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DOE Final Report (1994–1995)

Overview

Since 1993, the GANG laboratory has been co-directed by David Hoffman, Rob Kusner and Peter Norman. With Hoffman's departure to MSRI, Kusner and Norman have directed the lab. Since then, a great deal of mathematical research has been carried out here by them and by GANG faculty members Franz Pedit and Nate Whitaker. Also, new communication tools, such as the GANG Webserver (pioneered by GANG technical director Dave Oliver), have been developed. GANG has trained and supported nearly a dozen graduate students, and at least half as many undergrads in REU projects. The GANG Seminar continues to thrive, making Amherst a site for short and long term visitors to come to work with the GANG. This year GANG has been very fortunate to have hosted visiting faculty members Ken Brakke and John Sullivan in the spring semester, as well as postdoctoral associates Bo Guan and Ed Thayer all year. Long term visitors this past summer included Karsten Brauckmann and Robert Phelan, along with Brakke, Guan and Sullivan. And the future is even more promising, since our department has recently hired several new faculty members with whom GANG can expect substantial collaboration: George Kamberov (geometric nonlinear PDE); Markos Katsoulakis (geometric evolution equations and interacting particle methods in PDE); and Eyal Markman (algebraic geometry and integrable systems). We sincerely hope the DOE will consider our efforts worthy of future support.

The projects carried out at GANG are well represented by the GANG Preprint Series and Seminar Visitors (see Section 8), as well as the accompanying summary videotape (*Work at GANG: 1991-95* on 1/2" VHS). Some of the highlights of recent or ongoing research at GANG include:

- CMC surfaces (Brauckmann, Kusner, Pedit)
- Minimal surfaces (Kusner, Norman, Thayer)
- Fluid dynamics (Whitaker)
- Harmonic Maps (Pedit)
- Isometric immersions (Pedit)
- Knot energies (Kusner, Sadun, Sullivan)
- Foam structures (Brakke, Kusner, Phelan, Sullivan)
- High dimensional soap film singularities (Brakke, Sullivan)
- Elastic curves and surfaces (Kusner, Norman, Pedit, Sullivan)
- Self-similar curvature evolution (Ilmanen, Kusner, Sullivan)
- Integrable systems and theta functions (Norman, Pedit)
- Fully nonlinear geometric PDE (Guan)
- Geometric chemistry and biology (Kusner, Whitaker)

GANG is hosting the Five Colleges' 1995-96 Special Geometry Year, and in particular, senior visitor Ulrich Pinkall, Professor of Mathematics and Director of the SFB 288 at the Technische Universität, Berlin, Germany. Pinkall is working with Kusner, Norman, Oliver, Pedit and other faculty and students on geometric and computing issues surrounding minimal and CMC surfaces, completely integral systems, harmonic maps, and loop groups.

1 Geometric Variational Problems

Both theoretical and computational aspects of variational problems in low dimensional geometry and topology have been investigated by Kusner and his collaborators at GANG.

Kusner continues his work on the moduli spaces of CMC surfaces. His joint paper with Rafe Mazzeo and Dan Pollack, accepted for publication in *GAFA (Geometric and Functional Analysis)*, establishes a fundamental result about the space $\mathcal{M}_k$ of complete embedded CMC surfaces with k ends: $\mathcal{M}_k$ is a finite dimensional analytic variety, with generic dimension $3k - 6$. The new ideas Kusner introduced here also permitted a significantly simpler proof of the analogous constant scalar curvature result of Mazzeo, Pollack and Karen Uhlenbeck, about to appear in the *Journal of the AMS*. More-recently Kusner and Pollack have begun to understand the lagrangian structure of $\mathcal{M}_k$ in an invariant way in terms of moment mappings. Kusner reported on this work at the Burlington AMS meeting in August 1995.

Kusner and Schmitt completed and submitted for publication their paper on "The Spinor Representation of Minimal Surfaces". This is a new analytic setup in terms of which certain minimal surfaces, such as those with planar ends, have a particularly convenient representation. In this way we obtain a complete classification of genus zero minimal surfaces with planar ends (and of the corresponding Willmore surfaces), along with families of higher genus examples. The program MESH has been used to depict some of these.

