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Validation Issues for SSI Codes
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Abstract

The paper describes the results of a recent work which was performed to verify computer code predictions in the SSI area. The first part of the paper is concerned with analytic solutions of the system response. The mathematical derivations are reasonably reduced by the use of relatively simple models which capture fundamental ingredients of the physics of the system motion while allowing for the response to be obtained analytically. Having established explicit forms of the system response, numerical solutions from three computer codes are presented in comparative format.

Introduction

It is not unusual to encounter situations where results from different computer codes are different while they presumably represent the solution for the same problem. It has also been recognized that explaining such differences has often lead to controversies. A sizable computer code validation effort has been spent by both the industry as well as regulatory agencies. Particularly in the SSI area, ongoing validation efforts seem to be primarily motivated by the importance to show that codes can predict very close a set of recorded data. That importance, however, sometimes tends to overcome the significance of validating methodologies which leads to a better understanding of the underlying phenomenological processes. Assuming that codes used in such validation programs are sufficiently competent to model with fairly equal level of detail the behavior of the SSI system, they should yield fairly equal response solutions. It is reasonable then to expect that all codes would produce results that are either, close to the data (as a package) or far from the data (again as a package). That type of validation continues to draw interest, but is, in our opinion, not very useful. This paper is concerned with a more fundamental issue: given a set of SSI system parameters and input, show that a set of different computer codes produce the same system response which can be also obtained analytically. Obviously, this task is "self-validating" since the analytic solution is the point of convergence. Sometimes, it is tempting to view this task as a trivial one. Experience, however, has shown that it is not. Three codes were used for response computations: two SSI codes CARES (Ref. 5) and DIGES (Ref. 3) and a standard general purpose finite element code (ANSYS, Ref. 4). To make the task reasonably meaningful, a simplified SSI system is considered which models the superstructure by a one mass type single mode. Assuring that a given building at a given site can be sufficiently represented by such a simple system, is basically a question of modeling, in fact not of interest to the present discussion. It has been recognized, that simplified systems are of significant value, primarily because they allow for explicit descriptions of the physics of the problem. It has also been recognized that simple systems could be troublesome (Ref. 2). The approach of the paper is essentially motivated by the common knowledge that while the flexural motion of the system can, in most cases, be represented by conventional spectral decomposition, this is not the case with its total motion.

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System Amplifications

The system considered is shown in Figure 1. One can call it a 2-dof SSI system, although it isn't really different than the conventional 2-dof system of structural dynamics except that the bottom spring and damper represent properties of the foundation medium. We are seeking expressions of system amplifications. Accordingly, one can use spectral decomposition on the basis of the classical modal solution of the system, a solution that is readily available. This, however, would require the knowledge of two modal damping ratios. The latter can be derived to represent some average between the dissipation in the structure and that of the underlying medium. Such approach is proven to be very effective mainly because it facilitates the use of modal superposition in conjunction with standard finite element codes for obtaining solutions of SSI problems. The price of this convenience is that the system response could be computed incorrectly (Ref. 1). Spectral decomposition in the complex domain is more appropriate but had less success since complex eigenvalues are less descriptive than real ones, or at least they appear to be as such. Using neither real nor complex eigensolutions for expanding the response functions, direct solutions seem to be in order. Let's denote with small letters the kinematical parameters of the 2-dof SSI system when they are represented as functions of time and use capital letters for the corresponding frequency-dependent amplitudes. Equilibrium of the structural mass yields:

\[ \ddot{u}_s + 2\xi_s \omega_s y_s + \omega_s^2 y_s = 0 \]  

(1)

Since the forces between the two masses are self equilibrating we can view the motion of the foundation mass as a result of the imposition of the inertial force created by the motion of the structural mass. The resulting equilibrium is expressed as:

\[ m_f \ddot{u}_f + c_f \dot{y}_f + k_f y_f + m_s \ddot{u}_s = 0 \]  

(2)

Consider now steady-state conditions. Solving Eq. 1, we obtain:

\[ Y_s = H_s U_f ; \quad H_s(\omega) = \frac{\omega_s^2}{1 - \Omega_s^2 + i2\xi_s\Omega_s} ; \quad \Omega_s = \frac{\omega}{\omega_s} \]  

(3)

Again for steady-state conditions, Eq. 2 with the aid of Eq. 3 yields:

\[ U_f = H_f X_g ; \quad H_f(\omega) = \frac{K_f}{-\omega^2 [m_s(1+H_s) + m_f] + K_f} ; \quad K_f = k_f + i\omega c_f \]  

(4)

In view of Eqs. 3, 4 the total system response can be put into the form:

\[ \begin{bmatrix} U_s \\ U_f \end{bmatrix} = \begin{bmatrix} 1 + H_s \\ 1 \end{bmatrix} H_f X_g \]  

(5)

where the transfer functions \( H_s, H_f \) are given by Eqs. 3 and 4 respectively.

