The High Energy Behavior of the Forward Scattering Parameters—\(\sigma_{\text{tot}}, \rho, \text{ and } B\)

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Paper presented by Martin M. Block
at the
MultiParticle/94 Conference, Vietri sul Mare, Italy, September, 1994

Abstract

Utilizing the most recent experimental data, we reanalyze high energy \(\bar{p}p\) and pp data, using two distinct (and dissimilar) analysis techniques: (1) asymptotic amplitude analysis, under the assumption that we have reached ‘asymptopia’, and (2) an eikonal model whose amplitudes are designed to mimic real QCD amplitudes. The former gives strong evidence for a \(\log (s/s_0)\) dependence at current energies and not \(\log^2(s/s_0)\), and demonstrates that odderon are not necessary to explain the experimental data. The latter gives a unitary model for extrapolation into true ‘asymptopia’ from current energies, allowing us to predict the values of the total cross section at future supercolliders. Using our QCD-model, we obtain \(\sigma_{\text{tot}}(16 \text{ TeV}) = 109 \pm 4 \text{ mb}\) and \(\sigma_{\text{tot}}(40 \text{ TeV}) = 124 \pm 4 \text{ mb}\).

* Work partially supported by Department of Energy contract DA-AC02-76-ER02289 Task B.
† Work partially supported by Department of Energy contract DE-AC02-76ER00088 and the University of Wisconsin Research Committee with funds granted by the Wisconsin Alumni Research Foundation.
‡ Work partially supported by the Natural Sciences and Engineering Research Council of Canada and the FCAR of the Province of Quebec.
§ Work supported by Department of Energy contract W-31-109-ENG-38.
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I Introduction

Recently, the CDF group has announced new cross sections at $\sqrt{s} = 546$ GeV and 1080 GeV. The result at 1800 GeV is at variance with their earlier value (announced at the ‘Blois’ meeting at Elba). Only the new results of CDF have been included in Table 1, along with the recent precision remeasurement of $\rho$ at $\sqrt{s} = 546$ GeV by UA4/2 at the SppS CERN Collider, the reanalysis of the $\sqrt{s} = 1800$ GeV Tevatron Collider data by E710 of $\sigma_{tot}$ and $B$, as well as the new E710 measurement at the Tevatron of $\sigma_{tot}$ and $B$ at $\sqrt{s} = 1020$ GeV. These results complete the information we will have on high energy forward scattering parameters until the LHC is operative. Although the CDF and E710 results at 1800 GeV disagree, we see that the inclusion of both of them does not change the conclusions found in our earlier analysis[1], given at Multiparticle/93 in Aspen. We present two approaches to

Table 1: Recent Experimental Results at High Energies

<table>
<thead>
<tr>
<th></th>
<th>$\sigma$ (fb)</th>
<th>$B$ (GeV/c)$^{-2}$</th>
<th>$\rho$ (fb)</th>
<th>$\sqrt{s}$ (GeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>E710</td>
<td>72.2 ± 2.7</td>
<td>16.72 ± 0.44</td>
<td>0.134 ± 0.069</td>
<td>1800</td>
</tr>
<tr>
<td>E710</td>
<td>61.6 ± 5.7</td>
<td>16.2 ± 0.70</td>
<td></td>
<td>1020</td>
</tr>
<tr>
<td>CDF</td>
<td>80.0 ± 2.2</td>
<td>16.98 ± 0.25</td>
<td></td>
<td>1800</td>
</tr>
<tr>
<td>CDF</td>
<td>61.9 ± 1.5</td>
<td>15.28 ± 0.58</td>
<td></td>
<td>546</td>
</tr>
<tr>
<td>UA4/2</td>
<td></td>
<td></td>
<td>0.135 ± 0.02</td>
<td>546</td>
</tr>
</tbody>
</table>

interpret the high energy data:

(i) Model 1—An analytic asymptotic amplitude analysis,

(ii) Model 2—A QCD-inspired model.

