RESUMMATION OF GLUON RADIATION
AND THE TOP QUARK PRODUCTION CROSS SECTION * †

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ABSTRACT

A calculation of the total cross section for top quark production in hadron-hadron collisions is presented based on an all-orders perturbative resummation of initial-state gluon radiative contributions to the basic quantum chromodynamics subprocesses. For $p\bar{p}$ collisions at center-of-mass energy $\sqrt{s} = 1.8$ TeV and a top mass of 175 GeV, we obtain $\sigma(t\bar{t}) = 5.52^{+0.07}_{-0.05} pb$. Cross sections are provided as a function of top mass at CERN LHC energies.

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A calculation of the total cross section for top quark production in hadron-hadron collisions is presented based on an all-orders perturbative resummation of initial-state gluon radiative contributions to the basic quantum chromodynamics subprocesses. For \( p \bar{p} \) collisions at center-of-mass energy \( \sqrt{s} = 1.8 \) TeV and a top mass of 175 GeV, we obtain \( \sigma(tt) = 5.52^{+0.67}_{-0.48} \) pb. Cross sections are provided as a function of top mass at CERN LHC energies.

The quest for the top quark \( t \) reached fruition in the past year with the observation of \( tt \) pair production in proton-antiproton collisions at the Fermilab Tevatron\(^4\). A deserving question is the quantitative reliability of theoretical computations of the total cross section, as a function of top mass, based on the main production mechanisms in perturbative quantum chromodynamics (pQCD). In this paper, we discuss the motivation for incorporating the effects of initial-state gluon radiative corrections, and we present our all orders resummation of these contributions.\(^2\)

At lowest order in perturbation theory, two QCD partonic subprocesses contribute to \( p + \bar{p} \rightarrow t + \bar{t} + X \).

They are quark-antiquark annihilation: \( q + \bar{q} \rightarrow t + \bar{t} \) and gluon-gluon fusion: \( g + g \rightarrow t + \bar{t} \). Short-distance partonic cross sections based on these lowest order \( O(\alpha_s^2) \) subprocesses and on the next-to-leading \( O(\alpha_s^3) \) subprocesses have been investigated thoroughly.\(^3,4\) A full \( O(\alpha_s^3) \) calculation, for \( n \geq 4 \), does not exist. The physical top quark cross section is obtained from a convolution of the perturbative short-distance subprocess cross sections with parton distributions that specify the probability densities of the quarks, antiquarks, and gluons of the incident \( p \) and \( \bar{p} \). Our work addresses improvements in the reliability of calculations of the subprocess cross sections.

The motivation for this work begins with the observation that the size of the \( O(\alpha_s^2) \) terms in the \( gg \) and \( gg \) partonic cross sections are much larger than their \( O(\alpha_s^2) \) counterparts in some kinematic regions, notably in the near-threshold region of small \( \eta \). The variable \( \eta = \hat{s}/4n^2 - 1 \), where \( \hat{s} \) is the square of the energy of parton-parton subprocess and \( n \) denotes the mass of the top quark. Variable \( \eta \) measures the "distance" above the partonic production threshold. In both the \( gg \) and the \( gg \) channels, for a top mass of 175 GeV, the size of the \( O(\alpha_s^2) \) term exceeds that of the \( O(\alpha_s^3) \) term for \( \eta \approx 0.1 \), and the ratio grows as \( \eta \) decreases.\(^2\) Therefore, the important notion underlying perturbation theory, that successive terms in the perturbation series should be smaller, is not valid at small \( \eta \), i.e., in the region near production threshold. This region of phase space is important for top quark production at the Tevatron. Owing to the large mass of the top quark, relative to the \( pp \) center of mass energy \( \sqrt{s} \), the near-threshold region contributes significantly when the convolution integral, mentioned above, is done over the full of range of \( \eta \). Confidence in the results of a perturbative calculation of the overall \( tt \) cross section requires, therefore, an appropriate understanding of the origin of the large next-to-leading order enhancement of the partonic cross sections near threshold. (In the \( gg \) channel, the ratio of the \( O(\alpha_s^2) \) and \( O(\alpha_s^3) \) terms exceeds unity for large \( \eta \) also. The \( gg \) channel and the large \( \eta \) region are important for bottom quark production at the Tevatron\(^5\), but not for top quark production.)