The paper "Möbius Energies for Knots and Links, Surfaces and Submanifolds" by Kusner and Sullivan was accepted for the *Proceedings of the Georgia International Topology Conference*, held every eight years, at which Kusner was a principal lecturer. This past spring, Kusner and Sullivan were also working on their paper about "Electrons on the Sphere and the Möbius Energy of Hopf Links", which concerns the Morse theory of the Coulomb energy and the (equivariant) homotopy type of the configuration space of n distinct points on $\mathbb{CP}^1$. Note that this space also arises in braid theory and algebraic geometry: when $n = 2g + 2$, it gives a presentation of the moduli space of hyperelliptic genus g Riemann surfaces.

This past summer, Brauckmann worked with Kusner and Sullivan using the Brakke "Evolver" at GANG to compute (highly unstable) CMC surfaces, such as the gyroid companions, using discretizations of the squared mean curvature integral (shifted, so that zero energy implies CMC). This is part of a continuing collaboration with material scientists at MIT who are interested in such surfaces as the interfaces in diblock copolymers. Tom Ilmanen also worked with Brakke, Kusner and Sullivan using the Evolver to compute high genus surfaces which evolve self-similarly under the mean curvature flow. And Brauckmann has also been working with GANG graduate student Jorgen Berglund who is completing his dissertation this year (under Kusner's direction) on explicit constructions of embedded CMC surfaces via their conjugate cousin minimal surfaces in S^3 .

Kusner and Sullivan have been investigating minimal foams, especially those related to what chemists call "tetrahedrally close packed (TCP)" structures. We are interested in both the efficiency (isoperimetric ratios, e.g.) as well as mechanical properties (stress-strain data), and proving analytic bounds on these. Much of this originated from Kusner's participation in a physics workshop at Les Houches, France in January 1995. Collaborating with us were Brakke, Phelan and Andy Kraynik from Sandia National Labs in New Mexico. Sullivan made a presentation on TCP structures at the Burlington AMS meeting in August 1995. Kusner and Sullivan are completing their paper (with Fred Almgren of Princeton) in which they give an analytic comparison between the Kelvin and new Phelan-Weaire foams, showing the latter is more efficient.

Related to the preceding topic (and to changed knot energies) is Kusner's work with chemists Paul

Lahti and C. Peter Lillya of UMass. Their paper "New Surface Allotropes of Carbon", published in *Chemical Physics Letters*, describes the simplest "negatively curved" carbon surface molecule, C_{32} -trousers (in contrast to the "positively curved" C_{60} -fullerene), and computes basic structural data for this and related doubly and triply periodic molecules. Several chemists have shown interest in synthesizing C_{32} -trousers, which should have useful electronic and mechanical properties.

2 Soliton Geometry

Since Pedit was on research leave this past academic year, most of his work was carried out at the SFB 288 at TU-Berlin. Pedit's work over the past year focused on the study of integrable systems and their appearances in differential geometry.

There are two themes Pedit followed: first, the study of isometric immersions of space forms and, corresponding to them via their Gauss maps, special maps (curved flats) into symmetric spaces. Pedit finished two papers (joint with Dirk Ferus) both of which are available as SFB 288 preprints: "Isometric immersions and Soliton theory" which will be published in the *Mathematische Annalen*, and "Curved flats into symmetric spaces" which is submitted for publication. Further work in progress in this area is the study of curves in symmetric spaces, in particular the 2-sphere. The curved flat theory (every curve in a symmetric space is a curved flat) gives a hierarchy (indexed by the genus of the corresponding spectral curve, called the type) of integrable systems producing curves on the 2-sphere. For elliptic spectral curves one recovers elastic curves. Computer experiments indicate that every closed curve is a smooth limit of finite type curves. This is best understood from an integrable discrete analogue and its semi-classical limit. Experiments have been done at the SFB 288, where Pedit co-advised a Ph.D student (Uwe Schwarz) with Ulrich Pinkall, and will be carried out at GANG by undergraduates under Peter Norman. A long term project will be to link those investigations to the conjecture that there is a CMC torus along any closed curve inside an arbitrarily small epsilon tube around the curve. Another application of this study of curves is their appearance as boundary curves for the Plateau problem for minimal (and CMC) surfaces in the 3-sphere. This approach has been taken up by Jorg Richter, a GANG graduate student for the coming academic year.

