\nu}{1-\nu} & 0 \\ \frac{\nu}{1-\nu} & 1 & 0 \\ \frac{\nu}{1-\nu} & \frac{\nu}{1-\nu} & 0 \\ 0 & 0 & \frac{1-2\nu}{2(1-\nu)} \end{bmatrix} \cdot \begin{bmatrix} \frac{\partial u_1}{\partial x_1} \\ \frac{\partial u_2}{\partial x_2} \\ \frac{\partial u_1}{\partial x_2} + \frac{\partial u_2}{\partial x_1} \end{bmatrix} \quad (13)$$

$$-\left(\alpha\Delta T + \frac{1}{3H}\Delta P\right) \frac{E}{1-2\nu} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}$$

Assuming no body forces, the conservation of fluid mass and energy and the static force balance equations (Eqs. 1, 5, and 8) may be rewritten in terms of pressure, temperature, and deformation so that the fluid flow is modeled by:

$$\frac{\partial A_m}{\partial t} - \nabla \cdot \frac{\rho k}{\mu} \nabla P - \nabla \cdot \frac{\rho^2 k}{\mu} \dot{\mathbf{g}} + q_m = 0, \quad (14)$$

the heat transfer is predicted by:

$$\frac{\partial A_e}{\partial t} - \nabla \cdot \frac{\rho h k}{\mu} \nabla P - \nabla \cdot \frac{\rho^2 h k}{\mu} \dot{\mathbf{g}} - \nabla \cdot \kappa \nabla T + q_e = 0, \quad (15)$$

and the force equilibrium is described by:

$$\begin{aligned} \frac{E(1-\nu)}{(1+\nu)(1-2\nu)} \frac{\partial^2 u_1}{\partial x_1^2} + \frac{E}{2(1+\nu)} \frac{\partial^2 u_1}{\partial x_2^2} + \frac{E}{2(1+\nu)(1-2\nu)} \frac{\partial^2 u_2}{\partial x_1 \partial x_2} \\ - \frac{E}{1-2\nu} \left(\alpha \Delta \frac{\partial T}{\partial x_1} + \frac{1}{3H} \Delta \frac{\partial P}{\partial x_1} \right) = 0 \end{aligned} \quad (16)$$

$$\begin{aligned} \frac{E(1-\nu)}{(1+\nu)(1-2\nu)} \frac{\partial^2 u_2}{\partial x_2^2} + \frac{E}{2(1+\nu)} \frac{\partial^2 u_2}{\partial x_1^2} + \frac{E}{2(1+\nu)(1-2\nu)} \frac{\partial^2 u_1}{\partial x_1 \partial x_2} \\ - \frac{E}{1-2\nu} \left(\alpha \Delta \frac{\partial T}{\partial x_2} + \frac{1}{3H} \Delta \frac{\partial P}{\partial x_2} \right) = 0 \end{aligned} \quad (17)$$

These four equations describe the thermal, hydraulic, and mechanical behavior of the rock and fluid within porous media. They must be solved simultaneously throughout the continuum in order to determine the pressure, temperature, and displacements at any point within the medium.

2.2 Finite Element Formulation

In FEHM the finite element equations are formulated using the Galerkin method. The fluid flow domain, Ω , is divided into elements connected at nodes (Figure 1). The pressure, temperature, and deformation at any point in the domain are linearly interpolated from the values at the nodes using bilinear basis functions, $[N]$:

$$P = [N]\{P\}, h = [N]\{h\}, A_m = [N]\{A_m\}, \quad (18)$$

$$A_e = [N]\{A_e\}, u_k = [N]\{u_k\}, \text{ etc.}$$

where P represents the value of the pressure anywhere in the domain and $\{P\}$ represents the matrix of pressures at the nodes. The basis functions, $[N]$, are used so that the pressure at any point is influenced only by the pressure values at the adjacent nodes; the influence of a node being 100% at that node and linearly decreasing to 0% as a neighboring node is approached. The approximations to the actual values of these parameters using basis functions become more accurate as the size of the elements is decreased.

Figure 1. Discretization into elements

The Galerkin method is a special case of a general approach called the method of weighted residuals (Pinder and Gray, 1977). In the Galerkin method Eqs. 14-17 are rewritten as a series of linear equations (Eq. 18). Since the actual values of pressure, temperature, and deformation are approximated by equations involving the nodal values, there is some error involved. This error is called the residual, $\mathfrak{R}$. The Galerkin method involves minimizing this residual. This is done by forcing the weighted residual to zero where the weighting functions are the basis functions. Green's theorem is used to replace second order partial differentials with conductance terms. Thus the Galerkin equations of conservation of fluid mass is written as (Zyvoloski, 1983):

$$\begin{aligned} \int N_i N_j dV \frac{\partial}{\partial t} \{A_m\} + \int \nabla N_i \frac{\rho k}{\mu} \nabla N_j dV \{P\} + \oint \frac{\partial N_i}{\partial y} N_j \frac{\rho^2 k}{\mu} dV + \{q_m\} \\ = \int N_i \mathfrak{R}_m dV = 0, \end{aligned} \quad (19)$$

the conservation of energy is written as (Zyvoloski, 1983):

$$\begin{aligned} \int N_i N_j dV \frac{\partial}{\partial t} \{A_e\} + \int \nabla N_i \frac{\rho h k}{\mu} \nabla N_j dV \{P\} + \oint \frac{\partial N_i}{\partial y} N_j \frac{\rho^2 h k}{\mu} dV \\ + \int \nabla N_i k \nabla N_j dV \{T\} + \{q_e\} = \int N_i \mathfrak{R}_e dV = 0, \end{aligned} \quad (20)$$

and the force balance equations are written as:

$$\begin{aligned} \int \left(\frac{\partial N_i}{\partial x_1} C \frac{\partial N_j}{\partial x_1} + \frac{\partial N_i}{\partial x_2} C \frac{(1-2\nu)}{2(1-\nu)} \frac{\partial N_j}{\partial x_2} \right) dV \{u_1\} + \int \left(\frac{\partial N_i}{\partial x_1} C \frac{\nu}{(1-\nu)} \frac{\partial N_j}{\partial x_2} + \frac{\partial N_i}{\partial x_2} C \frac{(1-2\nu)}{2(1-\nu)} \frac{\partial N_j}{\partial x_1} \right) dV \{u_2\} - \\ \int_{d\Omega} F_s dS - \int \frac{dN_i}{dx_1} C \frac{(1+\nu)}{(1-\nu)} \left(\alpha N_j \{\Delta T\} + \frac{1}{3H} N_j \{\Delta P\} \right) dV = \int N_i \mathfrak{R}_f dV = 0 \end{aligned} \quad (21)$$

and

$$\int \left(\frac{\partial N_i}{\partial x_2} C \frac{\partial N_j}{\partial x_2} + \frac{\partial N_i}{\partial x_1} C \frac{(1-2\nu)}{2(1-\nu)} \frac{\partial N_j}{\partial x_1} \right) dV \{u_2\} + \int \left(\frac{\partial N_i}{\partial x_2} C \frac{\nu}{(1-\nu)} \frac{\partial N_j}{\partial x_1} + \frac{\partial N_i}{\partial x_1} C \frac{(1-2\nu)}{2(1-\nu)} \frac{\partial N_j}{\partial x_2} \right) dV \{u_1\} -$$

$$\int N_j \rho g x_2 dV - \int_{d\Omega} F_s dS - \int \frac{dN_i}{dx_2} C \frac{1+\nu}{1-\nu} (\alpha N_j \{\Delta T\} + \frac{1}{3H} N_j \{\Delta P\}) dV = \int N_i \mathcal{R}_f dV = 0 \quad (22)$$

where $C = \frac{E(1-\nu)}{(1+\nu)(1-2\nu)}$, F_s represents surface forces on the elements,

and $d\Omega$ indicates integration over the boundary surface.

For computational efficiency, the quadrature rules used for the fluid mass per unit volume and energy per unit volume in Eqs. 19 and 20, $\int N_i N_j dV \frac{\partial}{\partial t} \{A_m\}$ and $\int N_i N_j dV \frac{\partial}{\partial t} \{A_e\}$, use weights at the nodes rather than the usual Gauss quadrature weights for the interior of the element. This causes the capacitance matrix to be diagonal. The accuracy is less than Gauss quadrature, but is still second order. The other terms in Eqs. 19-22 are also handled in a way that is non-traditional but at least as well as traditional finite elements. In FEHM the nonlinear and purely geometric parts are separated. This separation is explained in detail in Fung *et.al.* (1994) and is valid over lower order elements. The nonlinear part uses average transmissibility, $D = \frac{k\rho}{\mu}$, between two nodes (Zyvoloski, *et.al.*, 1991). This is usually taken to be the upstream nodal value. The method thus requires parameters to be input at nodes rather than elements. The result is a much more stable code for solving nonlinear problems while still retaining much of the geometric flexibility of finite elements. The method has been used in FEHM since 1982 (Zyvoloski, 1983) and has been extensively

verified (Dash, *et.al.*, 1995). Eqs. 19-22 are solved iteratively in the numerical code using the Newton-Raphson method for the four unknowns; P , T , u_1 and u_2 .

3.0 Dual Porosity Method

Groundwater flow modeling in a uniform porous medium has reached a high level of development in the past three decades. However, actual reservoirs are not necessarily composed of a uniform porous medium. A three-scale hierarchy may exist; the porous matrix, the fracture flow, and the entire reservoir. The smallest scale, that of the pores, may be of the order of microns. In the fracture scale the spacing of the fractures may be of the order of meters. And the reservoir scale may be of the order of kilometers (Figure 2).

Figure 2. Hierarchical flow in a fractured reservoir system

Under saturated flow conditions, fluid flow through the fractures dominates the system. Simple averaging of the matrix and fracture permeabilities is usually not sufficient to model the system. Bear (1993) classifies fracture flow problems into four types:

- the very near field - interest is focused on fluid flow through a single fracture with possible transport into the surrounding porous medium

- the near field - interest is focused on a relatively small domain where the fractures are well defined either deterministically or statistically
- the far field - transport occurs simultaneously in two different continua, the fractures and the porous medium. The fracture material is described by averaged properties.
- the very far field - the properties of the porous medium and the fractures are averaged into a single continuum with averaged properties.

Numerical modeling of the behavior of reservoirs containing fractures has been done using an effective continuum, a dual porosity model, a dual porosity/ dual permeability model, a fracture network model and a model of discrete fractures. These methods are illustrated in Figure 3. Each method has its positive and negative features. Some methods ignore the matrix contribution to the fluid storage and fluid flow regime and others ignore the distinct behavior of fractures. Both of these features can contribute to fractured reservoir behavior. The choice of model will depend on the actual relative contributions of the fracture fluid flow and matrix fluid flow and storage.

- effective continuum method - (Figure 3a) this method models the fracture system and the surrounding porous matrix as a single continuum with averaged properties. If fractures are not modeled within the finite element mesh discretely, some interactive behavior

between the matrix and the fractures is lost. If fractures are modeled discretely the nodal geometry is usually very complicated.

Figure 3. Illustration of various methods used in modeling of fractured reservoirs

- dual porosity method - (Figure 3b) this method models the fracture and matrix systems as separate but connected systems. The fracture system has fluid flow along the length of the continuum. The matrix only has fluid flow to and from its corresponding node in the fracture system. Matrix nodes cannot communicate with each other. This method models fracture fluid flow and the fluid storage in the matrix. It tries to capture the important features of each system without using complex geometry and large numbers of nodes to model discrete fractures. It also has fewer unknowns than a dual porosity/ dual permeability model and thus saves solution time.
- dual porosity/ dual permeability method - (Figure 3c) this method is similar to the dual porosity system but allows fluid flow between matrix nodes. It models the fracture fluid flow as well as the matrix fluid flow and storage. The negative aspect is that an extra unknown is found in the solution matrix.
- fracture networks - (Figure 3d) a fracture network models fluid flow within fracture systems by modeling a series of easily defined (usually parallel) fractures. The fractures are not usually meant to represent actual fractures but fracture behavior. The porous matrix fluid storage is ignored. Complex discrete fracture geometry does not have to be defined in the input.
- discrete fractures - (Figure 3e) this method defines and models individual fractures which are defined by the mesh. The behavior of

the porous matrix is ignored. While individual fractures are well modeled the input geometry can be extremely complex.

The dual porosity method has been developed to numerically model fracture fluid flow in the far field where individual discrete fractures cannot be treated effectively (Warren and Root, 1963). The method does not model discrete fractures but, rather, fracture system behavior. The fracture system is modeled as an equivalent continuum where each node or element may represent a number of fractures or a fraction of a fracture. The dual porosity also models the matrix storage system. The method applies to systems where the equivalent fracture medium REV (representative elementary volume) is smaller than the size of the element. That is, the method applies when there are many fractures found within an element and the fracture permeability tensor does not significantly change as the element is increased in size (the REV is about 10-20 times the fracture spacing (Wei, *et.al.*, 1995)).

In the dual porosity method, the fracture system and the porous matrix between fractures are handled separately (Douglas and Arbogast, 1990). The reservoir is divided into elements. The entire set of elements is duplicated two or more times (Figure 4). This allows the definition at each point in the domain of two or more porosities. One entire set of elements represents the fracture material with "averaged" fracture properties. Because the elements possess "averaged" properties the actual fractures are not necessarily represented by particular elements or nodes. The fracture material is modeled as if it were effectively a continuous medium with porous flow. A second set of elements represents the porous matrix. And an optional third set of elements (or more) may be used to add nodes between the matrix and

fracture systems to model more complicated, transient flows. The fluid flow between the matrix and the fracture systems, however, remains 1-dimensional. In the modified FEHM code the permeability of the fracture elements is a function of the effective stress across the model "fracture" at each node.

Figure 4. Dual porosity model

The dual porosity method has been developed to efficiently model the fracture dominated groundwater system in a far field configuration. It models fractured systems in a way that is computationally tractable and yet sufficiently precise for engineering purposes. The dual porosity method is applicable to both finite difference and finite element methods. Model results have agreed well with porous media conditions. Good descriptions of this model may be found in Douglas and

Arbogast (1990) and Showalter (1991). Gerke and van Genuchten (1993) give dual porosity equations involving unsaturated flow and solute transport.

3.1 Physical Interpretations

Consider a small element of the porous matrix. It is surrounded on all sides by fractures. The bulk of the fluid flow is through the fractures. The surrounded matrix volume, called a cell, is isolated from its neighboring cells. It cannot communicate with neighboring cells, that is, transport fluid directly to them. It can only transport fluid to and from surrounding fractures (Figure 5). However, this fluid transported to and from fractures can be important to the overall behavior of the reservoir. The matrix cells act as fluid reservoirs attached to the fractures.

Figure 5. Dual porosity model showing isolation of matrix cells

One way to represent the dual porosity concept is to consider the total system as a porous medium. The fractures represent the voids in the porous medium. The cells (really the porous matrix) can be thought of as the "solids" of the porous medium. In this case, however, the solids are also porous but with much smaller pores.

Another way to represent the dual porosity concept is to consider a cylinder with uniform cross section which contains an absorbing material. The fluid flows through pores along the axis of the cylinder but can be absorbed or desorbed by the material fixed in the cylinder. The absorbing material acts as a fluid reservoir. It represents the primary porosity matrix material. The pores represent the dual porosity fracture material. The exchange between the absorbent and the pores would represent the fluid flow from the fracture to matrix system.

3.2 Mathematical Formulation

Consider a fluid flowing through a reservoir. Darcy's Law describes single phase flow within a single porosity system and is given in Eq. 4. Conservation of fluid mass states that:

$$\frac{\partial}{\partial t} \phi \rho + \nabla \cdot (\rho \mathbf{v}) + q_m = 0 \quad (23)$$

where q_m is an external sink or source term. Now consider dual porosity flow and let $\rho_m(x, t)$ represent the density of the fluid in the matrix (an average over the cell since there is no fluid flow within the cell) and $\rho_f(x, t)$ that in the fractures. The fluid flow in the fractures and the matrix is described separately.

Eqs. 23 -26 are modifications applicable to FEHM of the dual porosity governing equations described in Douglas and Arbogast (1990). The fluid flow in the fractures is described by:

$$\frac{\partial}{\partial t} \Phi \rho_f - \nabla \cdot \left(\frac{k_f}{\mu} (\nabla P_f + \vec{g} \rho_f) \right) + q_m + q_{matrix} = 0 \quad (24)$$

where q_{matrix} is the macroscopically distributed matrix source term representing the fluid flow from the matrix to the fractures. The term q_{matrix} is the sum of the normal components of fluid flow from the matrix cell through the cell boundary:

$$- \int_{cellboundary} \left[\frac{k_m}{\mu} (\nabla P_{m,f} + \vec{g} \rho_m) \right] \cdot \vec{n} dx_{boundary} = q_{matrix} \quad (25)$$

In each cell of the matrix, fluid behavior can be described by:

$$\frac{\partial}{\partial t} \Phi \rho_m - q_{matrix} = 0 \quad (26)$$

because it is assumed that there is no fluid flow within or between matrix cells. This dual porosity model is derived from physical considerations and is called the microscopic model. The two systems, fracture and matrix, are linked by assuming continuity of density and of fluid mass flux between the two domains. Solving the equations of fluid flow in the matrix elements is not difficult since there is no cell-to-cell flow.