The origin of the large threshold enhancements in the subprocess cross sections may be traced to initial-state gluon radiation.\(^3\) After the cancellation of soft singularities between the contributions from real gluon emission and virtual gluon exchange terms, and proper factorization of collinear divergences, there remain terms at \( O(\alpha_s^2) \) that are proportional to \( \ln(1-z) \). The variable \( z = 1 - 2k_g \cdot p_t/m^2 \) where \( p_t \) and \( k_g \) are the four-vector momenta of the produced top quark and the gluon radiated into the final state in the \( 2 \rightarrow 3 \) process. The limit \( z \rightarrow 1 \) corresponds to zero momentum carried by the gluon.

The partonic cross section may be expressed generally as

\[
\hat{s}_{ij}(\eta, m^2) = \int_{z_{\text{min}}}^{1} dz \left[ 1 + H_{ij}(z, \alpha) \right] \hat{s}_{ij}(\eta, m^2, z). \tag{1}
\]

In the near-threshold region,\(^6\)

\[
H_{ij}(z, \alpha) \simeq 2\alpha C_{ij} \ln^2(1-z)
+ \alpha^2 \left[ 2C_{ij} \ln(1-z) - \frac{4}{3} C_{ij} b_2 \ln^3(1-z) \right]. \tag{2}
\]

We work in the \( \overline{\text{MS}} \) factorisation scheme in which the \( q, \bar{q} \), and \( g \) densities and the next-to-leading order partonic cross sections are defined unambiguously. In Eq. (1),
the lower limit of integration \( z_{\min} = 1 - 4(1 + \eta) + 4/(1 + \eta) \) and \( ij \in \{ qg, gg \} \) denotes the initial parton channel. We set \( \alpha \equiv \alpha_s(m^2)/\pi \). Symbol \( \delta^{(0)}_{ij}(\eta, m^2, z) = d(x(0)_{ij}(\eta, m^2, z))/dz \), where \( \delta^{(0)}_{ij} \) is the lowest order partonic cross section expressed in terms of inelastic kinematic variables\(^6\) to account for the emitted radiation. The integration in Eq. (1) is over the phase space of the radiated gluons, parametrized through the dimensionless variable \( z \). In Eq. (2), \( C_{ij} \) is the color factor for the \( ij \) production channel.

Equation (2) approximates the near-threshold behavior of the partonic cross section. It manifests the logarithmic behavior \( \ln(1-z) \) mentioned above. Explicit calculations\(^3\) of the complete \( \mathcal{O}(\alpha_s^2) \) cross section provide the \( 2\alpha C_{ij} \ln^2(1-z) \) term. The terms proportional to \( \alpha^2 \) are appropriated from \( \mathcal{O}(\alpha_s^2) \) computations of massive lepton-pair production,\(^7,8,9\) based on the assumption of universality of leading logarithmic contributions. As in other hard-scattering processes, where large logarithmic contributions are present near threshold, the goal of gluon resummation in \( t \bar{t} \) production is to sum the series in \( \alpha^n \ln^m(1-z) \) to all orders. Resummation, studied extensively for massive lepton-pair production,\(^7,8,9\) is important both for theoretical understanding of the perturbative process and for stability of the quantitative predictions.