II Asymptotic Amplitude Analysis

In spite of the fact that there are excellent arguments [2] that the energy region in which present experiments are conducted—even at the Tevatron Collider—is too low to be considered asymptotic, we will consider here the consequences of assuming the opposite. This allows us to test specific hypotheses using a well-defined phenomenological analysis. We caution the reader that we don't believe we are in ‘asymtopia’ and thus don’t believe the analysis is applicable as a true asymptotic analysis. We do believe that present day energies are too low to make a truly asymptotic analysis. Nonetheless, we feel that such an analysis is valuable as a guideline to what is and is not happening at present energies.

We apply a “standard” asymptotic analytic amplitude analysis procedure[2] to the now-available data on $\sigma_{tot}$, the total cross section and $\rho$, the ratio of the real to the imaginary portion of the forward scattering amplitude, in the energy region $\sqrt{s} = 5$ to 1800 GeV. The data are parameterized in terms of even and odd analytic amplitudes. Consistent with all
asymptotic theorems, this allows use of even amplitudes varying as fast as \( \log^2(s/s_0) \) and odd amplitudes (the ‘Odderon’ family) that do not vanish as \( s \to \infty \).

We show here only the large \( s \) limit of the even and odd amplitudes that are used[2]. We make five fits to the data:

(i) Fit 1: \( \log^2(s/s_0) \) energy dependence for the cross section, with no Odderon amplitude,

(ii) Fit 2: \( \log^2(s/s_0) \) energy dependence for the cross section, with an Odderon amplitude whose cross sectional dependence is \( \log s \), the most rapid behavior allowed by asymptotic theorems,

(iii) Fit 3: \( \log^2(s/s_0) \) energy dependence for the cross section, with an Odderon amplitude whose cross sectional dependence is constant,

(iv) Fit 4: \( \log(s/s_0) \) energy dependence for the cross section, with no Odderon amplitude,

(v) Fit 5: \( \log(s/s_0) \) energy dependence for the cross section, with an Odderon amplitude whose cross sectional dependence is constant, the most rapid behavior allowed by asymptotic theorems for this choice of even amplitude.

In all cases, an odd amplitude which vanishes with increasing energy is also employed, as well as an even amplitude that mimics Regge behavior.

II.1 \( \log^2(s) \) Energy Behavior

We introduce \( f_+ \) and \( f_- \), the even and odd (under crossing) analytic amplitudes at \( t = 0 \), and define the pp and pp forward scattering amplitudes by \( \tilde{f}_{pp} = f_+ + f_- \) and \( f_{pp} = f_+ - f_- \), giving total cross sections \( \sigma_{\text{tot}} \) and the \( \rho \)-values

\[
\sigma_{pp} = \frac{4\pi}{p} \text{Im} f_{pp}, \quad \sigma_{pp} = \frac{4\pi}{p} \text{Im} f_{pp}, \quad \rho_{\tilde{p}p} = \frac{\text{Re} f_{\tilde{p}p}}{\text{Im} f_{\tilde{p}p}}, \quad \text{and} \quad \rho_{pp} = \frac{\text{Re} f_{pp}}{\text{Im} f_{pp}}. \tag{1}
\]

We parameterize the ‘conventional’ even and odd amplitudes \( f_+ \) and \( f_- \) by:

\[
\frac{4\pi}{p} f_+ = i \left( A + \beta \left[ \log \left( \frac{s}{s_0} \right) - i \frac{\pi}{2} \right]^2 + c s^{\mu-1} e^{i\pi(1-\mu)/2} \right) \tag{2}
\]

\[
\frac{4\pi}{p} f_- = -Ds^{\alpha-1} e^{i\pi(1-\alpha)/2}. \tag{3}
\]

The parameter \( \alpha \) in Eq (3) turns out to be about 0.5, and thus this odd amplitude vanishes as \( s \to \infty \).