A similar model can be derived using mathematical homogenization. Suppose there is a system with some property that is periodic over the whole volume such as porosity or permeability in a fractured reservoir. Homogenization is a mathematical procedure which derives a set of equations that describes the general trend of this property by looking at the limit as the period of the periodic property tends to zero.

Consider a single porosity flow system with discontinuous porosity and permeability defined over the domain. Let all the matrix cells be identical and periodic with period e , the dimension of the cell. In this model the normal fluid flow out from the cell must equal the flow into the fracture. Fluid pressure must be continuous over the fracture/cell boundary. If the cell size, e , becomes infinitely small, a set of equations similar to Eqs. 24, 25 and 26 can be derived. This is called the macroscopic model. There are some small differences between the microscopic and macroscopic models but these differences disappear when the models are numerically solved with finite difference or finite element methods. Thus similar models can be derived from physical considerations and from mathematical homogenization theory.

4.0 Hydraulic and Mechanical Behavior of Rock Joints

When groundwater flows through a saturated, fractured rock the fluid flow behavior is influenced by the properties of the fractures. Various models have been proposed describing the flow through the fractures though no one model yet appears completely satisfactory. The fracture flow models all attempt to describe the fracture permeability in terms of an equivalent aperture of the fracture analogous to fluid flow between two parallel plates. Flow is assumed to be laminar. The aperture of the fracture in turn is a function of the effective stress across the fracture. The effective stress is a function of the fluid pressure in both the matrix and the fracture systems, the temperature change, and change in mechanical stress. All these factors will influence the permeability of the fractures.

Consider a single fracture. If the walls of the fracture are smooth, flat, and parallel, the velocity of the fluid between the plates can be described as (Kundu, 1990):

$$v = \frac{y}{2\mu} \frac{dp}{dx} (y - b) \quad (27)$$

where $\frac{dp}{dx}$ is the fluid pressure gradient, b is the distance between the plates, and y is the coordinate perpendicular to the plates (Figure 6).

Figure 6. Fluid flow between two parallel plates

Now consider the fluid flow, Q , through a channel of width w :

$$Q = w \int_0^b v dy \quad (28)$$

or

$$Q = \frac{wb^3}{12\mu} \frac{dp}{dx} \quad (29)$$

Eq. 29 is referred to as the cubic law of fracture flow. Flow through an equivalent porous medium of height, b , width, w , and intrinsic permeability, k , is (Strack, 1989):

$$Q = -\frac{kbw}{\mu} \frac{dp}{dx} \quad (30)$$

Equating Eqs. 29 and 30 results in an equation for the equivalent intrinsic permeability of the channel as a function of the distance between the plates:

$$k = \frac{b^2}{12} \quad (31)$$

4.1 The Equivalent Aperture

Flow through fractures does not exactly follow the cubic law because a fracture is not smooth or parallel. Much has been done to more accurately describe the fluid flow through a fracture. Tsang (1992) describes the usage of the term "equivalent aperture" in fracture flow. One usage is that aperture which would permit a given flow rate through a fracture at a given pressure drop.

Barton, *et.al.* (1985) uses Eq. 31 to model fracture conductivity but with the following modification. The aperture used for Eq. 31 is the conducting or equivalent aperture, that through which flow actually occurs. Barton, *et.al.* (1985) found that the conducting aperture is related to the mechanical or physical fracture aperture empirically by:

$$b = \frac{b_m^2}{JRC^{2.5}} \quad (32)$$

where JRC is the joint roughness coefficient. Zhao and Brown (1992) relate the effective and mechanical fracture apertures by:

$$b = \frac{b_m}{\sqrt[3]{JCF}} \quad (33)$$

where JCF is a joint condition factor that reflects fracture roughness, surface mismatch, wall strength, detritus, loading history, sample disturbance, temperature, and sample size. In tests on granite cores with a maximum effective normal stress of 40 MPa, the values of the JCF ranged from 0.15 to 9.17 for natural and induced fractures.

4.2 Fracture Permeability

Gangi (1978) derived a theoretical description of the dependence of permeability on the effective stress across the fracture. He assumed that the fracture asperities (protuberances from the fracture wall into the stream flow) are rods of different heights, identical Young's moduli, and cross-sectional areas proportional to their heights. These rods hold the fracture open. As the effective stress across the fracture increased, the rods deform allowing the aperture to decrease. As the aperture decreases, more and more rods come in contact with both fracture surfaces thus increasing the stiffness of the fracture. Assuming that the fluid flows between two smooth parallel plates whose aperture is a function of the variable fracture stiffness the intrinsic permeability of the fractured rock is:

$$k = k_o \left(1 - \left(\frac{\sigma'_n}{\sigma_o} \right)^m \right)^3 \quad (34)$$

where k_o is the zero-stress permeability, m and σ_o are constants, and σ'_n is the effective normal stress. A plot of Gangi's relation for various values of m is shown in Figure 7.

Figure 7. Relationship of fractured rock permeability to effective normal stress (from Gangi, 1978)

Walsh (1981) derived an equation for fracture permeability based on an analogy to two-dimensional heat flow. He assumed that the effective permeability of a fracture is a function of the permeability of the asperities and of the voids. Parallel plate fluid flow theory was used to determine the permeability of the voids. Walsh's equation for the permeability of a fracture is:

$$k = k_o \left[1 - \sqrt{2} \frac{2h}{b_o} \ln \left(\frac{\sigma'_n}{\sigma'_o} \right) \right]^3 \left[\frac{2 - b(\sigma'_n - \sigma'_o)}{2 + b(\sigma'_n - \sigma'_o)} \right] \quad (35)$$

where h is the root mean square value of the asperity height distribution. The first term in brackets represents the influence of aperture on the fracture permeability. The second term in brackets represents the influence of the tortuosity. Walsh stated that the influence of the tortuosity is smaller than the influence of the apertures except at very high stresses. Thus he neglected the tortuosity term. Wash stated that Eq. 35 shows good agreement with experimental data.

Swan (1983) assumed that the roughness of a fracture can be modeled as hemispheres holding the fracture walls apart. The peaks are sufficiently far apart that they deform independently and only normal loading is considered. Swan found that the distribution of peak heights is best described by a Gaussian distribution. The hydraulic conductivity of the fracture can be described as:

$$k = k_o(c - m \cdot \ln(\sigma'_n))^2 \quad (36)$$

where c and m are experimentally determined constants.

Raven and Gale (1985) studied fractures in five granite cores of various diameters. The cores were placed in uniaxial compression to a maximum stress of 30 MPa and closure across the fracture and fluid flow through the fracture were determined for various conditions. From closure, displacement across the fractures was calculated. The results are shown in Figure 8. Note that the stress/strain behavior of the fracture is non-linear. As the stress increases, the fracture closes which brings more fracture asperities in contact with both fracture walls.

This distributes the load across more area which decreases the deformation rate. It also increases the fluid flow path tortuosity. Hysteresis is apparent in Figure 8. This may be the result of seating of the fracture or crushing of the asperities during loading.

Figure 8. Fracture and rock displacement as a function of normal stress (from Raven and Gale, 1985)

Figure 9 shows the fluid flow through the fracture as a result of increasing stress in Raven and Gale's study. Note that as the stress increases the amount of fluid flow per unit head loss decreases. However, the flow per unit head loss

approaches a limit. This limit appears to hold for all fracture flow studies and is thought to be due to fracture channels remaining open even under high stresses. In Figure 9 the flow rate per unit head loss is shown as a function of the aperture. The fracture aperture is calculated from the measured closure. The cubic law is also plotted. Note that the fluid flow rate does not follow the cubic law well. At high apertures (low stress) the flow is slower than predicted by the cubic law. At low apertures (high stress) the flow is faster than the cubic law predicts and approaches a limiting value. This limiting value is called the residual flow.

Figure 9. Fracture flow per unit head loss as a function of normal stress (from Raven and Gale, 1985)

Figure 10. Fracture aperture and flow rate (from Raven and Gale, 1985)

Pyrak-Nolte, *et.al.* (1987) studied fracture flow and deformation of granite cores containing fractures. A uniaxial stress of up to 85 MPa was applied to the cores. The displacement across the fractures was determined by the difference between displacements measured across the fracture and that measured in the same length of intact rock adjacent to the fracture. Pyrak-Nolte, *et.al.* found that the fluid flow rate is dependent on the load history. However, the trend of large changes in flow at low stress and a limiting value of flow at high stress is independent of the load history.

Figure 11 shows the flow per unit head loss as a function of effective stress for three samples. Pyrak-Nolte, *et.al.* used a power relationship to describe the flow through a fracture:

$$Q = Q_o + Db^{n+1} \frac{\partial P}{\partial x} \quad (37)$$

where Q_o is the limiting flow through the aperture under high stress, b is the difference between the fracture displacement due to the applied stress and the maximum possible displacement of the fracture (roughly equivalent to the aperture at the applied stress), and n and D are constants. A linear least-squares fitting routine was applied to the data to determine the constants in Eq. 37. The value of n ranged between 6.6 and 8.8 which is very different from the power for the cubic law.

Figure 11. Flow per unit head loss as a function of effective stress for three samples (from Pyrak-Nolte, *et.al.*, 1987)

Using Eq. 37, a general relationship between the intrinsic permeability and the fracture aperture can be written as:

$$k = k_o + Cb^n \quad (38)$$

where Q_o is the residual flow through a fracture at high stress with a residual permeability, k_o , and C and n are constants.

Zhao and Brown (1992) plotted the fracture permeability as a function of effective normal stress for experimental data from granite cores. In Figure 12 they compare these with the permeability models of Gangi, Walsh, Gale, and Swan. It does not appear that any one model is significantly better than another.

Figure 12. Fracture permeability as a function of effective normal stress for various models (from Zhao and Brown, 1992)

4.3 Aperture as a Function of Effective Normal Stress

Eq. 29 shows the flow through a fracture as a function of the aperture. The normal and shear stress across the fracture determines the size of the aperture. The stress/deformation behavior of a fracture is non-linear and relationships between the stress and deformation are needed. According to Bandis, *et.al.* (1983) the normal stiffness of fractures is influenced by:

- initial physical contact area, amplitude, and vertical distribution of the apertures,

- fracture wall roughness,
- strength and stiffness of the apertures,
- thickness, type, and properties of any infilling present.

Bandis, *et.al.* (1983) developed an empirical relationship between fracture deformation and effective normal stress for fractures with interlocked surfaces:

$$\sigma_n = \frac{-\Delta V}{\frac{1}{K_{ni}} - \frac{1}{K_{ni} V_m} \Delta V} \quad (39)$$

where σ_n is the normal stress across the fracture, ΔV is the fracture closure due to that stress, K_{ni} is the initial fracture stiffness, and V_m is the maximum fracture closure. Equating the closure with the change in the fracture aperture, $\Delta b = -\Delta V$, and the normal stress with the change in the effective normal stress since some initial stress state with an initial aperture, Eq. 39 can be rewritten as:

$$\Delta b = \frac{\Delta \sigma'_n}{K_{ni} - \frac{\Delta \sigma'_n}{V_m}} \quad (40)$$

Note that the denominator of Eq. 40 is the stiffness of the fracture which increases with increasing compression. Bandis, *et.al.* give empirical

relationships between V_m and K_{ni} and the joint roughness coefficient (JRC) and the mean joint compressive strength (JCS). Data is presented in Bandis, *et.al.* for the values of V_m and K_{ni} for several types of rocks under three loading cycles.

For fractures whose walls have been dislocated (or mismatched) Bandis, *et.al.* modeled the fracture deformation as:

$$\Delta b = c(\log(\sigma_n) - n) \quad (41)$$

where n and c are empirical constants. Figure 13 shows the stress/deformation curves for joints in the same rock but with interlocked and mismatched joints. Note the presence of hysteresis.

Figure 13. Comparison of deformation as a function of stress for intact rock and interlocked and dislocated fractures, ΔV_t = total rock deformation, ΔV_j = net fracture closure (Bandis, *et.al.*, 1983)

Zhao and Brown (1992) tested fractured granite cores for changes in fracture aperture with effective normal stress. The data was fitted to a logarithmic function:

$$b_m = b_{o,m} \left(1 - n \cdot \ln \left(\frac{\sigma'_n}{\sigma'_o} \right) \right) \quad (42)$$

where n is a constant. The data showed a good fit with Eq. 42.

4.4 Aperture as a Function of Shear Stress

As shearing takes place in a fracture, dilation can occur when asperities separate in order to move past each other. Barton, *et.al.* (1985) developed an empirical relationship between shear stress and the normal change in fracture aperture:

$$\Delta b = \frac{-\tau}{K_{si}} \left(\frac{1}{1 - \frac{\tau}{\sigma_{peak}}} \right) \tan \left(0.5 JRC \log \frac{JCS}{\sigma'_n} \right) \quad (43)$$

The resulting values for the mechanical aperture change due to shear stress can be added to that due to normal stress. The resulting conductivity can then be calculated.

Barton, *et.al.* (1985) indicate that for an idealized, *in-situ* block test the shear dilation for a fracture with a peak shear strength of 3.6 MPa and an effective normal stress of 2 MPa is approximately zero.

4.5 Effective Normal Stress Across a Fracture

Eq. 39-43 are functions of the effective normal stress across the fracture opening defined by:

$$\sigma'_n = \sigma_n + sP \quad (44)$$

where s is a constant (recall that tensile stresses are positive). As the overburden stress, σ_n , is decreased the effective stress across the fracture decreases and the fracture closes. As the fluid pressure, P , is increased the effective stress across the fracture increases and the fracture is pushed open. Thus the effective normal stress is dependent on both the overburden stress and the fluid pressure. The concept of effective stress is useful in several geotechnical areas and the value of s depends on the process being studied. In soils engineering s is always equal to one and Eq. 44 is referred to as the classic effective stress law (Warpinski and Teufel, 1992).

Walsh (1981) derived a value for s used in determining the permeability of fractures:

$$s = 1 + \frac{V_p \beta_r}{\frac{\partial V_p}{\partial \sigma}} \quad (45)$$

Walsh concludes that the value of s is about 0.9 for joints with polished surfaces and can be as low as 0.6.

In fracture theory, s is determined experimentally by holding the fracture aperture at a constant value while changing σ_n and P . Boitnott and Scholz (1990) defined s as:

$$s = \left. \frac{\partial \sigma_n}{\partial P} \right|_b \quad (46)$$

They measured fracture behavior in cores of silicate glass and rock. They found that the value of s for the glass decreased from a value of one for increasing overburden pressure (holding the fluid pressure constant) while the behavior of s was indeterminate but near one for the rock samples. In all cases s approached one at low overburden pressure. Boitnott and Scholz suggest the value of s is dependent on the loading path.

5.0 Modifications to FEHM

FEHM, an existing heat transfer, fluid mass transfer and stress finite element code was modified to allow the coupled stress capability to be used with the dual porosity feature. It was also modified to allow improved coupling by allowing the fracture system permeability in a dual porosity feature to change in response to changes in the effective normal stress across a fracture. The effective normal stress across a fracture is a function of the fluid pressure in the fracture and in the matrix, matrix temperature, and matrix stress. Using a dual porosity model of the fracture system decreases the number of necessary nodes to describe a fracture system and decreases the time spent describing the input mesh geometry. Implicit coupling of the heat transfer, fluid mass transfer, and stress was used in order to improve convergence behavior.

5.1 Assumptions

Several simplifying assumptions were made to allow modification of FEHM.

These are:

- 2-dimensional geometry
- orthogonal finite element grid
- horizontal orientation so that gravity is ignored
- saturated flow is present in both the matrix and fracture systems

- two perpendicular fracture systems can be modeled. Variation of fracture properties can occur from node to node
- the maximum fracture aperture change is set to 1.0 m and the minimum fracture aperture change is an input parameter, V_m
- normal stress is modeled but shear stress is ignored across the fracture
- the equivalent fracture aperture is used

5.2 Fracture Permeability

The cubic law was used in many coupled fracture flow studies (Asgian, 1989, Zhao and Brown, 1992, Millard *et.al.*, 1995) and was chosen as the model to be implemented in the coupled FEHM code. The Barton-Bandis model (1985) of fracture aperture as a function of stress is used when the effective stress is compressive. When in the tensile region, the stiffness of the fracture is held constant. The aperture behavior is shown in Figure 14. Eqs. 31, 40 and 44 can be combined to yield:

$$k_f = c \left(b_{initial} + \frac{\sigma_n - \sigma_{n, initial} + P_f - P_{f, initial}}{K_{ni} - \frac{\sigma_n - \sigma_{n, initial} + P_f - P_{f, initial}}{V_m}} \right)^a \quad (47)$$

where c is a constant usually equal to 1/12, a is generally chosen to be equal to 2 (as in the cubic law), and σ_n is the normal stress in the matrix material

surrounding the fracture. σ_n is therefore a function of P_m and T_m as well as u_1 and u_2 (see Eq. 13). Therefore the fracture system permeability is a function of P_f , P_m , T_m , u_1 , and u_2 . In finding the fracture aperture, the stress state at a node is found from the strains which are the derivatives of the displacements.

