FIFTH TECHNICAL PROGRESS REPORT
ON
HYDRODYNAMIC MODELS FOR SLURRY BUBBLE COLUMN REACTORS

OCTOBER 1995

U.S. DEPARTMENT OF ENERGY GRANT
DE-FG22-94PC94208

DIMITRI GIDASPOW
PRINCIPAL INVESTIGATOR
DEPARTMENT OF CHEMICAL AND ENVIRONMENTAL ENGINEERING
ILLINOIS INSTITUTE OF TECHNOLOGY
CHICAGO, ILLINOIS 60616

U.S.DOE PATENT CLEARANCE NOT REQUIRED PRIOR TO PUBLICATION OF THIS REPORT
ABSTRACT

HYDRODYNAMIC MODELS FOR SLURRY BUBBLE COLUMN REACTORS

The objective of this investigation is to convert our “learning gas-solid-liquid” fluidization model into a predictive design model. The IIT hydrodynamic model computes the phase velocities and the volume fractions of gas, liquid, and particulate phases. Model verification involves a comparison of these computed velocities and volume fractions to experimental values.

This report includes a paper submitted for review for presentation at the Second International Symposium on Numerical Methods for Multiphase Flows, ASME Fluids Engineering Division, San Diego, CA, July 7-11, 1996. We found a severe stability restriction that all codes with chemical reaction must satisfy to have meaningful results. This is explained in the paper.

Next quarter report will present our experimental measurements of granular temperature of Air Products catalyst.

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.
MULTIPHASE FLOW IN SLURRY BUBBLE COLUMN REACTORS
AND SOLID PROPELLANT ROCKETS

Ron Pape
IIT Research Institute
Chicago, Illinois

Dimitri Gidaspow & Steven Wu
Department of Chemical and environmental Engineering
Illinois Institute of Technology
Chicago, Illinois

ABSTRACT
A two-dimensional, transient computer code for solving a generalization of Navier-Stokes equations for reacting multiphase flow was developed and tested for two applications: production of methanol in an Air Products slurry bubble column reactor and generation of particles in a rocket motor.

For the slow catalytic methanol production the conventional ICE technique produced numerical solutions in agreement with Air Products pilot plant results and IIT’s hydrodynamics experiments. The code predicted the measured methanol production, the observed vortices and the catalyst viscosity obtained from measurements of granular temperature using a digital camera.

However, for the rapid propellant combustion the conventional ICE technique proved inadequate. In such problems the absolute error grows without bounds for explicit and for implicit numerical schemes, as, for example, determined by von Neumann stability analysis. An analysis of the relative error showed how to finite difference the rate of reaction to obtain numerically stable solutions.

INTRODUCTION
For the past 20 years at IIT we have been developing the theory and computer codes for multiphase flow and fluidization. The recent version of the theory is based on the concept of granular temperature - the random oscillating kinetic energy pioneered by Professor Stuart Savage of McGill and collaborators[1] and on the generalization of the Navier-Stokes equations for multiphase flow. In this theory the particulate phase has its own equation of state, viscosity, etc. Gidaspow and Huilin have recently[2] verified that for a dilute particle gas mixture the particle pressure equals the bulk density times the granular temperature. In other words, the particulate phase obeys an analogue of an ideal gas law. The particulate viscosity was obtained from measurements of granular temperature. Gidaspow[3] reviewed this theory.

The computer codes we developed are based on the Los Alamos K-FIX code[4] which uses the ICE method of solution. Syamlal and others of Morgantown incorporated some of the kinetic theory into the commercial FLUENT code. Most of our research was without reaction. When we tried simulators with rapid reaction we ran into numerical problems. This led us to perform the stability analysis reported in this paper. We believe this analysis should be useful for codes such as KIVA that are widely used for engine combustion simulations[5].

