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Induced Magnetic-Field Effects in Inductively Coupled Plasmas

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In inductive plasma sources, the rapid spatial decay of the electric field arising from the skin effect produces a large radio frequency (RF) magnetic field via Faraday's law. We previously determined that this magnetic field leads to a reduction of the electron density in the skin region, as well as a reduction in the collisionless heating rate. The electron deficit leads to the formation of an electrostatic potential which pulls electrons in to restore quasineutrality. Here we calculate the electron density including both the induced and electrostatic fields. If the wave frequency is not too low, the ions respond only to the averaged fields, and hence the electrostatic field is oscillatory, predominantly at the second harmonic of the applied field. We calculate the potential required to establish a constant electron density, and compare with numerical orbit-code calculations. For times short compared to ion transit times, the quasineutral density is just the initial ion density. For timescales long enough that the ions can relax, the density profile can be found from the solution of fluid equations with an effective (ponderomotive-like) potential added. Although the time-varying electrostatic potential is an extra source of heating, the net effect of the induced magnetic and electrostatic fields through trapping, early turning, and direct heating is a significant reduction in collisionless heating for parameters of interest.

I. Introduction

Inductively coupled discharges have emerged as promising high-density plasma sources for materials processing applications.¹ These are typically cylindrically symmetric devices with a coil at one end or wrapped around the side. Typical plasma parameters are plasma densities $n \sim 10^{11} - 10^{12} \text{ cm}^{-3}$, electron temperatures $T \sim 3 \text{ eV}$, ion temperatures $T_i \ll T$, and neutral pressures $P \sim 1 - 100 \text{ mTorr}$ (which implies that collisions with neutrals predominate over charged particle collisions). Industrial discharges have traditionally been operated at a frequency of 13.56 MHz, but lower frequencies are also now used. Typical device dimensions are of the order of 10 cm, while the skin depth for the induced RF fields is of the order of 1 cm.

In theoretical analyses and models of these devices, the interaction between the applied radio frequency (RF) and the plasma is usually treated by considering the induced electric field, neglecting the concurrent induced magnetic field. On this basis, descriptions have been developed for collisional and collisionless heating²⁻⁴ as well as the resultant plasma transport.⁵⁻⁷

In a previous paper⁸ we describe several important changes that occur when the induced magnetic fields are taken into account. These changes include a change in the direction of the heating, a reduction in the electron density in the RF skin region, and an associated reduction in the heating rate. These effects are especially pronounced at an RF frequency low compared to the electron transit frequency and/or at high power.

The reduction in electron density gives rise to a time-varying electrostatic potential which is the subject of the present paper. We develop a prescription for its determination, and show that it has several important consequences: It restores quasineutrality, and hence, if the RF frequency is not too low, it leads to a density which is approximately constant over an RF cycle; this implies that the potential itself oscillates at the second (and higher) harmonics of the applied RF frequency. On a longer timescale the ions respond to the average of the potential, and the result is that the quasineutral plasma is pushed back away from the wall (the quasineutral density is reduced near the wall). This leads to better confinement of the plasma: when transport processes, sources and sinks are included, one finds a higher ratio of density of plasma at the interface of the central and skin-field regions

to that at the center than one would find in the absence of the RF B field effects.

The electron density reduction from the RF magnetic field may be regarded as resulting from a ponderomotive force, as noted in Ref. 8. But our calculation applies to a regime quite different from the usual ones which rest variously upon assumptions of slowly varying wave amplitudes, weak nonlinearities, and/or fluid treatments. The effect is explicitly kinetic, requiring a non-Maxwellian trapped-electron population, arbitrarily nonlinear, and applies to strongly varying wave amplitudes. Also the work in this paper serves to identify the source of the mass redistribution as due to a real electrostatic potential arising from the modified electron dynamics, and the potential evolution on the wave timescale is explicitly followed. Because of the nonlinearities of the problem, and the requirement of instantaneous quasineutrality, the average of the potential so obtained can be quite different from that obtained by demanding only time-averaged quasineutrality. Finally, by following the fully nonlinear dynamics on the RF timescale, we gain important insights into the collisionless heating mechanism as modified by the induced magnetic field.

In this paper we consider the inductive fields as known functions of space and time. We defer to future work the calculation of a self-consistent vector potential which includes the dynamical effects discussed in Ref. 8 and here, though we note that some particle simulations addressing this issue have been done.⁹

This paper is organized as follows: In Sec. II, we generalize the results of Ref. 8, by deriving reduced orbit equations and general expressions for the energy gain and density reduction including the effect of an electrostatic potential. We restrict attention to the most important case of frequencies low compared to the electron skin transit time (Ref. 8 also considered the opposite limit). In Sec. III, we discuss the ambipolarity constraint and present analytic and numerical results for the ambipolar potential Φ vs. vector potential A for fixed density. In particular, the case where the ambipolar density n is set equal to the initial spatially and temporally constant ion density n_0 , is explored in some detail, including a numerical calculation of the heating rate. This case corresponds to times in an experiment or simulation longer than an RF period but short compared to the ion transit time across the skin layer, and also turns out to be relevant to the ion timescale when weak collisions are included. In Sec. IV, we consider the plasma dynamics on the ion timescale first in the collisionless limit. By integrating a set of ion fluid equations with the effect of the

averaged ambipolar potential included, we demonstrate the tendency to raise the density near the edge of the plasma but decrease it at the wall. Finally Sec. V is a discussion of our results, including a tentative explanation for the observation from Sec. III that adding the fluctuating ambipolar field does not appreciably alter the heating rate even though the electron density is restored to n_0 .

