Title:

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DYNAMICS IN LASER THROMBOLYSISRECEIVED
MAR 13 1996
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Submitted to:

SPIE International Symposium
Biomedical Optics - BIOS '96
San Jose, CA
January 28, 1996

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Numerical Studies of Bubble Dynamics in Laser Thrombolysis

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ABSTRACT

The applicability of modern numerical hydrodynamic methods for modeling the bubble dynamics occurring in Laser Thrombolysis is addressed. An idealized test problem is formulated and comparisons are made between numerical and analytical results. We find that approximately 23% of the available energy is radiated acoustically in one cycle with larger fractions likely to be radiated under more realistic conditions. We conclude that this approach shows promise in helping to optimize design parameters.

Keywords: laser thrombolysis, bubble dynamics, numerical methods, hydrodynamics, acoustic radiation

1. INTRODUCTION

We investigate aspects of bubble dynamics likely to play an important role in Laser Thrombolysis and other laser medical applications, particularly with respect to the partition of energy. We also investigate the accuracy of one of our shock physics computer codes, MESA-2D¹, when applied to a spherical cavity collapse test problem..

The compressible hydrodynamics associated with cavity motion in a fluid is described in Landau and Lifshitz². By postulating that the wavelength of sound generated by a deforming cavity is much less than a typical cavity dimension, one is led to the picture of a near-field region, where the fluid behaves in an incompressible manner, and a far field where outgoing acoustic waves transport energy to infinity. Reference 2 shows that, for a spherical cavity, the velocity potential in the near-field is $\phi = \dot{V}(t) / 4\pi r$; $V(t)$ is the cavity volume as a function of time, r is the radial distance from the cavity center and the dot denotes differentiation with respect to time. In the far field, ϕ must have the form of an outgoing spherical acoustic wave, specifically $\phi = -\dot{V}(t - r/c) / 4\pi r$. Further, the intensity of sound radiation is given by

$$I = \rho \dot{V}^2 / 4\pi c, \quad (1)$$

where I has dimensions of energy per unit time, and c is the fluid sound speed.

This physical picture can be used in conjunction with Cole's³ manipulation of the radial momentum equation to derive a self consistent generalization, through $O(1/c)$, of the incompressible equation of cavity motion (the Rayleigh-Plesset (R-P) Equation⁴). This generalized equation is essentially the Herring-Trilling Equation⁵.

2. GOVERNING EQUATIONS

The R-P Equation for a gas filled cavity is

$$\rho(R\ddot{R} + [3/2]\dot{R}^2) = P_g - P - 2\sigma/R - 4\mu\dot{R}/R, \quad (2)$$

where R is the cavity radius, P_g the gas pressure inside the cavity, P the fluid pressure at infinity, σ the fluid-cavity surface tension, and μ the fluid's viscosity coefficient. If we multiply Eq. 2 by $\dot{V}$, an energy conservation equation is obtained, which can be integrated once trivially if surface tension and viscosity are neglected and P_g is a function of cavity volume only. (We will always assume an adiabatic relationship, $P_g V^\gamma = \text{constant}$.) The resultant first-order ordinary differential equation can be integrated numerically to determine $R(t)$. This R-P solution will be compared to our computer model predictions.

Near maximum compression, the first order equation can be approximated as

$$\dot{R}^2 = \frac{2}{3} \frac{P_m V_m}{\rho(\gamma-1)} \left(1 - (V_m/V)^{\gamma-1}\right) \quad (3)$$

where P_m is the pressure, V_m the volume at collapse, and γ the gas specific heat ratio. Because most of the sound is radiated near maximum compression, we can use Eq. 3 in Eq. 1 to estimate the acoustic energy radiated in a half cycle (full collapse to full expansion, or vice versa):

$$L \equiv P_m V_m \frac{\sqrt{6} \gamma}{(\gamma-1)^{3/2}} \Gamma\left(\frac{2\gamma-1}{2\gamma-2}\right) \frac{\Gamma\left(\frac{3}{2}\right)}{\Gamma\left(\frac{5\gamma-4}{2\gamma-2}\right)} \sqrt{\frac{P_m}{\rho c^2}} \quad (4)$$

The Herring-Trilling Equation, the correct $O(1/c)$ generalization of Eq. 2, is

$$\rho \left[\left(1 - \frac{4\dot{R}}{3c}\right) \left(R\ddot{R} + \frac{3}{2}\dot{R}^2\right) - \frac{2\ddot{R}\dot{R}R}{3c} - \frac{R\dot{S}}{\rho c} \right] = P_g - P - 2\sigma/R - 4\mu\dot{R}/R, \quad (5)$$

where $S = P_g - 2\sigma/R - 4\mu\dot{R}/R$. Equation 5 incorporates the important effects of acoustic radiation.

