ABSTRACT. The purpose of this paper is to present results of a simplified approach to the dynamic finite element modeling of composite girder-slab bridges using a single beam element to represent the girder-slab cross section. Dynamic properties calculated with these simplified models are compared to experimental results and results obtained from more detailed shell element models. The method for modeling flexural behavior is first discussed followed by a discussion of modeling torsional behavior. The beam element models accurately calculated the mode shapes of the structure, but the associated resonant frequencies showed some error.

1. INTRODUCTION.

Forced and ambient vibration tests were performed on the I-40 bridge over the Rio Grande in Albuquerque, New Mexico during the summer of 1993. The results from these tests [1,2] showed that the bridge exhibited beam-like behavior. Subsequent to the experiments, detailed finite element models, using primarily shell elements, were developed and found to accurately predict the dynamic properties (natural frequencies and mode shapes) that were observed experimentally [3]. However, the detailed model suffers from complexity and size. A more simplified and practical model using a single beam element to represent the cross section of the bridge, which still accurately models the global dynamic response characteristics of the bridge, is desirable. Modeling of the I-40 Bridge using a simplified beam, finite element model vastly reduces the DOF compared to the shell element model. The development of such a model is presented in this paper.

Certain factors make the representation of the bridge by simple beam elements somewhat difficult. These factors include: 1. The composite (steel and concrete) nature of the bridge construction; 2. The presence of but a single axis of symmetry in the cross section; 3. The dynamic nature of the bridge response.

Implications of the above factors are that some flexural and torsional modes of response will be coupled, and that determination of the shear center, warping constant, sectorial moment, and the torsional constant of a thin-walled, open, noncircular composite cross section will be required.

The ABAQUS Finite Element Program [4] was selected for the refined-shell-element model of the I-40 Bridge as well as for the simplified beam-element modeling. The following are the input parameters (with units assuming mass as a derived unit, L = length, t = time, f = force) for the ABAQUS beam model (Beam General Section) which are relevant to the discussion here:

\[ A = \text{Beam cross-sectional area} \left( L^2 \right) \]
\[ I_{11}, I_{22} = \text{Area moments of inertia of the beam cross section about the centroidal axis} \left( L^4 \right) \]
\[ J = \text{Torsional constant} \left( L^4 \right) \]
\[ \Gamma_0 = \text{Sectorial moment} \left( L^4 \right) \]
\[ \Gamma_w = \text{Warping constant} \left( L^6 \right) \]
\[ \mu = \text{Mass density} \left( f^{-2} L^4 \right) \]
\[ E = \text{Young's modulus} \left( f^2 L^2 \right) \]
\[ G = \text{Shear modulus} \left( f L^2 \right) \]
\[ x_0, y_0 = \text{Shear-center location relative to centroid} \left( L \right) \]

A detailed summary of the beam-element formulation for thin-walled open sections can be found in the ABAQUS Theory Manual [4]. While it may not be possible to precisely model all aspects of the composite bridge with the above input parameters, the purpose of this paper is to demonstrate how the bridge can best be modeled.

2. BRIDGE GEOMETRY

The I-40 Bridge over the Rio Grande formerly consisted of twin spans (there are separate bridges for each traffic direction) made up of a concrete deck supported by two...
welded-steel plate girders and three steel stringers. Shear studs were not found when the concrete deck was removed. Loads from the stringers are transferred to the plate girders by floor beams located at 20-ft (6.1 m) intervals. Cross-bracing is provided between the floor beams. Figure 1 shows an elevation view of the portion of the bridge that was tested. The cross-section geometry of each bridge is shown in Fig. 2. It should be noted that the actual bridges have concrete crash barriers on either side of the concrete slab.

Each bridge is made up of three identical sections. Except for the common pier located at the end of each section, the sections are independent. A section has three spans; the end spans are of equal length, approximately 131 ft (39.9 m), and the center span is approximately 163 ft (49.4 m) long. Five plate girders are connected with four bolted splices to form a continuous beam over the three spans. The portions of the plate girders over the piers have increased flange dimensions, compared with the midspan portions, to resist the higher bending stresses at these locations. Connections that allow for thermal expansion as well as connections that prevent longitudinal translation are located at the base of each plate girder, where the girder is supported by a concrete pier or abutment. These connections are labeled "exp" and "pinned" in Fig. 1.