The derivatives of the displacements at a node are calculated numerically by

taking the average of the values on either side of a node of $\frac{u^{i+1} - u^i}{x^{i+1} - x^i}$ where

the superscripts, i and $i + 1$, refer to the values at the node and at its immediate neighbor. In the modified FEHM there is an arbitrary maximum limit of 1.0 m for the aperture change. The minimum limit, corresponding to the Barton-Bandis model (1985), is the maximum fracture closure, V_m .

Figure 14. Model of aperture as a function of effective stress showing increasing fracture stiffness with increasing compressive effective normal stress and constant fracture stiffness in the tensile region

The fracture permeability model was initially implemented in the code to create an explicit formulation. The fracture system permeability was updated at each time step based on the temperature, pressure, and stress state of the previous time step. In order to improve convergence capabilities, the implicit formulation of the coupled system then was developed and used.

5.3 Multiple Parallel Fractures

The dual porosity method models fracture systems, not single fractures. If a block of rock of height H and width W is assumed to contain n parallel, evenly

spaced fractures surrounded by an impermeable matrix, the flow through the rock fractures is:

$$Q = -\frac{k_f}{\mu} nbW \frac{\partial P}{\partial x} \quad (48)$$

where k_f refers to the permeability through a single fracture. The same flow through an equivalent continuum can be expressed as:

$$Q = -\frac{k_e}{\mu} HW \frac{\partial P}{\partial x} \quad (49)$$

where k_e refers to the effective permeability. Equating the two flows yields an expression for the effective permeability of a rock block containing fractures:

$$k_e = \frac{nb}{H} k_f \quad (50)$$

This would be the permeability used for the fracture material in a dual porosity model. Note that $\frac{H}{n}$ is equal to the fracture spacing.

5.4 Implicit Formulation

Initially, the FEHM matrix solution with the dual porosity simulation feature was written in terms of two unknowns, P_f , T_f , the fluid pressure and temperature in the fracture system. In FEHM's initial coding, the matrix pressure and temperature were also independent variables. However, solution for these variables used a backward substitution algorithm which was performed separately from the Newton-Raphson iteration used to solve the main system of

equations. The author added the stress coupling feature to the dual porosity capability by adding two unknowns to the formulation, u_1 , and u_2 , the x and y -displacements of the matrix system. Thus a solution matrix with four degrees of freedom for each pair of nodes of the problem was produced:

$$\begin{bmatrix} \frac{\partial M_f}{\partial P_f} & \frac{\partial M_f}{\partial T_f} & \frac{\partial M_f}{\partial u_1} & \frac{\partial M_f}{\partial u_2} \\ \frac{\partial E_f}{\partial P_f} & \frac{\partial E_f}{\partial T_f} & \frac{\partial E_f}{\partial u_1} & \frac{\partial E_f}{\partial u_2} \\ \frac{\partial S_1}{\partial P_f} & \frac{\partial S_1}{\partial T_f} & \frac{\partial S_1}{\partial u_1} & \frac{\partial S_1}{\partial u_2} \\ \frac{\partial S_2}{\partial P_f} & \frac{\partial S_2}{\partial T_f} & \frac{\partial S_2}{\partial u_1} & \frac{\partial S_2}{\partial u_2} \end{bmatrix} \cdot \begin{bmatrix} \epsilon_{P_f} \\ \epsilon_{T_f} \\ \epsilon_{u_1} \\ \epsilon_{u_2} \end{bmatrix} = \begin{bmatrix} \mathcal{R}_{P_f} \\ \mathcal{R}_{T_f} \\ \mathcal{R}_{u_1} \\ \mathcal{R}_{u_2} \end{bmatrix} \quad (51)$$

where ϵ is the error of P_f , T_f , u_1 , and u_2 at each time step, M_f refers to the equation of the conservation of mass in the fracture system, E_f refers to the equation of the conservation of energy in the fracture system, S_1 refers to the static force balance equation in the x -direction, and S_2 refers to the static force balance equation in the y -direction. The error is added to the previous values of P_f , T_f , u_1 , and u_2 to update them. $\mathcal{R}$ is the residual and should ideally be equal to zero but in reality is allowed to be less than some tolerance. The 4x4 matrix is called the Jacobian and consists of the derivatives of Eqs. 19-22. They are necessary to the implicit finite element formulation.

This problem formulation did not allow the dependence of fracture system permeability on the matrix permeability and temperature to be implicitly included. Therefore, a 6x6 system of equations was developed by the author to include the equation of the conservation of mass within the matrix system and the equation of the conservation of energy within the matrix system (Figure 15). To the best of the author's knowledge, no formulation of the coupled flow equations for heat transfer, fluid mass transfer, and force balance using a dual porosity formulation has previously been performed. The 6x6 formulation defined all six unknowns implicitly; P_f , T_f , P_m , T_m , u_1 , and u_2 :

$$\begin{bmatrix}
 \frac{\partial M_f}{\partial P_f} & \frac{\partial M_f}{\partial T_f} & \frac{\partial M_f}{\partial P_m} & \frac{\partial M_f}{\partial T_m} & \frac{\partial M_f}{\partial u_1} & \frac{\partial M_f}{\partial u_2} \\
 \frac{\partial E_f}{\partial P_f} & \frac{\partial E_f}{\partial T_f} & \frac{\partial E_f}{\partial P_m} & \frac{\partial E_f}{\partial T_m} & \frac{\partial E_f}{\partial u_1} & \frac{\partial E_f}{\partial u_2} \\
 \frac{\partial M_m}{\partial P_f} & \frac{\partial M_m}{\partial T_f} & \frac{\partial M_m}{\partial P_m} & \frac{\partial M_m}{\partial T_m} & \frac{\partial M_m}{\partial u_1} & \frac{\partial M_m}{\partial u_2} \\
 \frac{\partial E_m}{\partial P_f} & \frac{\partial E_m}{\partial T_f} & \frac{\partial E_m}{\partial P_m} & \frac{\partial E_m}{\partial T_m} & \frac{\partial E_m}{\partial u_1} & \frac{\partial E_m}{\partial u_2} \\
 \frac{\partial S_1}{\partial P_f} & \frac{\partial S_1}{\partial T_f} & \frac{\partial S_1}{\partial P_m} & \frac{\partial S_1}{\partial T_m} & \frac{\partial S_1}{\partial u_1} & \frac{\partial S_1}{\partial u_2} \\
 \frac{\partial S_2}{\partial P_f} & \frac{\partial S_2}{\partial T_f} & \frac{\partial S_2}{\partial P_m} & \frac{\partial S_2}{\partial T_m} & \frac{\partial S_2}{\partial u_1} & \frac{\partial S_2}{\partial u_2}
 \end{bmatrix} \cdot \begin{bmatrix} \epsilon_{P_f} \\ \epsilon_{T_f} \\ \epsilon_{P_m} \\ \epsilon_{T_m} \\ \epsilon_{u_1} \\ \epsilon_{u_2} \end{bmatrix} = \begin{bmatrix} \mathcal{R}_{P_f} \\ \mathcal{R}_{T_f} \\ \mathcal{R}_{P_m} \\ \mathcal{R}_{T_m} \\ \mathcal{R}_{u_1} \\ \mathcal{R}_{u_2} \end{bmatrix} \quad (52)$$

Figure 15. Six unknowns used in dual porosity configuration

At each time step the residual, $\mathfrak{R}$, is compared with an error tolerance using an l^2 norm. Thus convergence occurs when (Zyvoloski, 1991):

$$\left(\sum_i \mathfrak{R}_i^2 \right)^{\frac{1}{2}} \leq \text{tolerance} \bullet \mathfrak{R}_0 \quad (53)$$

where $\mathfrak{R}_0$ is the initial error. The tolerance is an input parameter.

Semiautomatic timestep control is used based on the convergence within the

Newton-Raphson iterations. If the code is unable to find a convergent solution within a given number of iterations, the time step is reduced and the procedure repeated.

Of the six governing equations in FEHM (conservation of mass for the fracture and matrix systems, conservation of energy for the fracture and matrix systems, and two force equilibrium), the permeability variable only occurs in the energy and mass balance equations. If the permeability is allowed to be a function of P_f , P_m , T_m , u_1 , and u_2 then new terms must be added to the Newton-Raphson equation which are the derivatives of the permeability (with respect to P_f , P_m , T_m , u_1 , and u_2) multiplied by the remaining part of the first terms of Eqs. 19 and 20. The new terms developed by the author for use in the modified FEHM are:

$$terms = \int_{volume} \nabla P \nabla N_j \frac{\rho \partial k}{\mu \partial \theta} dV \quad (54)$$

where N_j is a basis function, k is given in Eq. 47 and θ can be P_f , P_m , T_m , u_1 , and u_2 . The derivatives of k are:

$$\frac{\partial k_i}{\partial \theta} = \frac{cab^{a-1} K_{ni} \partial \sigma'_i}{\left(K_{ni} - \frac{\Delta \sigma'_n}{V_m} \right)^2 \partial \theta} \quad (55)$$

and the derivatives of the effective stress are:

$$\frac{\partial \sigma'_i}{\partial P_f} = 1 \quad (56)$$

$$\frac{\partial \sigma'_i}{\partial P_m} = -\frac{E}{1-2\nu} \beta \quad (57)$$

$$\frac{\partial \sigma'_i}{\partial T_m} = -\frac{E}{1-2\nu} \alpha \quad (58)$$

$$\frac{\partial \sigma'_i}{\partial u_i} = \frac{E(1-\nu)}{(1+\nu)(1-2\nu)} \frac{1}{\Delta x_i} \quad (59)$$

$$\frac{\partial \sigma'_i}{\partial u_j} = \frac{E\nu}{(1+\nu)(1-2\nu)} \frac{1}{\Delta x_j}, \quad i \neq j \quad (60)$$

where i represents either the x or y -direction. Eqs. 54 - 60 were developed by the author for use in FEHM modifications.

The use of Eqs. 19-22 in the fracture system gives an explicit formulation. The addition of terms given in Eqs. 54-60 to the Newton-Raphson formulation gives an implicit solution which reduces the number of iterations needed to reach solution in problems where the changing fracture system permeability is a dominant effect. The solution is implicit because all the unknowns are solved

simultaneously with a Newton-Raphson iteration which allows more efficient convergence.

5.5 Strain in Matrix as a Result of Changing Fracture Aperture

One of the considerations of allowing changing apertures in FEHM is that as the fracture opens, the stress in the surrounding rock decreases. This decreasing stress will act to close the fracture. This effect was formulated by the author for use in the modified FEHM as follows. Increasing the stress increases the strain. Since tension is positive and since strains superpose, the total strain perpendicular to the fracture can be written as:

$$\epsilon_t = \epsilon + \epsilon_f \quad (61)$$

where ϵ_t is the total matrix strain, ϵ is the matrix strain before aperture effects are included, and ϵ_f is the strain due to the expanding fracture. Thus the effect of the opening fracture on the surrounding rock stress can be treated by adding the change in nodal strain due to the fracture opening to the matrix strain when formulating the solution equations. In effect, this means subtracting the effects of fracture aperture change, Eq. 47, from Eq. 13 to include the effects of the opening fracture on stress. This results in:

$$\begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \end{bmatrix} = \frac{E(1-\nu)}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1 & \frac{\nu}{1-\nu} & 0 \\ \frac{\nu}{1-\nu} & 1 & 0 \\ \frac{\nu}{1-\nu} & \frac{\nu}{1-\nu} & 0 \\ 0 & 0 & \frac{1-2\nu}{2(1-\nu)} \end{bmatrix} \cdot \begin{bmatrix} \frac{\partial u_1}{\partial x_1} \\ \frac{\partial u_2}{\partial x_2} \\ \frac{\partial u_1}{\partial x_2} + \frac{\partial u_2}{\partial x_1} \end{bmatrix} - \left(\alpha \Delta T + \frac{1}{3H} \Delta P \right) \frac{E}{1-2\nu} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix} \quad (62)$$

$$\frac{E(1-\nu)}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1 & \frac{\nu}{1-\nu} & 0 \\ \frac{\nu}{1-\nu} & 1 & 0 \\ \frac{\nu}{1-\nu} & \frac{\nu}{1-\nu} & 0 \\ 0 & 0 & \frac{1-2\nu}{2(1-\nu)} \end{bmatrix} \cdot \begin{bmatrix} \frac{\Delta b_1}{\Delta x_1} \\ \frac{\Delta b_2}{\Delta x_2} \\ 0 \end{bmatrix}$$

To the best of the author's knowledge, Eq. 62 is the first time the last term of the equation has been added to the stress/strain equation with thermal and pore pressure effects in order to model the affect of changing fracture aperture on the stress in the surrounding rock mass.

With this modification of the matrix stress, the derivatives of the fracture apertures with respect to fracture temperature, fracture pressure, matrix temperature, matrix pressure, and the displacements need to be formulated.

Referring to Eq. 47, the derivative of the addition to the stress in the x -direction is:

$$\begin{aligned} & \frac{\partial}{\partial \theta} \left(\frac{E(1-\nu)}{(1+\nu)(1-2\nu)} \frac{\Delta b_1}{\Delta x_1} + \frac{E\nu}{(1+\nu)(1-2\nu)} \frac{\Delta b_2}{\Delta x_2} \right) \\ &= \frac{E(1-\nu)}{(1+\nu)(1-2\nu)} \left(\frac{1}{\Delta x_1} \right) \frac{K_{nx}}{\left(K_{nx} - \frac{\Delta \sigma'_1}{V_m} \right)^2} \frac{\partial \sigma'_1}{\partial \theta} + \frac{E\nu}{(1+\nu)(1-2\nu)} \frac{K_{ny}}{\left(K_{ny} - \frac{\Delta \sigma'_2}{V_m} \right)^2} \left(\frac{1}{\Delta x_2} \right) \frac{\partial \sigma'_2}{\partial \theta} \end{aligned} \quad (63)$$

where the derivatives, $\frac{\partial \sigma'_i}{\partial \theta}$ are defined in Eqs. 56-60. The stress in the y-direction is similarly defined.

5.6 Storage as a Result of Changing Fracture Aperture

In Eqs. 1 and 5 the change of fluid mass storage and energy storage with time is present. When a fracture opens, the fluid mass and energy storage volumes of the fracture system increase. Therefore Eqs. 1 and 5 were modified by the author:

$$\frac{\partial A_m}{\partial t} = \frac{\partial}{\partial t} \left(\frac{V}{V_o} \phi \rho \right) \quad (64)$$

$$\frac{\partial A_e}{\partial t} = \frac{\partial}{\partial t} \left(\frac{V}{V_o} [(1-\phi)\rho_r u_r + \phi \rho u] \right) \quad (65)$$

where V_o is the initial volume and V is the current volume of the fracture. For a fracture system with changing aperture V can be expressed as:

$$V = \left(1 + \frac{\Delta b_1}{b_{1,o}} \right) \left(1 + \frac{\Delta b_2}{b_{2,o}} \right) \Delta z \quad (66)$$

where Δz is the depth of the fracture. The derivatives of Eq. 64 and 65 are needed for the implicit formulation. The derivative of the mass storage was formulated by the author as:

$$\frac{\partial}{\partial \theta} \left(\frac{\partial A_m}{\partial t} \right) = \frac{\partial}{\partial t} \left[\frac{V}{V_o} \left[\rho \frac{\partial \phi}{\partial \theta} + \phi \frac{\partial \rho}{\partial \theta} \right] \right] + \phi \rho \left[\left(1 + \frac{\Delta b_1}{b_{1,o}} \right) \left(\frac{1}{b_{2,o}} \right) \frac{\partial}{\partial \theta} (\Delta b_2) + \left(1 + \frac{\Delta b_2}{b_{2,o}} \right) \left(\frac{1}{b_{1,o}} \right) \frac{\partial}{\partial \theta} (\Delta b_1) \right] \quad (67)$$

The derivatives, $\frac{\partial}{\partial \theta} (\Delta b_1)$ and $\frac{\partial}{\partial \theta} (\Delta b_2)$, can be described by:

$$\frac{\partial}{\partial \theta} (\Delta b_i) = \frac{K_{ni}}{\left(K_{ni} - \frac{\Delta \sigma'_i}{V_m} \right)^2} \frac{\partial \sigma'_i}{\partial \theta} \quad (68)$$

where θ represents P_f , P_m , T_m , u_1 , and u_2 and $\frac{\partial \sigma'_i}{\partial \theta}$ is described in Eqs. 56-60. The energy storage term is handled in a similar manner.