3. TEST PROBLEM DEFINITION

An empty cavity in an incompressible fluid will collapse to a singularity, as described by Rayleigh⁶. The empty cavity wall velocity will increase without bound and will eventually exceed the fluid's sound speed. At this point, compressibility will affect the collapse dramatically. A small amount of gas can be put in the cavity to cushion the collapse. With a gas fill, the wall velocity will increase to a maximum and then decrease to zero at maximum compression. This is followed by rebound, with the process repeating.

We choose as an initial condition a spherical cavity of radius 0.1 cm surrounded by water at 1.0 bar pressure filled with 0.025 bar of $\gamma = 1.4$ perfect gas at 300 K. The R-P solution implies that this cavity collapses to a radius of approximately 0.00944 cm with a gas pressure of about 505 bars and temperature of 5100 K. The maximum wall velocity is about 100 m/s. Even though this is much less than any relevant sound speed, Eq. 4 yields for these parameters an energy loss of about 20% of the available energy! Such large radiation losses occur because the cavity is radiating as a monopole rather than the familiar dipole from electromagnetic theory.

4. RUN CONDITIONS

MESA-2D is an Eulerian finite difference code. This means that the computational mesh is fixed in space and material flows through the mesh. With the gas cavity at $r = z = 0$ in cylindrical coordinates, we used a $\Delta r = \Delta z = 0.002$ cm square mesh out to $r, z = 0.2$ cm, twice the initial cavity radius. The mesh was then expanded at a 1.1 geometrical factor out to about 2.52 cm. Thus the total number of computational cells was $150 \times 150 = 2.25 \times 10^4$, with 50 cells across the initial cavity and about 5 cells across the collapsed radius. We used a rigid boundary at $r = r_{\max}$ and a transmissive boundary at $z = z_{\max}$. The gas was modeled as a perfect gas ($\gamma = 1.4$) and a SESAME⁵ tabular equation of state was used for the water. Run times to maximum compression were about two hours on a Cray YMP computer.

5. TEST PROBLEM COMPARISONS

Figure 1. shows a comparison of normalized cavity radius from the code calculation (crosses) and from the incompressible R-P solution (continuous lines). The cavity shape predicted by the code calculation is nearly spherical, but does exhibit some asymmetry. The crosses in Fig. 1 are "spherized" radii calculated from the gas volume. Note that the cavity collapses in about 94 μ s and then rebounds to its initial radius in the R-P case and to about 92% of its initial radius in the case of the code result. This differential corresponds to a loss of about 23% of the available energy in the numerical simulation.

If we expand these results near maximum compression, Fig. 2 results. Note that the numerical simulation reaches maximum compression slightly ahead of the R-P solution and that the former's peak compression is less than the latter's. The difference in peak compression implies a 13% loss of available energy for the code result. Recall that our acoustic radiation estimate accounted for about a 20% energy loss in a half cycle. We expect that the actual loss would be somewhat less than 20% because the R-P solution, which was used to arrive at the 20% estimate, exhibits more curvature near maximum compression than is observed in a realistic cavity collapse. Thus, the code results are consistent with our expectation as to the measure in which acoustic radiation modifies the R-P result.

One troubling aspect of Fig. 2 is that the cavity seems to be collapsing sooner in the code calculation than in the R-P solution, despite the loss of energy to infinity in the former. This is due to the cavity radius starting, at $t = 0$, with a small but finite velocity, $(P - P_g(0)) / \rho c$, as demanded by compressibility, in contrast to starting at zero velocity in the incompressible R-P solution. In fact, if we start the R-P solution off with this jump-off velocity, its collapse leads the computational result, as energy considerations require.

Fig. 1. Normalized cavity radius as a function of time.

6. CONCLUSIONS

Shock physics numerical methods are helping to increase fundamental understanding of Laser Thrombolysis processes including energy partitioning. These methods should prove useful in the design optimization of Laser Thrombolysis treatment protocols and engineering systems. Numerical code results on a simple cavity collapse test problem are consistent with our expectation as to the measure in which acoustic radiation modifies the R-P result. Acoustic radiation, because of its monopole character, is an effective energy loss mechanism in bubble dynamics. We found that, for our test problem, about 23% of available energy is radiated as sound during the first collapse-expansion cycle even though cavity wall velocity never exceeds about 100 m/s.

7. ACKNOWLEDGMENTS

This work was supported by a Cooperative Research and Development Agreement (CRADA) between Los Alamos National Laboratory (Department of Energy), Oregon Medical Laser Center, and Palomar Medical Technologies. The second author thanks A. Vogel, Medical Laser Center Lübeck, for discussions of bubble dynamics in medical laser applications.

Fig. 2. Normalized cavity radius as a function of time (expanded).

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