Anomalous Resistivity of a High-Z Plasma with Hydrogen Admixture

A. Toor
D. Ryutov

This paper was prepared for submittal to the
40th Annual Meeting of the American Physical Society Division of Plasma Physics
New Orleans, LA
November 15-20, 1998

November 6, 1998
DISCLAIMER

This document was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor the University of California nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or the University of California. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or the University of California, and shall not be used for advertising or product endorsement purposes.
Anomalous resistivity of a high-Z plasma with hydrogen admixture

A. Toor, D. Ryutov

Lawrence Livermore National Laboratory,
Livermore, CA 94551
ABSTRACT

Among microinstabilities that may affect the resistance of a $Z>>1$ plasma in fast $Z$ pinches, are the ion acoustic and the lower hybrid instabilities. We discuss effects of hydrogen ions on these instabilities and find that, by properly adjusting the hydrogen concentration, one can considerably increase the threshold current density for the onset of the instability. In addition to a strong Landau damping on hydrogen ions, there is a collisional stabilizing mechanism related to a collisional friction between the two ion species. Another interesting aspect of the stability analysis is related to the fact that the magnetization (a product of the gyrofrequency and the collisional frequency) of the heavy ions and the hydrogen ions is very different. We discuss possible ways of adding the hydrogen to high-$Z$ material. This is simple in case of gas-puff pinches, where the hydrogen could be added to the main gas before the puffing. For the wire arrays, one might try to saturate the assembled array by hydrogen prior to the main discharge. One more possibility is using interwoven thin wires of a main component (say, tungsten) and polymer CH fibers.

MOTIVATION:

- The current-driven microinstabilities may produce anomalously-high resistivity (thereby affecting the MHD behavior of the pinch, in particular, making the $m \neq 0$ R-T modes more unstable)

- The anomalous resistivity may reduce the current through a low-density blow-off plasma sometimes present inside and outside the imploding shell; this, in turn, may lead to formation of runaways, and generation of hard X-ray pulses (undesirable in fusion applications)

- The hydrogen admixture to a high-Z plasma may increase the instability threshold and thereby eliminate the aforementioned undesirable effects
The relative electron-ion velocity can easily exceed the ion thermal velocity:

\[ u = \frac{j}{en_e} \sim \frac{cB}{4\pi ne h} \sim \frac{I}{2\pi r^4}, \]

where \( I \) is the total pinch current, \( r \) is the pinch radius, and \( h \) is the thickness of the imploding shell. In “practical” units

\[ u(cm/s) \sim 10^4 \frac{AIMA}{Z^* \hat{m}(mg/cm)} (*), \]

where \( A \) is the atomic weight of plasma ions, \( Z^* \) is their average charge, and \( \hat{m} \) is the mass per unit length of the pinch. For \( A=180, Z^*=5, \hat{m}=2 \text{ mg/cm}, \) and \( I=20 \text{ MA}, \) one has \( u=3.6 \cdot 10^6 \text{ cm/s}, \) whereas the ion thermal velocity at \( T=30 \text{ eV} \) (typical for the run-in phase) is \( v_{Ti}=6 \cdot 10^5 \text{ cm/s}. \)
MOST IMPORTANT INSTABILITIES

- Lower-hybrid (LH) instability (low threshold)

- Ion-acoustic instability (higher threshold, higher effective collision frequency).
The plasma that one deals with in implosions of wire arrays is strongly collisional.

\[ v_{ii} (v_{ei}) = \text{ion-ion (electron-ion) collision frequency}; \quad \omega_{Bi} (\omega_{Be}) = \text{ion (electron) gyro-frequency}; \quad \omega_{pi} = \text{ion plasma frequency}. \]

Input parameters: \( A = 180, Z^* = 5, \) \( B = 2 \text{ MG}. \)

Point A: inside the current sheath; Point B: in a blow-off plasma.
“Canonical” ion-acoustic and lower-hybrid instabilities cannot develop inside the current sheath.