6.0 Program Modifications

In order to implement the coupled algorithms into FEHM, algorithms were added to calculate the permeability and its derivatives at each iteration. FEHM originally found the solution with a two degree of freedom solver because P_m and T_m were solved by separate algorithms. FEHM needed to have stress coupling added to the dual porosity feature and have the fracture system permeability be a function of P_f , P_m , T_m , u_1 , and u_2 . FEHM needed to be modified so that it could solve the equations implicitly with six degrees of freedom; P_f , P_m , T_f , T_m , u_1 , and u_2 . The following routines were created or modified from existing subroutines in order to implement the changes (Figure 16 shows the relationship of these routines. Note that only the modified routines are shown, not all the routines in FEHM):

Figure 16. Relationship of modified FEHM routines

- **main** - this is the first and main routine in FEHM. It updates the time steps. It also calls **fracdp** which in turn calls **fracin** which initializes the input values for the variable fracture aperture system (using macro FRAC). **Main** calls **stress** which calls the matrix building and solution routines.
- **fracdp** - calls **fracin**.
- **fracin** - reads and initializes the input values.
- **newt_hm2sd** - this routine is called from **newt_hm2s** which is called from **stress**. It implements the iterations within each timestep. **Newt_hm2sd** calls **thermw** which updates and finds derivatives for the porosity, volume, viscosity, density, and storage terms. **Newt_hm2sd** calls the matrix builder, **genhm2sd**; a normalizing routine, **normal6**; the solution routine, **solve_new6**; and the routine to update the pressure, temperature, and displacements, **uphm2sd**. **Newt_hm2sd** also calls the dual permeability routines, **duald**.
- **genhm2sd** - this routine is called from **newt_hm2sd**. It builds the matrices used in the solver. For each node, **genhm2sd** calls **geneqkdd** which calculates the pressure and temperature terms of the matrices and **streq2d** which calculates the stress terms of the matrices. It also calls **dsbn2d** which applies the force and displacement boundary conditions.

- **geneqkkd** - this routine calculates the pressure and temperature terms of Eq. 19 and 20. It also calls **fracder** and **fracderuv** to calculate the derivatives of permeability with respect to pressure and temperature.
- **streq2d** - this routine calculates the displacement terms of Eq. 21 and 22. It calculates the derivatives of the force equilibrium equation by calling **fracders** and **fracderuvs**.
- **uphm2sd** - this routine updates the pressure, temperature, and displacements after each iteration.
- **dsbn2d** - this routine applies the force and displacement boundary conditions to the solution matrix.
- **normal6** - this routine normalizes the terms of Eq. 52.
- **thermw** -this routine updates and finds derivatives for the porosity, volume, viscosity, density, and storage terms. **Thermw** calls **fracstor** which calculates the volume modification in the mass and energy storage terms due to changing fracture aperture and its derivatives.
- **solve_new6** - this routine and its subsequent subroutines solves Eq.52.
- **fracpnx** - this routine finds the fracture permeability and the stress across the fractures at each iteration.

- **fracder** - this routine finds the derivatives of the fracture permeability with respect to fracture temperature, fracture pressure, matrix temperature, and matrix pressure.
- **fracderuv** - this routine finds the derivatives of the fracture permeability with respect to the displacements.
- **fracders** - this routine finds the derivatives with respect to fracture temperature, fracture pressure, matrix temperature, and matrix pressure of the change in matrix displacement due to the change in fracture aperture.
- **fracderuvs** - this routine finds the derivatives with respect to the displacements of the change in matrix displacement due to the change in fracture aperture.
- **fracstor** - this routine updates the storage volume at each iteration. It calls **fracders** and **fracderuvs** to calculate the derivatives of the change in fracture storage. This routine also calls **fracpnx**.
- **duald** - this routine calculates terms used in the dual porosity formulation. It calls **dualfhd**.
- **dualfhd** - this routine calculates the terms used in the dual porosity formulation. It assumes that there is no communication layer and that

P_m and T_m are solved for directly along with the other four variables in the 6 dof solution routines.

7.0 Verification

Verification of the modified FEHM code is difficult due to the complexities of thermo-hydro-mechanical coupled processes. There is a lack of applicable analytical solution or experimental data for code verification and validation purposes (Noorishad, Tsang, and Witherspoon, 1992, Nguyen and Selvadurai, 1995, and Chan, *et.al.*, 1995). Complexities of the interactions of the coupled phenomena have generally allowed verification only through numerical cross-verification.

Various coupled code developers have tested their codes using several problems. Pine and Cundall (1985) developed a finite difference model explicitly coupling fluid mass transport with fracture permeability. The code was tested with an axisymmetric problem with shear behavior. Asgian (1989) developed a hybrid boundary element-finite difference program to model coupled stress and fluid mass in fractures. She did not compare her model results with pre-existing data. Kelkar and Zyvoloski (1990) modified FEHM to allow fluid flow through a single deformable fracture located along the edge of a porous medium. The code was tested using Wijesinghe's problem (Wijesinghe, 1986) which is described in Section 7.2. Noorishad, *et.al.* (1992) tested ROCMASII with Wijesinghe's problem and with a 2-dimensional problem involving a tunnel excavation. The tunnel excavation involved gravity and shear effects. GEOCRACK (Swenson, *et.al.*, 1995 and Sprecker, 1994) was verified using seven test cases to model the various couplings available. However, only two, Wijesinghe's problem and the Fenton Hill hot dry rock geothermal reservoir (described in Appendix 1) involved a coupled system with variable fracture permeability. The author knows of four test cases in the DECOVALEX project used to verify coupled code effects. Test BMT1 (Millard,

et.al., 1995) consists of a vertical section with a transient thermal source and the effects of gravity. Test BMT2 (Chan, *et.al.*, 1995) consists of a horizontal plane containing four fractures, no gravity, and was modeled with and without shear effects on the fracture aperture. Tests TC1:1 and TC1:2 (Makurat, *et.al.*, 1995) are 2-dimensional tests that involve transient force boundary conditions. Test TC3 (Ahola, *et.al.*, 1994) is of flow in a partially saturated porous medium.

These verification problems used by other code developers could not be used because they were not 2-dimensional, contained transient thermal boundary conditions, contained shear effects, or contained gravity forces. The DECOVALEX test case, BMT2 was not used to verify the modified FEHM code because it was judged that FEHM could not effectively model a coupled system with four distinct fractures due to a limitation on the number of elements within FEHM. Because of the large number of degrees of freedom (six) in the modified FEHM solution, problems involving large numbers of elements cannot be handled by the computing machines available to this author. The maximum number of nodes that could be used is approximately 800 using a Sparc Center 2000 with 256 megabytes of memory installed.

Test cases on non-fractured media that test FEHM coupled stress capabilities before and after modifications were implemented and establishing that force equilibrium in FEHM is correct are found in Appendix 2. Other capabilities of FEHM have been previously tested by Dash, *et.al.*, 1995. The modified FEHM code was verified with three cases. Case 1 tests the ability of FEHM to model flow through a fracture with a constant aperture. The temperature of the injection fluid is lower than that of the reservoir initially and cools the rock. Case 2 tests the ability of FEHM to model

changing fracture aperture due to increasing fracture fluid pressure. Case 3 is a model of the Fenton Hill hot dry rock project and is used to determine that FEHM with the modifications could model an actual fractured system with a reasonable time step (see Table 2 for a summary of the test cases). In Cases 1 and 2 the results of FEHM are compared with analytic solutions to verify that FEHM is working properly.

The input files for these test cases are contained in Appendix 5; however, a major part of the coordinate and element information is left out for the sake of brevity. A description of the use of FEHM is contained in Appendix 3.

Table 2: Test cases for FEHM

Case	Description	Phenomena tested
1	Single fracture, constant aperture	Dual porosity capabilities with thermal coupling
2	Single fracture, fluid pressure	Variable fracture capabilities with hydro-mechanical coupling
3	Hot dry rock reservoir	Convergence capabilities with useful time step size

7.1 Test Case 1: Single Fracture, Constant Aperture

Problem description: Test Case 1 consists of a 2-dimensional dual porosity system with a single horizontal fracture. This case tests the dual porosity capabilities of FEHM. The reservoir is 50 m high and 1000 m long with a unit thickness (Figure 17). Initially the reservoir has a temperature of 100 C and a pressure of 25 MPa. The left boundary of the fracture has a constant inflow of

$6.152 \times 10^{-3} \text{ kg sec}^{-1}$ per unit thickness of the reservoir. The pressure at the right boundary is constant at 25 MPa. At $t=0$ the fracture injection temperature is set to 90 C. The finite element results using constant fracture aperture are compared with the analytic solution presented by Gringarten, *et.al.* (1975).

Model Input: In FEHM with dual porosity capabilities, there are two overlapping element configurations; the matrix elements and the fracture system elements. Thus there are really twice the number of input nodes used for solution. In Case 1, there is a single line of nodes representing the fracture but contained within the fracture system configuration. In order to model a single fracture, the material properties of the fracture system at the fracture must have different properties than elsewhere.

The FEHM mesh consists of 287 nodes (Figure 17). The element size near the fracture is small to better model the effects around the fracture. This leads to a potential problem due to the aspect ratio of the elements. However, a significant increase in the number of nodes used in order to improve the aspect ratio could not be implemented due to memory limitations on computer systems available for this research. The initial stiffness of the fracture is set to $10^{30} \text{ MPa m}^{-1}$ so that the fracture aperture is constant. The injection temperature is changed to 90 C at $t=0$. Since this is a transient problem, the temperatures and displacements are modeled to 621 years to show transient behavior. Table 3 shows parameters used in Case 1. Table 4 shows the boundary conditions used.

The impermeable mass of rock surrounding the fracture transfers heat to the water. However, in a dual porosity system, the energy transfer through the

matrix cannot be transferred from matrix node to matrix node directly but must pass through the corresponding fracture nodes (Figure 5). The dual porosity model would not usually be chosen where conduction of heat is the dominant method of energy transfer. In order to use the modified FEHM on this problem the volume fraction of the fracture system not in the fracture needed to be set very high (0.99) in order to allow energy to be transferred from node to node in the rock mass. This increase in volume fraction is compensated for by multiplying the aperture factor for the fracture system not in the fracture, by what would have been the volume fraction. Note that this way of modeling the energy transfer would normally cause an error in the fluid storage. This is not the case in this model because of the low permeability of the fracture system nodes which are not in the fracture. Very little fluid gets into these nodes to be stored.

Figure 17. Model representation of Case 1

Table 3: Case 1 - Parameters used in model

Parameter	Value
length	1000.0 m
width	50.0 m
Young's modulus	1×10^3 MPa
Poisson's ratio	0.0
rock specific heat	$1046 \text{ J kg}^{-1}\text{C}^{-1}$
rock density	2650 kg m^{-3}
rock thermal conductivity	$2.59 \text{ W m}^{-1}\text{C}^{-1}$
fracture porosity	1.0
initial aperture	0.001 m
aperture power (for cubic law)	3
aperture factor	8.33×10^1
maximum aperture closure	0.001 m
fracture stiffness	$1. \times 10^{30} \text{ MPa m}^{-1}$
matrix permeability	$1. \times 10^{-20} \text{ m}^2$
matrix porosity	0.1
volumetric flow	$6.152 \times 10^{-3} \text{ kg sec}^{-1}$

Table 4: Case 1 - Boundary conditions used in model (l=left, r=right, t=top, b=bottom boundary)

Type	Boundary	Condition
stress	l,t,b	no normal displacement
fluid flow	r,l,t,b fracture node, l fracture node, r	no normal flow $Q = 6.125 \times 10^{-3} \text{ kg sec}^{-1}$ $P = 25 \text{ MPa}$
heat transfer	r,l,t,b fracture node, l fracture node, r	no normal transfer $T = 90 \text{ C}$ $T = \text{as numerically calculated}$

Results: The numeric results of FEHM are compared to the analytic results of Gringarten in Figure 18. As can be seen in Figure 18, the results of FEHM are in excellent agreement with the analytic results.

Figure 18. Case 1 - Comparison of analytic and FEHM results

Additional discussion: A problem similar to Case 1 was formulated by Swenson (1996) in order to find the response of a single fracture in an impermeable rock. The general configuration is shown in Figure 17. In this problem there is a difference of 150 C between the initial rock temperature and the injection temperature. The aperture of the fracture is allowed to change in response to the thermal contraction in the impermeable rock and to the fluid pressure in the fracture. This changes the permeability of the fracture and will affect the outlet temperature as a function of time. The parameters used in this

problem are shown in Table 5. Where applicable the parameters were from Swenson (1996). The fluid flow boundary conditions on the left side of the fracture were varied for different cases; sometimes a flow boundary condition and sometimes a pressure boundary condition was used.

Table 5: Problem with variable aperture

Parameter	Value
length	100.0 m
width	0.5 m - 200.0 m
Young's modulus	8.5×10^4 MPa
Poisson's ratio	0.25
rock specific heat	$803 \text{ J kg}^{-1}\text{C}^{-1}$
rock density	2716 kg m^{-3}
rock thermal conductivity	$2.57 \text{ W m}^{-1}\text{C}^{-1}$
fracture porosity	1.0
initial aperture	5.6×10^{-5} m
aperture power (for cubic law)	3
maximum aperture closure	0.001 m
fracture stiffness	$1. \times 10^8 \text{ MPa m}^{-1}$
matrix permeability	$1. \times 10^{-20} \text{ m}^2$
matrix porosity	0.1

When the model was run there were problems with convergence behavior. In addition, some results did not agree with Swenson's model results using GEOCRACK. This could be due to the use of the dual porosity method to model impermeable rock behavior. Dual porosity models are used when there is a

dominant flow system such as through a fractured system with storage of fluid within the matrix. In such a system heat transfer is predominantly by convection. However, in the problem formulated by Swenson the heat transfer through the impermeable rock was by conduction. In the dual porosity heat transfer formulation, conducted heat must be transferred from matrix node to the fracture system, travel through the fracture system, and then be transferred to the adjacent matrix node (Figure 5). This is a cumbersome way to transfer heat and may lead to error when conduction is the predominant method of heat transfer. Thus there may be problems with the FEHM results under these circumstances. This is worse when the temperature difference is large and when the thermal properties are such as to emphasize the effect of the temperature change as in the case of Swenson's problem. It is also worse when the change in fracture aperture is a large percentage of the original fracture aperture. Note that Swenson modeled this problem with the code, GEOCRACK. Since GEOCRACK is formulated to model fracture flow and heat transfer in impermeable rock mass it would more effectively model this problem. Other sources of differences in results between GEOCRACK and FEHM are:

- FEHM uses fluid density as a function of pressure and temperature while GEOCRACK uses a constant density
- FEHM models the fluid enthalpy to include the pressure work on the fluid while GEOCRACK does not include this term.

The original formulation of Case 1 was not significantly affected by this problem. This is because the temperature difference between the initial

temperature and the injection temperature was small. In addition, the thermal expansion coefficient was essentially zero and the fracture aperture was constant.

7.2 Test Case 2: Single Fracture, Fluid Pressure

Problem Description: Test Case 2 is taken from the semi-analytical similarity analysis of fluid flow through a single joint developed by Wijesinghe (1986). This problem has been used by Kelkar and Zyvoloski (1990), Noorishad, *et.al.* (1992), and Swenson, *et.al.* (1995) to test their coupled fracture codes. This test case consists of mass and force coupling in a variable aperture fracture. Wijesinghe's problem is a 1-dimensional flow coupled with deformation in a single joint surrounded by a solid. The solid extends to infinity normal to the fracture. The fracture is subjected to a uniform in-situ stress normal to the fracture which is large enough so that the fracture sides are prevented from losing contact with each other under the influence of the injected fluid pressure. Wijesinghe used the cubic law to model the joint permeability. Joint stiffness was constant.

The reservoir is 25 m long and 1 m high with a single horizontal fracture down the center. Initially the fracture has an aperture of 1.0×10^{-5} m. The fracture stiffness is 1.0×10^5 MPa. Initially there is a constant pressure of 21 MPa throughout the fracture and matrix system. At $t=0$, the fluid is injected into the fracture at a pressure of 21.9 MPa. This forces the fracture aperture to increase as the pressure front moves down the fracture. The conceptual model of the problem is shown in Figure 19.

Model Input: In FEHM with dual porosity capabilities, there are two overlapping element configurations; the matrix elements and the fracture system elements. Thus there are really twice the number of input nodes used for solution. In Case 2, there is a single line of nodes representing the fracture but contained within the fracture system configuration. In order to model a single fracture, the material properties of the fracture system at the fracture must have different properties than elsewhere.

There are 287 nodes in the fracture system (Figure 19). In order to model the fracture as a single line of nodes, every other node in the fracture system uses a very high fracture stiffness and very low fracture aperture. The input parameters are given in Table 6. The boundary conditions are given in Table 7.

In order to allow the area of fracture flow to change as the aperture changes, the aperture power used is 3 and the aperture factor of 1/12 is divided by the original aperture of the fracture. In order to prevent closing of the fracture, the minimum aperture closure is set to 1×10^{-30} m.

Figure 19. Model representation of Case 2

Table 6: Case 2 - Parameters used in model

Parameter	Value
length	25.0 m
width	1.0 m
rock density	2716 kg m ⁻³
rock specific heat	803 J kg ⁻¹ K
rock thermal conductivity	2.57 W m ⁻¹ K
Young's modulus	1 x 10 ³ MPa
Poisson's ratio	0.0
initial aperture	1 x 10 ⁻⁵ m
fracture porosity	1.0
aperture power (for cubic law)	3
aperture factor (for cubic law)	8333
minimum aperture	1 x 10 ⁻³⁰ m
fracture stiffness	1 x 10 ⁵ MPa m ⁻¹
matrix permeability	1 x 10 ⁻²⁰ m ²
matrix porosity	0.1

Table 7: Case 2 - Boundary conditions used in model (l=left, r=right, t=top, b=bottom boundary)

Type	Boundary	Condition
stress	l,t,b	no normal displacement
fluid flow	r,l,t,b fracture node, l fracture node, r	no normal flow P = 21.9 MPa P = 25 MPa
heat transfer	r,l,t,b	no normal transfer

Results: A comparison of Wijesinghe's solution with the results of FEHM is shown in Figure 20. The results are compared at 500 sec and 2000 sec. Figure 20 indicates that there is good agreement between the analytic and numeric results.

