CONF-95/280--1

NUMERICAL OPTIMIZATION OF SIBERIAN SNAKES AND SPIN ROTATORS FOR RHIC*

ALFREDO U. LUCCIO

AGS Department, Brookhaven National Laboratory

Upton, NY 11973-5000, USA

E-mail: luccio@bnldag.ags.bnl.gov

RECEIVED

MAY 05 1996

ABSTRACT

The structure of the four Siberian Snakes and eight Spin Rotators being designed for RHIC is discussed. These devices consist each of four helical dipoles. Results of orbit and spin tracking through the magnets are presented.

OSTI

1. Introduction

For the program of polarized protons in the Relativistic Heavy Ion Collider at Brookhaven, two Siberian snakes and four spin rotators per ring will be used. ¹ Snakes produce a complete spin flip. Spin rotators, in pairs, rotate the spin from the vertical direction to the horizontal plane at a collider intersection and back to the vertical after it. Snakes, 180° apart and with axes of spin precession at 90° to each other, avoid depolarization of the proton beam in traversing resonances. Rotators are needed to study proton collisions with the polarization in the horizontal plane.

Classical snakes and rotators consist of magnetic solenoids or of a sequence of magnetic dipoles with fields alternately directed in the radial and vertical direction. ² It has been also proposed to use helical magnets, or twisted dipoles, in which the field, transverse to the axis of the magnet, continuously rotates as we proceed along it. ³ After comparative studies, it was decided to adopt for RHIC an elegant solution with four helical magnets both for the snakes and the rotators, proposed by Shatunov and Ptitsin. ⁴ In order to simplify the construction of the magnets and to minimize costs we will use four identical super-conducting helical modules for each device.

Our snakes have four right-handed helices. Spin rotators, two right-handed and two left-handed helices. The maximum field is limited to 4 Tesla. While small bore helical undulators have been built or designed for free-electron lasers ⁵, large super-conducting helical magnets have never been built. In spite of this difficulty, our choice is dictated by distinctive advantages of helical over more conventional transverse structures

- i. The devices are modular, i. e. they can be built with arrangements of identical modules.
- ii. The maximum orbit excursion in the magnet is smaller.
- iii. Orbit excursion is independent from the separation between adjacent magnets.
- iv. They allow for an easier control of the spin rotation and the orientation of the

*Work Performed Under the Auspices of the U.S. Department of Energy

MASTER

spin precession axis.

To use the same standard cryostats of the RHIC dipoles, the length of the snakes or rotators should be limited to 12 meters. We have chosen 2.4 m as the length of each module, nominally separated by a distance of 32 cm. The magnet bore has been chosen as 10 cm. The magnet separation is not critical, because of the property (iii) above. The bore is adequate for the resulting orbit deformation, as it will be shown.

An approximate calculation of a four helix spin rotator can be made with the use of matrices that describe the spin rotation through the four modules. In a similar way the beam optical properties of the device can be estimated.⁶ However, since the fringe field of the magnets has an important effect both on the spin rotation and on the orbit, the optimization of the design has been performed numerically, by integration of the equations of motion and of the equations of spin through the magnetic field including the fringe field. To first order, the results agree well with the analytical estimates.

The field of an infinitely long helix has been first described analytically by Blewett and Chasman⁷ and then by Ptitsin.⁸ Other descriptions derive the field as a superposition of undulators⁹ or from the twisting of a cosine dipole.⁵ For the fringe field we have simply assumed that the field in the body of the magnet will continue outside with the same helical structure and decay as a $1/\cosh$ function in a distance equal to the radius of the magnet aperture. The actual field and its fringe will be better known when a magnet prototype is built and measured. In particular the fringe will depend by the method by which the super-conducting magnet coils are terminated. The tracking code can accept as input either an analytical or mapped (measured) field.