Remarks

Note the simplicity of Eqs. 5 as compared to available solutions expressed typically in terms of ratios involving lengthy forth order polynomials of frequency. In fact, one can prove by partial fractions that such solutions can be reduced to the solution given here. In addition to its apparent simplicity, Eq. 5 allows for physical interpretations. A couple of points of further interest. First, putting the equations of motion in matrix form, it leads to unsymmetrical matrices. This should not be confusing since there are several ways to derive them leading to various combinations, i.e., a) upper stiffness and damping with upper, lower or even symmetric mass matrix, b) all unsymmetric with stiffness and damping having zero diagonal elements, c) all symmetric with diagonal mass matrix (common case). It should be kept in mind that although such expressions appear different they have one thing in common, that is, they produce the same solution. In fact, the difference between them can be translated to different ways of viewing the system equilibrium. Another point of interest is that one can easily prove the validity of Eq. 5 using general substructure solutions.
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Consider, for example, Eq. 2.2.4-1 of Ref. 3. For the 2-dof SSI system shown in Figure 1, we have $A=1$, $T^2=m_t$ (because of normalization of structural modes is with respect to the mass) and $M_e=m_e$. Therefore:

$$A^TMA = m_s ; \Gamma H A^T = m_s H_s$$

which can be substituted into Eq. 2.2.4-1 of Ref. 3 to yield exactly Eq. 5.

Finally, the transfer function for the total foundation motion given by Eq. 4 can be, after simple manipulations, put into the following form:

$$H_f(\omega) = \frac{1}{1 - \Omega_f^2} ; \Omega_f = \frac{\omega}{\omega_f} ; \omega_f^2 = \frac{K_f}{m_{eff}}$$

which has the form of the transfer function of a 1-dof undamped system. Note, however, that despite the apparent similarity one should be careful not to be deceived, since the frequency ratio in the denominator is complex. To get Eq. 7, we have used what we call effective mass $m_{eff}$ which we define as:

$$m_{eff} = A(\omega)m_s ; \Re(\Lambda) = 1 + \rho + \Re(H_s) ; \Im(\Lambda) = -\Im(H_s) ; \rho = \frac{m_f}{m_s}$$

Note that $\Lambda$ has no units, and is a complex function of frequency.

Computer Solutions

Using Eq. 5 to represent the analytic solution of the system response, we compared it to the results from three computer codes currently used in engineering applications. Such comparison is shown in Figure 2 where the absolute values of the transfer function between the ground motion and the total response of the structural mass are compared. Since DIGES operates in the frequency domain for both deterministic as well probabilistic response analysis, such transfer functions are part of its solution. To obtain the corresponding values with CARES we divided output to input using a synthetic time history to represent the latter (this approach didn't require any code modification). Finally, the frequency analysis option of ANSYS, in which a displacement input of unit amplitude and imaginary phase was used. Phase angles from all codes compared equally well to the analytic solution. Figure 3 shows comparisons of in-structure spectra between DIGES and ANSYS. Starting with a given ground spectrum, DIGES converts it to a power spectrum and performs random vibration analysis to compute power spectra at the structural mass which are subsequently converted to in-structure spectra. In ANSYS, a synthetic time history generated from the ground spectrum was used and direct integration was performed to obtain in-structure spectra. In essence, Figure 3 demonstrates comparisons between direct generation and single time history analysis.

Acknowledgments

This work has been performed under the auspices of the US Department of Energy.

We thank Mr. J. Braverman for ANSYS computation.

Appendix-References

4. Swanson Analysis Systems Inc., "ANSYS Program".
Figure 1. 2-dof SSI System

Figure 2. System Amplifications
(—ANALYTIC, ⚫DIGES, □ANSYS, V CARES)

Figure 3. In-Structure Spectra
(—DIGES, —ANSYS)