Figure 20. Case 2 - Comparison of analytic and numeric apertures at 500 and 2000 days

7.3 Test Case 3: Hot Dry Rock Reservoir

Problem Description: Test Case 3 consists of a 2-dimensional, horizontal model of the Fenton Hill hot dry rock reservoir (see Appendix 1 for a brief description of the Fenton Hill hot dry rock reservoir). The Fenton Hill hot dry rock reservoir has been previously described and modeled by Swenson, *et.al.* (1995). The hot dry rock reservoir is 300m long, 600 m wide (Figure 21), and it is estimated to be 30 m high. The initial temperature of the reservoir is 230 C. The fracture system controls the fluid flow and before hydrofracturing was

essentially impermeable due to its great depth. The wells were subsequently pressurized to 62.7 MPa and shut off. During geothermal production the fluid was forced through the fracture system to the production well (at 45.0 MPa) which cools the rock and causes thermal contraction. This tends to slowly increase the fracture system permeability. The decreasing fluid pressure around the production well tends to decrease the permeability.

Model Input: The FEHM model consists of 370 nodes. Only half of the reservoir need be modeled due to symmetry. The parameters used in the model are given in Table 8. The boundary conditions are given in Table 9. For comparison, typical properties of granite such as might be found in a hot dry rock reservoir are found in Appendix 4. The initial pressure of the model is set at 62.7 MPa and the initial temperature is set to 230 C. These are conditions after the initial pressurization of the reservoir. At $t=0$, the injection well temperature is reduced to 70 C. The production well pressure is changed to 45.0 MPa. The simulation is run to 720 days, over twice the time the Fenton Hill reservoir was operated.

In order to allow the area of fracture flow to change as the aperture changes, the aperture power used is 3 and the aperture factor of 1/12 is divided by the original aperture of the fracture.

Some parameters of the Fenton Hill hot dry rock reservoir are not well known. In particular the fracture spacing, initial aperture, and fracture stiffness are only known in some range. The model was run to steady state without thermal deformation and those values of initial aperture and fracture stiffness which

resulted in a production well flow of 5.7 kg sec^{-1} (assuming a 30 m thick reservoir with symmetry) are used in the model. The model assumes two perpendicular fracture sets parallel to the mesh boundaries. A fracture spacing of 10 m is used. However, the matrix to fracture length used in the model is only 1 m to allow greater heat transfer to the matrix system (and consequently greater heat storage).

Figure 21. Model representation of Case 3

Table 8: Case 3 - Parameters used in model

Parameter	Value
length	600.0 m ^a
width	150.0 m ^a
Young's modulus	8.512 x 10 ⁴ MPa ^a
Poisson's ratio	0.25 ^a
rock density	2716 kg m ⁻³ a
rock specific heat	803 J kg ⁻¹ C ⁻¹ a
rock thermal conductivity	2.57 W m ⁻¹ C ⁻¹ a
fracture porosity	1.0
initial aperture	9.3 x 10 ⁻⁵ m
aperture power (for cubic law)	3
aperture factor (for cubic law)	896
minimum aperture	9.3 x 10 ⁻⁵ m
fracture stiffness	1.0 x 10 ⁸ MPa m ⁻¹
matrix permeability	1.0 x 10 ⁻²⁰ m ²
matrix porosity	0.01
rock thermal expansion coefficient	7.5 x 10 ⁻⁶ C ⁻¹ a

a. from Swenson, *et.al.*, 1995.

Table 9: Case 3 - Boundary conditions used in model (l=left, r=right, t=top, b=bottom boundary)

Type	Boundary	Condition
stress	r,l,t,b	no normal displacement
fluid flow	r,l,t,b injection well production well	no normal flow P = 62.7 MPa P = 45.0 MPa
heat transfer	r,l,t,b injection well production well	no normal transfer T = 70 C T = as numerically calculated

Results: The results of Case 3 at 720 days are shown in Figures 22 and 23. The production well flow after adjustment of the fracture properties is 5.7 kg/sec which is the actual field result. The temperature at the production well is 229 C at 270 days while the actual Fenton Hill reservoir showed a production well temperature of 230 C at 270 days of field testing. This indicates excellent agreement between FEHM results and field data for this single point at 270 days. This is the only temperature data point known for comparison.

The initial permeability of the modeled fracture system is $6.7 \times 10^{-15} \text{ m}^2$. After 720 days the permeability changes the near the injection well to the maximum amount of $6.9 \times 10^{-15} \text{ m}^2$ and near the top of the reservoir to the minimum amount of $6.6 \times 10^{-15} \text{ m}^2$. Thus the fracture permeability did not change much in the fracture system.

The average time step size is 10 days with an average of 4.0 Newton-Raphson iterations per time step. This is a reasonable time step size for running a problem over long periods of time.

Figure 22. Case 3 - Pressure distribution of hot dry rock reservoir at 720 days

Figure 23. Case 3 - Temperature distribution of hot dry rock reservoir at 720 days

Additional Discussion: In groundwater fracture systems the fracture aperture and thus the fracture permeability are rarely constant across a field. A second model roughly representing the Fenton Hill reservoir was formulated. This model is of a reservoir 600 m wide by 600 m long. The parameters used are the same as for Case 3 except for the initial fracture aperture and the distance between the two wells. In this run, the initial fracture apertures are randomized with a mean of 8×10^{-5} m and a standard deviation of 2×10^{-5} m. For simplicity the volume fraction and the constant, c , in Eq. 47 are homogeneous. The pressure and temperature distributions for these models at 720 days are shown in Figures 24 and 25. If the model is given a constant initial fracture aperture of 8×10^{-5} m, the temperature at the production well is 175 C at 720 days. However with a randomized initial fracture aperture, the production well temperature is 187 C, a smaller temperature drawdown. This indicates that the length of the flow path can have an influence on the temperature and pressure distribution. In addition, it indicates that the tortuosity effects that would be present in a 3-dimensional model may be important.

Figure 24. Case 3 - Pressure distribution of hot dry rock reservoir at 720 days with randomized initial aperture distribution

Figure 25. Case 3 - Temperature distribution of hot dry rock reservoir at 720 days with randomized initial aperture distribution

8.0 Sensitivity Analysis

When analyzing a geologic system, it is rare that all the parameters needed to clearly define a groundwater model are known. Some parameters may be known. Others may be bounded within a range or only estimated. Therefore, knowledge of the sensitivity of a system to changes in parameters is important. For this sensitivity analysis, a modified Case 1 was used as the base case. Case 1 consists of a reservoir 1000 m long and 50 m high containing a single fracture. The reservoir is initially at 100 C. At $t=0$, water is injected into the fracture at a temperature of 90 C. Case 1 was modified for this sensitivity analysis by changing the flow input boundary condition to a fixed pressure boundary condition of 25.048 MPa. The initial fracture stiffness of Case 1 was changed from 1×10^{30} MPa m^{-1} (a constant aperture condition) to 1×10^5 MPa m^{-1} . These changes do not alter the previous results of Case 1 and will allow the fracture permeability to change as the following parameters are varied. Note that, with the exception of the above changes, the input parameters were returned to those of Case 1 before each of the following runs.

The convergence capability of the model may be influenced by the value of parameters. In the following sensitivity runs the time step was a constant, 99 days, except where noted. The number of Newton-Raphson iterations per time step (NR) is shown to indicate the ability of the model to converge. Lower Newton-Raphson iterations indicate faster convergence.

8.1 Time Step

Finite element results are often sensitive to the time step used for the calculations. Too small a time step will cause longer computation times. Extremely small time steps cause instability. Too large a time step will result in errors in the output or even in a lack of convergence. The user does not usually specify the time step used in FEHM. The initial time step and the maximum time step are specified. The time step at every time is calculated within FEHM based on convergence criteria. Therefore, the time steps specified in Figure 26 are the average time steps over the entire run. Figure 26 indicates that results do depend upon the time step but little change occurs with a time step smaller than 936 days. Using this time step results in a reasonable computation time for the problem.

Figure 26. Comparison of outlet temperatures using varying time step size

8.2 Mesh Size

It is well known that increasing the nodal spacing will decrease the accuracy of results. The sensitivity of Case 1 to horizontal element size was tested by increasing the number of nodes in the x-direction resulting in a total number of 287, 147, 77, and 42 nodes. The results are shown in Figure 27. Figure 27 indicates that the results of Case 1 are not particularly sensitive to the number of nodes in the horizontal direction within the range tested. The number of nodes used in Case 1 was adequate.

Figure 27. Comparison of outlet temperatures using varying mesh size

The vertical nodal spacing was varied to determine the sensitivity of the problem to the vertical size. For this problem results were nearly identical. However, the vertical nodal spacing has been known to affect results for other problems. This occurs when FEHM is used to model the coupled behavior of a single fracture in an impermeable matrix under the influence of a high temperature gradient. In a dual porosity model, energy is transferred from the matrix system to the fracture system but not directly between matrix nodes. Energy can be conducted within the solid or convected with the fluid. Within an impermeable matrix, there is no fluid flow to facilitate convection. Therefore, the only method of energy transfer from a single matrix is through conduction. In reality,

the route of conduction is through the solid. In a dual porosity model, the energy is conducted from the matrix to the fracture system to the matrix in order for a temperature front to travel through the solid. This is a longer distance and results in inaccurate heat transfer. Larger elements surrounding a single fracture result in greater inaccuracy.

8.3 Coefficient of Thermal Expansion

The coefficient of thermal expansion was varied to determine the sensitivity of the problem to this parameter. Case 1 used a value of $\alpha = 1 \times 10^{-30} \text{ C}^{-1}$ in order to maintain a constant aperture. This was varied with the results shown in Figure 28. It can be seen that the temperatures decline rapidly as the coefficient of thermal expansion is increased. Because the matrix is cooling, the rock is contracting and the fracture aperture increases. The fracture opens more when there is a larger coefficient of thermal expansion. The larger opening allows the fluid to reach the outlet faster. Thus a given quantity of fluid gains less heat from the surrounding rock and reaches the outlet at a lower temperature.

Figure 28. Variation of outlet temperature with coefficient of thermal expansion

8.4 Matrix Permeability

Matrix permeability was varied from $1 \times 10^{-8} \text{ m}^2$ to $1 \times 10^{-20} \text{ m}^2$. There was no change in the outlet temperature distribution indicating that there is virtually no flow to the matrix.

8.5 Matrix Porosity

The matrix porosity controls the storage of heat and fluid in the system because of the large volume associated with the matrix. If larger amounts of water are stored in the matrix, more heat is available for transfer to the fracture fluid and

the outlet temperatures will be higher. This is shown in Figure 29. Note that in fractured reservoirs, the matrix porosity is usually less than one. In hot dry rock reservoirs the matrix porosity is extremely low because the reservoir is at great depth.

Figure 29. Comparison of outlet temperature for various matrix porosities

8.6 Initial Fracture Stiffness

The fracture stiffness is probably the most important parameter controlling the permeability of the fracture system (Figure 30). The fracture aperture change is proportional to the effective stress. The constant of proportionality is the fracture stiffness. The dominant feature of the fracture stiffness is the initial

fracture stiffness. As the stiffness is increased, the fracture aperture change decreases and there is less change in the fracture system permeability. Thus the system is less responsive to changes in fluid pressure or temperature when the initial fracture stiffness is high. In Case 1, this allows less flow to move down the fracture resulting in more energy exchange with the matrix and higher outlet temperatures. The effective stress in Case 1 contains the effects of decreasing temperature and increasing fracture pressure. Since the pressure difference across the fracture is small in this case, its effect is negligible. In order to emphasize the effect of the decrease in temperature, the coefficient of thermal expansion was changed to $1 \times 10^{-3} \text{ C}^{-1}$ for this comparison.

Figure 30. Comparison of outlet temperatures with varying initial fracture stiffness

8.7 Young's Modulus

Because the fracture pressure is so small in this problem, the effective stress is effectively a function of the thermal contraction. This means that the fracture aperture is effectively proportional to $\frac{E}{K_{ni}}$. Increasing the Young's modulus has the same effect as decreasing the initial fracture stiffness. Therefore, in Case 1, the effect of changing the Young's modulus is the inverse of that shown in Figure 30.

8.8 Temperature Difference

The difference in temperature was found to affect the convergence capability of FEHM during the modeling of Case 1. Large differences between the injection temperature and the initial temperature cause slower convergence and may require smaller time steps. This problem is worse when using extreme parameters such as large values of the thermal expansion coefficient that interact with the temperature difference to affect results. Figure 31 shows the results of Case 1 when the difference between the initial temperature and the injection temperature is increased. For these runs a coefficient of thermal expansion of $1.0 \times 10^{-4} \text{ C}^{-1}$ is used. Note that the vertical axis is dimensionless. The time step size is 99 days in each run. The number of Newton-Raphson iterations increases as the temperature difference increases. This indicates that the code has more trouble converging with larger temperature differences. If the temperature difference is large enough the problem may not continue to run with a reasonable time step size. The poorer convergence with larger temperature

difference is probably made worse by the fact that this problem is of a single fracture in an impermeable rock (see Section 7.2).

Figure 31. Comparison of outlet temperature for differences of injection temperature and initial rock temperature. The vertical axis is dimensionless temperature,

$$T' = \frac{T_{outlet} - T_{injection}}{T_{initial} - T_{injection}}$$

8.9 Combinations of Parameters

It has been found that, while changes in each individual parameter may not have a large effect, combinations of parameters can result in instability of the problem. Large temperature differences will increase instability. However, the

large temperature differences will have more effect on stability if there is a large coefficient of thermal expansion and a small initial fracture stiffness. Large differences will have more effect when using the modified FEHM to model single fractures in impermeable rock masses as the dual porosity system is designed to model fracture systems. In general, the following make a difference in the ability of FEHM to converge when under the influence of large temperature differences:

- problems with larger rock blocks surrounding a single fracture will converge better than problems involving smaller rock blocks when the thermal expansion coefficient is large and the boundary conditions are fixed displacement. Under unfavorable conditions the thermal expansion or contraction effects extend to the boundary.
- problems with smaller initial fracture stiffness and large thermal expansion coefficients will converge poorly under large temperature differences. The large amount of expansion or contraction in the rock causes larger changes in the fracture aperture which can result in instability. This problem is made worse by very small initial apertures because the change in aperture is a larger fraction of the initial state. It can be made worse by large pressure changes.

The user of FEHM should be aware of these limitations as they may affect the convergence properties of models. As an example, a problem similar to Case 1 was run to a final time of 10 days with the initial parameters of $\alpha = 1 \times 10^{-30} \text{ C}^{-1}$, $b_o = 1 \times 10^{-3} \text{ m}$, $\nu = .25$, $K_{ni} = 1 \times 10^5 \text{ MPa m}^{-1}$, the difference between the

injection temperature and initial temperature, $\Delta T = 90$ C, and the difference between the injection and outlet pressure, $\Delta P = 0.048$ MPa. Table 10 shows the cumulative effect of changing the above parameters on the convergence behavior of the problem. The convergence behavior is indicated by the total number of Newton-Raphson iterations to run the problem to 10 days. It is obvious that convergence would be difficult with certain combinations of parameters.

Table 10: Effect of combined parameters on convergence behavior

Cumulative change in parameters	Newton-Raphson Iterations
Initial case	34
Change α to $1 \times 10^{-2} \text{ C}^{-1}$	49
Change b_o to $1 \times 10^{-6} \text{ m}$	82
Change K_{ni} to $1 \times 10^4 \text{ MPa m}^{-1}$	364
Change ΔP to 5 MPa	1082

9.0 Conclusion

Dual porosity groundwater models have been developed to model fracture systems. Dual porosity models allow the bulk of groundwater flow to occur in a fracture system while the surrounding matrix controls the bulk of the fluid storage. FEHM is a mass, energy and force coupled finite element code that had dual porosity capabilities though this feature could not be implemented simultaneously with the force coupling capability. Before the current modifications, FEHM modeled a fracture system with a fixed permeability. Such a system was unable to couple the effects of thermal expansion and contraction and stress with the change in groundwater flow due to changing fracture permeability. It was unable to couple the effect of increasing fluid pressure due to changing fracture permeability.

In this research FEHM has been modified to allow the stress coupling capability to be implemented within the dual porosity feature. FEHM modifications also allow the fracture system aperture to change as a function of the fracture pressure, matrix pressure, matrix temperature, and stress in order to implicitly couple the hydraulic, thermal, and mechanical phenomena in a fracture system. The aperture is modeled as a function of the effective stress across the fracture as formulated by Bandis, *et.al.* (1983). This changing aperture causes changes in the fracture system permeability, storage, and in the surrounding stress. The fracture permeability is modeled using the cubic law (Kundu, 1990).

The modified FEHM using the dual porosity formulation can model coupled fractured systems without the difficulty of defining each fracture with distinct nodes yet can more fully model fracture system behavior than an effective continuum

model. The improvement over defining each fracture with distinct nodes saves time spent on fracture system model input. It also saves on computer solution time because the number of necessary nodes is less than if each fracture has to be defined in the mesh. The dual porosity model more effectively models the fracture and matrix storage interaction than fracture network or discrete fracture methods. The implicit formulation of the coupled solution in the modified FEHM is a major improvement over previous coupled codes with explicit coupled solutions because the implicit coupling exhibits faster convergence behavior. This coupled model may be used for 2-dimensional, saturated fractured reservoirs. Potential applications include analysis of hot dry rock geothermal extraction, geothermal subsidence, high level nuclear waste repositories, and petroleum production.

