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DAMAGE DETECTION AND MODEL REFINEMENT USING ELEMENTAL STIFFNESS PERTURBATIONS WITH CONSTRAINED CONNECTIVITY

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ABSTRACT

A new optimal update method for the correlation of dynamic structural finite element models with modal data is presented. The method computes a minimum-rank solution for the perturbations of the elemental stiffness parameters while constraining the connectivity of the global stiffness matrix. The resulting model contains a more accurate representation of the dynamics of the test structure. The changes between the original model and the updated model can be interpreted as modeling errors or as changes in the structure resulting from damage. The motivation for the method is presented in the context of existing optimal matrix update procedures. The method is demonstrated numerically on a spring-mass system and is also applied to experimental data from the NASA Langley 8-bay truss damage detection experiment. The results demonstrate that the proposed procedure may be useful for updating elemental stiffness parameters in the context of damage detection and model refinement.

NOMENCLATURE

$[M]$, $[D]$, $[K]$	Structural mass, damping, and stiffness matrices of "correct" model
$[M_u]$, $[D_u]$, $[K_u]$	Structural mass, damping, and stiffness matrices of nominal model
$[\Delta M]$, $[\Delta D]$, $[\Delta K]$	Structural mass, damping, and stiffness matrix perturbations
ω_j, $\{\phi_j\}$, $\{E_j\}$	Circular modal frequency, mode shape, and force error for j^{th} mode
$\{p\}$, $[P]$	Vector and diagonal matrix of element-level stiffness parameters
$[A]$	Stiffness connectivity matrix
E, I	Young's modulus, cross-sectional moment of inertia
n_d	Number of degrees of freedom
n_p	Number of elemental stiffness parameters

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INTRODUCTION

Modification of a structural finite element model (FEM) such that the FEM eigensolution matches the results of a modal vibration experiment is a subject that has received much attention in the literature in recent years. Methods for this type of FEM updating are applicable to problems such as model refinement, for better prediction of structural static and dynamic response, and structural damage detection, for location of cracks and failures in structures such as aircraft skin, bridge supports and offshore oil platforms.

One class of methods for correlating measured modal data with analytical finite element models is the minimization or elimination of "modal force error," which is the error resulting from the substitution of the analytical FEM and the measured modal data into the structural eigenproblem. Various methods have been developed to minimize or eliminate some measure of the error in the eigenproblem by perturbing the baseline values in the analytical model, such as the components of the stiffness, damping and mass matrices. One type of method, known as "sensitivity-based model update," uses the sensitivities of the modal response parameters of the FEM (such as modal frequencies and mode shapes) to the structural design parameters (such as Young's modulus, density, etc.) to iteratively minimize the modal force error (see, for example, Hemez and Farhat, [1], [2], [3]). Another type of method, known as "eigenstructure assignment," designs a controller which minimizes the modal force error. The controller gains are then interpreted in terms of structural parameter modifications (see, for example, Lim and Kashangaki [4]). Still another type of method, known as "optimal matrix update," solves a closed-form equation for the matrix perturbations which minimize the modal force error or constrain the solution to satisfy it (see, for example, Baruch and Bar Itzhack [5], Kabe [6], Berman and Nagy [7], Smith and Beattie [8], [9], Kaouk and Zimmerman [10], [11]) It is this type of method which is of interest in this paper. Much of the research done in optimal matrix update has focused on estimating perturbations to the mass and stiffness properties directly. In the context of structural damage detection and health monitoring, the perturbations to the stiffness properties are usually the most relevant. In this paper, only the perturbation of the structural stiffness properties will be considered.

Computing the stiffness property perturbations which eliminate the modal force error is often an underdetermined problem, since the number of unknowns in the perturbation set can be much larger than the number of measured modes and the number of measurement degrees of freedom. In this case, the property perturbations which satisfy the modal force error equation are non-unique. Optimal matrix update methods thus apply a minimization to the property perturbation to select a solution to the modal force error equation subject to constraints such as symmetry, positive definiteness, and sparsity. Typically, this minimization applies to either a norm or the rank of the perturbation property matrix or vector.

The main distinction between optimal update methods which minimize some measure of the stiffness property perturbations can be drawn based on two characteristics: First, the stiffness property which is varied, and second, the objective function that is used to select the solution. The stiffness properties can be categorized as the global stiffness matrix, the elemental stiffness matrices, or the elemental stiffness parameters (e.g. E, I, etc.). The objective functions are either the minimum of a norm of the property perturbation or the minimum of the rank of the property perturbation. Table (1) shows how several of the most widely known optimal matrix update procedures can be categorized according to these characteristics. The columns in this table categorize methods (and cite examples from the literature) according to which model parameter is used in the update procedure. The rows categorize the methods by whether a minimum norm (e.g. least-squares) or a minimum rank function is used as the objective of the optimization.