II. Orbit Equations and Electron Response

In this section we generalize the treatment of collisionless electron dynamics from Ref. 8 to include an electrostatic potential.

A. Geometry and equations

We consider the (x, y, z) slab coordinate system shown in Fig. 1, where a plasma fills the region $z > 0$. The electrons are specularly reflected at the $z = 0$ boundary to model an electrostatic sheath potential.

An RF coil for exciting an inductive electromagnetic field is assumed to exist to the left of the $z = 0$ boundary. The electromagnetic fields penetrating into the plasma have the form $\mathbf{E} = E_y(z, t)\mathbf{e}_y$, $\mathbf{B} = B_x(z, t)\mathbf{e}_x$, where E_y and B_x satisfy $\partial E_y/\partial z = \partial B_x/\partial t$ from Faraday's Law; we use MKS units. The electrons are assumed to originate from sources outside the field-containing skin layer.

Because the Hamiltonian corresponding to this electromagnetic field has no dependence on y , the canonical momentum $p_y = mv_y - eA_y$ is a constant of the motion,¹⁰ where $A_y(z, t)$ is the vector potential and e is the magnitude of the electron charge. Hence,

$$v_y - (eA_y/m) = \text{const} \equiv v_{y0} \quad (1)$$

where $\partial A_y/\partial t = -E_y$, $\partial A_y/\partial z = -B_x$ and, v_{y0} is the value of v_y far from the wall where A_y is negligible. Note that reflection off of the sheath at $z = 0$ leaves p_y unchanged. Thus, if E_y has the form of a skin field, *i.e.*, $E_y \rightarrow 0$ for $z = L \gg \delta$, where δ is the characteristic decay length of E_y with z , then an electron which passes into and back out of the RF field region conserves its value of v_y . The result is true regardless of field amplitude and shape; although, if the particle trajectory begins and ends a finite distance from the wall, then

for a sufficiently strong RF field, the initial and final A_y do not approach zero, and v_y will change according to Eq. (1).

This slab model can represent an idealized version of either top-coil or side-coil cylindrical inductively coupled plasmas; in the former case, z is the axial coordinate and x the radial, whereas in the latter case z is radial and x is axial. In both cases y represents the azimuthal direction. The idealization is that the fields be strictly azimuthally symmetric. However, for real inductive or helical-resonator sources with spiral windings of finite pitch and extent, there is no exact symmetry, and no strictly conserved momentum component. Nevertheless, the general results obtained in this paper should be approximately applicable to these systems if asymmetries are not too large.

B. Equations of Motion

The electron equations of motion for the prescribed inductive E_y and B_x fields plus the (so far arbitrary) electrostatic electric field $E_z = -\partial\Phi/\partial z$ are

$$m \frac{dv_z}{dt} = ev_y B_x + e \frac{\partial\Phi}{\partial z} \quad (2)$$

$$m \frac{dv_y}{dt} = -eE_y - ev_z B_x \quad (3)$$

As there are no forces in the x direction, v_x is constant. We neglect the influence of collisions during a sheath transit which would change v_x . We also neglect other effects which could break the constancy of p_y , such as plasma instabilities with a finite y (azimuthal) wave number, and asymmetries in the applied RF field. Note that plasma instabilities would lead to conservation of p_y summed over all particles that can interact with the wave, but no conservation for an individual electron.

Multiplying (2) by v_z and (3) by v_y and adding, gives the change in energy

$$\frac{d}{dt} \left(\frac{mv^2}{2} - e\Phi \right) = -eE_y v_y - e \frac{\partial\Phi}{\partial t} \quad (4)$$

Hence we see that the total (kinetic plus potential) energy of a particle can be changed by both the induced electric field and the time-varying part of the electrostatic field. The induced-field contribution is in general different with and without consideration of the induced magnetic field contributions because of differences in $v_y(t)$.

In what follows it will be useful to separate out the role of fluctuating and time-independent parts of Φ by writing $\Phi = \Phi_{dc} + \tilde{\Phi}$ and replacing v_z by $\mathcal{E}_z = (mv_z^2/2) - e\Phi_{dc}$. For the moment we leave the precise definition of this split undefined; it will be advantageous to make splits in which $\tilde{\Phi}$ has a non-zero average value. We obtain our fundamental dynamical equation by introducing this change of variables and Eq. (1) into Eq. (2), yielding

$$\frac{d\mathcal{E}_z}{dt} = v_z m R(z, t) \quad (5)$$

where

$$mR(z, t) \equiv -e \frac{\partial A_y}{\partial z} \left(v_{y0} + \frac{eA_y}{m} \right) + e \frac{\partial \tilde{\Phi}}{\partial z} \quad (6)$$

where v_{y0} is the initial value of v_y at $z = L$. This is the generalization of Eq. (5) of Ref. 8, and is again the starting point for our dynamical analysis.