Results with the explicit formulation of the variable fracture permeability behavior and the implicit formulation results were compared using several test cases. The number of time steps and the CPU time needed to run the implicit codes were usually less than or equal to that needed to run the explicit formulation. This indicates that the implicit formulation leads to more rapid convergence of solutions.

Test cases were run to verify the accuracy of FEHM. These results agreed well with analytic solutions. Test Case 1 was run to test the heat transfer capabilities of FEHM. The results of Case 1 agree well with analytic solutions. Test Case 2 was run to test the variable aperture modifications of FEHM. Results of Case 2 agreed well with the semi-analytic solution. Case 3 was run to test the capability of FEHM to model a realistic situation. The geothermal reservoir model results were in excellent agreement with the single drawdown temperature from Fenton Hill field

data. In addition, a reasonable time step size, 10 days, was used to model the Fenton Hill geothermal reservoir. This is a very reasonable time step size.

It is judged that FEHM with the variable aperture modification to the dual porosity capabilities does work well based on the results of the test cases. FEHM can be used to accurately and efficiently model fracture flow systems where mass, energy and stress effects are coupled. It should prove useful in further study of coupled fracture systems.

10.0 References

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11.0 Nomenclature

A = area (L^2)

A_e = energy per unit volume (M/T^2L)

A_m = fluid mass per unit volume (M/L^3)

b = fracture aperture (L)

b_o = initial fracture aperture at some reference stress, σ'_o , (L)

b_i = body force in i direction (ML/T^2)

b_m = mechanical aperture (L)

c = half width of beam (L)

c = specific heat ($L^2/T^2 K$)

$C = \frac{E(1-\nu)}{(1+\nu)(1-2\nu)}$ (ML/T^2)

e = matrix cell size (L)

E = Young's modulus ($M L / T^2$)

E_f refers to the equation of conservation of energy in the fracture system

E_m refers to the equation of conservation of energy in the matrix system

$\vec{f}_e$ = energy flux (M / T^3)

$\vec{f}_m$ = fluid mass flux ($M/L^2 T$)

F = force ($M L / T^2$)

F_s = surface boundary forces ($M L / T^2$)

$\vec{g}$ = constant gravitational vector, pointing downwards (L / T^2)

h = specific enthalpy of fluid (L^2/T^2)

H = Biot's physical constant (M/LT^2)

JCS = Barton's joint compressive strength

JRC = Barton's joint roughness coefficient

k = intrinsic permeability (L^2)

k_o = intrinsic permeability at some reference stress, σ'_o (L^2)
 k_f = intrinsic permeability of a fracture (L^2)
 k_m = intrinsic permeability of matrix (L^2)
 k_e = effective intrinsic permeability of a rock containing fractures (L^2)
 K_{ni} = initial normal stiffness of fracture ($M/L^2 T^2$)
 K_{si} = initial shear stiffness of fracture ($M/L^2 T^2$)
 l = length (L)
 m = fluid mass (M)
 M_f refers to the equation of conservation of mass in the fracture system
 M_m refers to the equation of conservation of mass in the matrix system
 N = finite element basis function
 P = fluid pressure ($M/L T^2$)
 P_f = fluid pressure in fracture system ($M/L T^2$)
 P_m = fluid pressure in matrix system ($M/L T^2$)
 q_e = energy source ($M/L T^3$)
 q_m = fluid mass source ($M/L^3 T$)
 Q = fluid flow (L^3/T)
 Q_o = irreducible fracture flow (L^3/T)
 r = radius (L)
 r_1 = inner radius (L)
 r_2 = outer radius (L)
 S = surface area (L^2)
 S_1 refers to the static force balance equation in the x-direction
 S_2 refers to the static force balance equation in the y-direction

t = time (T)

T = temperature (K)

T_f = temperature in fracture system (K)

T_m = temperature in matrix (K)

u = specific internal energy of fluid (L^2/T^2)

u_r = specific internal energy of rock (L^2/T^2)

u = rock displacement (L)

u_i = rock displacement in i direction (L)

u_r = radial displacement (L)

u_θ = angular displacement (L)

$\vec{v}$ = fluid velocity vector (L/T)

V = volume (L^3)

V_m = maximum fracture closure (L)

V_p = pore volume (L^3)

x_j = coordinate in j direction (L)

α = coefficient of thermal expansion (1/K)

θ = angle

θ = dummy variable representing the six unknowns in the FEHM formulation

β_r = rock compressibility ($L T^3 / M$)

ϵ_{ij} = strain tensor

K = effective thermal conductivity ($M L / K T^3$)

ρ = fluid density (M / L^3)

ρ_r = rock density (M / L^3)

ϕ = porosity

Φ = fracture porosity

$\mathfrak{R}$ = finite element residual

σ = stress (M/L T²)

σ_{ij} = stress tensor (M/L T²)

σ_n = normal stress (M/L T²)

σ'_n = effective normal stress (M/L T²)

σ'_o = reference effective normal stress (M/L T²)

σ_{peak} = peak shear strength (M/L T²)

τ = shear stress (M/L T²)

$d\Omega$ = boundary surface (L²)

ν = Poisson's ratio

μ = dynamic viscosity (M/L T)

12.0 Appendix 1: Fenton Hill HDR Project

Hot dry rock (HDR) energy extraction is based on the concept that the thermal energy of the earth's interior can be extracted and used directly for heating or converted to electrical energy. At depths where the temperature is of interest, the combination of temperature, pressure, and geochemical reactions tends to reduce the porosity and permeability of the rock. The rock permeability at these depths is too low to produce an economic source of indigenous geothermal water; thus the term dry rock (Smith, 1988). Thermal energy is removed by forcing surface water into the rock at these depths, forcing open fractures and creating a flow path. The water absorbs heat and is pumped to the surface for energy extraction (Figure 32).

The first field experiment of the HDR concept was performed in the Jemez Mountains of New Mexico by Los Alamos National Laboratory. Various other field studies have since been performed in the United Kingdom, France, Germany, Japan, and Sweden (Richards, *et.al.*, 1994). Los Alamos National Laboratory is continuing to study the behavior of the HDR geothermal reservoir at Fenton Hill, New Mexico.

Worldwide geothermal electric power production (not necessarily HDR) has increased from 1960 MW in 1980 to 5915 MW in 1993. It is expected to be 10,197 MW by the year 2000. In 1993, North America's share of this was 2594 MW. North America's share is expected to decrease sharply by the year 2000 because of the expanded production in Asia (Dickson and Fanelli, 1993). Investigation of the use of this geothermal energy source is important to the development of alternative energy sources.

The heat removal from the rock by the fluid will affect the amount of contraction in the rock and in the fluid as well as create buoyancy effects in the fluid. The fracture apertures will thus be influenced by thermal changes. At the Rosemanowes, Cornwall HDR project in Great Britain, it was found that the granite permeability was dependent on the injection well pressure (Figure 33 (Richards, *et.al.*, 1994)). Therefore the effects of heat and mass transfer are interrelated through the mechanisms of convection and thermal stress. A commercial scale HDR system for electrical generation requires 50 kg/s production flow rate for about 20 years without a large decrease in production fluid temperature (Richards, *et.al.*, 1994). It is desirable that the interaction of mass and energy in the fractured rock mass be predicted numerically before investing in expensive field exploration.

Figure 32. Hot dry rock geothermal reservoir concept

Figure 33. Fracture impedance as a function of injection well pressure (Richards,*et.al.*, 1994)

The Fenton Hill HDR reservoir is located in a fractured granite rock at a depth of 3660 m. The rock contains four joint sets which generally strike north-south; two sets dip at about 5° - 10° from vertical, the least dipping set is about 60° from vertical. Seismic investigation indicates that fracture spacing is about 10 to 40 m. Data indicates that fracture permeability can be modeled using the Gangi (1978) model and the cubic law. The initial mean temperature of the rock is 235 C. The temperature has not decreased significantly to date at Fenton Hill. Currently two wells are being used to investigate reservoir behavior. The injection well pressure is 62.7 MPa. The production well surface water pressure is 45.0 MPa. These well pressures result in a flow at the production well of 5.7 kg/sec. The reservoir is

thought to extend as a horizontal prism of width 300m, length 600 m, and thickness 30 m. *In-situ* perpendicular horizontal stresses are thought to be 22.5 MPa and 27.5 MPa. Far field permeability is extremely small. Under injection well pressurization the fractures are thought to open to 1×10^{-5} to 1×10^{-4} m. Because well pressure is maintained during energy production, fluid thermal contraction is not thought to be of immediate importance. However, thermal rock contraction will influence the fracture permeability over long periods of time. In addition, it is thought that fluid storage in the rock matrix within microcracks may have an important influence on reservoir behavior (Swenson, *et.al.*, 1995, DuTeau, 1995)

13.0 Appendix 2: Tests of Stress Coupling in FEHM

Cases A1 - A4 test FEHM stress coupled capabilities before and after modifications were implemented and establish that force equilibrium in FEHM is correct. The input files for these test cases are contained in Appendix 5; however, a major part of the coordinate and element information is left out for the sake of brevity.

13.1 Test Case A1: Bending Beam

Problem Description: Test Case A1 consists of a 2-dimensional bending beam with one end fixed and a force at the other end (Figure 34). This case tests the steady state stress-displacement capabilities of FEHM. The beam is 15 m long with a width of 1 m. Young's modulus is 5.0×10^5 Pa and the applied force is 8 N. The perpendicular deflection at the end of the beam is described by (Timoshenko and Goodier, 1970):

$$\Delta x = \frac{Fl^3}{2Ec^3} \quad (69)$$

where c is the half width of the beam.

Model Input: The numerical model consists of 130 elements in the vertical direction and 7 elements in the horizontal direction. A horizontal force of 1.0 N is applied to each of the nodes along the top boundary. Poisson's ratio is 0.0. The boundary conditions are given in Table 11.

Figure 34. Model representation of Case A1

Table 11: Case A1 - Boundary conditions used in model (l=left, r=right, t=top, b=bottom boundary)

Type	Boundary	Condition
stress	b t	no displacement horizontal force = 1 N at each node
fluid flow	r,l,t,b	no normal flow
heat transfer	r,l,t,b	no normal transfer

Results: The analytic solution of the bending beam problem results in a beam end deflection of 0.215 m. The deflection calculated by FEHM is 0.214 m. This results in an error of 0.5% which is excellent agreement.

13.2 Test Case A2: Hole in a Plate

Problem Description: Test Case A2 consists of a 2-dimensional plate, 9.75 m square, with a small hole of radius 0.5 m in the center (Figure 35). This test case tests the steady state, stress-displacement capabilities of FEHM. There is a horizontal force of 100 N on the vertical boundaries. Young's modulus is 5.0×10^5 Pa and Poisson's ratio is 0.25.

The radial and the angular displacements are (Timoshenko and Goodier, 1970):

$$\Delta u_r = \frac{\sigma}{2E} \left[r + \frac{r_1^2}{r} + \left(r - \frac{r_1^4}{r^3} + \frac{4r_1^2}{r} \right) \cos 2\theta - \nu \left[r - \frac{r_1^2}{r} - \left(r - \frac{r_1^4}{r^3} \right) \cos 2\theta \right] \right] \quad (70)$$

$$\Delta u_{\theta} = \frac{\sigma}{2E} \left[\sin 2\theta \cdot \left[-r - \frac{r_1^4}{r^3} - \frac{2r_1^2}{r} + \nu \left(-r + \frac{r_1^4}{r^3} + \frac{2r_1^2}{r} \right) \right] \right] \quad (71)$$

Model Input: The FEHM model consists of 204 nodes. Due to symmetry only one quarter of the problem is modeled. The quarter plate is 4.875 m in both directions. Forces are applied along the right boundary to simulate a stress of 100 N. The boundary conditions are given in Table 12.

Figure 35. Model representation of Case A2

Table 12: Case A2 - Boundary conditions used in model (l=left, r=right, t=top, b=bottom boundary)

Type	Boundary	Condition
stress	l,b r	no normal displacement stress = 20.51 N m^{-2}
fluid flow	r,l,t,b	no normal flow
heat transfer	r,l,t,b	no normal transfer

Results: Table 13 contains the results of both the analytic computations and the finite element program along a line at 45° from the horizontal. The difference (analytic result minus numerical result) and the percent difference (difference divided by analytic result) is also presented. Table 13 indicates that there is very good agreement between FEHM results and the analytic solution in the applied stress direction. The results of FEHM and the analytic solution are within 10% error in the direction perpendicular to the applied stress. This error gets larger as the boundary of the problem is approached which is to be expected. Close to the central hole the error is less than 5%. The comparison of analytic and FEHM results is plotted in Figure 36.

Table 13: Case A2 - Comparison of analytic and numerical results

Radius, m	Analytic x-displace- ment, m	Numerical x-displace- ment, m	Difference, m	Percent Difference	Analytic y-displace- ment, m	Numerical y-displace- ment, m	Difference, m	Percent Difference
.500	2.13E-04	2.08E-04	5.00E-06	2.35	-7.05E-05	-6.83E-05	-2.20E-06	3.12
.669	1.86E-04	1.84E-04	2.00E-06	1.08	-4.87E-05	-4.88E-05	1.00E-07	0.21
.837	1.86E-04	1.87E-04	-1.00E-06	0.54	-4.43E-05	-4.61E-05	1.80E-06	4.06
1.17	2.10E-04	2.14E-04	-4.00E-06	1.90	-4.86E-05	-5.16E-05	3.00E-06	6.17
1.51	2.47E-04	2.52E-04	-5.00E-06	2.02	-5.79E-05	-6.21E-05	4.20E-06	7.25
2.18	3.31E-04	3.38E-04	-7.00E-06	2.11	-7.96E-05	-8.46E-05	5.00E-06	6.28
2.86	4.22E-04	4.28E-04	-6.00E-06	1.42	-1.03E-04	-1.07E-04	4.00E-06	3.88
4.20	6.06E-04	6.08E-04	-2.00E-06	0.33	-1.50E-04	-1.49E-04	-1.00E-06	0.67
5.54	7.92E-04	7.87E-04	5.00E-06	0.63	-1.97E-04	-1.87E-04	-1.00E-05	5.08
6.22	8.87E-04	8.76E-04	1.10E-05	1.24	-2.21E-04	-2.04E-04	-1.70E-05	7.69
6.56	9.35E-04	9.21E-04	1.40E-05	1.50	-2.33E-04	-2.13E-04	-2.00E-05	8.58
6.90	9.83E-04	9.65E-04	1.89E-05	1.83	-2.45E-04	-2.22E-04	-2.30E-05	9.39

Figure 36. Case A2 - Comparison of analytic and FEHM results

The finite element stress code, ABAQUS (ABAQUS, 1989), was run with an identical mesh and stress/strain properties. The results of ABAQUS are identical with those of FEHM along the 45° line. This indicates that the differences between the analytic and numerical solutions are probably due to the size of the mesh and not to any inherent problems within FEHM.

13.3 Test Case A3: Hole in a Cylinder

Problem Description: Test Case A3 consists of a cylinder with a hole in the center (Figure 37). The entire cylinder is cooled 10 C and the cylinder contracts in response. This case was also run with the pressure increase of 10 MPa and the

temperature held constant in order to test the similar effect of a change in pressure in the cylinder. This case tests the steady-state thermal-mechanical and hydraulic-mechanical couplings in FEHM. The inner radius is 0.5 m, the outer radius is 6.5 m, the thermal expansion coefficient is $1 \times 10^{-5} \text{ C}^{-1}$, and Biot's constant is $1 \times 10^{-5} \text{ MPa}^{-1}$.

The radial displacement is (Timoshenko and Goodier, 1970):

$$u_r = \frac{\Delta T}{2} \alpha \frac{1 + \nu}{1 - \nu} \left[\frac{r^2 - r_1^2}{r} + (1 - 2\nu)r + \frac{r_1^2}{r} \right] \quad (72)$$

Model Input: The FEHM mesh consists of 221 nodes. Due to symmetry only one quarter of the configuration is modeled. Initially the temperature is 35 C. The cylinder temperature is then changed to 25 C and thermal contraction occurs. The boundary conditions are give in Table 14.

Figure 37. Model representation of Case A3

Table 14: Case A3 - Boundary conditions used in model (l=left, r=right, t=top, b=bottom boundary)

Type	Boundary	Condition
stress	l,b	no normal displacement
fluid flow	l,b, r ₁ , r ₂	no normal flow
heat transfer	l,b, r ₁ , r ₂	no normal transfer

Results: Along the x -axis the analytic and numerical displacements due to thermal contraction agree with 0% error. Along a line 45° to the x -axis the error is less than 0.2% as shown in Table 15. This small error is thought to be due to roundoff. When the pressure effect is tested the results are similar. Table 15 indicates there is excellent agreement of FEHM results and the analytic solution.