The second theme, related to the first, is the study of harmonic maps of Riemann surfaces into certain homogeneous spaces with principal applications to classical surface theory: constant mean curvature surfaces, Willmore surfaces and surfaces of constant Gauss curvature. Pedit published (jointly with Fran Burstall) the paper "Dressing orbits of harmonic maps" in the *Duke Mathematical Journal*. Pedit submitted one paper "On the meromorphic potential of harmonic maps into k -symmetric spaces" (jointly with Dorfmeister, McIntosh and Wu) which is also available as an SFB 288 preprint. Experimental work in this area is still difficult due to the complicated parameter space and delicate numerical calculations in loop groups. Work in progress includes a discussion of harmonic 2-spheres in Lie groups (first considered by Karen Uhlenbeck) via the Weierstrass representation obtained by Dorfmeister-Pedit-Wu. This part of the work also includes Martin Guest and Fran Burstall, both of whom have been GANG visitors and will visit GANG again in the Fall of 1995 to continue on the above research.

3 Numerical Fluid Dynamics and Mathematical Modelling

Whitaker's recent work includes developing accurate and efficient numerical methods for evolving interfaces between two fluids in a Hele-Shaw cell. Some of the most accurate methods for this problem were

published by Whitaker in the *Journal of Computational Physics*. He is continuing this work by improving the accuracy of a method presented there. The new results will include a convergence proof along with the implementation of a multi-grid strategy for improved accuracy and efficiency.

In collaboration with Frank Baginski, Whitaker has developed algorithms for numerically computing surfaces of constant Gauss curvature. This work has been submitted for publication to the journal, *Numerical Methods for Partial Differential Equations*.

In collaboration with C. Woodcock and R. Horowitz of the Zoology Department at UMass, Whitaker has presented a model of DNA double helix. This three-dimensional model possesses many of the quantitative and qualitative properties of DNA. These results recently appeared in the *Proceedings of the National Academy of Sciences*.

Whitaker is currently working with Bruce Turkington, a UMass colleague, on computing statistical equilibrium solutions to the equations describing the flow of an ideal fluid. Whitaker has developed an algorithm for finding a most probable flow by maximizing a nonlinear functional subject to the nonlinear constraints of the flow. The method is developed with a global convergence theory. This is, in general, difficult because of the nonuniqueness of critical points for this maximization problem. The algorithm is also highly efficient and accurate, accomplished by solving a simple dual problem at each iteration. The algorithm is used to model such physical phenomena as vortex layer roll-up and the merging of vortex patches, all in a periodic domain. This work has been accepted for publication in the journal, *SIAM Journal of Scientific Computing*. This algorithm has also been used to compute most probable solutions in a circular domain. These results are compared to some time-dependent numerical calculations and were recently published in the journal *Physics of Fluids A*. The algorithm above computes the most probable vorticity distribution for a vortex patch, i.e., the vorticity takes on only the values 0 and $\lambda > 0$. An algorithm has been developed recently to compute the most probable flow for a continuous distribution of vorticity. It is also accompanied by a global convergence theory. These results will be presented at the *International Conference on Vortex Methods* in August, 1995 in Montreal, Canada. The work has also been accepted to appear in the proceedings of the conference. Whitaker is currently visiting Dr. Joel Sommeria (Laboratoire de Physique, Ecole Normale Supérieure de Lyon) in Lyon, France. Whitaker is developing similar algorithms and applying them to related problems. Sommeria is one of the original proposers of the maximum entropy theory.

4 Embedded Minimal Surfaces

Hoffman spent this academic year at MSRI in Berkeley, California, where he is the Head of their Scientific Graphics Initiative. He also was a visiting Professor of Mathematics at Stanford University, where he taught an advanced graduate course and seminar on properly embedded minimal surfaces. Hoffman gave colloquia at UC Santa Cruz and Stanford University, and in the Undergraduate Summer Program at UC Berkeley, organized by Leon Henkin. He has co-organized conferences at MSRI (The Future of Mathematical Communication), and Stanford (Conference in Honor of Robert Osserman), where he also spoke. Hoffman traveled in June to Paris and Bonn to work with Hermann Karcher, Fusheng Wei, Harold Rosenberg and Antonio Ros.