STABILITY ANALYSIS

General
A stability analysis has been conducted on the finite difference form of the continuity, momentum and energy equations. To illustrate the method, the continuity equation will be discussed. When considering only convective and reaction terms, the continuity equation is

$$\frac{\partial (\rho_k \epsilon_k)}{\partial t} + \nabla (\rho_k \epsilon_k \mathbf{v}_k) = \dot{m}_k$$  \hspace{1cm} (1)
The finite difference form of the continuity equation in the IIT code is
\[(e_k \rho_k)^{n+1}_{j} - (e_k \rho_k)^{n}_{j} = \frac{\Delta t}{\Delta z_j} (e_k \rho_k) v_k^{n+1}_{j} \]
\[\frac{\partial}{\partial z_j} (e_k \rho_k) v_k^{n+1}_{j} + \partial z_j \nabla h_{kj}^{n+1} \]

The mass generation term can be represented in the following form:
\[m_{kj}^{n+1} = (e_k \rho_k)^{n}_{j} z_k \exp\left[-\frac{E_k}{RT_{kj}^n}\right] \]

To simplify the analysis, consider Cartesian coordinates, only the z-direction, and constant incremental distance. It is also assumed that all velocities are in the positive z-direction, corresponding to a specific donor cell differencing configuration. Temperature is also assumed constant. In this case, the donor cell flux is given by:
\[(e_k \rho_k v_k^{n+1}_{j} = v_k^{n+1}_{j} (e_k \rho_k) v_k^{n+1}_{j} - v_k^{n+1}_{j} (e_k \rho_k) v_k^{n+1}_{j}\]

Velocity will be taken as a constant, \(U\), and the following parameters are defined:
\[F_i = F_i^{n+1} = (e_k \rho_k) v_k^{n+1}_{j} \]
\[U = v_i = v_k^{n+1}_{j}, \quad m_i = (e_k \rho_k)^{n} M; \quad R = \frac{\partial}{\partial t} \]

With these substitutions, the continuity equation becomes:
\[F_i = F_i - (RU F_i - RU F_{i+1}) + M \partial F_i \]

(5)

Although, this is the form of the conservation of mass equation in the code, the formulation could also be fully explicit or fully implicit as given below.
Fully Explicit Form:
\[F_{j}^{n+1} = F_{j} - RU(F_{j}^{n} - F_{j+1}^{n}) + M \partial F_{j} \]

(6)

Fully Implicit Form:
\[F_{j}^{n+1} = F_{j} - RU(F_{j}^{n} - F_{j+1}^{n}) + M \partial F_{j} \]

Conservation of Momentum and Energy

In the IIT code, the conservation of momentum and energy equations are in mixed implicit - explicit form. With respect to terms considered here (convection and reaction) the momentum equation is fully explicit. For stability analysis we neglect pressure gradients, gravity, shear and drag terms in the momentum equation. In the energy equation, we neglect pressure gradients, convective heat transfer, conduction, viscous dissipation and gas particle drag dissipation. For momentum, \(F_j = (e_k \rho_k v_k^n)_{j} \) and M has identical meaning to that used in the continuity assessment. For the energy equation, \(F_j = (e_k \rho_k H_k^n)_{j} \) and M must be corrected by multiplying by the parameter \(\eta = \Delta H/\Delta k \).

Von Neumann Absolute Stability Analyses

It is recognized that all of the relations described above can be fully explicit, fully implicit or mixed form. Each of these forms is analyzed by applying a Von Neumann stability analysis. Separation of variables is accomplished by assuming F to be of the form:
\[F_j^n = \lambda^n e^{j \alpha} \]

(8)

Using this assumed form in the various model equations and solving for \(\lambda\) (the amplification factor), stability criteria are derived by observing that the amplification factor must be less than or equal to 1 for stability.