Asymptotic theorems by Eden and Kinoshita[2] prove that the difference of cross sections can not grow faster than \( \log^{7/2}(s) \), when the cross section grows as \( \log^7(s) \). Thus, odd amplitudes which do not vanish as \( s \to \infty \) for this case are:

\[
\frac{4\pi}{p} f_-^{(0)} = -e^{(0)}, \quad \frac{4\pi}{p} f_-^{(1)} = - \left[ \log \left( \frac{s}{s_0} \right) - i \frac{\pi}{2} \right] e^{(1)}, \quad \text{and} \quad \frac{4\pi}{p} f_-^{(2)} = - \left[ \log \left( \frac{s}{s_0} \right) - i \frac{\pi}{2} \right]^2 e^{(2)}.
\]

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The complete odd amplitude is formed by adding any one (or none) of the \( f^{(i)} \) to the conventional odd amplitude \( f_- \) of Eq (3). We then fit the experimental \( \rho \) and \( \sigma_{\text{tot}} \) data, for both pp and \( \bar{p}p \), for energies between 5 and 1800 GeV, to obtain the real constants \( A, \beta, s_0, c, \mu, D, \alpha, \epsilon \). The data used below 500 GeV are listed in [2], and the high energy points are from UA1, UA4, E710 and CDF[2]. We emphasize that what we really fit for the UA4 and CDF cross sections is the measured experimental quantity \( \sigma_{\text{tot}} \times (1 + \rho^2) \), which is appropriate for experiments that measure a ‘luminosity-free’ cross section, whereas for UA1 and the 1020 GeV point of E710, we fit the experimental quantity \( \sigma_{\text{tot}} \times \sqrt{1 + \rho^2} \), which was their experimentally measured quantity (they measured a ‘luminosity-dependent’ cross section).

II.2 Fitted Results for \( \log^2(s) \) Behavior

(i) Fit 1—This fit uses no Odderon in the odd amplitude and uses the even amplitude of Eq (2). The \( \chi^2/\text{d.f.} \) (\( \chi^2 \)/degree of freedom) for the fit was 2.03, a rather large number. The fitted constants are shown in Table 2, Fit 1—the computed curves are shown in Fig. 2a (for \( \sigma_{\text{tot}} \)) and Fig. 2b (for \( \rho \)). The most obvious features of the fit are

(a) the predicted value of the total cross section is much too high to fit the experimental values (E710 and CDF) at 1800 GeV,

(b) it predicts much too high a \( \rho \)-value at 546 GeV.

We conclude that a simple \( \log^2(s) \) fit does not fit the data.

(iii) Fit 2—We fit the data with an additional degree of freedom, by adding Odderon 2 to \( f_- \) of Eq (3), along with the even amplitude of Eq (2). The parameters are summarized as Fit 2, in Table 2. Again, we conclude that this combination doesn’t fit the data, since the high energy cross section predicted at 1800 GeV is much too high. Although the \( \rho \)-value predicted at 540 GeV is slightly lower, the \( \rho \) values predicted are still too high.

(iv) Fit 3—The odd amplitude added to the conventional \( f_- \) of Eq (3) was Odderon 1. The parameters are given as Fit 3 in Table 2. Again, the fit suffers from the same defect as the Odderon 2 fit, giving much too high a total cross section at 1800 GeV, as well as predicting a UA4/2 \( \rho \)-value which was much too high.

The addition of Odderon 0 can have no effect on the cross section. Since it turns out to have a negligible effect on \( \rho \), we will not consider it further.