Hoffman worked with Ed Thayer, a GANG post-doc, on problems involving periodic embedded minimal surfaces. Wei and Karcher and Hoffman completed the proof of embeddedness of the periodic genus one helicoid. The paper is now in final revision form. With James T. Hoffman (UMass), Meinhard Wohlgemuth and Edwin Thomas (MIT), Hoffman worked to develop and apply the software application

“TEMSim” (Transmission Electron Microscope Simulation) for use in analyzing the interface structure of compound polymers by comparison with simulations of mathematically defined periodic minimal and CMC surfaces. With Thomas, Hoffman is writing a survey article, intended for *Scientific American*, on mathematical interfaces in the study of compound polymers.

Hoffman began research with John MacCuan, a recent Stanford Ph.D., in two areas: connected and embedded minimal surfaces bounded by convex curves in parallel planes; embedded minimal surfaces in the ball that meet the boundary of the ball orthogonally. During the Fall Semester of 1994, Hoffman consulted with David Oliver on many technical and computer issues related to his research, including the preparation of a hyperTeX version of his monograph (with H. Karcher) entitled “Complete Embedded Minimal Surfaces of Finite Total Curvature”.

Hoffman has resigned from the Mathematics Department of the University of Massachusetts. He will continue to consult with David Oliver on research-related issues, will use some of the laboratory facilities from time to time (mostly over the internet) and will be available for research consultation with GANG faculty, graduate students and visitors to the project.

5 GANG Graphics and Mathematical Software

The following is a summary of our software development efforts. Most of the software tools described here are now utilized in research on a daily basis at the GANG. A number of these tools (including MESH, DISP, SOLSURF, and the GANGVideo tools discussed below) are available over the Internet from GANG computers via anonymous FTP and the World Wide Web.¹ Most of the papers and other documents discussed in this report are available for anonymous FTP or World Wide Web access. Those not available electronically are available via US Mail from GANG.

Jim Hoffman, GANG’s Scientific Programmer, authored a major new piece of software, “TEMSim”, for use by materials scientists who analyze geometric surfaces as models for material interfaces. TEMSim takes as input geometric descriptions of several classes of surface, and displays a simulated transmission electron microscope (TEM) image in addition to the geometry. Viewing parameters are under the control of the user through a graphical user interface written in Tcl/Tk. For release 1.0, Jim enhanced TEMSim’s light simulation model, added user-defined contrast controls, improved the documentation, and wrote a number of scripts to input different formats of data into TEMSim. Jim also wrote the new “NetEdit” program, an interactive editor for files input to Ken Brakke’s Surface Evolver.

Jim Hoffman replaced the older SOLVE graphical user interface for the SOLSURF program with a new interface built with Tcl/Tk. SOLSURF was also enhanced to output “network” (a new flexible storage data structure) files for other surface evolution programs. Jim enhanced the rendering capabilities of VPS/render which GANG uses to create photographic quality imagery, and added enhancements to VPS to support the network data structure. GANG’s MESH program was enhanced as well to output network files, and Jim continued to fix bugs found by users.

The Center’s MESH adaptive mesh generator has been updated for the new version of the SGI Irix operating system by Jim Hoffman. David Oliver wrote a graphical user interface, called MGEN, to assist users in building and “make”ing new MESH surface generators. J. Hoffman has written a suite of new software tools and interfaces to existing ones, that assist polymer scientists in viewing surfaces that are

¹The Center’s anonymous FTP software server is <ftp.gang.umass.edu> (IP address 128.119.46.8). The Center’s World Wide Web server URL is <http://www.gang.umass.edu/>.

used as “models” for certain polymer structures. Like MESH, these tools run on the SGI Irix operating system. These tools are being developed in collaboration with Polymer Scientists at MIT.

Brakke made major improvements to his Evolver program while visiting GANG. In addition to including new energies to allow users at GANG investigate specific mathematical problems, he developed a major new feature, the “Hessian method”, which allows users to investigate unstable critical structures quantitatively. For example, Kusner and Sullivan have made extensive use of the Hessian method in their study of Coulomb electrons on the sphere. Brakke has documented this in the penultimate version (1.99) of the “Evolver Manual”, available as the most recent GANG preprint.