As shown in Table (1), the majority of the early work in optimal matrix update used the minimum norm perturbation of the global stiffness matrix [5], [6], [8], [7]. The motivation for using this objective function is that the desired perturbation is the one which is "smallest" in overall magnitude. Later work by Kaouk and Zimmerman [10], as shown in the second row of Table (1), used the minimum rank perturbation of the global stiffness matrix. This was motivated by the application of damage detection, where the perturbations could be assumed to be limited to a few isolated locations. The minimum rank stiffness matrix perturbation can be thought of as the stiffness matrix perturbation with the smallest number of nonzero values. An extension of this work computes the perturbations at the element stiffness matrix level, to limit the computed perturbations to certain structural DOF. [11]

A common drawback of the methods listed in the first two columns of Table (1) is that the computed perturbations are made to stiffness matrix values at the structural DOF, rather than at the element stiffness parameter level. There are three main advantages to computing perturbations to the elemental stiffness parameters rather than to stiffness matrix entries: 1) The resulting updates have direct physical relevance, and thus can be more easily interpreted in terms of structural damage or errors in the FEM; 2) The connectivity of the FEM is preserved, so that the resulting updated FEM has the same load path set as the original; and 3) A single parameter which affects a large number of structural elements can be varied independently. This advantage is especially relevant, for example, in civil engineering applications, where a parameter such as the Young's modulus of concrete may be uniform throughout a number of elements but not precisely known. Previous techniques to compute perturbations at the element parameter level have been proposed by Chen and Garba [12] and Li and Smith [13]. These techniques use the sensitivity of the entries in the stiffness matrix to the elemental stiffness parameters so that the minimum norm criterion can be applied directly to the vector of elemental stiffness parameters. Thus the resulting update consists of a vector of elemental stiffness parameters that is a minimum norm solution to the optimal update equation. The stiffness parameters that are updated can be limited in the construction of the sensitivity matrix.

The method proposed in this paper computes perturbations to the element-level stiffness parameters by solving the optimal update equation for the stiffness matrix subject to a minimum *rank* objective function. For convenience, the new technique is termed the minimum rank elemental update (MREU). This method is designed to exploit the advantages of both the minimum rank solution technique and the computation of element-level stiffness perturbations. The remainder of the paper is organized as follows: First, the theory of optimal matrix update is summarized, including brief outlines of the existing methods. Second, the theoretical development of the MREU is presented, followed by a method for computing the required "stiffness connectivity matrix." Next, the method is demonstrated using a numerical example, followed by application to experimental data from a NASA truss damage-detection experiment. The results of the MREU are compared to the results of a minimum-norm elemental stiffness update using this experimental data set.

THEORY OF OPTIMAL MATRIX UPDATE

The basic theory of optimal matrix update techniques begins with the second-order structural equation of motion

$$[M]\{\ddot{x}\} + [D]\{\dot{x}\} + [K]\{x\} = \{F\} \qquad (1)$$

The eigensolution of this equation with no externally applied forces represents the free vibration of the structure. For the j-th structural vibration mode, this is expressed as

$$(-\omega_j^2[M] + i\omega_j[D] + [K])\{\phi_j\} = \{0\} \qquad (2)$$

Now presuming that the structural model matrices contain some error, perhaps because of modeling errors or changes in the structure such as damage, the model matrices can be related to the true matrices as

$$\begin{aligned} [M] &= [M_u] - [\Delta M] \\ [D] &= [D_u] - [\Delta D] \\ [K] &= [K_u] - [\Delta K] \end{aligned} \qquad (3)$$

where the matrices with subscript u are the nominal model matrices, and the Δ matrices are the matrix perturbations. Substituting Eq. (3) into Eq. (2) and moving the perturbation terms to the right side yields

$$\begin{aligned} &(-\omega_j^2[M_u] + i\omega_j[D_u] + [K_u])\{\phi_j\} \\ &\quad = (-\omega_j^2[\Delta M] + i\omega_j[\Delta D] + [\Delta K])\{\phi_j\} \end{aligned} \qquad (4)$$

Since all of the terms on the left side of Eq. (4) are known, the modal force error $\{E_j\}$ can be defined for each measured mode as

$$\{E_j\} = (-\omega_j^2[M_u] + i\omega_j[D_u] + [K_u])\{\phi_j\} \qquad (5)$$

Then the matrix perturbations can be computed by solving

$$(-\omega_j^2[\Delta M] + i\omega_j[\Delta D] + [\Delta K])\{\phi_j\} = \{E_j\} \qquad (6)$$

for $[\Delta M]$, $[\Delta D]$ and $[\Delta K]$. Under the assumptions $[\Delta M] = 0$ and $[\Delta D] = 0$, Eq. (6) simplifies to

$$[\Delta K]\{\phi_j\} = \{E_j\} \qquad (7)$$

As described in the introduction, examples of the methods used to solve Eq. (7) are categorized in Table (1). A brief summary of the mathematical formulations of these methods follows.

The minimum-norm perturbation of the global stiffness matrix was the approach used by Baruch and Bar-Itzhack [5], Kabe [6], Berman and Nagy [7], and Smith and Beattie [8]. As described by Smith and Beattie [8], this approach can be summarized as

$$\min\|\Delta K\| \qquad (8)$$

subject to the constraints of Eq. (7) and $[\Delta K]$ symmetric and sparse. Constraining the sparsity to be the same as the nominal FEM stiffness matrix has the effect of ensuring that no new load paths are generated by the updated model.