3. MODELING FLEXURAL BEHAVIOR

Flexural stiffness of the composite cross section is modeled by developing an "equivalent" or "transformed" cross-sectional area using well-known techniques as discussed in Popov [5]. This method provides an exact single-material representation of a multimaterial beam's flexural response subject to the limitations of strength-of-materials beam theory (linear elastic response, small deformations, etc.).

The resulting centroidal axis location found from the centroid of the transformed cross-sectional area is then the correct neutral axis for bending. Similarly, the area moment of inertia of the transformed section about this neutral axis is appropriate for modeling the flexural response. The process can be repeated about the orthogonal axis. Note that shear deformation and rotary inertia effects are not considered in this development.

For the beam model of the I-40 Bridge, this method provides the location of the centroidal axis and the two area moments of inertia about the centroidal axis, I_{11} and I_{22} (I_x and I_y, using axes labels in Fig. 2) for each of the two cross sections modeled (thick and thin flange sections), and, by implication, E. Detailed calculations for centroidal axis locations of the transformed bridge cross section and moments of inertia of the transformed sections about these axes are presented in [3] where the bridge has been modeled as an equivalent steel cross section. Numerical values of these parameters are summarized below in Section 5. The concrete deck is assumed to be uncracked, and reinforcing steel is ignored. Although this method will give the appropriate transformed cross-sectional area, this area will have to be adjusted based on mass distribution considerations for torsion discussed below.

Equivalent beam representations of the piers were also developed, however the reinforcing steel was neglected in these representations so a transformed section analysis was not necessary.
4. MODELING TORSIONAL BEHAVIOR

4.1. Torsional Rigidity

Expressions are readily available for the torsional rigidity of both individual and built-up, thin noncircular members made from the same material [6]. However, in the case of the I-40 Bridge, the cross section is composed of steel and concrete members. The torsion of a composite structural member is analogous to the case of linear springs in parallel. For this simple analogy, the torsional stiffness of the system is the sum of the individual stiffnesses. Therefore, the total torque is equal to the sum of the torques for each element, and the angle of twist per unit length, $\theta$, for each element is the same. Assuming linear elastic behavior, by the above analogy, the torsional rigidity of the $n$ individual members are additive, hence

$$\frac{T}{\theta} = \sum_{i=1}^{n} G_i J_i,$$

where $T$ is the applied torque, and $G_i$ and $J_i$ are the shear modulus and torsional constant for the $i$th material.

The cross section of the I-40 Bridge is modeled as a composite section using Eq. 1 to calculate torsional rigidity. Individual torsional constants are determined using handbook values or the well-known formula for a section built up from thin rectangular plates [6].

4.2. Shear Center Location

A closed form solution for the shear center location of the I-40 bridge is given in [3]. However, numerical procedures can also be used to determine the shear-center location. These procedures are outlined below where, in the interest of brevity, numerous steps of the development have been omitted. A detailed summary of this procedure that includes the omitted steps and examples where the method is applied to problems with closed-form solutions are presented in [3].

Consider, for illustrative purposes, the channel section shown in Fig. 3. Examine stresses caused by a shear force, $V_x$, acting in the $x$ direction through the shear center $(x_0, y_0)$. The coordinates $x_0$ and $y_0$ locate the shear center relative to the centroid of the cross section. Assuming thin-walled sections, which implies that the shear stress is constant across the thickness, equilibrium considerations require that

$$\tau_t = -\frac{V_x}{t_y} \int_{0}^{s} x_t \, ds.$$

If the shear force is applied through the shear center, the moment about the centroid caused by this force, $V_x y_0$, is resisted by the moment from the resultant shear stresses acting over their respective areas. Referring to Fig. 3, this equilibrium consideration can be used to develop an expression for the shear center location as

$$y_0 = \frac{l_{wx}}{l_y}.$$

Fig. 3 Channel section used to develop numerical procedure

Where $l_{wx}$ and is referred to as the warping product of inertia, and $l_y$ is the area moment of inertia about the $y$ axis. The warping product of inertia is defined as

$$l_{wx} = \int \omega x \, ds.$$  

$\omega$ is the double sectorial area (2A in Fig. 3), or unit warping constant with respect to the centroid, and is defined as

$$\omega = \int \rho \, ds.$$  

The above definitions are obtained from [6] for a general thin-walled section.