We conclude that an even amplitude varying as \( \log^2(s/s_0) \) does not fit the cross section data. We see that the experimental cross section does not rise as rapidly as \( \log^2(s/s_0) \), in the present-day energy region. The addition of an Odderon term does not change this conclusion.
<table>
<thead>
<tr>
<th>Parameters</th>
<th>Fit 1</th>
<th>Fit 2</th>
<th>Fit 3</th>
<th>Fit 4</th>
<th>Fit 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$ (mb)</td>
<td>40.3±.21</td>
<td>40.1±.25</td>
<td>41.6±.04</td>
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<td>$\beta$ (mb)</td>
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<td>.47±.02</td>
<td>.57±.01</td>
<td>7.7±.1</td>
<td>9.0±.7</td>
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<td>$s_0$ ((GeV)$^2$)</td>
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<td>198±24</td>
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<td>500</td>
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<tr>
<td>$D$ (mb(GeV)$^{2(1-\alpha)}$)</td>
<td>−40.6±1.9</td>
<td>−39.1±2.1</td>
<td>−37.2±1.7</td>
<td>−44.2±2.1</td>
<td>−43.3±2.1</td>
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<tr>
<td>$\alpha$</td>
<td>.46±.02</td>
<td>.47±.02</td>
<td>.48±.02</td>
<td>.44±.01</td>
<td>.45±.02</td>
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<tr>
<td>$c$ (mb(GeV)$^{2(1-\mu)}$)</td>
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<td>28.0±4.2</td>
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<tr>
<td>$\mu$</td>
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<td>.49</td>
<td>.83±.01</td>
<td>.85±.01</td>
<td></td>
</tr>
<tr>
<td>$\epsilon^{(2)}$ (mb)</td>
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<td></td>
<td>.040±.040</td>
<td>−.013±.042</td>
<td></td>
</tr>
<tr>
<td>$\epsilon^{(1)}$ (mb)</td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>$\chi^2$/d.f.</td>
<td>2.03</td>
<td>1.93</td>
<td>2.66</td>
<td>1.26</td>
<td>1.24</td>
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<tr>
<td>d.f.</td>
<td>82</td>
<td>81</td>
<td>81</td>
<td>82</td>
<td>81</td>
</tr>
</tbody>
</table>

Table 2: Results of fits to total cross sections and $\rho$-values, including Odderons. Fit 1, Fit 2 and Fit 3 correspond to an asymptotic cross section variation of $\log^2(s/s_0)$, with no Odderon, Odderon 2 and Odderon 1, respectively, whereas Fit 4 and Fit 5 correspond to an energy dependence of $\log(s/s_0)$, with no Odderon and Odderon 1, respectively.

### II.3 log(s) Energy Behavior

Since the experimental cross section in the energy region 5-1800 GeV did not vary as fast as $\log^2(s/s_0)$, we now consider an asymptotic variation that goes as $\log(s/s_0)$. We substitute for the even amplitude in Eq (2) a new amplitude $f_+$ varying as $\log(s/s_0)$:

$$\frac{4\pi}{p} f_+ = i \left( A + \beta \left[ \log \left( \frac{s}{s_0} \right) - i \frac{\pi}{2} \right] + c s^{\mu-1} e^{i(1-\mu)/2} \right).$$

We use the conventional odd amplitude of Eq (3), along with no Odderon or Odderon 1, in Fits 4 and 5, respectively. We make the important observation that since the energy variation of the cross section is now only $\log(s)$, Odderon 2 is not allowed by the asymptotic theorems.

### II.4 Fitted Results for log(s) Behavior

(i) Fit 4—The data are fitted with a $\log(s/s_0)$ cross section energy behavior, with no Odderon. The results are detailed in Table 2, and plotted in Fig. 3a and Fig. 3b. The fit is quite satisfactory, giving a $\chi^2$/d.f. of 1.26, fitting reasonably well to all cross section data over the entire range of energy. Most importantly, it now fits the UA4/2 $\rho$-value at 546 GeV, as well as the E710 $\rho$-value at 1800 GeV.
Using this fit, we obtain a cross section of \( \sigma_{\text{tot}} = 117.4 \pm 1.3 \text{ mb} \) at 40 TeV (where the error is statistical, and results from the errors in the fitted parameters) and a LHC cross section \( \sigma_{\text{tot}} = 104.4 \pm 1.0 \text{ mb} \). As we will comment on later, we believe that we are not yet in ‘asymptopia’, and we think that the cross section will ultimately rise faster than log(s). Hence, we consider these high energy cross section extrapolations lower limits to the real cross sections at these high energies.

