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Phase retrieval and time-frequency methods in the measurement of ultrashort laser pulses

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Abstract

We employ techniques from time-frequency analysis, image recovery, and phase retrieval to measure the time-dependent intensity and phase of an ultrashort laser pulse.

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The recovery of an optical field with respect to position when only the intensity can be measured is an important problem in image science. In this case *a priori* information in the form of constraints can be applied and advantage can be taken of the inherently two-dimensional nature of the problem in order to reconstruct the full complex field from the available information. A similar recovery problem also arises with *temporally* varying data. One such case is the measurement of the complete time-dependent intensity and phase of an ultrashort laser pulse. This problem is particularly difficult for two reasons. First, it is inherently one-dimensional, so phase-retrieval methods, so successful for the spatial problem, do not directly apply. Second, such pulses are shorter than all possible measuring devices, so even the intensity cannot be measured. Traditionally, optical scientists working with ultrashort laser pulses have had only partial diagnostics, typically the intensity autocorrelation and the spectral intensity of the pulse. These diagnostics are not enough to completely characterize the laser pulse.

Recently several techniques have become available to measure the time- (or frequency-) dependent intensity and phase of ultrashort laser pulses [1]. One of these, Frequency-Resolved Optical Gating (FROG), is rigorous and has achieved single-laser-shot operation [2]. FROG combines the concepts of time-frequency analysis in the form of spectrogram generation (in order to create a two-dimensional problem), and uses a phase-retrieval-based algorithm to invert the experimental data to yield the intensity and phase of the laboratory laser pulse. In FROG it is easy to generate a spectrogram of the unknown signal, and inversion of the spectrogram to recover the signal is the main goal.

Because the temporal width of a femtosecond laser pulse is much shorter than anything achievable by electronics (or any other technology), FROG uses the pulse to measure itself. In FROG, the laser pulse is split into two replicas of itself by a partially reflecting beamsplitter, and the two replicas interact with each other in a medium with an instantaneous nonlinear-optical response. This interaction generates a signal field that is then frequency-resolved using a spectrometer. The spectrum of the signal field is measured for all relevant values of the temporal delay between the two pulses.

The FROG technique can utilize several different nonlinear optical interactions [3]. In the polarization-gate (PG) geometry, the form of the signal field can be written as

$$E_{sig}(t,\tau) = E(t)|E(t-\tau)|^2 \tag{1}$$

where $E(t)$ is the pulse electric field (written in complex notation and with the optical carrier frequency removed) and τ is the time delay between the two replicas of the pulse. The signal field, or FROG trace, is then given by

$$I_{FROG}(\tau,\omega) = \left| \int_{-\infty}^{\infty} dt\, E_{sig}(t,\tau) \exp(-i\omega t) \right|^2 . \tag{2}$$

Note that the FROG trace is a type of self-gating spectrogram [4], in which the gate function is actually the intensity envelope of the pulse to be measured (in other FROG

geometries, the gate function will have a different form, but will still be a function of the pulse). In the language of Cohen's general class of time-frequency distributions [5], the kernel associated with the FROG trace is signal dependent. This fact makes the usual inversion techniques unusable, and other methods must be sought.

The FROG pulse retrieval problem can be recast as a phase retrieval problem by rewriting Eq. (2) as

$$I_{FROG}(\tau,\omega)=\left|\int_{-\infty}^{\infty} dt\, d\Omega\, E_{sig}(t,\Omega)\exp(i\omega t+i\tau\Omega)\right|^2, \tag{3}$$

where Ω is the conjugate variable associated with an inverse Fourier transform with respect to τ. Equation (3) is the canonical form of the two-dimensional phase retrieval problem. We can then apply variants of the iterative-Fourier-transform or error-reduction phase-retrieval algorithm [6] to find the signal field. Once we have found the signal field, it is easy to find the pulse field, as the nonlinear relationship embodied in Eq. (1) admits only one solution for $E(t)$ (except for trival constant phase and temporal offsets).

In phase-retrieval problems, one needs a constraint on the form of the signal field. Usually, this is some sort of support constraint. However, in FROG, we have a much more stringent constraint in Eq. (1). An admissable signal field must satisfy this relationship, i.e. it must be generatable from a one-dimensional physically-realizable function.

In the FROG reconstruction algorithm, we perform the usual magnitude replacement in the Fourier domain (where the experimental FROG trace is measured), as is common in restoration-from-magnitude problems. In the signal domain, we apply a variety of techniques to enforce the nonlinear-optical constraint of Eq. (1). In early versions of the algorithm [7], we noted that simply integrating boths sides of Eq. (1) with respect to τ would yield a function proportional to the electric field. We therefore performed such an integration of the signal field after magnitude replacement in order to generate a new version of the field $E(t)$, which was then used to generate a new signal field for the next iteration of the algorithm. This version of the algorithm had the advantage of being fast, but stagnated on pulses with significant intensity substructure.