Table 15: Case A3 - Comparison of analytic and numerical results with change in temperature

Radius, m	Analytic displacement, m	Numerical displacement, m	Difference, m	Percent Difference
0.500	6.25E-5	6.25E-5	0	0
1.797	2.25E-4	2.25E-4	0	0
2.997	3.75E-4	3.75E-4	0	0
4.068	5.08E-4	5.09E-4	0.01e-4	0.2
5.008	6.26E-4	6.26E-4	0	0
5.819	7.27E-4	7.27E-4	0	0
6.500	8.12E-4	8.13E-4	0.01e-4	0.1

13.4 Test Case A4: Solid Cylinder

Problem Description: Test Case A4 consists of a solid, infinitely long cylinder initially at temperature, $T_o = 25$ C. This test case tests the transient thermal-mechanical coupling of FEHM. The cylinder radius is 6.5 m. A constant temperature of 10 C is applied to the outer boundary of the cylinder. The inner temperature and radial displacement of the cylinder are a function of time. The conceptual model of the cylinder is shown in Figure 38.

The temperature change of an infinite cylinder with a fixed boundary temperature, T_1 , can be expressed as (Timoshenko and Goodier, 1970):

$$T = T_1 + (T_o - T_1) \sum_{n=1}^{\infty} A_n J_o\left(\beta_n \frac{r}{r_2}\right) e^{-p_n t} \quad (73)$$

where T_o is the initial temperature of the cylinder, T_1 is the boundary temperature, β_n are the roots of the equation $J_o(\beta) = 0$, J_o is the Bessel function of zero order, and the constants p_n are given by:

$$p_n = \frac{\kappa \beta_n^2}{c \rho r_2^2} \quad (74)$$

The density of a material increases with decreasing temperature and can be expressed as:

$$\rho = \rho_o(1 - \alpha\Delta T) \quad (75)$$

The mass of the material within the cylinder can be expressed as $m = \rho V$ and, since the mass before and after thermal expansion is constant,

$$\rho_o \cdot \pi r_o^2 = \int_0^r 2\pi x \cdot \rho_o(1 - \alpha\Delta T) dx \quad (76)$$

where ρ_o and r_o represent conditions before the expansion takes place. Eq. 76 can be solved numerically to find the radial displacement (difference in radii before and after expansion).

Model Input: The FEHM input consists of 205 nodes. Due to symmetry only one quarter of the configuration is modeled. Initially the temperature of the cylinder is 25 C. The cylinder boundary temperature is changed to 10 C at $t=0$. Since this is a transient problem, the temperatures and displacements are examined at 5 days. The temperature distribution used as input for the solution of Eq. 76 for each radius of interest is obtained from FEHM output at $t= 5$ days. The parameters used in Case A4 are found in Table 16. The boundary conditions are given in Table 17.

Figure 38. Model representation of Case A4

Table 16: Case A4 - Parameters used in model

Parameter	Value
initial temperature	25 C
boundary temperature	10 C
outer radius	6.5 m
density	7750 kg m ⁻³
specific heat	500 J kg ⁻¹ C ⁻¹
thermal conductivity	50 W m ⁻¹ C ⁻¹
Young's modulus	1.0 x 10 ³ MPa
Poisson's ratio	0.5
linear thermal expansion coefficient	1.0 x 10 ⁻⁴ C ⁻¹

Table 17: Case A4 - Boundary conditions used in model (l=left, r=right, t=top, b=bottom boundary)

Type	Boundary	Condition
stress	l,b	no normal displacement
fluid flow	l,b, outer radius	no normal flow
heat transfer	l,b outer radius	no normal transfer T = 10 C

Results: The results of Eq. 73 are compared to the FEHM results at $t=5$ days in Table 18. The largest percent error is 2.32% which shows very good agreement. Using the temperature results from Table 18, the analytic solution to the radial displacements are calculated. The results are compared to those of FEHM in Figure 39. Excellent agreement is shown.

Table 18: Case A4 - Comparison of FEHM and analytic temperatures at 5 days

Radius, m	Temperature, Analytic, C	Temperature, FEHM, C	Percent error
6.5000	10.000	10.000	0.00
5.3496	12.732	12.963	-1.81
4.4867	14.833	15.178	-2.32
3.8252	16.354	16.699	-2.11
3.2212	17.611	17.913	-1.72
2.6748	18.604	18.842	-1.28
2.1858	19.358	19.529	-0.88
1.6969	19.971	20.076	-0.52
1.2655	20.389	20.443	-0.26
0.8916	20.655	20.675	-0.10
0.5752	20.809	20.808	0.00
0.2876	20.891	20.881	0.05
0.0000	20.918	20.912	0.03

Figure 39. Case A4 - Comparison of analytic and numeric results of radial displacements at 5 days

14.0 Appendix 3: Using FEHMS

A users manual for the original FEHM code has been published; FEHMN 1.0: Finite Element Heat and Mass Transfer Code (Zyvoloski, *et.al.*, 1991). The following pertain to the use of the dual porosity with variable fracture aperture system. The input instructions for macros STRESS and FRAC are shown in Tables 19 and 20. Other macros are found in the user manual.

- macro CTRL, UPWGT=1.0 should always be used for variable fracture aperture systems. In the macro, SOL, INTG=-1 should be used.
- macro DUAL - The macro DUAL lets the user model the dual porosity system. The dual porosity properties need to be input for the number of fracture nodes (The total number of nodes tracked in the system is three times the number of fracture nodes; the fracture nodes, the matrix nodes, and the communication nodes which are ignored when a variable fracture aperture system is used). The parameter VOLFD1 is equal to the fracture aperture divided by the distance between model nodes times the number of fractures per node. VOLFD2 is equal to 1-VOLFD1. APUVD is the distance fluid travels from fracture to matrix and is usually taken as the half fracture spacing. To model only the fracture system and ignore matrix effects, APUVD should be a large number.
- macro FRAC - The macro FRAC lets the user model the variable fracture aperture system. It must be used in conjunction with the

macros DUAL and STRESS. In the first group of input parameters, AIO is the initial fracture aperture in the vertical fractures. STIFX is the initial fracture stiffness for a vertical set of fractures and it effects the vertical fracture system. VMINX is the maximum fracture closure for vertical fractures. Note that the value of VMINX effects how fast the fracture stiffness changes with stress. SIGFOX is the initial state of stress (not effective stress) in the x -direction (Eq. 47). The parameters affecting the vertical fracture system will affect the vertical fracture permeability but they are the influenced by the horizontal stress system.

BIO, STIFY, VMINY and SIGFOY are defined the same way.

However, they apply to a horizontal fracture system and affect the horizontal fracture permeability.

In the second group of input parameters, the power used in Eq. 47 must be input as AFI (a value of 2 for the cubic law). CFI is the multiplying constant used in Eq. 47 (a value of $1/12$ for the cubic law). Note that if the model is to use a changing aperture to also calculate the areas surrounding nodes (a common effect), AFI should be 3 and CFI should be $1/12$ divided by the initial fracture aperture. FINX is input as the number of vertical fractures associated with each node. FINY is the number of horizontal fractures associated with each node. FTOLL is the tolerance that would be accepted when calculating the stress. It is used to remove the effect of error due to numerical calculations.

In order to model a horizontal fracture system STIFX should be a very large number and VMINX should be very small. AIO should also be very small. If a horizontal fracture with a constant aperture is needed STIFY should be very large and VMINY should be very small.

The following are the input instructions for macro frac:

Control statement frac (required for variable fracture aperture problem)

Group 1 - JA, JB, JC, AIO, STIFX, VMINX, SIGFOX, BIO, STIFY, VMINY, SIGFOY

Group 2 - JA,JB, JC, FTOLL, AFI, CFI, FINX, FINY

Table 19: Input for control statement frac

Input Variable	Format	Description
JA	integer	first node to be assigned the property
JB	integer	last node to be assigned the property
JC	integer	loop increment for singing property
AIO	real	initial vertical fracture aperture (m)
STIFX	real	initial vertical fracture stiffness (MPa m ⁻¹)
VMINX	real	maximum vertical fracture closure (m)
SIGFOX	real	initial horizontal stress state (MPa)
BIO	real	initial horizontal fracture aperture (m)
STIFY	real	initial horizontal fracture stiffness (MPa m ⁻¹)

Table 19: Input for control statement frac

Input Variable	Format	Description
VMINY	real	maximum horizontal fracture closure (m)
SIGFOY	real	initial vertical stress state (MPa)
FTOLL	real	tolerance of stress due to fracture opening. Only the last value input is used.
AFI	real	power of cubic law
CFI	real	coefficient of cubic law
FINX	real	number of horizontal fractures assigned to node
FINY	real	number of vertical fractures assigned to node

- **macro PERM** - The macro PERM allows the user to input the matrix and fracture permeabilities. The user needs to input values for three times the number of fracture nodes; one set for the fracture nodes, one set for the matrix layer, and one set for the communication layer. Note that input for the communication layer is necessary even though there is no communication layer used in these modifications for variable aperture. The permeability for the fracture is used to calculate the matrix permeability using VOLFD2 but is never actually used for the fracture system permeability. The fracture system permeability is calculated during an iteration using the cubic law.
- **macro STRESS** - The macro STRESS allows the user to input the stress information. Since this macro was already used for input into FEHM, there are some input variables that are residual, that is, not

needed for the current application. For the variable fracture aperture application the user should use a 2-d planar problem. IHMS should be set to 3 to obtain a hydro-thermo-mechanical coupled solution.

The following are the input instructions for macro STRESS:

Control statement stress (required for variable fracture aperture problem)

Group 1 - DAYSTR, ICYCS, COFTOL, IPSPS, ISTRS

Group 2 - JA, JB, JC, YOUNG, POISON, ALPHAD, BULKD

Group 3 - JA, JB, JC, KQ, VALUE

Group 4 - IPCHNG

Group 5 - IHMS, ICPST

Group 6 - NOMASS, NOHEAT, NOYDIS

Group 7 - LOCALX, LOCALY

Group 8 - FUPWT, FDNWT

Group 9 - W0, FPOR, FRIC

Group 10 - NVFC

Group 11 - NVFCL(iiii, iiii=1,NVCF)

Group 12 - JA, JB, JC, PHILO, TEMPO

Table 20: Input for control statement stress

Input Variable	Format	Description
JA	integer	first node to be assigned the property
JB	integer	last node to be assigned the property
JC	integer	loop increment for singing property
DAYSTR	integer	residual value; use 0
ICYCS	integer	residual value; use 1
COFTOL	integer	residual value; use 0
IPSPS	integer	controls type of stress problem =0; 2-d plain stress =1; 2-d plain strain =2; 2-d radial =3; 3-d
ISTRS	integer	controls type of stress solution =0; no stress =1; coupled mechanical-hydraulic-thermal =-1; stress only
YOUNG	real	Young's modulus of matrix material (MPa)
POISSON	real	Poisson's ratio of matrix material
ALPHAD	real	coefficient of linear thermal expansion of matrix material ($^{\circ}\text{C}^{-1}$)
BULKD	real	coefficient of pore pressure effect of matrix material (Biot's constant) (MPa^{-1})

Table 20: Input for control statement stress

Input Variable	Format	Description
KQ	integer	type of stress boundary condition =+1; specified x-displacement =+2; specified y-displacement =+3; specified z-displacement =-1; specified x-force =-2; specified y-force =-3; specified z-force
VALUE	real	amount of force or displacement
IPCHNG	integer	residual value for subroutine sptnew (at present use only ipchng =0) =0; no action >0; open and read from file inst.new the following - JA, JB, JC,PCHNG, TCHNG<0; read from the current FEHM's input file the following TPTCH, PCHMIN, PCHMAX, TCHMIN, TCHMAX
IHMS	integer	determines type of coupling in solution =0; stress only =1; hydraulic and mechanical coupling =2; thermal and mechanical coupling =3; hydraulic, thermal, and mechanical coupling
ICPST	integer	residual value; use 1
NOMASS	integer	=0; mass balance equation is solved =1; mass balance equation is not solved
NOHEAT	integer	=0; energy equation is solved =1; energy equation is not solved
NOYDIS	integer	residual value; use 0 =0; y-displacement is solved =1; y-displacement equation is not solved
LOCALX	integer	residual value; use 1 non-local terms are dropped from the x-displacement equation

Table 20: Input for control statement stress

Input Variable	Format	Description
LOCALY	integer	residual value; use 1 non-local terms are dropped from the y-displacement equation
FUPWT	integer	residual value; use 1 value of upstream weighting in the coupled flow terms for the distinct fracture nodes
FDNWT	integer	residual value; use 0 value of downstream weighting in the coupled flow terms for the distinct fracture nodes.
W0	real	residual value; use 1 residual distinct fracture aperture
FPOR	real	residual value; use 1 fracture porosity
FRIC	real	residual value; use 1 friction factor in the fracture flow coefficient
NVFC	integer	residual value; use 0 number of nodes in the distinct fracture
NVFC1	integer	residual value; use 0 list of NVFC nodes numbers that are in the distinct fracture
PHILO	real	specified initial pressure of fracture nodes (MPa)
TEMPO	real	specified initial temperature of fracture nodes (C ⁻¹)

15.0 Appendix 4: Properties of Granite

The following are properties of granite such as might be found in a hot dry rock reservoir. These properties are from Wang, *et.al.*, (1981) and Rummel (1991).

- thermal conductivity = $2.5\text{-}3.2 \text{ W m}^{-1} \text{ C}^{-1}$
- thermal expansion coefficient = $9.2 \times 10^{-6} \text{ C}^{-1}$ (in range 20-50 C)
- density = $2600 - 2660 \text{ kg m}^{-3}$
- specific heat = $836 - 837 \text{ J kg}^{-1} \text{ C}^{-1}$
- Young's modulus = $38 - 69 \text{ GPa}$
- intrinsic permeability = $4.9 \times 10^{-17} \text{ m}^2$ at atmospheric pressure and $9.9 \times 10^{-19} \text{ m}^2$ at 100 MPa

16.0 Appendix 5: Test Cases - Input Files

16.1 Case A1: Bending Beam

```
****Case A1 - 6/8/94 k bower - beam problem
node
  6
4 1041 1043 1045 1047 1048
sol
  +1 +1
init
  20. 25. 25. 0. 0. 25. 0. 0.
pres
  01 1048 01 20. 25. 1
  00 00 00 0. 0. 0
rock
  1 1048 1 2500. 1000. .5
  0 0 0 0. 0. 0. 0.
cond
  1 1048 01 2.7e+02 2.7e+02 0.0e-00
  0 0 0 0. 0. 0.
perm
  01 1048 1 8.33e-08 8.33e-08 0. 0. 0. 0.
  0 0 0 0. 0. 0. 0. 0.
time
  .578704e-6 1.0 0015 5 1994 01
  0 0 0 0
ctrl
  90 1.e-08 30
  1 1048 1 8
  0 0 0 0
  1. 0.0 0.5
  20 3.00 1.e-8 0.500
  1 0
iter
  1.e-5 1.e-5 1.e-5 1.e-09 1.1
  0 0 0 0 0
ivfc      ! activate call to vfcal
strs      ! activate coupled stress module
0. 1 0. 0 -1 ! daystr,icycs,coftol,ipsps,istrs
1 1048 1 5.e5 0.0 1.e-30 1.e-30 ! ja,jb,jc,young,poison,alphad,bulkd
```



```

0 0 0 0. 0. 0. 0.
01 08 1 02 0.
01 08 1 01 0.
1041 1048 1 -1 -1.0000
0 0 0 0 0
0          ! ipchng, no action if = 0
0 1        !ihms, icpst
coord n/a
1048
1 1.00000 0.00000 0.00000
2 0.85714 0.00000 0.00000
3 0.71429 0.00000 0.00000
4 0.57143 0.00000 0.00000
5 0.42857 0.00000 0.00000
6 0.28571 0.00000 0.00000
7 0.14286 0.00000 0.00000
8 0.00000 0.00000 0.00000
.
.
.
1046 0.28571 15.00000 0.00000
1047 0.14286 15.00000 0.00000
1048 0.00000 15.00000 0.00000
0 0.00000 0.00000 0.00000
elem
4 910 0
1 9 10 2 1
2 10 11 3 2
3 11 12 4 3
.
.
.
908 1045 1046 1038 1037
909 1046 1047 1039 1038
910 1047 1048 1040 1039
0 0 0 0 0
stop

```

16.2 Case A2: Hole in a Plate

Case A2 - 4/8/94 - k bower - hole in plate problem

```

node
204
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28
29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53
54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71 72 73 74 75 76 77 78
79 80 81 82 83 84 85 86 87 88 89 90 91 92 93 94 95 96 97 98 99 100 101 102
103 104 105 106 107 108 109 110 111 112 113 114 115 116 117 118 119 120
121 122 123 124 125 126 127 128 129 130 131 132 133 134 135 136 137 138 139
140 141 142 143 144 145 146 147 148 149 150 151 152 153 154 155 156 157 158
159 160 161 162 163 164 165 166 167 168 169 170 171 172 173 174 175 176 177
178 179 180 181 182 183 184 185 186 187 188 189 190 191 192 193 194 195 196
197 198 199 200 201 202 203 204
sol
1 1
init
20 25 25 0 0 25 0 0
pres
1 204 1 20 25 1
0 0 0 0 0
rock
1 204 1 2500. 1000. 1.
0 0 0 0 0 0 0
cond
1 204 1 2.7e02 2.7e02 0
0 0 0 0 0 0
perm
1 204 1 8.33d-8 8.33e-8 0
0 0 0 0 0 0
time
.578704e-6 1 1 1 1994 1
0 0 0 0
ctrl
200 1.e-15 40
1 4 1 3
0 0 0 0
1. 0.0 0.5
40 1.00 1.e-8 0.500
1 0
iter
1.e-6 1.e-6 1.e-2 1.e-10 1.1
0 0 0 0 0 0 0
ivfc

```

strs

0 1 0 0 1

1 204 1 5.e5 0.25 1.e-30 1.e-30

0 0 0 0. 0. 0. 0.