The minimum-rank perturbation approach of Kaouk and Zimmerman [10] can be summarized as

$$\min(\text{rank}([\Delta K])) \tag{9}$$

subject to the constraints of Eq. (7) and $[\Delta K]$ symmetric and positive definite. An extension of this method partitions the perturbation matrix so that only the elemental stiffness matrices associated with certain structural DOF are updated. [11]

The minimum-norm, element-level update procedures presented by Chen and Garba [12] and Li and Smith [13] incorporate the connectivity constraint between the element-level stiffness parameters and the entries in the global stiffness matrix directly into Eq. (7) to get

$$\frac{\partial}{\partial p}([K]\{\phi_j\})\{\Delta p\} = \{E_j\} \tag{10}$$

which is then solved for minimum-norm $\{\Delta p\}$ (in the underdetermined case) or the least-squares error (in the overdetermined case).

The MREU technique uses an approach similar to that presented by Chen and Garba [12] and Li and Smith [13], by including the connectivity constraint directly into the modal force error equation, but uses a minimum-rank solution, as presented by Kaouk and Zimmerman [10] to solve for the elemental parameters. The derivation of the MREU is presented in the following section.

DERIVATION OF MINIMUM RANK ELEMENTAL PARAMETER UPDATE

The derivation of the MREU technique begins with the parameterization of the $(n_d \times n_d)$ global stiffness matrix $[K]$ as

$$[K] = [A][P][A]^T \tag{11}$$

where the $(n_d \times n_p)$ matrix $[A]$ is defined as the "stiffness connectivity matrix," and the $(n_p \times n_p)$ diagonal matrix $[P]$ has the elemental stiffness parameters of the $(n_p \times 1)$ vector $\{p\}$ as its diagonal entries. Mathematically, this is defined as

$$\text{diag}([P]) = \{p\} \tag{12}$$

Assuming that the global stiffness matrix is a linear function of the elemental stiffness parameters, and assuming $[A]$ is independent of $[P]$, Eq. (11) can be perturbed to get

$$[K + \Delta K] = [A][P + \Delta P][A]^T \tag{13}$$

Expanding Eq. (13) and subtracting Eq. (11) from it yields the parameterization of the perturbed global stiffness matrix $[\Delta K]$,

$$[\Delta K] = [A][\Delta P][A]^T \tag{14}$$

The connectivity constraint can be enforced by substituting Eq. (14) into Eq. (7) to get

$$[A][\Delta P][A]^T\{\phi_j\} = \{E_j\} \tag{15}$$

To put Eq. (15) in the proper form for minimum rank solution, first perform a minimum norm solution for $[\Delta P][A]^T\{\phi_j\}$ to get

$$[\Delta P][A]^T\{\phi_j\} = [A]^T([A][A]^T)^{-1}\{E_j\} \tag{16}$$

in the underdetermined case and

$$[\Delta P][A]^T\{\phi_j\} = ([A]^T[A])^{-1}[A]^T\{E_j\} \tag{17}$$

in the overdetermined case. The solution of Eq. (16) or Eq. (17) for symmetric minimal rank $[\Delta P]$ is given as in Ref. [10] as

$$[\Delta P] = [Y][H][Y]^T \tag{18}$$

where

$$[H] = ([Y]^T[A]^T\{\phi_j\})^{-1} \tag{19}$$

and

$$[Y] = [A]^T([A][A]^T)^{-1}\{E_j\} \tag{20}$$

in the underdetermined case and

$$[Y] = ([A]^T[A])^{-1}[A]^T\{E_j\} \tag{21}$$

in the overdetermined case. It should be noted that since $[A]$ is typically very sparse, it is more efficient to use the algorithms from a sparse linear algebra library (such as those contained in MATLAB [14]) to compute $[Y]$.

In the case where the entire FEM global DOF set is used in the analysis, $[A]$ is independent of the elemental stiffness parameters $[P]$. However, when a reduced DOF set is used, such as that obtained using Guyan condensation [15], the matrix $[A]$ becomes a function of the stiffness parameters $[P]$, so $[\Delta P]$ must be computed iteratively. The consequences of this case are not addressed in this paper.

COMPUTATION OF STIFFNESS CONNECTIVITY MATRIX

The stiffness connectivity matrix, as defined in Eq. (11), provides a transformation from the elemental stiffness parameters to the global system DOF. It can be computed from a sensitivity analysis of the global stiffness matrix using the following procedure. Recognizing that $[\Delta P]$ is diagonal, Eq. (14) can be rewritten in tensor form (as shown in Ref. [16]), so that the (i, j) entry in the global stiffness matrix can be parameterized as

$$\Delta K_{ij} = \lfloor A_{i1}A_{j1} \quad A_{i2}A_{j2} \quad \dots \rfloor \begin{Bmatrix} \Delta p_1 \\ \Delta p_2 \\ \dots \end{Bmatrix} \qquad (22)$$

This relationship can also be expressed using the sensitivity of the (i, j) entry in the global stiffness matrix to the β^{th} elemental stiffness parameter, i.e.

$$\Delta K_{ij} = \frac{\partial K_{ij}}{\partial p_\beta}(\Delta p_\beta) \qquad (23)$$

Comparing Eq. (23) to Eq. (22) yields an equivalence between the entries of the sensitivity matrix and the stiffness connectivity matrix, which can be written

$$A_{i\beta}A_{j\beta} = \frac{\partial K_{ij}}{\partial p_\beta} \qquad (24)$$

If the finite elements used in the model are linear functions of the elemental stiffness parameters, this sensitivity can be computed using a finite difference approach as

$$\frac{\partial K_{ij}}{\partial p_\beta} = \frac{\Delta K_{ij}}{\Delta p_\beta} \qquad (25)$$

The stiffness connectivity matrix $[A]$ can then be obtained by computing the sensitivity matrix using Eq. (25) and then solving Eq. (24) for each entry in the connectivity matrix.