(ii) Fit 5—The data are fitted with a log(s) cross section energy behavior, along with Odderon 1. The results are given in Table 2. This fit (as is Fit 4) is quite satisfactory, giving a \( \chi^2/\text{d.f.} \) of 1.24. Indeed, it is almost indistinguishable from fit 4.

We find that the experimental cross sections and \( p \)-values in the energy domain 5-1800 GeV can be reproduced using a log(s/s_0) energy variation. Further, the introduction of an Odderon amplitude is not needed to explain the experimental data. Thus, we conclude that the Odderon hypothesis is irrelevant, since the experimental data do not require the introduction of non-vanishing odd amplitudes.

### III Too Low an Energy for ‘Asymptopia’?

Indications that we may still be far from asymptopia come from experimental measurements of the elastic differential cross section, \( \frac{d\sigma}{dt} \), out to intermediate \( t \)-values. Consider the \( t = 0 \) parameters, the slope \( B \) and the curvature parameter \( C \), defined as:

\[
B = \left[ \frac{d}{dt} \left( \log \frac{d\sigma}{dt}(t) \right) \right]_{t=0} \quad \text{and} \quad C = \frac{1}{2} \left[ \frac{d^2}{dt^2} \left( \log \frac{d\sigma}{dt}(t) \right) \right]_{t=0}.
\]

Thus, we can parameterize the elastic scattering cross section for intermediate \( t \) values as:

\[
\frac{d\sigma}{dt}(t) = \left[ \frac{d\sigma}{dt}(t) \right]_{t=0} e^{B|t|+Ct^2},
\]

for the four-momentum transfer interval \( 0 \leq |t| \leq t_{\text{max}} \), where \( t_{\text{max}} \approx 0.5(\text{GeV/c})^2 \).

At low energies, at the ISR[2] and the SppS [2], a substantial positive curvature \( C \) was found. In a recent experiment, the E710 group[2] measured \( B = 16.26 \pm 0.23(\text{GeV/c})^{-2} \) and \( C = 0.14 \pm 0.70(\text{GeV/c})^{-4} \), indicating that the curvature is going through zero. This suggests that the sign of the curvature is about to change to the negative curvature associated with a variety of asymptotic theories, including an expanding disk [2] and the Critical Pomeron[2]. We recall to the reader that for the particular case of a disk with sharp edges, the curvature (which is negative for all energies) is given by \( C = -\frac{R}{192} \) and the slope by \( B = \frac{R^2}{4} \), where \( R \) is the radius of the disk.

One can define the threshold to ‘asymptopia’ as the energy where the curvature \( C \) goes through zero. Figure 1, taken from Block, Halzen and Margolis[2], compares their predictions for the elastic differential scattering cross section \( \frac{d\sigma}{dt} \) vs. \( |t| \) for \( \bar{p}p \) at \( \sqrt{s}=1800 \text{ GeV} \) with the experimental points from the E710 measurement[2]. There is good agreement, both in shape and magnitude. As we will see, this model provides a good fit to current experimental data for the total cross section \( \sigma_{\text{tot}} \). This also implies that the Tevatron energy, \( \sqrt{s}=1800 \text{ GeV} \), is on the threshold of, but not in, ‘asymptopia’.

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Figure 1: The differential elastic scattering cross section $\frac{d\sigma}{dt}$ vs. $|t|$, for pp at $\sqrt{s}=1800$ GeV. The solid curve is the theoretical prediction based on a QCD model of soft interactions, and the experimental points are from the E710 collaboration.