1. Weak-field limit

In the weak-field (linear) limit, one can solve Eq. (5) for the change in $\mathcal{E}_z$ by dropping the quadratic term in R and by integrating along unperturbed orbits $z_0(t)$ where z_0 satisfies

$$t = \int_{z_t}^{z_0} dz' / v_z^0, \quad ,$$

$v_z^0 = \sigma(v_{z0}^2 + 2e\Phi_{dc}/m)^{1/2}$ with $\sigma = -1$ or $+1$ for $t < 0$ or > 0 , respectively. The magnitude of the unperturbed z velocity at “infinity” (far from the wall) is $v_{z0}(> 0)$, and z_t denotes the turning point in the zero-order field (reflecting off of the potential Φ_{dc} or at the sheath). Here $t = 0$ corresponds to the time of reflection, and the potential Φ_{dc} is taken to be just that which one would have in the absence of the RF fields. We do not consider here local trapping by Φ_{dc} since, for conventional discharge plasmas, the sign of the electric field precludes such a possibility. As noted in Ref. 8, this calculation is useful not only for the lowest-order single-particle dynamics, but also as the fundamental building block for the quasilinear diffusion coefficient $D_{zz} = \langle (\Delta \mathcal{E}_z)^2 \rangle / (2m^2 v_z^2 \Delta t)$, and hence the quasilinear heating rate $m \int (\partial/\partial v_z)(v_z D_{zz}) f d^3v$. The result is

$$\Delta \mathcal{E}_z = \int_{-\infty}^{\infty} e v_z^0 \left(-v_{y0} \partial A_y / \partial z + \partial \tilde{\Phi} / \partial z \right) dt. \quad (7)$$

where A_y and $\tilde{\Phi}$ are evaluated along the unperturbed orbit.

The linear response is a sum of responses to the separate inductive and electrostatic fields. For the particular case of exponentially decaying, sinusoidally varying fields, $A_y =$

$(E_{y0}/\omega) \exp(-z/\delta) \cos(\omega t + \phi)$, $\tilde{\Phi} = \Phi_0 \exp(-z/\delta_\phi) \cos(\omega_\phi t + \phi')$, and $\Phi_{dc} = 0$, the inductive and electrostatic contributions become, respectively,

$$(\Delta \mathcal{E}_z)_A = \frac{-2ev_{y0}E_{y0}\nu_{||} \sin \theta_t}{\omega^2 + \nu_{||}^2} \quad (8)$$

where $\theta_t = (\omega t + \phi)_t$ and $\nu_{||} = v_{z0}/\delta$, and

$$(\Delta \mathcal{E}_z)_\Phi = \frac{2e\Phi_0\nu'_{||}\omega_\phi \sin \theta'_t}{\omega_\phi^2 + \nu_{||}'^2}, \quad (9)$$

where $\theta'_t = (\omega_\phi t + \phi')_t$ and $\nu'_{||} = v_{z0}/\delta_\phi$.

If we substitute these expressions into the quasilinear heating rate formula, and anticipate the result that the electrostatic potential oscillates at even harmonics of the applied inductive field (since the electron density reduction which drives the oscillatory $\tilde{\Phi}$ is only a function of $|A|$), then we find that the heating rate is just the sum of the heating rates for the inductive and electrostatic fields, since the cross term averages to zero. For a Maxwellian electron distribution, the heating rate for the inductive field is that given by Eq. (39) of Ref. 8 (which, because the distribution is isotropic, turns out to be the same as that given in Ref. 4), namely:

$$\frac{d(n\langle \mathcal{E} \rangle)}{dt} = \frac{ne^2\delta^2 E_{y0}^2}{2\sqrt{\pi}v_t m L} [(1 + \alpha) \exp(\alpha) E_1(\alpha) - 1] \quad (10)$$

where $v_t^2 = 2T/m$, $\alpha = (\delta\omega/v_t)^2$, and E_1 is the exponential integral. The electrostatic term is

$$\left(\frac{d(n\langle \mathcal{E} \rangle)}{dt} \right)_\Phi = \frac{ne^2\Phi_0^2\omega_\phi^2}{\sqrt{\pi}v_t\nu_{\phi t}^2 m L} [(1 + \alpha_\phi) \exp(\alpha_\phi) E_1(\alpha_\phi) - 1] \quad (11)$$

where $\alpha_\phi = (\delta_\phi\omega_\phi/v_t)^2$ and $\nu_{\phi t} = v_t/\delta_\phi$.

2. Arbitrary amplitude, low frequency limit

For arbitrary field amplitudes, one can solve Eq. (5) for the orbit in the rapid-transit, low-frequency limit $\tau_{||} \equiv \oint dz/v_z \ll \omega^{-1}$, or when the opposite inequality is satisfied. We restrict attention here to the low-frequency limit, where the induced magnetic-field effects are strongest; the high-frequency result can be derived following the calculation in Ref. 8. To proceed, we expand R in time about the instant where the electron turns (which we continue to designate as $t = 0$),

$$R(z, t) = R(z, 0) + R_t t + \dots \quad (12)$$

where $R_t \equiv [\partial R / \partial t]_{t=0}$. Note that now the turning of the electron orbit can be done by the sheath at $z = 0$, by the electrostatic potential Φ or by the action of the RF B-field itself. Equation (5) then becomes

$$\frac{d\mathcal{E}_z}{dz} \approx R(z, 0) + R_t \int_{z_t}^z dz' / v_z + \dots, \quad (13)$$

where z_t denotes the turning point. To leading order ($\omega\tau_{||} \rightarrow 0$),

$$\begin{aligned} v_z(z) = v_z^0 &\equiv \sigma \left[v_{z0}^2 + v_\phi^2 - 2 \int_z^L R_A(z', 0) dz' \right]^{1/2} \\ &= \sigma \left[v_{z0}^2 + v_\phi^2 - 2v_c v_{y0} - v_c^2 \right]^{1/2}, \end{aligned} \quad (14)$$

where $R_A = (-e\partial A_y / m\partial z)(v_{y0} + eA_y/m)$ is the inductive part of R , where

$$v_c(z) \equiv eA_y(z, 0)/m \quad (15)$$

and where $v_\phi^2 \equiv 2e\Phi/m$ (Φ being the total potential).