It should be noted that the stiffness connectivity matrix $[A]$ is equivalent to that defined by Peterson, et. al. (REF disassembly paper). In that paper, $[A]$ is computed algebraically using element-level expressions for the connectivity. The matrix obtained using the above described sensitivity computations should be equivalent to the matrix obtained using the algebraic method.

NUMERICAL EXAMPLE

To demonstrate the numerical implementation of the MREU procedure, it is applied to the spring-mass system shown in Figure (1). The following example demonstrates that an unknown set of stiffness parameter perturbations can be computed using only the mass and stiffness matrix of the nominal model and the measured modal frequencies and mode shapes of the perturbed system. Consider the nominal model of the system to have the parameters

$$\begin{aligned} \{k_1, k_2, k_3, k_4\} &= \{1, 1, 1, 1\} \\ \{m_1, m_2, m_3\} &= \{1, 1, 1\} \end{aligned} \qquad (26)$$

which has the mass and stiffness matrices

$$[K_u] = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \qquad [M_u] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad (27)$$

A sensitivity analysis of this stiffness matrix $[K_u]$ to the four stiffness parameters $\{k_1, k_2, k_3, k_4\}$ yields a connectivity matrix $[A]$ of

$$[A] = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix} \quad (28)$$

Now consider a perturbed set of stiffness parameters and perturbed stiffness matrix

$$\{k_1, k_2, k_3, k_4\} = \{1, 0.8, 1, 1\}$$

$$[K] = \begin{bmatrix} 1.8 & -0.8 & 0 \\ -0.8 & 1.8 & -1 \\ 0 & -1 & 2 \end{bmatrix} \quad (29)$$

Computing the modal parameters of the perturbed system using the perturbed stiffness matrix from Eq. (29) and the nominal mass matrix from Eq. (27) yields

$$\{\omega_1, \omega_2, \omega_3\} = \{0.7586, 1.3703, 1.7740\}$$

$$\{\phi_1, \phi_2, \phi_3\} = \begin{bmatrix} 0.4715 & -0.7815 & 0.4086 \\ 0.7218 & 0.0758 & -0.6880 \\ 0.5067 & 0.6193 & 0.5998 \end{bmatrix} \quad (30)$$

Assume that only the first mode is measured, so that ω_1 and $\{\phi_1\}$ from Eq. (30) are known. The modal force error $\{E_1\}$ for this mode is computed using by substituting $[K_u]$, $[M_u]$, ω_1 and ϕ_1 into Eq. (5) to get

$$\{E_1\} = \begin{Bmatrix} -0.0501 \\ 0.0501 \\ 0 \end{Bmatrix} \quad (31)$$

The MREU equation can then be formed as in Eq. (16) to get

$$[\Delta P]\begin{Bmatrix} 0.4715 \\ -0.2503 \\ 0.2151 \\ 0.5067 \end{Bmatrix} = \begin{Bmatrix} 0 \\ -0.0501 \\ 0 \\ 0 \end{Bmatrix} \quad (32)$$

Solving Eq. (32) using Eq. (18), Eq. (19), and Eq. (20) and taking the diagonal entries of $[\Delta P]$ yields the parameter perturbations

$$\{\Delta k_1, \Delta k_2, \Delta k_3, \Delta k_4\} = \{0, 0.2, 0, 0\} \quad (33)$$

which are exactly the perturbations between the nominal stiffness parameters of Eq. (26) and the perturbed stiffness parameters of Eq. (29).

EXPERIMENTAL APPLICATION

To demonstrate the validity of the MREU procedure, the method is applied to data from the NASA Dynamic Scale Model Technology (DSMT) program of Langley Research Center. [17] The structure is an eight-bay truss mounted in a cantilevered configuration, as shown in Figure (2). A series of modal tests was performed on the structure with various structural members removed to simulate different instances of damage. The data sets from this test have been analyzed by many different researchers (see, for example, Ref. [10] and Ref. [13]). This data set is used to demonstrate the validity of the MREU procedure because it is known to be well-characterized.

The structure was modeled in ABAQUS [18], using 32 Nodes and 109 truss rod elements, for a total of 96 DOF. The element parameters selected for perturbation are the Young's moduli of the longerons and battens in bays 6, 7, and 8 (where bay 8 is closest to the cantilever) and the diagonals in bay 6, for a total of 26 perturbed parameters. The first damage case studied involved the removal of longeron 46 in bay 8 (denoted damage case "a" by Kashangaki) and the second damage case involved the removal of two members in bay 6 -- longeron 35 and diagonal 99 (denoted damage case "o" by Kashangaki). The relevant elements for these two damage scenarios are shown in Figure (2).

A minimum norm procedure was implemented to use as a basis for comparison to the MREU procedure. The minimum-norm algorithm computes the least-squares solution to Eq. (10) to obtain the parameter perturbation vector $\{\Delta p\}$, and thus is similar to the methods presented by Chen and Garba [12], and Li and Smith [13].

To validate the implementation of the MREU and the minimum-norm element stiffness parameter update procedures, the first FEM mode for damage case 46 were used to compute the elemental Young's moduli perturbations. The results of this update are shown in Figure (3) for the MREU procedure and in Figure (4) for the minimum-norm update procedure. These results are both nearly perfect, and so the algorithms and the stiffness connectivity matrices can be considered to be correct.