IV An Eikonalized QCD Model

We have used a QCD-inspired, eikonalized model[2] and found that asymptotically[2], there is a critical impact parameter $b_c$ such that $\sigma_{\text{tot}} = 2\pi b_c^2 = 2\pi \left(\frac{J-1}{\mu_{\text{soft}}}\right)^2 \log^2 \frac{s}{s_0}$. The coefficient of the $\log^2(s/s_0)$ term is given in terms of parameters describing the gluon density of the nucleon, rather than the pion mass which sets the scale of the $\log^2 s$ coefficient in the original Froissart bound. The physical origin of the rising cross section for a disk is seen to be the increasing number of soft gluons at small $x$, where the gluon structure function behaves as $x^{-J}$. The large number of gluons turns the proton into a disk with radius $\mu^{-1} \approx 0.8 \text{(GeV)}^{-1}$.

In this model the eikonal behaves asymptotically as $s^{J-1}$. We find $J-1 \approx 0.05-0.06$, not very different from the power behavior of $s^{0.086}$ as given by Donnachie and Landshoff’s Pomeron amplitude[2].

After including quarks as well as gluons, contributions to the total cross section of a constant term, Regge type terms and a log $s$ term also appear, which render the QCD model more complicated. They are however essential when one attempts to apply these ideas to the transitional energy regime of present experiments. In general, the impact parameter amplitude $a(s, b)$ is given by $a(s, b) = \frac{1}{2}(1 - e^{-\chi(s, b)})$, where $b$ is the transverse distance in impact parameter space, and $\chi(s, b)$ is the eikonal[2]. The nuclear amplitude $f_N(s, t)$ is given by $f_N(s, t) = 2 \int b \, db \, J_0(b\sqrt{-t})a(s, b)$, and the total cross section and the differential elastic scattering cross section are given by $\sigma_{\text{tot}} = 4\pi \text{Im} f_N(s, t)$, and $\frac{d\sigma}{dt} = \pi |f_N(s, t)|^2$, respectively. For analyticity, we must have even and odd eikonals $\chi_{\text{even}}(s, b)$ and $\chi_{\text{odd}}(s, b)$, where $\chi = \chi_{\text{even}} + \chi_{\text{odd}}$. As $s \to \infty$, $\chi_{\text{odd}}(s, b)$, which parameterizes the difference between the $pp$ and $p\bar{p}$ scattering amplitudes, vanishes. The eikonal function $\chi$ is

$$2\chi(b, s) = W(b)a(s) = P(b, s),$$
\[ P = P_{gg} + P_{qg} + P_{qq} \quad \text{and} \quad W_{ij}(b) = \frac{\mu_i^2}{\tilde{Q}_{ij}} (\mu_i b)^3 K_3(\mu_i b). \]

We compute \( P_{gg} \) exactly as \( P_{gg}(b, s) = W_{gg}(b) \sigma_{gg}^{QCD}(s) \), where, using \( \tau = \tilde{s}/s \),

\[ \sigma_{gg}^{QCD}(s) = \int d\tau F_{gg}(x_1 x_2 = \tau) \sigma_{gg}(\tau s), \quad \text{and} \]

\[ F_{gg} = \int dx_1 dx_2 f_g(x_1) f_g(x_2) \delta(x_1 x_2 - \tau). \]

We use the gluon structure function \( f_g(x) \sim \frac{(1-x)^5}{x} \), with \( J \), in Regge language, being the Pomeron intercept, and \( \sigma_{gg}(\tilde{s}) = \frac{9 \pi \alpha_s^2}{\tilde{s}^2} \theta(\tilde{s} - m_0^2) \). The high energy behavior is controlled by

\[ \lim_{\tau \to \infty} \int_{m_0^2/s}^{1} d\tau F_{gg}(\tau) \sim \int_{m_0^2/s}^{1} d\tau \frac{-\log \tau}{\tau^J} \sim \left( \frac{s}{m_0^2} \right)^{J-1}, \] (4)

which gives rise to the asymptotic energy dependence of the cross section for the disk. To reproduce the cross section at lower energies one must consider the \( qq \) and \( qg \) contributions. Using toy structure functions \( f_q \sim x^{-1/2}(1-x)^3 \) and \( f_g \sim x^{-1}(1-x)^5 \), we find

\[ P_{qq} = W_{qq}(\mu_{qq} b)\left[ m_0 \log \frac{s}{s_0} + \mathcal{P}(\frac{m_0}{\sqrt{s}}) \right] \] (5)

and

\[ P_{qg} = W_{qg}(\mu_{qg} b)\left[ a' \log \frac{s}{s_0} + \mathcal{P}'(\frac{m_0}{\sqrt{s}}) \right]. \] (6)