To zero order in $\omega\tau_{||}$, there is no change in the energy in a round trip through the RF field region because $v_c(L) \rightarrow 0$. However, Eq. (14) gives the leading-order expression for the turning point, $z_t = \max(0, z_0)$, where z_0 is the position at which $v_z^0 = 0$. Equation (14) also provides the orbits that we use in evaluating the next-order expression for the energy change and the lowest-order expression for the electron density. Note also that (14) depends only on the local potential A_y , not on its spatial shape.

To next order in $\omega\tau_{||}$, we obtain the change in z energy as:

$$\Delta\mathcal{E} = \Delta\mathcal{E}_z = 2m \int_{z_t}^L R_t dz \int_{z_t}^z dz' / v_z^0. \quad (16)$$

Interchanging the order of integration and putting in the specific forms for R and R_t , the previous equation can be written in the form:

$$\Delta\mathcal{E} = 2m \int_{z_t}^L \frac{d\hat{v}_z}{dz} \frac{\hat{v}_z}{v_z^0} \left[\frac{(v_{y0} - eA_y/m)(\partial A / \partial t) - \partial \Phi / \partial t}{(v_{y0} - eA_y/m)(\partial A / \partial x) - \partial \Phi / \partial x} \right]_{t=0} dz, \quad (17)$$

where $\hat{v}_z = (2\mathcal{E}_z/m)^{1/2}$. In the particular case where Φ can be expressed as a function of A_y (which applies both to the analysis of timescales short compared to ion transit times in Sec. 3 and to our model for the ion timescale with collisions discussed in Sec. 4), then this expression reduces to a form not quite as simple as that obtained neglecting the electrostatic potential in Ref. 8, namely

$$\Delta\mathcal{E} = 2m \int_{z_t}^L \frac{dv_z^0}{dz} \frac{\hat{v}_z}{v_z^0} \left[\frac{\partial A_y / \partial t}{\partial A_y / \partial z} \right]_{t=0} dz. \quad (18)$$

For our model with an exponentially decaying vector potential, if we exclude any Φ_{dc} so that $\hat{v}_z = v_z^0$, then we obtain

$$\Delta\mathcal{E} = -2m\delta\omega(v_{z0} - v_{zt})\tan\theta_t, \quad (19)$$

where $v_{zt} = v_z^0(z_t)$ is obtained from Eq. (14) with $\sigma = 1$ (i.e., $v_{zt} \geq 0$) for a particle which reflects off of the sheath (which we are treating here as a reflecting wall, rather than as part of the electrostatic potential), while $v_{zt} = 0$ for a particle which is turned by the induced magnetic field and the electrostatic field before reaching the sheath. In this model, we have, as in the case with no electrostatic potential, no RF amplitude dependence for a particle which turns before the sheath. Thus, not only has the energy gain saturated as a function of field amplitude, but the energy gain doesn't depend on the strength of the electrostatic potential. However, there is still Φ dependence for particles which are turned by the sheath, and the transition boundary between particles which are turned by the RF fields and those that are not is shifted. Specifically, the criterion for turning in the inductive field before reaching the sheath is that, at $z = 0$, $(v_z^0)^2 < 0$, or

$$\left(\frac{\delta}{\rho_z}\right)^2 + \frac{2v_{y0}}{v_{z0}}\frac{\delta}{\rho_z} - \frac{2e\Phi}{mv_{z0}^2} > 1, \quad (20)$$

where $\rho_z = mv_z/eB$, and we notice that for positive Φ fewer electrons turn. It can be verified that the weak-field limit of Eq. (19) is equal to the low-frequency limit of Eqs. (8)-(9). Also, note that the reduction of the energy gain to the integral of a total derivative is in general not possible for finite Φ_{dc} .

At the same time that positive Φ reduces the number of electrons turning before the sheath, it increases the number which trap in the wave. Trapping occurs for local (v_y, v_z) such that $v_{z0}^2 < 0$. The quasineutral solutions we will describe in Secs. 3 and 4 have the property that $\Phi(t)$ increases from its minimum value as does $|A|$. Hence, an electron which becomes trapped as A increases remains so until A decreases to approximately the same level. The electric fields are thus approximately opposed as the electron approaches and recedes from the region where it is trapped. While in the trapped state, its longitudinal adiabatic invariant $J = \int v_{\parallel} dz$ is approximately conserved [the departure being of order $\exp(-\alpha\nu_{\parallel}/\omega)$,¹¹ with $\alpha \sim 1$]. Hence we expect the energy gain of trapped particles to be small in the rapid-transit limit.

In Ref. 8, we presented results from a number of test-particle simulations for the expo-

nential field that established the validity of Eq. (19) for the single-particle energy change for the case $\Phi = 0$.

We do not give an analytic form distribution-function-integrated form for the heating rate, since electrons for which the rapid-transit ordering is violated play a critical role in limiting the density integral, as is seen already in the weak-field form, Eqs. (10)-(11).