The updates resulting from applying the two algorithms to experimentally measured mode number 1 for damage case 46 are shown in Figure (5) and Figure (6). In this damage case, element #9 corresponds to longeron 46. Thus, a perfect result would have 100% change in E for element 9 and 0% change in E for all other elements. The result of the MREU, shown in Figure (5), has a clear indication of nearly 100% reduction in E for element 9, as well as changes of 30% and less for surrounding members. However, the minimum norm update fails to locate the damaged member. It is interesting to note that for damage case 46, going from FEM modes to measured modes introduces some error into the MREU solution (compare Figure (3) and Figure (5)), but causes the minimum-norm solution to go from nearly perfect to completely wrong (compare Figure (4) and Figure (6)).

The application of the MREU and minimum norm update techniques applied to damage case 35/99 are shown in Figure (7) and Figure (8), respectively. Measured modes 1 and 4 are used for these updates. The damaged members correspond to elements #1 and #21 in these two plots. As with damage case 46, the results from the MREU technique show two clear peaks near 100% at the correct member numbers, but also show some "smearing" at adjacent members. However, the minimum norm technique again shows completely wrong results. It is interesting to note that the smearing which occurs in the MREU result consists of perturbations primarily to longerons rather than battens or diagonals. This demonstrates the insensitivity of these particular modes to changes in the stiffnesses of most battens and diagonals.

A known characteristic of any minimum rank update, as reported by Zimmerman, et. al. [19] is that the number of modes used in the update is equal to the rank of the computed update parameters. Thus, using two modes will give an updated parameter matrix or vector with rank two. However, since the MREU takes the diagonal of the solution to the minimum-rank equation, it is not strictly constrained to follow this rule. However, the results seen from the application of the MREU technique to this experimental data set seems to support the assertion that the best results are obtained when the number of modes equals the expected rank of the perturbation matrix. For example, consider the damage 46 MREU result using only mode 1 shown in Figure (5) as compared to the damage 46 MREU result using modes 1 and 4, shown in Figure (9). The 2-mode update of Figure (9) still shows nearly 100% stiffness reduction in element #9, but the smearing effect is increased and more elements have a significant stiffness change. Likewise, comparison of the damage 35/99 MREU result using modes 1 and 4 in Figure (7) to the damage 35/99 MREU result using only mode 1 in Figure (10) shows that the stiffness changes in the damaged members (100% change in elements #1 and #21 on the plots) are detected more accurately using 2 modes, but with more smearing. Thus, the optimal number of modes to use in the MREU seems to be equal to the expected rank of the elemental stiffness perturbation vector.

As a final note on these experimental results, it is widely assumed that sensor DOF 45 in the DSMT 8-bay damage data sets contains erroneous measurements. No attempt to correct this erroneous sensor measurement was made in this analysis.

CONCLUSIONS

A new optimal matrix update method was introduced and demonstrated on both numerical and experimental data. The method computes a minimum-rank vector of perturbations to the element-level stiffness parameters while constraining the connectivity of the global stiffness matrix. The motivation for the method was shown in the context of existing optimal matrix update methods. The method was able to locate the damaged members using data from the NASA Langley Research Center 8-bay truss damage detection experiments for both single-member and multiple-member damage cases. The results indicate that the method may be useful for modification of dynamic finite element models in the context of damage detection and model refinement.

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REFERENCES

[1] Hemez, F.M. and Farhat, C., "Structural Damage Detection via a Finite Element Model Updating Methodology," *International Journal of Analytical and Experimental Modal Analysis*, Vol. 10, No. 3, July 1995, pp. 152–166.

[2] Farhat, C. and Hemez, F.M., "Updating Finite Element Dynamic Models Using an Element-By-Element Sensitivity Methodology," *AIAA Journal*, Vol. 31, No. 9, September 1993, pp. 1702–1711.

[3] Hemez, F.M., *Theoretical and Experimental Correlation between Finite Element Models and Modal Tests in the Context of Large Flexible Space Structures*, Ph. D. Dissertation, Research Report CU-CAS-93-18, Center for Aerospace Structures, University of Colorado, Boulder, CO, August 1993.

[4] Lim, T.W. and Kashangaki, T. A-L., "Structural Damage Detection of Space Truss Structures Using Best Achievable Eigenvectors," *AIAA Journal*, Vol. 32, No. 5, May 1994, pp. 1049–1057.

[5] Baruch, M. and Bar Itzhack, I.Y., "Optimum Weighted Orthogonalization of Measured Modes," *AIAA Journal*, Vol. 16, No. 4, April 1978, pp. 346–351.

[6] Kabe, A.M., "Stiffness Matrix Adjustment Using Mode Data," *AIAA Journal*, Vol. 23, No. 9, September 1985, pp. 1431–1436.

[7] Berman, A. and Nagy, E.J., "Improvements of a Large Analytical Model Using Test Data," *AIAA Journal*, Vol. 21, No. 8, August 1983, pp. 1168–1173.

[8] Smith, S.W. and Beattie, C.A., *Model Correlation and Damage Location for Large Space Truss Structures: Secant Method Development and Evaluation*, Final Report, NASA-CR-188102, Jan. 1991.