\( \mathcal{P} \) and \( \mathcal{P}' \) are polynomials in \( m_0/\sqrt{s} \). Since we only attempt to decipher the high energy behavior, we replace (5) and (6) by

\[ P_{qq} = W(\mu_{qq} b)[a + b \frac{m_0}{\sqrt{s}}] \quad \text{and} \quad P_{qg} = W(\sqrt{\mu_{qq} \mu_{qg} b})[a' + b' \log \frac{s}{m_0^2}], \] (7)

as suggested by standard Regge arguments. Since the coefficients in Eqs. (7) are treated as free parameters, this low energy parameterization can also accommodate elastic and diffractive contributions. We insure the correct analyticity properties of our model amplitudes by substituting \( s \to s \ e^{-i\pi/2} \) throughout.

We also introduce a crossing odd amplitude, suggested by Regge theory, as

\[ P_{odd} = W(\mu_{odd} b)\alpha'' \frac{m_0}{\sqrt{s}} e^{-i\pi/4}. \]

The data were simultaneously fit for \( \sigma_{tot} \), \( \rho \) and \( B \), for both pp and \( \bar{p}p \) collisions, in the energy region \( 15-1800 \) GeV (including the new Tevatron results and the new UA4/2 \( \rho \)-value), and the results are shown in Figs. 4a-c, respectively. As seen, the experimental data are well reproduced by the model, with a \( \chi^2/d.f. = 1.58 \), with six fitted parameters.

High energy predictions from our QCD fits, tightly bounded by the log \( s \) and Regge limits, are shown in Table 3. The quoted errors are the statistical errors due to the errors in the fitted parameters.

The Regge pole model[2], in which the scattering amplitude grows as a power of \( s \), i.e., \( s^{0.86} \), violates unitarity and thus can not be applicable in this form at sufficiently high energies. We regard the cross sections, \( \sigma_{tot} \approx 115 \) mb at \( \sqrt{s} = 16 \) TeV and \( \sigma_{tot} \approx 135 \) mb at \( \sqrt{s} = 40 \) TeV, predicted by this model as upper limits, and are so listed in Table 3.

We see from Table 3 that the central SSC value of 124 mb is bracketed from above by 135 mb, which is the Regge pole upper limit, and from below by 117 mb, which is the extrapolation of an asymptotic log \( s \) fit. Similarly, at the LHC energy, the Regge pole upper limit of 115 mb and the log \( s \) lower limit of 104 mb tightly bracket our QCD prediction of 109 mb.
Table 3: Collider cross section predictions, with upper and lower bounds, using ‘QCD’.

<table>
<thead>
<tr>
<th>√s (TeV)</th>
<th>Lower Bound (log s) (mb)</th>
<th>QCD (Eikonal) (mb)</th>
<th>Upper Bound (Regge) (mb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LHC 16</td>
<td>104 ± 1</td>
<td>109 ± 4</td>
<td>115</td>
</tr>
<tr>
<td>SSC 40</td>
<td>117 ± 1</td>
<td>124 ± 4</td>
<td>135</td>
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</table>

V Conclusions

We note that in actual numerical fits, these two very different models give virtually indistinguishable results in the energy region in which data are available, a similarity which must disappear at sufficiently high energies. Although both models easily accommodate the data at present energies, the physics of each model is significantly different. Most importantly, these two models ascribe the rise of the total cross section as due to

(i) an asymptotic term, which (the data will show) behaves as \( \log(s/s_0) \), where \( s_0 \) is a scale for \( s \), for Model 1,

(ii) the dramatic increase of the number of soft partons, for Model 2,

respectively.