This spring and summer, Brakke, Kusner and Sullivan in collaboration with George Francis at Illinois, have been creating a new “Minimax Sphere Eversion” animation, using the Evolver and visualization software developed at GANG, the Geometry Center and NCSA.

During his visit to GANG, Sullivan redeveloped his “Voronoi Cell Solver” for constructing foam structures, instrumental in work he has been doing with Brakke, Kraynik, Kusner and Phelan. Sullivan has also developed a number of useful Evolver scripts for “aligning” Evolver datafiles, especially electrons on S^2 and foams.

For a six month period this year, David Oliver was on leave at the Technische Universität of Berlin, SFB 288. There, Oliver participated in the initial development of Oorange (Object-oriented analysis, numerics & geometry environment), a completely new environment for scientific (especially mathematical) visualization. This project is being directed by Ulrich Pinkall. The project team includes Charlie Gunn (formerly of The Geometry Center, co-developer of Geomview) and Konrad Polthier (formerly of Universität Bonn, co-developer of Grape), along with most of the key developers of SFB-288’s AVS-based geometry computing environment. Oorange is being designed from the ground up to support the sharing of software and data among cooperating research teams (enhanced by a native link to the World Wide Web). Oorange also provides powerful new object-oriented computing tools which promise to simplify the process of software development for mathematics. Oorange is based entirely on well-maintained public domain software tools. The initial public release of Oorange is expected in late September 1995 for SGI’s Irix operating system. Oorange is designed to be portable to the Linux operating system so that it can be deployed on inexpensive PC hardware. GANG will be the major Oorange software repository in the USA, and Oliver will continue to be a member of the primary Oorange development team. While in Berlin, Oliver was responsible for developing Oorange’s dynamic loading capability (which allows Oorange to incorporate new computation, viewing and data objects without re-compilation), the interface to the World Wide Web, the Oorange help and documentation systems (HTML browser and public browser interface), and the Oorange “shells”.

David Oliver also put extensive effort into designing, building and populating GANG’s World Wide Web server. A large percentage of the Center’s published and publishable work resides there for access by the mathematical community.

6 Description of the Computation and Visual Analysis Facility

6.1 Facility Description

The GANG Center maintains a visual computing facility in the Lederle Graduate Research Center. This facility serves the principal investigators along with collaborators, visiting scientists and graduate students (typically, about 20 people), offering computation, computer graphic, videographic and software

development services to these researchers. This section describes the current facility's hardware and software environment.

The GANG facility centers on a fiber-optic backbone network interconnecting a trio of Silicon Graphics Iris workstations. This network operates at 100Mb/s using the Fiber Distributed Data Interface (FDDI). We had expected to connect this network to that of the Computer Networks and Performance Evaluation Group in the Department of Computer Science during this year. However, Oliver's leave to Berlin and local factors prevented this. Work is now proceeding and the net will also include a small group of workstations in the Department of Polymer Science and Engineering. An additional Silicon Graphics workstation will be added within the next several months.

A "gateway" supports physical connection of the FDDI network to the slower-speed copper network that serves the GANG's smaller machines. In turn, the GANG's network is connected via the gateway to other campus research laboratories and computing facilities via the campus backbone, recently upgraded with an FDDI trunk and high-performance routers. A T1 (1.4Mb/s) link connects the campus to the NSFNet (via our regional network, NEARNet).

The Silicon Graphics workstations on the high-speed backbone have high-performance CPUs (two with 64-bit architecture) and graphics processors, large memories, and a large secondary storage pool. One of these machines is a multi-processor visualization "server". Two of these machines have direct connection to the GANGVideo video recording hardware system. GANG also operates a Sun "development server" that serves the GANG's smaller client workstations, acts as "home" for the GANG's network services, and manages a number of peripheral devices. This system has both high-resolution monochrome and color graphics consoles, and runs a number of Sun-specific mathematical packages. The color console is attached to the GANGVideo hardware.

6.2 GANGVideo Distributed Video Services

The GANG Center takes a distributed, interactive approach to the use of broadcast video and production of scientific video animations. The GANGVideo hardware system is network accessible through a set of media control libraries that form the basis for both graphical user interfaces and command line programs. These software tools allow researchers to produce live animations, create frames or sequences for archiving, and browse the video library. Recently, an animation scripter was added as well.