2. Formalism

The helical field used in the calculation is the Blewett-Chasman field expanded to third order in the Cartesian coordinates x (radial) and y (vertical). The longitudinal coordinate is z , the axis of the helix. The components of the field are

$$\begin{cases} b_x \approx \left[-1 - \frac{1}{4}(3u^2 + v^2) \right] \sin kz + \frac{1}{2}v \cos kz \\ b_y \approx \left[1 + \frac{1}{4}(u^2 + 3v^2) \right] \cos kz - \frac{1}{2}uv \sin kz \\ b_z \approx -\sqrt{2} \left[v + \frac{1}{4}(u^2v + v^3) \right] \sin kz - \sqrt{2} \left[u + \frac{1}{4}(u^3 + uv^2) \right] \cos kz \end{cases}, \quad (1)$$

with

$$b = B/B_0 \quad \begin{cases} u = kx/\sqrt{2} \\ v = ky/\sqrt{2} \end{cases} \quad k = 2\pi/\lambda. \quad (2)$$

B_0 is the maximum field on axis, λ the field rotation wavelength. Within the magnet aperture the above expansion is very accurate, when compared with the complete analytical expression in terms of Bessel functions. It should be noted that the longi-

tudinal component of the field is zero on axis but is not negligible along the deformed trajectory followed by a particle through the magnet.

The differential equation of motion in a pure magnetic field is

$$\frac{d\vec{\beta}}{dt} = \vec{\beta} \times \vec{\Omega}, \quad (3)$$

where $\vec{\beta}$ is the relativistic velocity factor and the definitions hold

$$\vec{\Omega} = e\vec{B}/m\gamma \quad \beta^2 = 1 - 1/\gamma^2 \quad \gamma = E/mc^2. \quad (4)$$

The BMT differential equation for the spin precession is ¹⁰

$$\frac{d\vec{S}}{dt} = C_1[\vec{S} \times \vec{\Omega}] + C_2(\vec{\beta} \cdot \vec{\Omega})[\vec{S} \times \vec{\beta}], \quad (5)$$

with the constants

$$C_1 = 1 + G\gamma, \quad C_2 = -\frac{G\gamma^2}{1 + \gamma}, \quad G = \frac{1}{2}g - 1 = 1.7928. \quad (6)$$

In Equation 5 the spin is treated as an ordinary three-dimensional vector. Using z as the independent variable, and with the following definitions

$$\frac{dz}{dt} = \beta_z c, \quad \beta_z = \frac{\beta}{\sqrt{1 + x'^2 + y'^2}}, \quad \begin{cases} x' = \beta_x/\beta_z \\ y' = \beta_y/\beta_z \end{cases}, \quad (7)$$

the scalar equations of motion are

$$\begin{cases} (dx/dz)' = x'y'\Omega_x - (1 + x'^2)\Omega_y + y'\Omega_z \\ (dy/dz)' = (1 + y'^2)\Omega_x - x'y'\Omega_y - x'\Omega_z \\ dx/dz = x' \\ dy/dz = y' \end{cases}. \quad (8)$$

Similarly, the scalar spin equations are

$$\begin{cases} dS_x/dz = p_x S_y - p_y S_x \\ dS_y/dz = p_x S_z - p_z S_x \\ dS_z/dz = p_y S_x - p_x S_y \end{cases}, \quad (9)$$

with

$$\begin{cases} p_x = C_1\Omega_x/\beta_z + C_2x'\Gamma \\ p_y = C_1\Omega_y/\beta_z + C_2y'\Gamma \\ p_z = C_1\Omega_z/\beta_z + C_2\Gamma \end{cases}, \quad \Gamma = \beta_z(x'\Omega_x + y'\Omega_y + \Omega_z), \quad S_x^2 + S_y^2 + S_z^2 = 1. \quad (10)$$

Fig. 1. 4-Helix Spin Rotator. Field at $\gamma = 27$. The longitudinal field is on scale $\times 10$.

The integration is performed by a code, Snig¹¹ employing a Hamming Predictor-Corrector third order solver with automatic adjustment of the integration step.