Heins [9] presents a numerical procedure for evaluating $l_{wx}$ and $l_y$ assuming the cross section can be idealized as a series of straight, connected segments, as shown in Fig. 4. At some distance $s$ in Fig. 4, there exists an element $ij$ of thickness $t_i$ and length $L_{ij}$. From Eq. 5 it is seen that the parameter $\omega$ can be represented as

$$\omega = \omega + \rho \omega L_{ij}$$

and that it varies linearly over the segment $ij$. Therefore, over segment $ij$ $\omega(x)$ can be expressed as

$$\omega(x) = \omega + \frac{(\omega - \omega)(x - x_i)}{(x_j - x_i)}.$$

It is also noted from examination of Fig. 4 that

$$ds = \frac{dx}{\cos \alpha_{ij}}, \quad \text{and} \quad \cos \alpha_{ij} = \frac{x_j - x_i}{L_{ij}},$$

and the integral expression for the warping product of inertia given by Eq. 4 can be written as

$$l_{wx} = \frac{1}{3} \sum_{k=1}^{n} \left( \omega x_i + \omega x_j \right) L_{ij} + \frac{1}{6} \sum_{k=1}^{n} \left( \omega x_i + \omega x_j \right) L_{ij}.$$
is evaluated for an element $ij$. The normalized unit warping of the cross section is

$$W_n = \frac{1}{A} \int \omega_0 ds - \omega_0 = \frac{1}{2A} \sum_{i} \left( \omega_{0i} + \omega_{0j} \right) t_{i-j} - \omega_0$$

(13)

where $A$ is total cross-sectional area, $\Sigma t_{ij} L_{ij}$.

The warping constant, $I_{\omega}$, is defined as

$$I_{\omega} = \int W^2_n ds = \frac{1}{3} \sum_{i} \left( W^2_{ni} + W_{n} W_{n,j} + W^2_{nj} \right) t_{i-j}$$

(14)

where $b$ is the entire length of the section.

Finally, the warping statical moment, $S_w$, is

$$S_w = \int W_n ds = \frac{1}{2} \sum_{i} \left( W_{ni} + W_{nj} \right) t_{i-j}.$$  

(15)

Note that the warping constant, $I_{\omega}$, is identical to the warping constant, $I_{\omega}$ required in the input data for the ABAQUS program. Further, the warping statical moment, $S_w$, is identical to the sectorial moment, $\Gamma_0$, required for ABAQUS.

Spreadsheet calculations of the two torsional constants, $I_{\omega}$ and $S_w$, of the transformed I-40 Bridge cross section (neglecting the stringers) using Heins' numerical procedure are summarized in [3]. The calculations resulted in the following values for the torsional constants needed to model the I-40 Bridge:

Thin-flange section: $I_{\omega} = 3.35 \times 10^{10}$ in$^2$ (8.99 m$^6$)

$S_w = \text{Approx. 0}$

Thick-flange section: $I_{\omega} = 5.06 \times 10^{10}$ in$^2$ (13.6 m$^6$)

$S_w = \text{Approx. 0}$

At this point, only the mass density and cross-sectional area of the bridge are yet unspecified in the ABAQUS input.

5. Modeling the Mass Distribution

For the torsional portion of the coupled beam response to be correct, the generalized torsional mass must be input properly. This implies that the mass polar moment of inertia about the center of mass must be correct. The area polar moment of inertia in ABAQUS is internally computed from the user-supplied area moments of inertia as

$$I_{\omega} = I_{11} + I_{22}$$

(16)

Based on axes shown in Fig. 2.)

Note that the warping constant, $I_{\omega}$, is identical to the warping constant, $I_{\omega}$ required in the input data for the ABAQUS program. Further, the warping statical moment, $S_w$, is identical to the sectorial moment, $\Gamma_0$, required for ABAQUS.