[9] Smith, S.W., "Application Strategies for Structure Identification with Optimal-Matrix Updates," *Proceedings of the 11th International Modal Analysis Conference*, Orlando, FL, February 1993.

[10] Kaouk, M. and Zimmerman, D.C., "Damage Assessment Using a Generalized Minimum Rank Perturbation Theory," *AIAA Journal*, Vol. 32, No. 4, April 1994, pp. 836–842.

[11] Kaouk, M. and Zimmerman, D.C., "Structural Health Assessment Using a Partition Model Update Technique," *Proc. 13th International Modal Analysis Conference*, Nashville, TN, February, 1995, pp. 1673–1679.

[12] Chen, J.-C. and Garba, J.A., "On-Orbit Damage Assessment for Large Space Structures," *AIAA Journal*, Vol. 26, No. 9, Sept. 1988, pp. 1119–1126.

[13] Li, C. and Smith, S.W., "Hybrid Approach for Damage Detection in Flexible Structures," *Journal of Guidance, Control, and Dynamics*, Vol. 18, No. 3, May-June 1995, pp. 419–425.

[14] *MATLAB Users Manual*, The Mathworks, Inc., 1994.

[15] Guyan, R.J., "Reduction of Stiffness and Mass Matrices," *AIAA Journal*, Vol. 3, No. 2, p. 380.

[16] Peterson, L.D., Doebling, S.W., and Alvin, K.F., "Experimental Determination of Local Structural Stiffness by Disassembly of

Measured Flexibility Matrices," *Proc. of 36th AIAA/ASME Structures, Structural Dynamics and Materials Conference*, April 1995, AIAA Paper No. 95-1090.

[17] Kashangaki, T.A.-L., *On-Orbit Damage Detection and Health Monitoring of Large Space Trusses - Status and Critical Issues*, NASA Technical Memorandum 104045, March 1991.

[18] *ABAQUS User's Manual, Version 5.4*, Hibbitt, Karlsson & Sorensen, Inc., 1994.

[19] Zimmerman, D.C., Kaouk, M., and Simmermacher, T., "On the Role of Engineering Insight and Judgement for Structural Damage Detection," *Proc. of 13th International Modal Analysis Conference*, Nashville, TN, February 1995, pp. 414–420.

Table 1. Characteristics of Optimal Matrix Update Methods

Criteria	Global Matrix	Elemental Matrix	Elemental Parameter
Min. Norm	Ref. [5] Ref. [6] Ref. [7] Ref. [8]	?	Ref. [12] Ref. [13]
Min. Rank	Ref. [10]	Ref. [11]	**This Paper**