The Center's primary graphics computers are connected to a Lyon Lamb Real Time Converter (RTC) which supports full-screen and arbitrary-window video output in a variety of video formats. Output can be directed to a laserdisk recorder (Sony LVR5000), a 3/4" Umatic-SP tape recorder (Sony BVU-950), or a 1/2" VHS tape (Mitsubishi HR8000A). The laserdisk provides essentially permanent video frame storage. Up to 50 minutes of video can be recorded on each (write-once) LVR disk, either as still frames or sequences. These disks form the core GANG Geometry Database. A graphical database "browser" is part of the GANGVideo software.

The GANG has a secondary high-resolution viewing capability, using "slave" monitors and high-resolution switching/distribution equipment. Though limited to the laboratory workspace, the live output of three graphics workstations can be viewed through these, accommodating larger groups.

6.2.1 GANG Software

GANG staff have developed a number of software tools that are used by the Center's associated researchers for differential geometry research and scientific visualization.

MESH integrates with user-written surface-generation software to compute the triangulation of minimal surfaces and surfaces of constant mean curvature. SOLSURF computes the zero-set of arbitrary functions of three variables. DISP creates a number of visual representations from N-dimensional arrays of data. The Visual Programming System (VPS) is a suite of programs for creating and visualizing surface descriptions. All of the Center's geometry computation programs use the IVIEW 3D geometry viewer, allowing real-time manipulation of geometry data as it is computed. All programs work in concert with the GANGVideo software suite. Most GANG software runs only on the SGI Irix operating system.

GANGVideo, described above, is a distributed video control environment that allows internetworked workstations to access and control video production equipment for traditional frame-by-frame animation, or interactive viewing and recording.

GANG is participating in the development of Oorange, a new mathematical visualization environment from the Technical University of Berlin. GANG will be Oorange's US distribution site, and will continue to develop the Oorange environment in cooperation with TU-Berlin.

With the exception of IVIEW (incorporated in application programs) and Oorange (not yet released) software mentioned here is available via anonymous FTP from GANG (on server <ftp.gang.umass.edu>) and via the World Wide Web. GANG software generally carries either the Berkeley-style copyright, or the Free Software Foundation's "copyleft".

7 Research by Undergraduates and the GANG Graduate Seminar

Engaging undergraduate students in mathematics is an integral part of the activity at GANG. During the last two years at least four undergraduates, Matt Tagliani, Alvin Kho, Wei Shen and James Lawrence, have made substantial contributions to GANG, producing tools for doing classical differential geometry that are used in undergraduate courses. They have also worked on a wide variety of particular research problems: detecting critical configurations of points on the sphere under a Coulomb potential; discrete analogues of constant Gauss curvature surfaces; surfaces which minimize the integral of squared mean curvature while keeping surface area, volume, and the integral of mean curvature fixed; and the shapes attained by strands of compactified DNA.

This year Kusner and Norman led the GANG seminar on the topic of elastica, that is, curves which are critical for the total squared curvature. Students prepared and presented background material on elastica and the use of theta functions in differential geometry. Questions on elastica were explored computationally from several points of view: by considering elastica as energy minimizers, by explicitly parametrizing these curves via theta series, by using numerical techniques from ordinary differential equations and dynamical systems. In particular, gradient flow for total squared curvature on the space of closed immersed curves in the plane was considered. Last February, Huayong Yang (one of Kusner's Ph.D. students) proved (and presented to the Seminar) that there is unique elastic "figure 8" curve (up to rescaling), and hence that the total squared curvature is a perfect Morse function. In addition, a number of visiting researchers gave talks extending the students' work. Later in the Spring semester, other topics were introduced by Brakke and Sullivan, including variational methods for constructing and understanding films and foams via (branched) covering spaces.

8 The GANG Preprint Series and Seminar Visitors

The breadth and depth of research carried out at GANG is well represented by the GANG Preprint Series and the GANG Seminar visitors.