A more powerful version of the algorithm incorporates the method of generalized projections onto sets [8]. In this case, the signal-domain constraint is implemented by starting with the current signal field (after magnitude replacement, which has also been shown to be a projection), and moving to the closest signal field that satisfies the constraint of Eq. (1). This is done by minimizing a distance metric (Euclidian norm) between the current signal field and an arbitrary point inside the constraint set (this arbitratry point is found by allowing $E(t)$ to vary arbitrarily and then generating the signal field through Eq. (1)) [9]. This version of the algorithm is quite powerful, and has been able to retrieve all test pulses that we have input so far. The generalized-projections-based algorithm also performs better on noisy data (inevitable in experiments) than the original algorithm.

A variant of FROG occurs when we allow the gate function to be another unknown function, i.e. we use an unknown gate pulse to measure an unknown signal pulse. In this case, the signal field of Eq. (1) becomes

$$E_{sig}(t,\tau)=E_s(t)\left|E_g(t-\tau)\right|^2. \tag{4}$$

The inversion problem now bears a resemblence to blind deconvolution as well as phase retrieval. Under certain cirumstances we can recover both the signal field and the magnitude squared of the gate field from a single experimental trace. We call this the Twin Recovery of E-field Envelopes using FROG, or TREEFROG. The TREEFROG

algorithm, which is based on blind-deconvolution algorithms [10], updates the signal and gate fields on alternate iterations.

We can also allow for more complicated form of the signal field. This is necessary because the form of Eq. (1) implies an instantaneous response of the medium used to generate the signal field, while most nonlinear optical materials have a more complicated response. We can include these effects by writing a generalized signal field

$$E_{sig}(t,\tau) = f[E(t),\tau] \tag{5}$$

where f is an arbitrary function of the electric field and the time delay between the two replicas fo the field. Using the method of generalized projections, we can construct an algorithm to retrieve the field $E(t)$ given knowledge of the function f. In the future, we also hope to be able to perform the converse problem: to retrieve the form of f given knowledge of the field. In this way, we could use FROG-related techniques to characterized nonlinear optical material responses, a problem of considerable interest in the laser science field.

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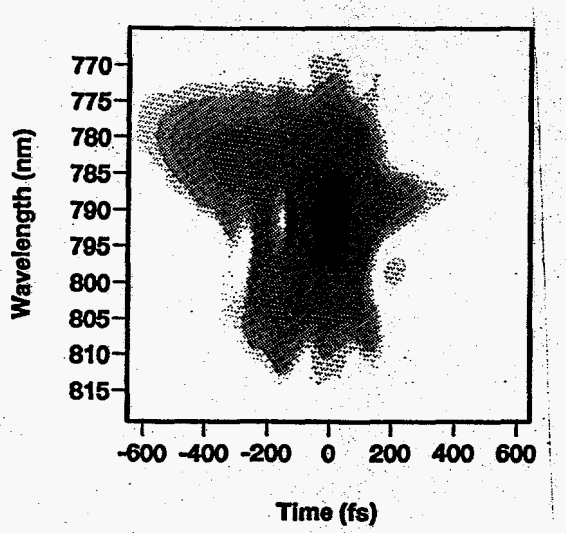

Fig. 1. A FROG trace of an ultrashort laser pulse. The FROG trace is a type of self-referenced spectrogram, where the signal forms its own window.

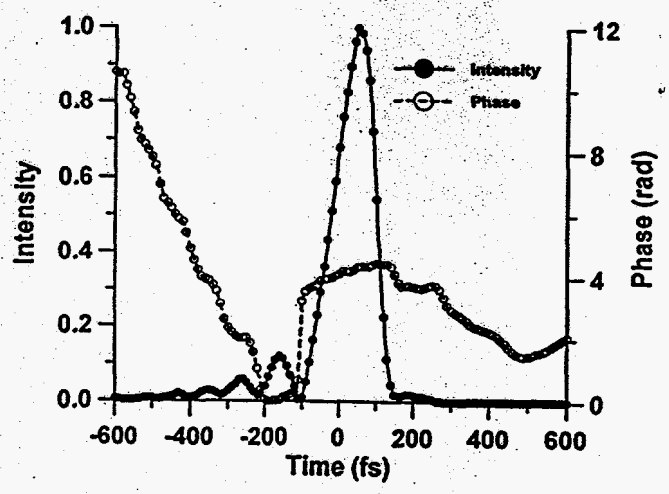

Fig. 2. The intensity and phase of the laser pulse that created the FROG trace of Fig. 1. A phase-retrieval-based algorithm was used to invert the FROG trace. Data is courtesy of Dr. Kent Wilson and Dr. Bern Kohler of UCSD.