In resummation procedures, the large logarithmic contributions are exponentiated into a function of the QCD running coupling evaluated at a variable momentum scale, \( q \), that is a measure of the radiated gluon momentum. A common limitation of many methods is that an infrared singularity is encountered in the soft-gluon limit \( z \to 1 \), associated with the logarithmic behavior of \( \alpha(q^2), \alpha(q^2) \propto \ln^{-1}(q^2/\Lambda_Q^2) \). This divergence of the integrand at the upper limit necessitates introduction of an undetermined infrared cutoff cutoff \( \mu_e \) that prevents the integration over \( z \) from reaching the Landau pole of the QCD running coupling constant. The cutoff has a related effect of eliminating a portion of the integration over the partonic subenergy when the convolution with parton densities is done to obtain the physical cross section. The presence of an extra scale spoils the renormalization group properties of the overall expression. Moreover, dependence of the resummed cross section on this undetermined cutoff is important numerically.\(^5\)

The principal-value method of resummation\(^8\) (PVR) has an important technical advantage in that it does not depend on arbitrary infrared cutoffs, as all Landau-pole singularities are by-passed by a Cauchy principal-value prescription. Because extra undetermined scales are absent, the method also permits an evaluation of the perturbative regime of applicability of the method, i.e., the region of the gluon radiation phase space where perturbation theory should be valid. The method has been tested successfully in massive lepton-pair production.\(^9\)

To illustrate how infrared cutoffs are avoided in the PVR method, it is useful to express in moment space the exponent that resums the \( \ln(1-z) \) terms:

\[
E(n, m^2) = - \int_0^1 dz \frac{z^{n-1} - 1}{1-z} \int \frac{dy}{\lambda} g[\lambda/m^2] . \tag{3}
\]

The function \( g(\lambda) \) is calculable perturbatively, but the behavior of \( \alpha \) leads to divergence of the integral when \( \lambda m^2 \to \Lambda_Q^2 \). To avoid the divergence, a cutoff can be introduced in the integral over \( z \) or directly in momentum space, in the fashion of Laenen et al.\(^6\) In the principal-value redefinition of resummation, the singularity is avoided by replacement of the integral over the real axis \( z \) in Eq. (3) by an integral along a contour \( P \) in the complex plane:

\[
E^{PV}(n, m^2) \equiv - \int P \frac{z^{n-1} - 1}{1-z} \int \frac{dy}{\lambda} g[\lambda/m^2] . \tag{4}
\]

The function \( E^{PV}(n, m^2) \) is finite since the Landau pole singularity is by-passed. In Eq. (4), all large soft-gluon threshold contributions are included through the two-loop running of \( \alpha \).

Equations (3) and (4) have identical perturbative content, but, when expanded in power series in \( \alpha(m^2) \) and in \( \Lambda_Q/m \), they manifest differences in their inverse power (high-twist) terms. Since the inverse power content is not a prediction of perturbative QCD, neither expression is \( a \) priori preferable, except for the attractive finiteness of Eq. (4). In our application of principal-value resummation to top quark production, we choose to use the result only in the region of phase space in which the perturbative content dominates. Thus, the high-twist content of Eq. (4), and the difference between the high-twist components of Eqs. (4) and (3), are not matters of phenomenological significance.

After inversion of the Mellin transform, the resummed partonic cross sections according to PVR, including all large threshold corrections, can be written in the form of Eq. (1), but with Eq. (2) replaced by

\[
\mathcal{H}^{PV}_{ij}(z, \alpha) = \int_0^{\ln(1-z)} d\xi e^{\delta^{(0)}_{ij}(\xi, \alpha)} \sum_{j=0}^{\infty} Q_j(z, \alpha) . \tag{5}
\]