3. Spin Rotators

Spin rotators for RHIC rotate the spin by 90° , from the vertical y to the horizontal $x - z$ plane. For experiments, collisions between counter streaming protons in the machine interactions will be done at every energy, from injection, $\gamma = 27$, to the maximum energy, $\gamma = 250$. In the collisions one preferably wants the spin in the longitudinal, or z . An arrangement that satisfies these requirements consists of four super conducting helical magnets, alternately right-handed and left-handed, with a maximum field close to 4 Tesla. Geometric parameters for the rotator are in Table 1. Fig. 1 shows the three components of the magnetic field, and Fig. 2 the orbit at injection energy. The magnetic field shown is evaluated along the orbit. The tracked spin with Snig in a rotator is shown in Fig. 3.

Table 1. Spin Rotators for RHIC, including fringe field.

Number of Helical Magnets	4		
Total length [m]	10.56		
Magnet bore [mm]	100		
Helical magnets			
Length [m]	Field ^c [Tesla]	Field rotation ^a [deg]	Field orientation ^b [deg]
2.40	1.191	+345	7.5
2.40	3.864	-345	187.5
2.40	3.864	+345	7.5
2.40	1.191	-345	187.5

^a '+' : right-handed, '-' : left-handed helix.

^b Field angle respect to the vertical at magnets' entry.

The twist of the helix in each magnet shown in Table 1 is 345° . According to the original theory⁴, in each magnetic module the transverse field should rotate a

Fig. 2. Spin Rotator. Orbit at $\gamma = 27$.

Fig. 3. Spin Rotator. $\gamma = 27$. The three components of the spin.

full 360° as one progresses to the other side. If the fringe field is taken into account, because of the anti-symmetric structure of the field in a spin rotator, the field integrals cannot be made equal to zero, shown in Fig. 1. The result is that a particle entering in the magnet on axis will not in general emerge on axis. We solved the problem by making the field rotation $< 360^\circ$, so the field integrals can be exactly compensated at least for one beam energy.

In RHIC the direction of a rotator beam line is at an horizontal angle $\phi = 3.674$ mrad with the direction of the adjacent insertion.¹² The spin, after the rotator, will perform a further precession in the horizontal plane. Then, to achieve longitudinal polarization at the insertion, the spin should emerge from the rotator in the horizontal plane and at an angle $G\gamma\phi$ or $G\gamma\phi \pm \pi$ with the rotator longitudinal axis z , in the positive direction (forward) or negative (backward), respectively. The corresponding angles for radial polarization are $G\gamma\phi \pm \pi/2$ in the negative direction of the axis x (inward) or positive (outward), respectively.

The spin rotation at the rotators' exit depends on beam energy. There, we must have $S_y = 0$. Fig. 4 shows the loci of $S_y = 0$ on a $B_1 - B_2$ plane, with the B 's the value of the field in the two pairs of helices.^a From the diagram of Fig. 4(a) we

^aA similar diagram, calculated with matrices has been presented by V.Ptitsin. Ours is calculated by *tracking* and includes fringe fields.

Fig. 4. (a) Spin Rotator. Loci of $S_y = 0$ vs the field in the two pairs of helices. The angle of spin in the horizontal plane is indicated. (b) Field [T] in the two pairs of magnets to achieve longitudinal polarization in the interaction point of RHIC, vs. beam energy.

can find the values of the field needed to provide a longitudinal or radial polarization at the interaction at different proton energies in RHIC. The figure shows that the spin rotator provides a longitudinal forward and a radial outward polarization for all proton energies in the range, with field not greater than 4 Tesla, but not all angles for longitudinal backwards and radial inwards. The values of the field and of other parameters for longitudinal forward polarization (the most important) are given in Table 2 and Fig. 4(b).

Table 2. Spin Rotators for longitudinal polarization at the interaction.