C. Electron density modification

In Ref. (8) it was shown that the presence of the RF magnetic field leads to a reduction in the electron density during either half of the RF cycle. This leads to the formation of a positive potential during either half cycle which acts to counter the magnetic field effect and thus restore quasineutrality. In this section we recalculate the density including a fluctuating electrostatic potential. In this calculation it is convenient to set $\Phi_{dc} = 0$, reabsorbing any steady part into $\tilde{\Phi}$. In the low-frequency limit, an expression for the electron density reduction can be obtained by starting from the leading-order expression (14) for the electron velocity. From Eq. (14), the local electron phase space at any instant is filled with particles which are either trapped by the electromagnetic field and prevented from reaching $z = L$, or untrapped, according to whether

$$v_{z0}^2 = v_z^2 + v_\phi^2 + 2v_c v_y - v_c^2 < 0 \quad (21)$$

or the inequality is reversed, respectively.

We separately calculate the density of trapped and untrapped electrons. By switching variables of integration from v_y to $-v_y$ in Eqs. (23) and (27), it is straightforward to show that the density for either group is a function only of $|A_y|$; hence, without loss of generality, we consider A_y and hence v_c to be positive. For the untrapped electrons, in the limit where the collision frequency ν_c is small compared to $1/\tau_{||}$, the distribution function at any position is related to the distribution f_0 at $z = L$ (where the RF fields are negligible) via constancy of the canonical momentum p_y and through Eq. (14); that is,

$$f_u(v_x, v_y, v_z) = f_0(v_x, v_{y0}, v_z^0) \quad (22)$$

where $v_{y0} = v_y - v_c$, and v_z^0 is given in terms of (v_y, v_z) by Eq. (14). In the absence of collisions, v_x is unchanged. If f_0 is taken as Maxwellian in $\mathbf{v}_0$, then f_u is Maxwellian

in v but with the normalization shifted by $e\Phi$, $f_u = \hat{n}\pi^{-3/2}v_t^{-3}\exp(-v^2/v_t^2)$, where $\hat{n} = n_0 \exp(e\Phi/T)$, and n_0 is the density $z = L$. Note that $\hat{n}$ is the density which would be obtained were all of velocity space at the local position filled with this distribution function. The untrapped density, n_u , is reduced below $\hat{n}$ because the untrapped region of velocity space is only a portion of the total. Specifically,

$$n_u = \hat{n}\pi^{-1} \int_{ut} d^2u \exp(-u_y^2 - u_z^2) \quad (23)$$

where “ ut ” denotes the untrapped region, where $u_z^2 + \varphi + 2u_y\hat{A} - \hat{A}^2 > 0$, $\hat{A} \equiv |v_c|/v_t = e|A_y|/mv_t$, and $\varphi \equiv (v_\phi/v_t)^2 = e\Phi/T$. Performing the integration over u_y gives

$$n_u = n_0\pi^{-1/2} \int_0^\infty du_z \exp(-u_z^2) \operatorname{erfc} \left(\frac{\hat{A}^2 + \varphi - u_z^2}{2\hat{A}} \right) \quad (24)$$

An alternative form which is also useful is obtained by reversing the order of integrations,

$$n_u = n_0\pi^{-1/2} \int_{-\infty}^{\hat{A}^\dagger} du_y \exp(-u_y^2) \operatorname{erf} \left(\hat{A}^2 + \varphi - 2\hat{A}u_y \right)^{1/2} \quad (25)$$

where $\hat{A}^\dagger = \hat{A} + \varphi/\hat{A}$.

The trapped distribution function is calculated by exploiting conservation of the longitudinal adiabatic invariant: assuming as usual that $\nu_c \ll \omega \ll \nu_\parallel$, the electromagnetic fields establish a well that traps electrons while preserving their longitudinal adiabatic invariant $J = \int dz p_z$. It was shown in Ref. (8) that this implies

$$f_{tr}(v_x, v_y, v_z) = f_0(v_x, v_y, 0) \quad (26)$$

The derivation and the result are unaltered by addition of the electrostatic potential, so long as the minimum of the potential over an RF cycle is zero or negative so that there is no continuously trapped portion of phase space (which, if it existed, would slowly fill in due to collisions, in contrast to the transiently trapped portions which are emptied out and freshly refilled in each RF cycle). If f_0 is Maxwellian, the trapped density is

$$n_{tr} = n_0\pi^{-1} \int_{tr} du_y du_z \exp[-(u_y - \hat{A})^2] \quad (27)$$

where “ tr ” denotes the region where $u_z^2 + \varphi + 2u_y\hat{A} - \hat{A}^2 < 0$. This integral can be expressed in closed form as

$$n_{tr} = n_0 \frac{(2\hat{A})^{1/2} \Gamma(5/4)}{\pi} \left[-\hat{A}^* \exp(-\hat{A}^2/4) {}_1F_1 \left(\frac{5}{4}, \frac{3}{2}, \frac{\hat{A}^2}{4} \right) + \pi^{1/2} \mathcal{L} \left(\frac{1}{4}, -\frac{1}{2}, -\frac{\hat{A}^2}{4} \right) \right] \quad (28)$$

where $\hat{A}^* = \hat{A} - \varphi/\hat{A}$, Γ is the gamma function, ${}_1F_1$ is the hypergeometric function, and $\mathcal{L}$ is the Laguerre function. The total density is $n = n_u + n_{tr}$ and is a function of $\hat{A}$ and φ .