We conclude:

(i) there is no experimental evidence from forward scattering parameters for the existence of ‘odderons’, i.e., odd amplitudes that do not vanish with increasing energy.

(ii) the data at present energies indicate that the total cross section is rising as \( \log(s/s_0) \), and not as \( \log^2(s/s_0) \), which is probably an indicator that we are not yet in ‘asymptopia’.

Our QCD fit predicts \( \sigma_{\text{tot}}(16 \text{ TeV}) = 109 \pm 4 \text{ mb} \) and \( \sigma_{\text{tot}}(40 \text{ TeV}) = 124 \pm 4 \text{ mb} \), results which are tightly bracketed from above by Regge predictions and from below by a \( \log(s) \) fit.
References


[2] For a complete bibliography, see ref. [1].

Figure Captions

Figure 2a—The total cross section $\sigma_{tot}$, in mb, for $\bar{p}p$ and $pp$ scattering vs. the energy, $\sqrt{s}$, in GeV, for Fit 1, described in Table I. The fit was made with a $\log^{2}(s)$ energy variation, and no Odderon. The crosses are for the $\bar{p}p$ experimental data and the circles indicate $pp$ data. The dot-dashed curves are for $\bar{p}p$, and the solid curves for $pp$. The $pp$ cosmic-ray lower limit[2] is appended to the curve, but is not used in the fit.

Figure 2b—The $\rho$-value for $\bar{p}p$ and $pp$ scattering vs. the energy, $\sqrt{s}$, in GeV, for Fit 1, described in Table I. The fit was made with a $\log^{2}(s)$ energy variation, and no Odderon. The crosses are for the $\bar{p}p$ experimental data and the circles indicate $pp$ data. The dot-dashed curves are for $\bar{p}p$, and the solid curves for $pp$.

Figure 3a—The total cross section $\sigma_{tot}$, in mb, for $\bar{p}p$ and $pp$ scattering vs. the energy, $\sqrt{s}$, in GeV, for Fit 4, described in Table I. The fit was made with a $\log(s)$ energy variation, and no Odderon. The crosses are for the $\bar{p}p$ experimental data and the circles indicate $pp$ data. The dot-dashed curves are for $\bar{p}p$, and the solid curves for $pp$. The $pp$ cosmic-ray lower limit[2] is appended to the curve, but is not used in the fit.

Figure 3b—The $\rho$-value for $\bar{p}p$ and $pp$ scattering vs. the energy, $\sqrt{s}$, in GeV, for Fit 4, described in Table I. The fit was made with a $\log(s)$ energy variation, and no Odderon. The crosses are for the $\bar{p}p$ experimental data and the circles indicate $pp$ data. The dot-dashed curves are for $\bar{p}p$, and the solid curves for $pp$.

Figure 4a—$\sigma_{tot}$, the calculated and measured total cross sections in mb, for $\bar{p}p$ (dashed curve and crosses) and $pp$ (full curve and circles), vs. $\sqrt{s}$, the energy, in GeV.

Figure 4b—$\rho$, the calculated and measured ratios $Re f(0)/Im f(0)$, for $\bar{p}p$ (dashed curve and crosses) and $pp$ (full curve and circles), vs. $\sqrt{s}$, the energy, in GeV.

Figure 4c—$B$, the calculated and measured nuclear slope parameters in $(GeV/c)^{-2}$, for $\bar{p}p$ (dashed curve and crosses) and $pp$ (full curve and circles) vs. $\sqrt{s}$, the energy, in GeV.
Fig. 4a: \( \sigma \) vs. \( \sqrt{s} \) (GeV)

Fig. 4b: \( P \) vs. \( \sqrt{s} \) (GeV)

Fig. 4c: \( B \) vs. \( \sqrt{s} \) (GeV)

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