The leading large threshold corrections are contained in the exponent \( E_{ij}(z, \alpha) \), a calculable polynomial in \( z \). The functions \( \{Q_j(z, \alpha)\} \) arise from the analytical inversion of the Mellin transform from moment space to the physically relevant momentum space expressed in Eq. (5). These functions are produced by the resummation and are expressed in terms of successive derivatives of \( E: F_k(z, \alpha) \equiv \partial^k E(z, \alpha)/k! \partial^k z \).
The functional form of $E_{ij}$ for $t\bar{t}$ production is identical to that for $t\bar{t}$ production, except for the identification of the two separate channels, denoted by the subscript $ij$. However, only the leading threshold corrections are universal. Final-state gluon radiation as well as initial-state/final-state interference effects produce sub-leading logarithmic contributions that differ for processes with different final states. Among all $\{Q_j\}$ in Eq. (5), only the very leading one is universal. This is the linear term in $P_1$, which turns out to be $P_1$ itself. Since we intend to resum only the universal leading logarithms, we retain only $P_1$. Hence, Eq. (5) can be integrated explicitly, and the resummed version of Eq. (1) is
\begin{equation}
\hat{\sigma}_{ij}^{PVR}(\eta, m^2) = \int_{z_{min}}^{z_{max}} dz \hat{E}_{ij}(\ln(z^{-1}), o)\hat{E}_{ij}(\eta, m^2, z). \tag{6}
\end{equation}

The upper limit of integration in Eq. (6) is set by the boundary between the perturbative and high-twist regimes. To characterize a region in momentum space as high-twist, one must convert to momentum space through inversion of the Mellin transform, Eq. (5). Specification of the boundary is realized by the constraint that all $\{Q_j\}$, $j \geq 1$ be small compared to $Q_0$. This constraint can be shown to correspond to
\begin{equation}
P_1 \left( \ln \left( \frac{1}{1-z} \right), o \right) < 1. \tag{7}
\end{equation}

As remarked above, we accept only the perturbative content of principal-value resummation, and our cross section is evaluated accordingly. Specifically, we use Eq. (6) with the upper limit of integration, $z_{max}$, calculated from Eq. (7). The upshot is an effective threshold cutoff on the integral over the scaled subenergy variable $\eta$, but one that is not arbitrary. The cutoff restricts the region of applicability of resummation to the part of phase space in which the perturbative content of Eq. (5) is the dominant content. For the top mass $m = 175$ GeV, we determine that the perturbative regime is restricted to $\eta \geq 0.007$ for the $qq$ channel and $\eta \geq 0.05$ for the $gg$ channel. The difference reflects the larger color factor in the $gg$ case. (One could attempt to apply Eq. (6) all the way to $z_{max} = 1$, i.e., to $\eta = 0$, beyond the perturbative regime of Eq. (7), but one would then be using a model for non-perturbative effects, the one suggested by PVR, far beyond the knowledge justified by perturbation theory.)

Because we restrict application of our method of resummation to the perturbative regime our final result does not rely much upon the PVR method to by-pass infrared renormalons and associated infrared problems. In this respect, the presence of arbitrary infrared cutoffs in previous resummations is superfluous, as all necessary information about infrared sensitivity (i.e., the perturbative regime) can be obtained by examination of the perturbative asymptotic properties of the resummation.

The resummation procedure includes only the leading threshold $\ln^2(1-z)$ piece of the full $O(\alpha_s^2)$ calculation. To restore the full content of the complete next-to-leading calculation, $\hat{\sigma}_{ij}^{(0+1)}$, we define our final resummed cross sections for each production channel through the improved prediction
\begin{equation}
\hat{\sigma}_{ij}^{\text{final}}(\eta, m^2) = \hat{\sigma}_{ij}^{PVR}(\eta, m^2) + \hat{\delta}_{ij}^{(0+1)}(\eta, m^2) - \hat{\delta}_{ij}^{(0+1)}(\eta, m^2) \bigg|_{PVR}. \tag{8}
\end{equation}

The last term in Eq. (8) is the part of the next-to-leading order partonic cross section included in the resummation.