1 12 1 2 0

160 204 4 1 0

97 97 1 -1 25.

85 85 1 -1 50.

73 73 1 -1 75.

61 61 1 -1 100.

49 49 1 -1 96.875

37 37 1 -1 70.3125

25 25 1 -1 35.156

13 13 1 -1 23.4375

1 1 1 -1 11.72

0 0 0 0 0

0

3 1

0 0 0

0 0

1 0

1 1 1

0

0

1 204 1 20. 25.

0 0 0 0 0

coor n/a

204

1	4.87500	0.00000	0.00000
2	4.64474	0.00000	0.00000

.

.

.

202	0.46875	4.87500	0.00000
-----	---------	---------	---------

203	0.23437	4.87500	0.00000
-----	---------	---------	---------

204	0.00000	4.87500	0.00000
-----	---------	---------	---------

0	0.00000	0.00000	0.00000
---	---------	---------	---------

elem

4 176 0

1	13	14	2	1
---	----	----	---	---

2	14	15	3	2
---	----	----	---	---

3	15	16	4	3
---	----	----	---	---

4 16 17 5 4

172 199 200 196 195
173 156 201 197 152
174 201 202 198 197
175 202 203 199 198
176 203 204 200 199
0 0 0 0 0

stop

16.3 Case A3: Hole in Cylinder

Case A3- 9/28/94, k.bower -- t/u,v coupling problem, hole in cylinder

node

26

1 2 3 4 5 6 7 8 9 10 11 12 13 105 106 107 108 109
110 111 112 113 114 115 116 117

sol

1 1

init

20 25 00 0 0 00 0 0

pres

1 221 1 20. 25. 1

0 0 0 0 0 0

rock

1 221 1 2500. 1000. 1.

0 0 0 0 0 0

cond

1 221 1 2.7e02 2.7e02 0

0 0 0 0 0 0

perm

1 221 1 8.33d-8 8.33e-8 0

0 0 0 0 0 0

time

.578704e-6 1 10 1 1994 1

0 0 0 0

ctrl

200 1.e-15 8

1 221 1 2

```

0 0 0 0
1. 0 0.5
40 1.00 1.e-8 0.500
1 0
iter
1.e-6 1.e-6 1.e-6 1.e-10 1.0
0 0 0 0 0 0
ivfc
strs
0 1 0 1 1 ! plain stress
1 221 1 1.e03 0.25 1.e-05 1.e-30
0 0 0 0. 0. 0. 0.
1 13 1 2 0
209 221 1 1 0
0 0 0 0 0
0
1 1
0 0 0 ! nomass,noheat,noydis
0 0 ! localx, nlocaly
1 0 ! fupwt, fdnwt
1.e-4 1 1 !w0, fpor, fric
0 ! nvfc
0 ! nvfcl
001 221 1 20. 35.
0 0 0 0 0
coord n/a
221
1 6.50000 0.00000 0.00000
2 6.17568 0.00000 0.00000
3 5.81892 0.00000 0.00000
4 5.42973 0.00000 0.00000
.
.
.
218 0.00000 2.41351 0.00000
219 0.00000 1.79730 0.00000
220 0.00000 1.14865 0.00000
221 0.00000 0.50000 0.00000
0 0.00000 0.00000 0.00000
elem
4 192 0
1 14 15 2 1

```

2	15	16	3	2
3	16	17	4	3
4	17	18	5	4

```

190 218 219 206 205
191 219 220 207 206
192 220 221 208 207
0 0 0 0 0
stop

```

16.4 Case A4: Solid Cylinder

Case A4 - 1/5/95, k.bower, t,uv coupling, solid cylinder,transient
node

205

```

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40
41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60
61 62 63 64 65 66 67 68 69 70 71 72 73 74 75 76 77 78 79 80
81 82 83 84 85 86 87 88 89 90 91 92 93 94 95 96 97 98 99 100
101 102 103 104 105 106 107 108 109 110
111 112 113 114 115 116 117 118 119 120
121 122 123 124 125 126 127 128 129 130
131 132 133 134 135 136 137 138 139 140
141 142 143 144 145 146 147 148 149 150
151 152 153 154 155 156 157 158 159 160
161 162 163 164 165 166 167 168 169 170
171 172 173 174 175 176 177 178 179 180
181 182 183 184 185 186 187 188 189 190
191 192 193 194 195 196 197 198 199 200
201 202 203 204 205

```

sol

1 1

init

20. 25. 00 0 0 00 0 0

flow

```

1 1 1 20.000 -10.00 100.
14 194 12 20.000 -10.00 100.
0 0 0 0 0

```

hflx

1 1 1 10. 100.
14 194 12 10. 100.

rock

1 205 1 7.75e3 500. .001
0 0 0 0 0 0

cond

1 205 1 50 50 0.
0 0 0 0 0 0

perm

1 205 1 8.33e-8 8.33e-8 0
0 0 0 0 0 0

time

5.e-5 05. 100 5 1994 1
0 0 0 0

ctrl

200 1.e-05 8
1 205 1 2
0 0 0 0
1. 0 0.5
100 2.00 1.e-8 1.0

1 0

iter

1.e-5 1.e-5 1.e-5 1.e-9 1.1
0 0 0 0 0 0

ivfc

strs

0 1 0 1 1 ! plain strain
1 205 1 1.e03 0.50 1e-4 1.e-30
0 0 0 0. 0. 0. 0.
1 13 1 2 0
13 13 1 1 0
194 205 1 1 0
0 0 0 0 0
0

3 1

0 0 0 ! nomass,noheat,noydis
0 0 ! localx, nlocaly
1 0 ! fupwt, fdnwt
1 1 1 !w0, fpor, fric
0 ! nvfc

```

0          ! nvfc1
1 205 1 20. 25.
0 0 0 0 0
coord n/a
205
    1    6.50000    0.00000    0.00000
    2    5.34956    0.00000    0.00000
    3    4.48673    0.00000    0.00000
    4    3.82522    0.00000    0.00000
.
.
.
202    0.00000    1.26549    0.00000
    203    0.00000    0.89159    0.00000
    204    0.00000    0.57522    0.00000
    205    0.00000    0.28761    0.00000
    0     0.00000    0.00000    0.00000
elem
4 192 0
    1    14    15    2    1
    2    15    16    3    2
    3    16    17    4    3
.
.
.
190    203    204    192    191
    191    204    205    193    192
    192    205    13    13    193
    0     0     0     0     0
stop

```

16.5 Case 1: Single Fracture, Constant Aperture

Case 1**** gringarten #1, single fracture - K.Bower, 8/23/95

node

7

1 41 42 82 83 123 124

dual

1

1 287 1 .99

1 41 1 6.2e-4


```

0000
  1 287 1 .01
  1 41 1 .99938
0000
  1 287 1 1e1
0000
frac
  1 287 1 1.e-30 1e30 1.e-30 0.0 1.e-30 1e30 1.e-30 0.0
  1 41 1 1.e-30 1e30 1.e-30 0.0 1.e-03 1e30 1.e-3 0.0
000000000000
  1 287 1 1.e-20 3 8.33e1 1.e-30 1.e-30
  1 41 1 1.e-20 3 8.33e1 1.e-30 1.
000000000
sol
  +1 -1
init
  25. 100. 0. 0. 0. 0. 0. 0.
flow
  41 41 1 -.006152 -90.0 0.
  1 1 1 25.0 -100. 100.
  00 00 00 0.0 0
hflx
  41 41 1 90. 1000.

rock
  1 861 1 2650. 1046 .1
  1 41 1 2650. 1046 1.
0 0 0.0. 0. 0.
cond
  1 861 01 2.59 2.59 00
  0 0 0 0. 0. 0.
perm
  01 861 1 1e-20 1e-20 0
  1 41 1 1e-20 1e-20 0
0000.0.0.
time
1.e0 1.9e05 5500 15 1994 01
0
ctrl
  50 1.e-05 22
  1 861 1 5
  0 0 0 0

```

```

1. 0.0 1.0
40 2.00 1e-06 1e2
1 0
iter
1.e-5 1.e-5 1.e-5 1.e-6 1.1 0
0 0 0 0 0
ivfc          ! activate call to vfc
strs          ! activate coupled stress module
0. 1 0. 1 1   ! daystr,icycs,coftol,ipsps,istrs*****
1 287 1 1e03 0 1e-30 1e-30   ! ja,jb,jc,young,poison,alphad,bulkd
0 0 0 0. 0. 0. 0.
41 287 41 1 0.
1 41 1 2 0.
247 287 1 2 0.
0 0 0 0 0
0          ! ipchng, no action if = 0
3 1          ! ihms, icpst
0 0 0          ! nomass,noheat,noydis
0 0          ! localx, nlocaly
1 0          ! fupwt, fdnwt
1.e-4 1 1          ! w0, fpor, fric
0          ! nvfc
0          ! nvfcl
1 287 1 25 100
0 0 0 0 0
coord n/a
287
1 1000.00000 0.00000 0.00000
2 975.00000 0.00000 0.00000
3 950.00000 0.00000 0.00000
.
.
.
284 75.00000 50.00000 0.00000
285 50.00000 50.00000 0.00000
286 25.00000 50.00000 0.00000
287 0.00000 50.00000 0.00000
0 0.00000 0.00000 0.00000
elem
4 240 0
1 42 43 2 1
2 43 44 3 2

```

3	44	45	4	3
4	45	46	5	4

237	283	284	243	242
238	284	285	244	243
239	285	286	245	244
240	286	287	246	245
0	0	0	0	0

16.6 Case 2: Single Fracture, Fluid Pressure

Case 2 **** Wijesinghe, #1 - K.Bower, 6/26/95 283 100.00000 50.00000
0.00000

node

14

124 125 126 130 135 140 145 150 155 160 161 162 163 164

dual

1

1 287 1 1.4e-4

0 0 0 0

1 287 1 .99986

0 0 0 0

1 287 1 1e8

0 0 0 0

frac

1 287 1 1.e-30 1.e60 1.e-60 00.0 1.e-30 1.e60 1.e-60 00.0

124 164 1 1.e-30 1.e60 1.e-60 00.0 1.e-05 1.e05 1.e-60 00.0

0 0 0 0 0 0 0 0 0 0

1 287 1 1.e-30 3 8333 1.e-30 1.e-30

124 164 1 1.e-30 3 8333 1.e-30 1.

0 0 0 0 0 0 0

sol

+1 -1

init

21. 20. 0. 0. 0. 0. 0. 0.

pres

164 164 1 21.9 20. 1

0 0 0 0 0 0

flow

```

164 164 1 21.9 -20. 1000
124 124 1 21.0 -20. 1.e1
00 00 00 0.0.0
hflx
164 164 1 20. 100.
0 0 0 0
rock
1 861 1 2716. 803 .1
124 164 1 2716. 803 1.
0 0 0 0.0. 0. 0.
cond
1 861 01 2.57 2.57 0.00
0 0 0 0. 0. 0.
perm
1 861 1 1.e-20 1.e-20 0.
0 0 0 0.0.0.
time
1e-6 2.315e-2 50000 100 1994 01

ctrl
50 1.e-07 22
1 861 1 8
0 0 0 0
1. 0.0 1.
40 2.00 1e-28 5e-03
1 0
iter
1.e-5 1.e-5 1.e-7 1.e-7 1.1 0
0 0 0 0 0 0
ivfc ! activate call to vfcal
strs ! activate coupled stress module
0. 1 0. 1 1 ! daystr,icycs,coftol,ipsps,istrs*****
1 287 1 1.e3 0. 1e-30 1e-30 ! ja,jb,jc,young,poison,alphad,bulkd
0 0 0 0.0.0.0.
41 287 41 1 0.
1 41 1 2 0.
247 287 1 2 0.
0 0 0 0 0
0 ! ipchng, no action if = 0
3 1 ! ihms, icpst
0 0 0 ! nomass,noheat,noydis
0 0 ! localx, nlocaly

```

```

1 0          ! fupwt, fdnwt
1.e-4 1 1    !w0, fpor, fric
0           ! nvfc
0           ! nvfcl
1 287 1 21. 20.
0 0 0 0 0
coor n/a
287
    1 25.00000 0.00000 0.00000
    2 24.37500 0.00000 0.00000
    3 23.75000 0.00000 0.00000
    4 23.12500 0.00000 0.00000
.
.
.
284 1.87500 1.00000 0.00000
285 1.25000 1.00000 0.00000
286 0.62500 1.00000 0.00000
287 0.00000 1.00000 0.00000
0 0.00000 0.00000 0.00000
elem
4 240 0
    1 42 43 2 1
    2 43 44 3 2
    3 44 45 4 3
    4 45 46 5 4
    5 46 47 6 5
.
.
.
236 282 283 242 241
237 283 284 243 242
238 284 285 244 243
239 285 286 245 244
240 286 287 246 245
    0 0 0 0 0
stop

```

16.7 Case 3: Hot Dry Rock Reservoir

```
Case3 **** Fenton Hill #1 - K.Bower, 10/20/94
node
  34
  2 6 7 8 9 10 11 12 13 14 15 16 17 20 23 24 25 26 27 35 40 45
50 55 60 65 70 75 80 85 90 95 100 105
cont
free 100000 720
dual
1
  1 370 1 9.3e-06
0 0 0 0
  1 370 1 .99999069
0 0 0 0
  1 370 1 1.e0
0 0 0 0
frac
  1 370 1 9.3e-05 1.e08 9.3e-05 00.0 9.3e-05 1.e08 9.3e-05 00.0
0 0 0 0 0 0 0 0 0 0 0 0
  1 370 1 1.e-20 3 896 1.7 1.7
0 0 0 0 0 0 0 0 0 0 0 0
sol
  +1 -1
init
  62.7 230. 0. 0. 0. 0. 0. 0.
flow
  13 13 1 45.0 -230 1.
  25 25 1 62.7 -70 100.
  00 00 00 0. 0. 0
pres
  25 395 370 62.7 70. 1
0 0 0 0 0
hflx
  25 25 1 70 100000.

rock
  1 1110 1 2716. 803 .01
  1 370 1 2716. 803. 1.
  0 0 0 0. 0. 0. 0.
cond
  1 1110 01 2.57e0 2.57e0 0.00
```

```

1 370 1 2.57e0 2.57e0 0.0
0 0 0 0. 0. 0.
perm
01 1110 1 1.e-20 1.e-20 0.
0 0 0 0. 0. 0.
time
10. 720 9000 1 1994 01
0
ctrl
50 1.e-06 25
1 1110 1 8
0 0 0 0
1. 0.0 1.0
25 2.00 1e-8 10
1 0
iter
1.e-5 1.e-5 1.e-5 1.e-06 1.1 0
0 0 0 0 0
ivfc          ! activate call to vfcal
strs          ! activate coupled stress module
0. 1 0. 1 1    ! daystr, icycs, coftol, ipsps, istrs*****
1 370 1 8.5e04 0.25 7.5e-06 1.e-30 ! ja,jb,jc,young,poison,alphad,bulkd
0 0 0 0. 0. 0. 0.
1 334 37 1 0.
1 37 1 2 0.
334 370 1 2 0.
37 370 37 1 0.
0 0 0 0 0
0          ! ipchng, no action if = 0
3 1          ! ihms, icpst
0 0 0          ! nomass,noheat,noydis
0 0          ! localx, nlocaly
1 0          ! fupwt, fdnwt
1.e-4 1 1          ! w0, fpor, fric
0          ! nvfc
0          ! nvfcl
1 370 1 62.7 230.
0 0 0 0 0
coord n/a
370
1 600.00000 0.00000 0.00000
2 583.33333 0.00000 0.00000

```

3	566.66667	0.00000	0.00000
4	550.00000	0.00000	0.00000
5	533.33333	0.00000	0.00000
6	516.66667	0.00000	0.00000
7	500.00000	0.00000	0.00000

362	133.33333	150.00000	0.00000
363	116.66667	150.00000	0.00000
364	100.00000	150.00000	0.00000
365	83.33333	150.00000	0.00000
366	66.66667	150.00000	0.00000
367	50.00000	150.00000	0.00000
368	33.33333	150.00000	0.00000
369	16.66667	150.00000	0.00000
370	0.00000	150.00000	0.00000
0	0.00000	0.00000	0.00000

elem

4 324 0

1	38	39	2	1
2	39	40	3	2
3	40	41	4	3
4	41	42	5	4
5	42	43	6	5
6	43	44	7	6
7	44	45	8	7
8	45	46	9	8
9	46	47	10	9
10	47	48	11	10
11	48	49	12	11

318	363	364	327	326
319	364	365	328	327
320	365	366	329	328
321	366	367	330	329
322	367	368	331	330
323	368	369	332	331
324	369	370	333	332

0 0 0 0 0

stop