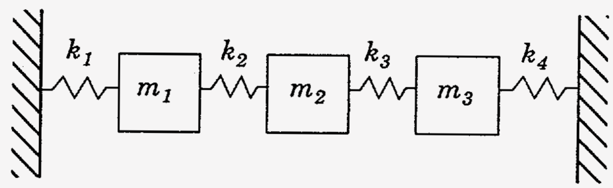

Figure 1. Spring-Mass System for Numerical Example

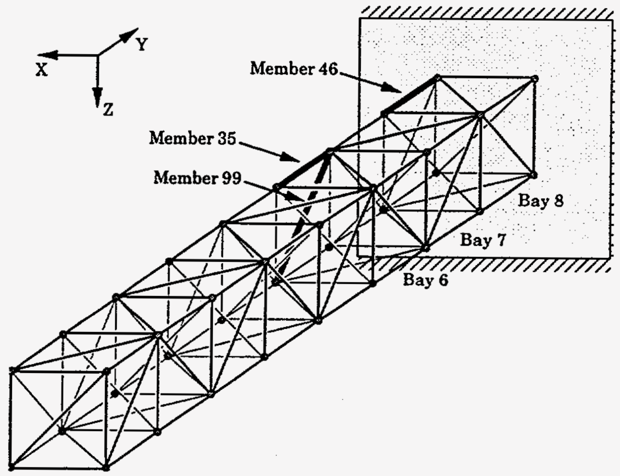

Figure 2. The NASA DSMT Eight Bay Truss

DISCLAIMER

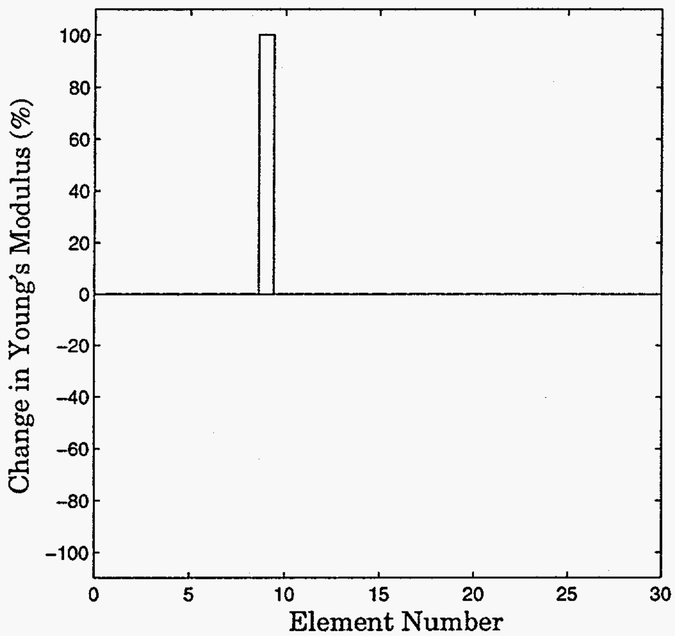

Figure 3. MREU Solution for Damage Case 46, FEM Mode 1

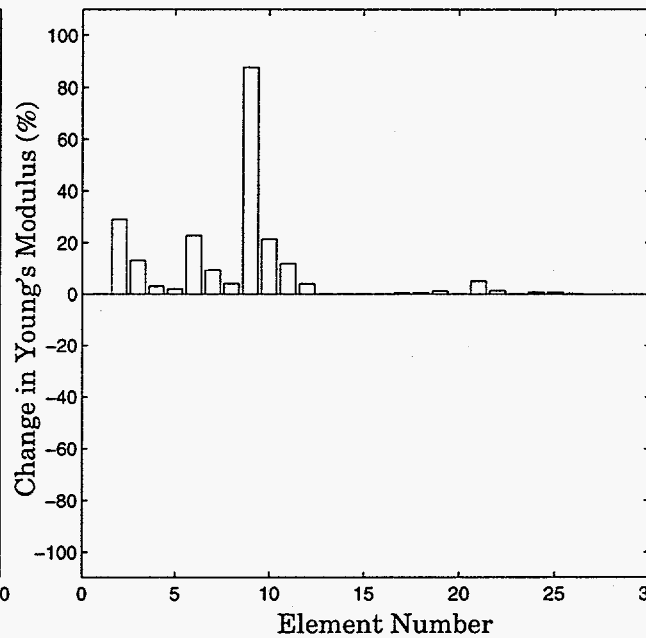

Figure 5. MREU solution for Damage Case 46, Measured Mode 1

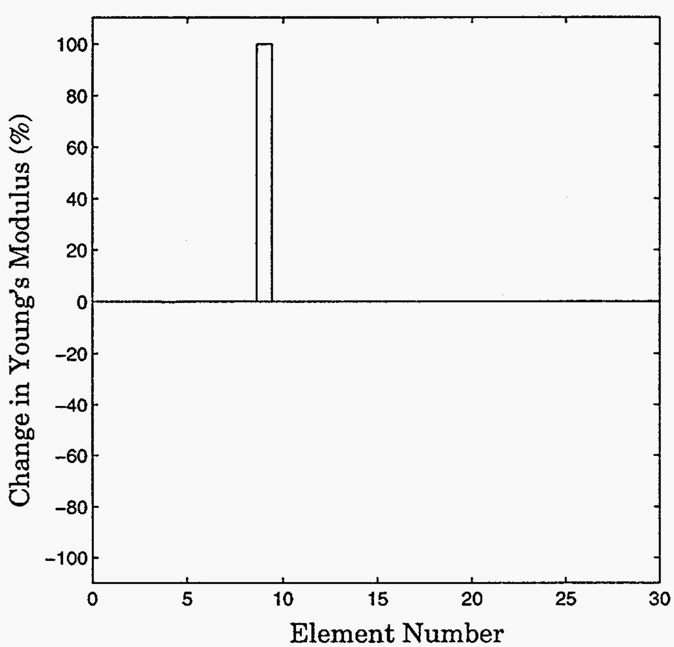

Figure 4. Minimum Norm solution for Damage Case 46, FEM Mode 1

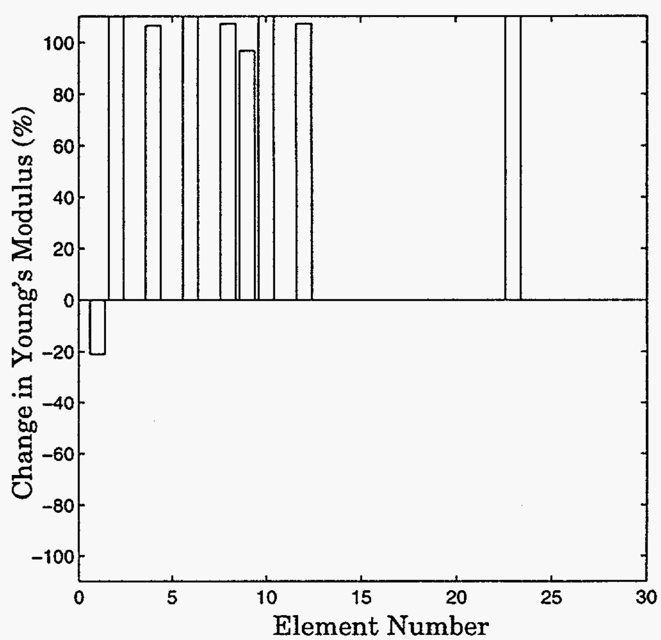