In the remainder of this report, we present our results for the physical inclusive total cross section for $t\bar{t}$ production for a top-mass range $m \in \{150, 250\}$ GeV, including a discussion of the remaining theoretical uncertainties. We obtain the physical cross section by convoluting Eq. (8) with $CTEQ3M$ parton densities and summing over both channels. In Fig.1 we show the top-mass dependence of the physical cross section for $p\bar{p} \rightarrow (t\bar{t})X$.

![Graph showing the cross section as a function of top mass](image)

Figure 1: Physical cross section for $p\bar{p} \rightarrow (t\bar{t})X$ at $\sqrt{s} = 1.8$ TeV as a function of top mass. Data from the CDF and D0 collaborations are plotted. Shown are calculations for three choices of the scale $\mu/m = 0.5$ (dashed), 1 (solid), and 2 (dotted)

The behavior of the physical cross section as a function of the renormalisation/factorization scale $\mu$ is mild in the range $\mu/m \in \{0.5, 2\}$. We consider the variation in this range to be a good measure of the theoretical perturbative uncertainty. Our prediction for the inclusive total $t\bar{t}$ production cross section at the Tevatron is
\begin{equation}
\sigma_{\text{final}}^{t\bar{t}}(m = 175 \text{ GeV}) = 5.52_{-0.45}^{+0.07} \text{ pb}. \tag{9}
\end{equation}
We define the central value (5.52 pb) to be that obtained with $\mu/m = 1$. The upper and lower limits correspond to the maximum and minimum values of the cross section in the range $\mu/m \in \{0.5, 2\}$. The cross section is insensitive to the choice of parton densities. Repeating the same analysis with the $MRS(A')$ densities\textsuperscript{11}, we obtain

$$\sigma_{\text{final}}(m = 175 \text{ GeV}) = 5.32^{+0.08}_{-0.41} \text{ pb}. \hspace{1em} (10)$$

The central values in Eqs. (9) and (10) are about 10% larger than that of Laenen et al\textsuperscript{6} but within the quoted uncertainties. Our calculated cross sections agree with the CDF and D0 measurements\textsuperscript{1}, within the current experimental error bars.

The bands of perturbative uncertainty quoted in Eqs. (9) and (10) are relatively narrow. On the other hand, we noted in discussing Eq. (8) that $z_{\text{max}} < 1$, meaning that there is a reasonable range of $\eta$ near threshold in which perturbative resummation does not apply. Perturbation theory is not justified in this region. Correspondingly, further strong interaction enhancements of the $t\bar{t}$ cross section may arise from physics in this region. We know of no reliable way to estimate the size of such non-perturbative effects and, therefore, cannot include such uncertainties in the estimates of the perturbative uncertainty of Eqs. (9) and (10).

Turning to $pp$ scattering at the energies of the Large Hadron Collider (LHC) at CERN, we note a few significant differences from $p\bar{p}$ production at the energy of the Fermilab Tevatron. The dominance of the $q\bar{q}$ production channel at the Tevatron is replaced by $gg$ dominance at the LHC. Owing to the much larger value of $\sqrt{s}$, the near-threshold region in the subenergy variable is relatively less important, reducing the significance of initial-state soft gluon radiation. Lastly, physics in the region of large $\sqrt{s}$, where straightforward next-to-leading order QCD is also inadequate, may become significant for $t\bar{t}$ production. Using the approach described in this paper, focussed on the resummation of initial-state gluon radiation, we present predictions in Fig. 2 for LHC energies of 10 and 14 TeV.

In summary, we present a calculation of the total cross section for $t\bar{t}$ pair production in perturbative QCD including resummation of initial-sate gluon radiation to all orders in $\alpha_s$. Two advantages of the principal-value method of resummation are the well-defined perturbative domain of applicability and the absence of arbitrary infrared cutoffs. Both $q\bar{q}$ and $gg$ production channels are included in the calculation. At $\sqrt{s} = 1.8$ TeV, our final resummed cross section is approximately 10% greater than the pure next-to-leading order result and in agreement with data.

References