The validity of these equations for the $\varphi = 0$ case was established in Ref. 8, where the curve n/n_0 vs. $\hat{A}$ was plotted along with numerical data for several frequencies obtained from the orbit code described earlier. In this study, an ensemble of particles chosen from an incident Maxwellian were followed. The result is reproduced here as Fig. 2. We present a test of the results including the electrostatic potential in the next section.

It is straightforward to derive asymptotic expansions for n for small $\hat{A}$ and φ , large $\hat{A}$ and φ , and the hybrid case of small $\hat{A}$ but φ much larger [$\mathcal{O}(1)$]. The latter case may be of particular interest for ion-time-scale kinetic simulations with no collisions. The results are:

$$n \approx n_0 \left[1 + \varphi - c_0 \left(\hat{A}^{3/2} + c_1 \varphi \hat{A}^{1/2} \right) \right], \quad \hat{A} \ll 1 \quad (29)$$

$$n \approx n_0 c_\infty \exp(-\hat{A}^2/4)/\hat{A}, \quad \hat{A} \gg 1. \quad (30)$$

where $c_0 = 2^{5/2}(3\pi)^{-1}\Gamma(5/4)$, $c_1 = \Gamma(3/4)/\Gamma(5/4)$, and $c_\infty = 2^{3/2}\pi^{-1/2}$. The small- $\hat{A}$, finite- φ cases are of different form for positive and negative φ , and the expansions on which they are based assume that $|\varphi| \gg \hat{A}$:

$$n \approx n_0 \left[e^\varphi \operatorname{erfc}(\varphi^{1/2}) + 2(\varphi/\pi)^{1/2} - \hat{A}^2 \pi^{-1/2} \varphi^{-3/2} (4\varphi + 1/2) + \mathcal{O}(\hat{A}^4) \right], \quad \varphi > 0 \quad (31)$$

and

$$n \approx n_0 \left\{ e^\varphi + (2\hat{A}/\pi) \exp[-(\hat{A} - \varphi/\hat{A})^2/4] \left[(\hat{A} - \varphi/\hat{A})^{-3/2} + (-\hat{A} - \varphi/\hat{A})^{-3/2} \right] \right\}, \quad \varphi < 0. \quad (32)$$

We note that in the latter case there is an extremely weak dependence on the inductive fields; the density reduction is almost entirely the result of repelling of electrons by the negative potential.

III. Self-consistent potential profiles

A. Wave timescale

For $\varphi = 0$, Eqs. (24)-(28) predict an electron density reduction which is a function of the instantaneous $|A_y|$. (This is modified with corrections of order $1/\omega\tau_{||}$). But the ions

are much less mobile than the electrons. Hence a charge imbalance would develop, with a structure that has second and higher harmonics of A_y , as shown in Ref. 8. However, because the skin depth δ is many Debye lengths and the wave frequency is small compared to the electron plasma frequency, the plasma must be quasineutral ($n_e \approx n_i$) even on the timescale of the RF cycle. Hence the electrostatic potential must adjust over the course of the RF cycle to maintain this condition.

Furthermore, for parameters of interest to discharge plasmas, the ions are essentially immobile on the time scale of the RF oscillations. For example, in an Argon plasma, even ions which have already reached the Bohm velocity in a plasma with a 3 eV electron temperature move only 1.4 mm in a half cycle of a 1 MHz wave (compared to skin depths $\gtrsim 1$ cm). Hence to a good approximation the electrostatic potential must set up so that the plasma is quasineutral with a density that is *constant* during the RF cycle.

If we start out a real plasma or a computer simulation with uniform density, then for times short compared to ion transit times, the constant quasineutral density is the initial one, namely $n = n_0 = \text{const.}$ (in space and time). From the asymptotic formulas (29) and (29), we see that the potential required to maintain $n = n_0$ is approximately

$$\varphi \approx c_0 \hat{A}^{3/2} + \dots, \quad \hat{A} \ll 1, \quad (33)$$

and

$$\varphi \approx \hat{A}^2 - \hat{A} \left[(1/2) \ln(2\hat{A}/\pi) \right] + \dots, \quad \hat{A} \gg 1. \quad (34)$$

For intermediate $\hat{A}$ we can solve for φ by numerical root finding, setting the sum of Eqs. (24) and (28) to n_0 . The result is shown in Fig. 3. Over the plotted range ($0 < \hat{A} < 4$) a good fit is given by

$$\varphi \approx 0.122 \hat{A}^{3/2} + 0.838 \hat{A}^2 \quad (35)$$

An even better fit, within 2% over the range $0.1 < \hat{A} < 15$, is given by the formula

$$\varphi \approx \hat{A}(1 + 0.94\hat{A})/(1 + 1/0.36\hat{A})^{1/2}. \quad (36)$$

However, this formula is not as easy to average as (35); we retain both for the following discussion. Also it should be noted that at the largest $\hat{A}$ fitted, 2% is actually not good enough to ensure a specified value of n/n_0 , as φ itself is very large. One can remedy this

by carrying the asymptotic expansion to one higher order, but the difference is typically smaller than finite $\omega\tau_{\parallel}$ corrections.