Figure 6. Minimum Norm solution for Damage Case 46, Measured Mode 1

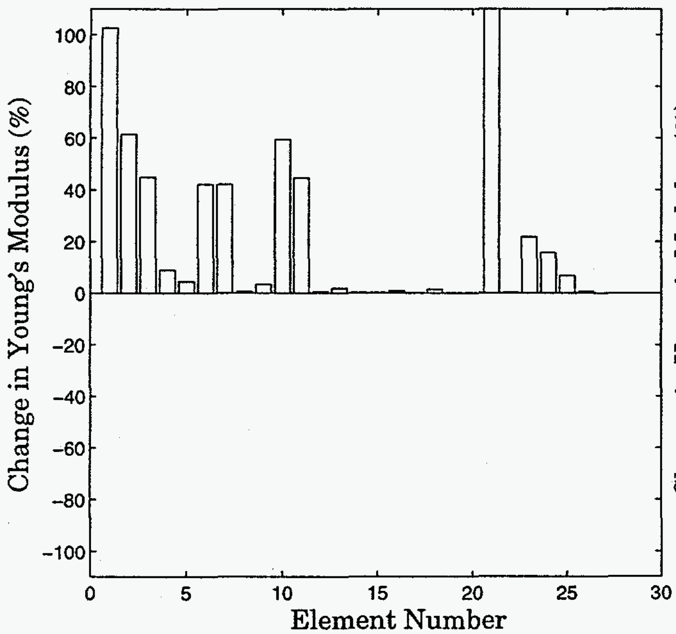

Figure 7. MREU solution for Damage Case 35/99, Measured Modes 1 and 4

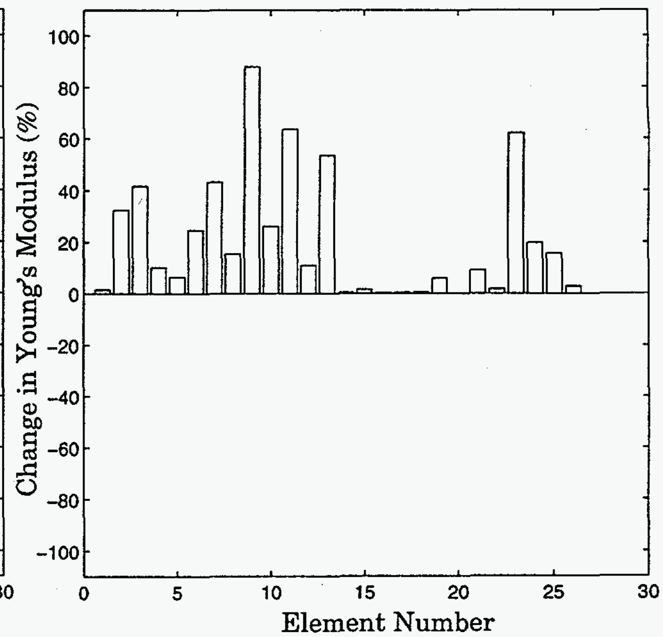

Figure 9. MREU solution for Damage Case 46, Measured Modes 1 and 4

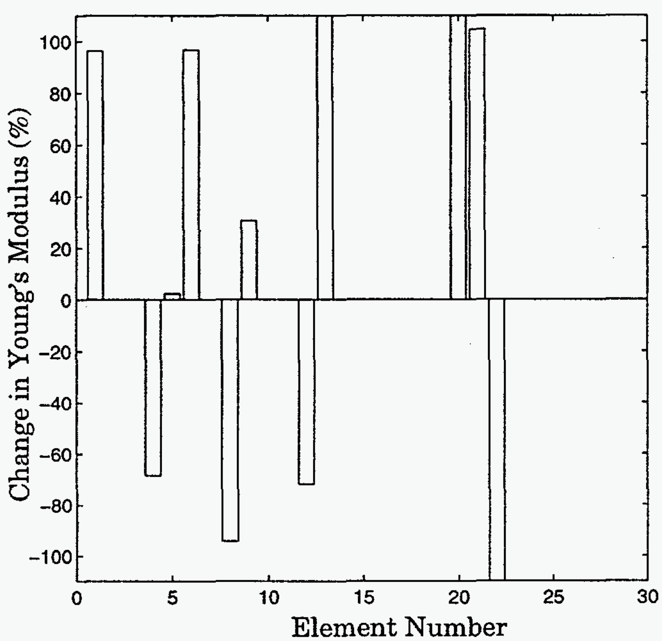

Figure 8. Minimum Norm solution for Damage Case 35/99, Measured Modes 1 and 4

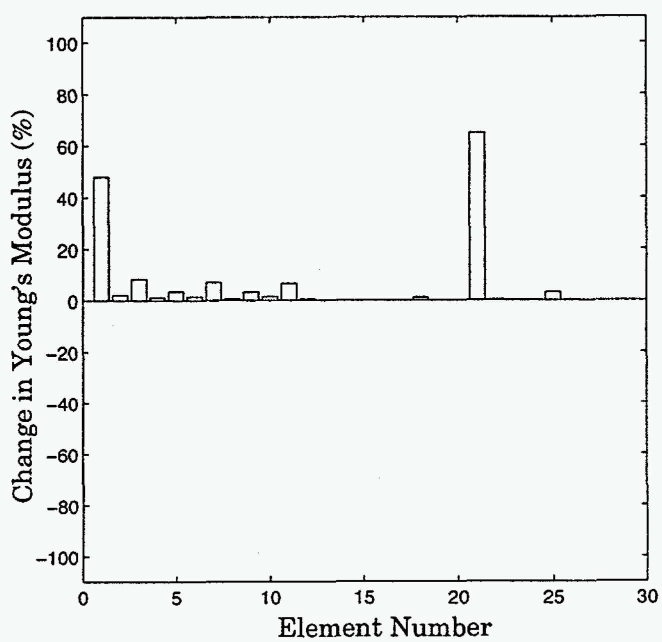

Figure 10. MREU solution for Damage Case 35/99, Measured Mode 1