Beam Energy, γ	27	50	75	100	150	200	250
$G\gamma\phi$	10.19	18.70	28.31	37.75	56.63	75.49	113.23
Field [T]							
B_1	2.047	2.281	2.528	2.752	3.082	3.257	3.354
B_2	2.654	2.543	2.439	2.373	2.421	2.679	3.005
Max. orbit [mm]							
horizontal	24.10	14.50	10.71	8.74	6.53	5.17	4.26
vertical	10.02	5.19	3.46	2.83	2.11	1.67	1.38
Field integrals [T-m]							
$\int B_x ds$	15.01	15.39	15.85	16.35	17.55	18.93	20.28
$\int B_y ds$	15.01	15.40	15.85	16.36	17.56	18.94	20.30
$\int B_z ds$	0.552	0.312	0.240	0.208	0.170	0.144	0.125
Orb. lengthening [mm]							
	1.252	0.378	0.178	0.107	0.055	0.036	0.026

4. Siberian Snakes

Siberian snakes for RHIC rotate the spin by 180° from the vertical up to the vertical down. The axis of precession makes an angle $\psi = \pm 45^\circ$ with the beam. This angle should be slightly adjustable to compensate for misalignments and possible spin rotation created by the solenoidal fields in the detectors. A snake that satisfies these

Fig. 5. 4-Helix Snake. Field at $\gamma = 27$. Axis at 45° . Longitudinal field is on scale $\times 10$.

requirements consists of four super-conducting helical magnets, right-handed, with a maximum field of 4 Tesla.

Snake parameters are in Table 3. Fig. 5 shows the three components of the magnetic field, Fig. 6 the orbit, and Fig. 7 the spin precession at injection energy. The symmetry of the Siberian snake is such that the fringe field would cancel in any case, but in order to simplify the construction of the snakes and rotators, a solution was chosen with all magnetic modules identical in both devices (apart from their right- or left-handedness).

Table 3. Siberian Snakes for RHIC, including fringe field.

Number of Helical Magnets		4	
Total length [m]		10.56	
Magnet bore [mm]		100	
Helical magnets			
Length [m]	Field ^c [Tesla]	Field rotation ^a [deg]	Field orientation ^b [deg]
2.40	1.191	+345	7.5
2.40	3.864	+345	187.5
2.40	3.864	+345	7.5
2.40	1.191	+345	187.5
Max. orbit excursion ^{c,d} [mm]		14.7 (hor), 31.5 (vert)	
Total field integral [T-m]		22.77	
Orbit lengthening [mm]		1.82	

^a '+': right-handed, '-': left-handed helix.

^b Field angle respect to the vertical at magnets' entry.

^c For axis of spin precession at 45° .

^d @injection, $\gamma = 27$

All helical magnets are powered by separate power supplies. This allow for an adjustment of the spin tune to $1/2$ and for the small changes in the direction of the precession axis, as shown in Fig. 8.

Fig. 6. Snake. Orbit at $\gamma = 27$.

Fig. 7. Snake. $\gamma = 27$. The three components of the spin.

Fig. 8. Snake. Tunability of the Snake's precession axis angle.

5. Optical transfer matrix

We have calculated the optical transfer matrix up to the third order with Snig.¹³ This was done by tracking a number of particles (typically 35) with coordinates randomly extracted inside a phase-space ellipse with the appropriate Twiss parameters for RHIC. The output of the tracking is then fitted with a third order polynomial in x , y , x' and y' and the coefficients of a first and second order matrix are found. The first order matrix elements are shown in Eq. 11. They are in good agreement with the corresponding approximate analytical calculations.⁶ We use more particles than are strictly needed, and the fitting is repeated several times with all possible combinations of the particles. So, the matrix elements shown are in reality averages with statistical variance.

$$\begin{pmatrix} x \\ x' \\ y \\ y' \end{pmatrix} = \begin{pmatrix} .9825 & 11.91 & -.0050 & -.0541 \\ -.0029 & .9825 & 0. & -.0038 \\ .0039 & .0541 & .9486 & 11.72 \\ 0. & .0049 & -.0085 & .9486 \end{pmatrix} \times \begin{pmatrix} x_0 \\ x'_0 \\ y_0 \\ y'_0 \end{pmatrix} \quad (11)$$

$$\text{Beam within limits: } \begin{cases} -25.117 < x_0[\text{mm}] < 28.792 \\ -0.216 < x'_0[\text{mrad}] < 0.234 \\ -20.970 < y_0[\text{mm}] < 23.367 \\ -0.236 < y'_0[\text{mrad}] < 0.198 \end{cases}$$