A classical Von Neuman absolute stability analysis [6] was applied to the three differencing equations. By this technique it is found that the fully explicit form requires that M be negative in order to have absolute stability in some regime of RU. With the implicit form, absolute stability imposes strange restrictions on MSt, depending on the value of RU. For example, when RU is 0, MSt must exceed 2 for absolute stability. The mixed form is found to have the restriction:
\[-2 \leq MSt \leq 0 \]

The implications of these results would be devastating, if true. In multiphase reactive flow modeling, whenever there is consumption of a phase (negative M) there is simultaneous generation in another phase (positive M). Negative M is not a problem, but absolute stability restrictions are a severe problem when M is positive. This is a direct consequence of the form of the equations. Analytic solution of the simplified equations for the duration of one time step yields:
\[F_j^{n+1} = F_j^n e^{M \Delta t} \]

(10)

If time (or \(\Delta t\)) were allowed to increase without bound, \(F_j^{n+1}\) would also be unbounded, and this is reflected in the amplification factor.

Relative Stability

Morton and Richtmeyer [6] point out that the absolute stability criteria is too restrictive for situations of the type described here. Instead, stability should be assessed relative to the desired analytic solution for the configuration of interest. As indicated above, the maximum change in F due to the reaction term will be:
\[F_j^{n+1} = F_j^n e^{M \Delta t} \]

(11)

This is the maximum change due to reaction, and it is the analytic solution used to assess relative stability. For assessment of relative stability, we have \(|\lambda| \leq e^{M \Delta t}\) instead of \(|\lambda| \leq 1\), as was required for absolute stability.

Fully Explicit Numerical Scheme. For the fully explicit scheme, including reaction and donor cell
differencing with a positive velocity, the finite difference equation is
\[ F_j' = F_j - RU (F_j - F_{j-1}) + M \delta t F_j \]  
(12)
A Von Neumann stability analysis leads to the following amplification factor for absolute stability.
\[ |\lambda| = \frac{1 + 2(R^2U^2 - RU - RU_M)\delta t}{a + b} \geq 1 \]  
(13)
When relative stability is evaluated, we have:
\[ \frac{|\lambda|}{M\delta t} \leq 1 \]  
(14)
By plotting amplification factor as shown in equation 13 or 14 versus \( M\delta t \) for different values of RU, the region of stability can be identified. Figure 1 is a plot of absolute and relative stability for the fully explicit scheme for the specific case of \( RU = 0 \). In all cases with the explicit scheme, absolute stability requires that \( M\delta t \) be negative. When \( RU = 0 \), with relative stability all positive values of \( M\delta t \) have amplification factors less than or equal to 1, and are therefore stable. If either absolute or relative amplification factor is less than or equal to 1 in a range, the range will be stable. Figure 2 maps the bound of stability in the RU versus \( M\delta t \) plane. All values to the right of the curve are stable. The rectangle at the bottom has been selected as a practical bound. We choose the following bounds for computations:
\[ RU \leq 0.5 \]  
(15)
\[-1 \leq M\delta t \leq 1 \]  
(16)
**Fully Implicit Numerical Scheme.** For the fully implicit scheme, including reaction and donor cell differencing with a positive velocity, the finite difference equation is
\[ F_j' = F_j - RU (F_j' - F_{j-1}) + M \delta t F_j \]  
(17)
A Von Neumann stability analysis leads to the following amplification factor for relative stability.
\[ |\lambda| = \frac{1}{\sqrt{a^2 + b^2}} \leq e^{M\delta t} \]  
(18)
where \( a = 1 + RU(1 - \cos \theta) - M\delta t \)
and \( b = RU \sin \theta \).
Figure 3 is a map of the primary stable zone for the fully implicit scheme. Absolute stability is achieved for all positive values of \( M\delta t \), but there are no severe problems when \( M\delta t \) is near +1. Relative stability has this problem also. This is due to a singularity that occurs because of the \(-M\delta t \) term in the denominator for the amplification factor. Singularities show up at other \( M\delta t \) values for other values of RU. In all cases the implicit scheme has problems for positive reaction terms. Based on absolute stability, all negative reaction terms are acceptable.