During this period the GANG Preprint Series III has been completed. The second half of Series III includes:

- D. Kim and R. Kusner (with G. Stengle). **Torus Knots Extremizing the Conformal Energy**
- D. Hoffman and W. Rossman. **Limit Surfaces of Riemann Examples**
- J. Bolton, F. Pedit and L. Woodward. **Minimal surfaces and the affine Toda field model**
- F.E. Burstall and F. Pedit. **Harmonic maps via Adler-Kostant-Symes theory**
- D. Hoffman, H. Karcher, Fusheng Wei. **The Genus One Helicoid and the Minimal Surfaces that led to its discovery**
- F. E. Baginski and N. Whitaker. **Numerical Solutions of Boundary Value Problems for K -Surfaces in R^3**
- Five Colleges RGI/REU (R. Kusner, P. Norman, and J. Sullivan). **Möbius Energy Minimizing Knot and Link Catalogue**
- E. C. Thayer. **Periodic Surfaces and Infinite Genus Riemann Surfaces**
- M. Callahan, D. Hoffman, and H. Karcher. **A Family of Singly-Periodic Minimal Surfaces Invariant Under a Screw Motion**
- E. C. Thayer and M. Wohlgemuth. **New Doubly-Periodic Minimal Surfaces**
- M. Traizet. **New Triply Periodic Minimal Surfaces**
- R. Kusner and J. M. Sullivan. **Möbius Energies for Knots and Links, Surfaces and Submanifolds**
- F. Pedit and H. Wu. **Discretizing constant curvature surfaces via loop group factorizations: the discrete sine- and sine-Gordon equations**
- J. Dorfmeister, F. Pedit, and H. Wu. **Weierstrass type representation of harmonic maps into symmetric spaces**
- Robert Kusner and Nick Schmitt. **The spinor representation of minimal surfaces**

And in October 1994, the GANG Preprint Series IV began:

- Rob Kusner, Rafe Mazzeo and Daniel Pollack. **The Moduli Space of Complete Embedded Constant Mean Curvature Surfaces**
- Rob Kusner, Paul Lahti and C. Peter Lillya. **New Surface Allotropes of Carbon**
- Edward C. Thayer. **Complete Minimal Surfaces in Euclidean 3-Space**
- F.E. Burstall and Franz Pedit. **Dressing Orbits of Harmonic Maps**
- Bo Guan. **Mean Curvature Motion of Non-parametric Hypersurfaces with Contact Angle Condition**
- Bo Guan. **On the Existence of Regularity of Hypersurfaces of Prescribed Gauss Curvature with Boundary**
- Bo Guan and Yanyan Li. **Monge-Ampere Equations on Riemannian Manifolds**
- Bo Guan. **The Dirichlet Problem for Monge-Ampere Equations in Non-Convex Domains**
- Jorgen Berglund and Wayne Rossman. **Minimal Surfaces with Catenoid Ends**
- Karsten G. Brauckmann. **The Family of Constant Mean Curvature Gyroids**
- Bo Guan. **The Dirichlet Problem for Monge-Ampère Equations in Non-Convex Domains and Spacelike Hypersurfaces of Constant Gauss Curvature**
- John Sullivan. **Geometry and the Imagination at UMass Amherst, Spring 1995**

Ken Brakke. The Surface Evolver Manual, version 1.99

All of the above preprints are available via the GANG Webserver, most have been submitted for publication, and many have been accepted in archival journals or conference proceedings.

Over the past year GANG has hosted a variety of visitors who have lectured in the GANG Seminar and worked with GANG faculty, staff, and students. These visitors include Gudlaugur Thorbergsson (Notre Dame), Walter Seaman (Iowa), Ron Perline (Drexel), Wayne Rossman (Sendai), Andrew Kraynik (Sandia), Esfandiar Navayazdani (TU Berlin), Bertram Kostant (MIT), Lorenzo Sadun (UT Austin), Ken Brakke (Susquehanna), J. M. Landsberg (Columbia), Ara Basmajian (Oklahoma), Karen Uhlenbeck (UT Austin), Herman Gluck (Penn.), Emma Previato (Boston), John Sullivan (Minnesota), W. H. Meeks, III (UMass), Alexander Bobenko (TU Berlin), Janet Talvacchia (Swarthmore), Meinhard Wohlgemuth (MIT), Henry McKean (Courant), Bruce Kleiner (Penn.), Karsten Brauckmann (Bonn), Bo Guan (Tennessee), Tom Ilmanen (Northwestern), George Kamberov (Penn.), and Robert Phelan (Dublin).