The final spin values are also calculated and averaged, to determine how the spin rotation is affected by the finite size of the beam. They are in Table 4. Results also show that the spin reversal is not affected by the size of the beam. 6. Conclusions

Table 4. Spin tracking in a 4-Helix Snake

	S_x	S_y	S_z
start spin	0	1	0
final spin	$-8.27 \cdot 10^{-5}$	-1.	$4.07 \cdot 10^{-4}$
with variance	$5.17 \cdot 10^{-13}$	$2.15 \cdot 10^{-9}$	$3.61 \cdot 10^{-12}$
precession axis angle [deg]	135.086 (horiz.)		-0.010 (vert.)
with variance	$1.00 \cdot 10^{-6}$		$5.87 \cdot 10^{-11}$

Spin rotators and Siberian snakes for RHIC can be built using 4 helical magnets. Although super-conducting helical magnets of this size have never been built, they can be derived, by twisting, from the super-conducting RHIC cosine dipoles. We found that the fringe fields are important for orbit and spin dynamics, and were taken into

account in the calculations, using a plausible model for the fringes. Only magnetic measurements on the prototypes will allow a final optimization.

Prototypes are currently being built and measured. The code Snig was written with the option to integrate the equations of motion and the equations of spin using either a synthetic magnetic field, or a measured field, or a calculated map (e.g. by Tosca3D ¹⁴).

7. Acknowledgments

The four helix structure has been first proposed by Vadim Ptitsin and Yuri Shatunov of Novosibirsk. Basic parameters for the Snakes and Rotators are the result of many discussions with them and with Ernest Courant, Steve Peggs, Thomas Roser, Mike Syphers, Erich Willen

8. References

1. RSC Collaboration (T.Roser Spokeperson), *Conceptual Design for the Acceleration of Polarized Protons in RHIC*, May 24, 1993, Rev. February 15, 1995
2. J.Collins, S.F.Heppelman, R.W.Robinett (Eds.), *Polarized Collider Workshop* University Park, PA, 1990. AIP Conference Proceedings No. 223, New York 1990. Also: A.Luccio and M.Conte, Proc.IEEE 1993 PAC. Washington, DC, May 17-20, 1993, Vol.I, p.438.
3. E.D.Courant, lecture notes, BNL, 1987.
4. V.I.Ptitsin and Yu.M.Shatunov, *Helical Spin Rotators and Snakes*, Proc. Third Workshop on Siberian Snakes and Spin Rotators, (A.Luccio and T.Roser Eds.) (Upton, NY, Sept. 12-13, 1994) BNL-52453, p.15.
5. S.Caspi, Rept.LBL-35928, Berkeley, CA, July 1994.
6. E.D.Courant, Rept. RHIC/AP/47, Upton, NY, 1995.
7. J.P.Blewett and R.Chasman, *J.Ap.Phys.* 48 (1977) 2692.
8. V.I.Ptitsin, Rept. RHIC/AP/41, Upton, NY, October 1994.
9. A.Luccio, in *Halbach Festschrift*, LBL-PUB-754, Vol.I, Feb. 1995, p.65.
10. J.D.Jackson, *Classical Electromagnetism* Wiley, NY, 2.nd ed.1962, p.558.
11. A.U.Luccio, Rept. BNL-52461, Upton, NY, April 1995.
12. RHIC Design Manual, BNL Report, Upton, NY, May 1994.
13. A.U.Luccio and F.Pilat, Proc. IEEE 1995 PAC, Dallas, TX, 5/1-5, 1995.
14. Tosca Ref. Manual, Vers. 6.4, Vector Field Ltd., Kidlington, England, 1993.

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.