**Mixed Form.** A third numerical scheme is considered. If the stability problems are due to a singularity caused by the term \((-M\delta t)\) in the amplification factor denominator, the reaction term should be kept in the numerator. This is accomplished by always accounting for reaction explicitly, regardless of the form of the overall scheme. Therefore, consider all terms implicit with the reaction accounted for explicitly:
\[ F_j' = F_j - RU (F_j' - F_{j-1}) + M \delta t F_j \]  
(19)
A Von Neumann stability analysis leads to the following amplification factor for relative stability.
\[ |\lambda| \leq \frac{|\lambda|}{M\delta t} \leq e^{M\delta t} \]  
(20)
where \( a = 1 + RU(1 - \cos \theta) \),
\[ b = RU \sin \theta \]
\[ c = 1 + M\delta t \]
It can be shown that the maximum of the magnitude of the amplification factor is independent of RU. For negative reaction terms, absolute stability controls, and \( M\delta t \) must be greater than -2 for stability. For positive reaction terms relative stability controls, and there is no restriction on positive reaction terms. This is shown in figure 4. The mixed form is the least restrictive of the numerical schemes with respect to \( M\delta t \). The primary conclusion, however, is that the reaction term should be handled explicitly, regardless of the form of the overall numerical scheme being employed.

**INTERPRETATION OF RESULTS**

The stability guidelines presented above are applied to two problems, and the implications discussed. The problems assessed here are: (1) a fluidized bed reactor, and (2) a solid propellant combustion chamber.

**Fluidized Bed Reactor.** Simulations of Synthesis of Methanol in Air Products and Chemicals' Slurry Reactors

The governing equations for the hydrodynamic model and the chemical reactions were reported by Gidaspow, et al (1995) [7]. Table 1 shows the reactor's operating conditions. Table 2 shows the material balances. Table 3 shows the comparison of the simulation and Air Products' RUN E8.1 [8]. Figure 5 shows the comparison of methanol concentration profiles from IIT's hydrodynamic model, one-dimensional model and Viking's model [9]. The hydrodynamic model predicted much better results than the other two models did. Figure 6 shows a transient gas flow pattern in the reactor. Figure 7 shows the transient solid and gas volume fraction, thermal temperature, granular temperature and viscosity profiles. The blocks in Figure 6 and 7 represent heat exchangers.

The reaction rates and flow velocities are both extremely low in this system. For this case, the stability parameters can be summarized as
\( \delta t = 10^{-3} \) s  
\( RU = 8.02 \times 10^{-4} < 0.5 \)
\( \delta x = 1.9 \) cm  
\( M = 1.54 \times 10^{-9} \) s\(^{-1} \)
\( U = 15.24 \) cm/s  
\( |M\delta t| = 1.54 \times 10^{-6} \) < 1
Clearly, both stability criteria RU and $|M_\Delta t|$ are easily satisfied in this problem.

**Solid Rocket Combustion Chamber.** The simulation of a solid rocket combustion chamber is an ongoing investigation, and the stability analysis has been conducted to provide the insight necessary for proper setup of the simulations being done. A preliminary result is shown in figure 8. A number of computer experiments have been done using this configuration. One series of runs helps to clarify the meaning of the stability criteria. These simulations all used one microsecond time steps and 1 cm cell dimensions. Velocities were low in this configuration, so the RU criteria was easily satisfied. Therefore, the results for three specified constant reaction rates could be compared.

<table>
<thead>
<tr>
<th>Case</th>
<th>Excessive Rate</th>
<th>Critical Rate</th>
<th>Low Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M \Delta t$</td>
<td>$10^7$</td>
<td>$10^6$</td>
<td>$10^5$</td>
</tr>
<tr>
<td>Result</td>
<td>Immediate Failure</td>
<td>Ran 14 time-steps (borderline)</td>
<td>Did not Fail</td>
</tr>
</tbody>
</table>

ACKNOWLEDGEMENT

The numerical analysis presented in the paper was supported in part by IIT Research Institute and is being done as part of a PhD research investigation of Ron Pape. The modeling of the slurry bubble column reactor is supported by a DOE UCR grant DE-FG22-94PC94208 and is part of the PhD thesis of Steven Wu.