We have tested this result by running the orbit code with this specified relationship between φ and $\hat{A}$. The measure of success is the degree to which the temporal and spatial variation of the electron density is reduced compared to the case with no electrostatic potential. The results are shown in Fig. 4; comparing Figs. 4a ($\varphi = 0$) and 4b (analytic φ) we see that the analytic φ does approximately restore $n = n_0$. For this case, $\omega\tau_{\parallel} = 0.06$. Other cases (not shown) for 2 MHz and 5 MHz give a measure of the finite $\omega\tau_{\parallel}$ effects where we keep $\hat{A} = 2.7$. They show a progressive reduction of $n(0)/n_0$ including φ , with $n(0)/n_0 \approx 0.6$ for 5 MHz.

Inclusion of φ also effects the heating rates and distribution function near the wall. In Ref. 8, we found that the RF B-field strongly reduced the heating at low frequency and/or large fields. This was partially attributed to the density reduction. From the orbit code, we find that the heating rate is nearly unchanged in spite of the fact that many more electrons now reach the large RF field region near the wall. For the 1 MHz example shown in Fig. 4, the reduction of the collisionless heating from the quasilinear rate, Eq. (10), is a factor of 0.18 for the case $\varphi = 0$ and a factor of 0.15 when φ is taken from Eq. (35). Thus, the extra electrons that are present near the wall are not appreciably heated. The distribution function near the wall does show a significantly larger adiabatic quiver component for the finite φ case, with mean energy in the (v_x, v_y) components rising by an average factor of 3.3, compared to a factor of 1.4 for the $\varphi = 0$ case. Outside the strong-field region, the mean energies rapidly drop to near the injection values. A diagnostic in the orbit code measuring the ionization rate for Argon shows a 12-fold increase at the wall for the finite φ case owing to large electron quiver motion; for $\varphi = 0$, the increase is only a factor of 1.6. The quiver motion will also be partially converted to irreversible heating by collisions, but this should always be less than the result obtained neglecting the RF B-field.

B. Ion dynamics

The ions respond approximately to the time average of the RF fields. While the inductive fields average to zero, the average $\langle\Phi\rangle$ of the electrostatic potential computed above does

not; it is positive. This average positive potential tends to push ions away from the wall into the interior region, thus providing better overall particle confinement. Because of the reduction in density, the potential does not have to be as big to draw in enough electrons to restore quasineutrality; hence the average is reduced, and the $\Phi(t)$ that emerges from the electron density analysis can be negative as well as positive. This is illustrated by the dashed curve of Fig. 3, which is a plot of $\varphi(\hat{A})$ for $n/n_0 = 0.3$. It is in this context that the case of finite potential and small $\hat{A}$ is interesting: the most negative potential is achieved when $\hat{A}$ vanishes during the RF cycle. The minimum, in our $\omega\tau_{\parallel} \ll 1$ approximation, is simply $T_e \ln(n/n_0)$.

In fact, for a simple model in which the ion momentum balance is assumed to be between the average electrostatic force and the ion pressure gradient (neglecting inertia and friction), then the average potential required to achieve a sizeable change in the ion density is of order of T_i/e where T_i is the ion temperature, while the variation in Φ is of order T_e for $\hat{A} \sim 1$. Thus, in this simple model, the self-consistent potential on the ion dynamics timescale is one in which the constant ambipolar density is enough below n_0 that $\langle\varphi\rangle \approx 0$.

A more realistic description of the ion dynamics is achieved by solving Eqs. (24) and (28) for $\Phi[n, |A_y(t)|]$, averaging over time (holding n fixed) to yield $\langle\Phi\rangle$ as a function of n and the amplitude of A_y . This result is substituted into the ion momentum equation

$$m_i n \frac{\partial u_i}{\partial t} + m_i n u_i \frac{\partial u_i}{\partial z} + n e \frac{\partial \Phi}{\partial z} = -\nabla P_i, \quad (37)$$

where m_i , u_i , and P_i are ion mass, velocity, and pressure, respectively. The resultant equation, coupled with the continuity equation and an equation of state, can be solved either dynamically (on the ion timescale) or in steady state. This approach makes sense even on the assumed long timescale of electron collisions, so long as the minimum of the potential over an RF cycle does not become positive relative to the field-free interior; otherwise part of the electron phase space is continuously trapped and slowly fills in with a Maxwellian distribution, (instead of the constant-longitudinal-invariant one) requiring modification to the calculations of Sec. 2c. The calculation can also be extended to very low frequencies ($\ll 1$ MHz for typical discharge parameters), where ion dynamics on the RF cycle begin to be important, simply by not averaging Φ and following the RF dynamics numerically.

A useful simplification to this prescription follows if we approximate the family of curves

$\varphi(\hat{A})$ for various n 's as being vertically displaced from one another; from Fig. 3 it is evident that this is a fairly good approximation. We can then write $\Phi = \Phi_{dc} + \tilde{\Phi}$ with $e\Phi_{dc} = T_e \ln n/n_0$, and $\tilde{\Phi}(A)$ determined from Eqs. (24) and (28) with $n_{tr} + n_u = n_0$. [Now $\tilde{\Phi} = \tilde{\Phi}(A)$ as assumed in constructing the simplified energy gain expression (18).] In particular, for the simple fit to $\Phi(\hat{A})$ given by Eq. (35) and a sinusoidally varying $\hat{A}$ with magnitude A_m , time averaging gives