There exists a collisional (dissipative) instability of the ion-acoustic type. It develops when the relative velocity $u$ exceeds the phase velocity of the perturbations in the ion rest-frame. Its growth-rate is:

$$\frac{\gamma}{v_{ei}} = \left(\frac{ku}{\omega} - 1\right) \frac{A_m m_p \omega^4 \rho_{De}^4}{2Z* T^2},$$

where $m_e (m_p)$ is the electron (proton) mass, and $\rho_{De}$ is the electron Debye radius.

For the mode with $\omega/k \sim$ a few $v_{Ti}$, the growth rate is $\sim 10^{-2} v_{ei} \sim 3 \cdot 10^{11}$ s$^{-1}$. This mode saturates when the effective collision frequency becomes 2-3 times higher than the initial Coulomb collision frequency. In other words, it can potentially lead to the increase of the pinch resistance by a factor of 2-3 compared to the “classical” value.
This mode can be stabilized by a small admixture of hydrogen. A relative motion of the hydrogen ions with respect to the main component in the wave field causes enhanced dissipation of the wave. The amount of hydrogen required for stabilization is:

\[
\frac{n_H}{n_Z} > 10^{-2} \frac{Z^*}{\mu_{\text{mag}}} \frac{m_p}{A \sqrt{m_e}}
\]

For the standard Z-pinch parameters, the r.h.s. is approximately equal to $10^{-2}$. 
Consider the lower-hybrid instability for a lower-density and hotter blow-off plasma. Here, the relative velocity \( u \) can be considerably smaller than the ion thermal velocity (a general expression (*) is not valid for the blow-off plasma; \( u \) should be evaluated here on the case-by-case basis). The growth rate of the lower-hybrid instability in this case is (Krall; Davidson; Drake; Guzdar):

\[
\gamma \sim \omega_{LH} \left( \frac{u}{v_{Ti}} \right)^2
\]

where

\[
\omega_{LH} = \omega_{Be} \sqrt{\frac{Z \ast m_e}{A m_p}} \quad (**)
\]
In the case where hydrogen admixture is present, the expression for the lower-hybrid frequency changes. In this case,

\[ \omega_{LH} = \omega_{Be} \sqrt{\frac{m_e}{m_p}} \frac{n_H + (Z^2 / \Lambda)n_z}{n_H + Z*n_z} \]

We, however, assume that the hydrogen concentration is small enough,

\[ n_H < (Z^2 / \Lambda)n_Z \]

so that Eq. (**) remains valid.
At this relatively small hydrogen density, the hydrogen affects only the damping rate of the oscillations. Its contribution to the damping rate is:

\[ \gamma_H = \sqrt{\frac{\pi Z^*}{A}} \frac{\omega_{LH}}{(k\rho_e)^3} \frac{n_H}{n_Z} \]

where \( \rho_e \) is the electron gyro-radius. For the most dangerous perturbations with \( k \sim 0.5 \rho_e \), the stabilization occurs at

\[ \frac{n_H}{n_Z} > \left( \frac{u}{v_{Ti}} \right)^2 \sqrt{\frac{A}{200Z^*}} \]
How to add hydrogen to the bulk of the wires?

- Saturate an assembled array with hydrogen prior to discharge.

- Interweave thin wires of a main component (say, tungsten) with polymer $C_mH_n$ fibers.

- Use, instead of hydrogen, some other light admixture (Li or Be) in the form of an alloy with the main component (the theory would require some minor modifications in this case).
CONCLUSIONS

- Current-driven microinstabilities for fast Z-pinches have been considered.

- Within the thickness of the imploding shell, the plasma is so strongly dominated by collisions that "canonical" lower-hybrid and ion-acoustic instabilities cannot develop.

- In this area, only residual dissipative instability of the ion-acoustic type remains.

- It can increase the plasma resistivity by a factor 2-3. It can be easily stabilized by a few percent admixture of the hydrogen, which adds damping via collisional friction against the main component.

- Lower-density and hotter blow-off plasma is more vulnerable with respect to classical current-driven microinstabilities.
CONCLUSIONS (CONTINUED)

- Small admixture of hydrogen has a favorable effect also in this situation. Stabilization comes from additional Landau damping.

- We have derived simple expressions for the amount of hydrogen required for stabilization.