**REFERENCES**

2. Gidaspow, D. and Huilin, L., AIChE meeting, Nov. 1995 Miami Beach, to be preprinted.

**NOMENCLATURE**

E—activity energy of reaction
F = $\rho \phi$
H—enthalpy
$\varepsilon$—volume fraction
$\dot{m}$—rate of production
$M = z_\varepsilon \exp \left[ -E_\varepsilon / RT_\varepsilon \right]$
$R = \delta t / \delta r$
t—time
T—temperature
$U_\nu$—velocity in $r$ direction
$U_\nu$—velocity in $z$ direction
Greek letters:
$\lambda$—eigenvalue
$\varepsilon$—volume fraction
$\rho$—density
Subscripts:
i,j—cell index
k—phase k
Superscripts:
n—the nth time step

**DISCLAIMER**

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.
Table 1 Reactor Operating Conditions:

<table>
<thead>
<tr>
<th></th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>2-D reactor</td>
<td>22.5&quot;x320&quot;</td>
</tr>
<tr>
<td>pressure</td>
<td>735.2 psig</td>
</tr>
<tr>
<td>temperature</td>
<td>250.3 °C</td>
</tr>
<tr>
<td>gas velocity</td>
<td>15.24 cm/s</td>
</tr>
<tr>
<td>catalyst density</td>
<td>3.011 g/cm³</td>
</tr>
<tr>
<td>catalyst diameter</td>
<td>50 μm</td>
</tr>
<tr>
<td>liquid density</td>
<td>0.70025 g/cm³</td>
</tr>
<tr>
<td>gas type</td>
<td>CO-rich</td>
</tr>
</tbody>
</table>

Table 2 Material Balance:

<table>
<thead>
<tr>
<th></th>
<th>Inlet</th>
<th>outlet*</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>% mol</td>
<td>kgmol/hr</td>
</tr>
<tr>
<td>CO</td>
<td>51</td>
<td>87.9</td>
</tr>
<tr>
<td>CO₂</td>
<td>13</td>
<td>22.4</td>
</tr>
<tr>
<td>H₂</td>
<td>35</td>
<td>60.3</td>
</tr>
<tr>
<td>CH₃OH</td>
<td>0</td>
<td>0.0</td>
</tr>
<tr>
<td>N₂</td>
<td>1</td>
<td>1.7</td>
</tr>
<tr>
<td>M (kg/kgmol)</td>
<td>20.98</td>
<td></td>
</tr>
<tr>
<td>F (kgmol/hr)</td>
<td>172.3</td>
<td></td>
</tr>
<tr>
<td>F (kg/hr)</td>
<td>3614</td>
<td></td>
</tr>
</tbody>
</table>

* Taken at the vapor-slurry interface.
** Time averaging flowrate.

Table 3 Comparison of Simulation and Air Products' RUN E-8.1:

<table>
<thead>
<tr>
<th></th>
<th>CO conv. (%)</th>
<th>gas holdup (%)</th>
<th>slurry height (inches)</th>
<th>total catalyst (kg)</th>
<th>CH₃OH (gmol/hr/kg)</th>
<th>net CH₃OH (TPD)</th>
</tr>
</thead>
<tbody>
<tr>
<td>RUN E-8.1</td>
<td>13.50</td>
<td>29.5</td>
<td>200</td>
<td>567</td>
<td>20.50</td>
<td>10.03</td>
</tr>
</tbody>
</table>
Figure 1. Stability For Explicit Form with Reaction and for RU = 0

Figure 2. Stability Map for Fully Explicit Form
Figure 3. Stable Regions for Implicit Scheme with Reaction

Figure 4. Absolute and Relative Stability for Mixed Form
Fig. 5 Comparison of methanol mole fraction profiles from IIT's hydrodynamic model, one-dimensional model and Vikings' model.

Figure 6 Computed Gas Transient Flow Pattern.
Figure 7  Simulation of Air Products Slurry Bubble Column Reactor (RUN E8.1) at 12 seconds from Start-up.
Figure 8  ROCKET MOTOR (1 ms after start-up)

Solid Velocity (m/s)

Solid Volume Fraction:

Initial thickness=0.35cm

Temperature(°K):

300  2500