$$\langle \varphi \rangle \approx 0.0677 A_m^{3/2} + 0.419 A_m^2. \quad (38)$$

This provides a simple explicit formulation with which to determine the self-consistent density. For a simple steady-state model with $T_i = \text{const}$, $n = n_0$ at the midplane (redefined here to be $z = 0$), a symmetric vector potential $A = A_0 \cosh(z/\delta)$, and an assumed constant ionization rate adjusted to make the velocity equal to the Bohm velocity at the wall, we obtain the density profiles shown in Fig. 5 with and without the φ term. The flattening of the density is evident. The parameters chosen for this example are rather typical for experimental conditions: $\delta/\lambda_i = .05$, where λ_i is the ion mean free path, T_i/T_e assumed negligible, total ion collision frequency/ionization rate = 2, and $A_m \approx 1.7$ at the wall. For a frequency of 2 MHz and a skin depth of 1 cm, the choice of λ_i corresponds to an electron collisionality $\nu_e/\omega \approx 0.3$. Decreasing δ/λ_i to be further into the collisionless regime makes the flattening even more pronounced.

IV. Discussion and conclusions

We have shown that, in an inductive plasma source, the electron density reduction brought about by orbit modification (including turning of orbits) created by the induced magnetic field gives rise to an electrostatic potential. This potential acts in such a way as to restore quasineutrality and keep the density nearly constant over an RF cycle. The time averaged part of this potential pushes the ions (*i.e.* the plasma mass) away from the wall by the coils.

The steady-state part of the potential acts to bolster the density an RF skin depth from the wall. This can either improve or degrade source uniformity, depending on whether the density is already peaking near the coils as it would in a short inductively coupled source.

In Sec. 3 it was noted that the orbit-code studies with the potential added to restore

quasineutrality resulted in a heating rate that was not much different, even a bit lower, than the heating rate without the electrostatic potential. This is at first sight surprising: the electron density in the absence of Φ was significantly suppressed below n_0 , and the potential restored $n = n_0$. Additionally, we expect the inductive and electrostatic contributions to the heating to add, since the acting RF fields are at different frequencies. Furthermore, inspection of the quasilinear, distribution-function-averaged form of the heating rate (10) and (11) or simply comparing the induced and electrostatic electric fields for with the approximate ambipolar solution $\varphi \sim \hat{A}^2$ valid for $\hat{A} \gtrsim 1$ (see Sec. 3), would lead us to expect that the heating rate would be dominated by the electrostatic contribution for $(\nu_{\parallel}/\omega)\hat{A} > 1$ which is easily satisfied for low-frequency (~ 1 MHz) inductive plasma sources. However, one must also consider that the inductive contribution averaged over the electrons in the skin region is itself lowered by presence of the electrostatic field, because, as noted in Sec. 2, the wave-trapped electrons, which constitute much of the density for $\hat{A} \gtrsim 1$, experience little heating. In fact the untrapped density is lowered by the presence of a positive incremental potential, because these electrons are accelerated by the potential. It is thus feasible that the total heating rate, including inductive and electrostatic contributions and a much larger electron density, could be lower than with no Φ . energy. A conjecture that we will explore is that the heating rate is given roughly by the untrapped density at the sheath times the quasilinear heating rate. In the computed example (Fig. 4), this heating reduction occurs for a flat quasineutral density profile. The heating will be further modified by the redistribution in the density as found in the solutions of Sec. 4.

The formulas derived in this paper provide a prescription useful for an ion-time-scale fluid code at low pressure (electron collision frequency small compared to the transit frequency through the skin layer). In particular the electron momentum equation is replaced by the relation $e\Phi/T = \ln(n/n_0) - \langle \varphi(\hat{A}) \rangle$ with $\varphi(\hat{A})$ given by Eq. (35) or (36).

For higher pressures, the trapped phase space starts to fill in by collisions, reducing the ponderomotive-like effects described here and restoring the heating efficiency (the trapped particles start to participate). But at the same time the distribution function expressed as a function of collisionless constants of the motion can start to vary with z . This enables standard fluid ponderomotive effects¹² to begin to be seen. In the local fluid limit, the ponderomotive potential and density reduction for an inductive discharge¹³ are qualitatively

similar to the results obtained here. The heating in that limit would be the standard collisional result evaluated with the ponderomotively redistributed density. The transition between these regimes will be the subject of a future report.

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Figures

Fig. 1. Coordinate system and typical RF field variation for an inductive plasma source with an RF coil to the left of $z = 0$.

Fig. 2. Density ratio, $n(0)/n_0$, versus $\hat{A} = eA/mv_t$ for a 3 eV Maxwellian with frequencies $f_{RF} = 2, 5$, and 13.56 MHz.

Fig. 3. Normalized fluctuating potential φ vs. $\hat{A}$ for fixed $n = n_0$ (solid curve) and $n = 0.3n_0$ (dashed curve).

Fig. 4. Orbit code results for density as a function of time and distance from the sheath, $(\omega t, z)$, for $f_{RF} = 1$ MHz and $E_{y0} = 100$ V/m. In (a), $\varphi = 0$, and in (b), $\varphi(t)$ has the values predicted to make $n = n_0$ from Eq. (35). Maximum $\hat{A} = 2.7$ and maximum $\varphi = 6.7$. Injected distribution at $z = 5$ cm is a 3 eV Maxwellian.

Fig. 5. Density profiles with (solid curve) and without (dashed curve) RF B-field effects including averaged self-consistent potential. The regions denoted by δ correspond to the $1/e$ fall-off distance of the ponderomotive potential at either end of the discharge.

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5

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