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Thick-flange section: $I_{\omega} = 5.06 \times 10^{10}$ in$^2$ (13.6 m$^6$)

$S_w = \text{Approx. 0}$

At this point, only the mass density and cross-sectional area of the bridge are yet unspecified in the ABAQUS input.
The quantity $pLlp$, where $L$ is the length of the beam and $p$ is the mass density, then provides the generalized torsional mass necessary to correctly model the torsional vibration response. A confounding factor is that $I_{11} (I_y)$ and $I_{22} (I_z)$ are input about the centroid of the cross section which, for a composite cross section, is not, in general, coincident with the center of mass.

The procedure utilized is to calculate the center of mass of the composite steel/concrete beam cross section, determine the polar moments of inertia about the center of mass for both steel and concrete components in the cross section, and then use the following equation to determine an equivalent mass density, $p_{eq}$, for torsional vibrations:

$$p_{eq} = \frac{\mu p_{pc} + \mu_s p_{ps}}{I_{11} + I_{22}}, \quad (16)$$

where $I_{11}, I_{22} =$ Transformed area moments of inertia supplied to ABAQUS,
$
\mu_s =$ Mass density of steel,
$\mu_c =$ Mass density of concrete,
$I_{ps} =$ Polar area moment of inertia of steel cross section about the center of mass, and
$I_{pc} =$ Polar area moment of inertia of concrete cross section about the center of mass.

Finally, the area of the equivalent beam is specified such that

$$A = \frac{A_c \mu_c + A_s \mu_s}{p_{eq}}, \quad (17)$$

where $A_c =$ the area of concrete forming the cross section, and $A_s =$ the area of steel forming the cross section.

Numerical values of input parameters used for the simplified beam element model of the I-40 Bridge are summarized in Table 1. Because the plate girders have increased flange dimensions at locations near the interior piers, two sets of beam properties were developed.

### 6. BOUNDARY CONDITIONS

When a beam element is used, the node points that define the element are typically located at the centroid of the cross section. The simplified beam element model of the I-40 Bridge was defined in this manner. Such a model poses problems when trying to simulate the connectivity of the plate girder to the piers and abutment. The modeling of these interfaces can significantly influence the calculated mode shapes and resonant frequencies.

The following boundary conditions and kinematic constraints were applied to the beam element model of the I-40 Bridge:

1. The node of the beam representing the plate girder corresponding to the abutment end was fixed against translation in the $x$, $y$, and $z$ direction and against rotation about the $y$ and $z$ axes. The coordinate axes can be seen in Figs. 1 and 2.

2. The nodes at the ends of the beam elements representing the base of the piers were fully fixed against translation and rotation in all directions.

3. Translation in the $x$ and $y$ directions of the nodes representing the top of the pier were constrained to similar translations of the node on the beam centroidal axis directly above the pier.

4. Rotations in the $y$ and $z$ directions of the nodes representing the top of the pier were constrained to similar rotations of the node on the beam centroidal axis directly above the pier.

5. Translation in the $z$ direction of the nodes representing the top of the pier was constrained to rotation about the $x$ axis of the node on the beam centroidal axis directly above the pier. The constraint equation was

$$\delta_z = h \theta_x, \quad (18)$$

where $\delta_z$ is translation in the $z$ direction of the node at the top of the pier, $h$ is the distance from the centroidal axis of the beam to the top of the pier, and $\theta_x$ is the rotation about the $x$ axis of node on the beam directly above the pier. The distance $h$ can be determined from the centroid locations

<table>
<thead>
<tr>
<th>TABLE 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Summary of Input Values for the Simplified Beam Element Model of the I-40 Bridge</td>
</tr>
<tr>
<td><strong>PARAMETER</strong></td>
</tr>
<tr>
<td>Cross-Sectional Area, $A$</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Moments of Inertia $I_{xx}$</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Moments of Inertia $I_{yy}$</td>
</tr>
<tr>
<td>Moments of Inertia $I_{yy}$</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Torsional Constant, $J$</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Shear Center, $y_0$</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Warping Constant, $T_w$</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Sectorial Moment, $I_0$</td>
</tr>
<tr>
<td>Elastic Modulus, $E$</td>
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<tr>
<td></td>
</tr>
<tr>
<td>Shear Modulus, $G$</td>
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<tr>
<td></td>
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<tr>
<td>Mass Density, $\mu$</td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>

1Distance from centroidal axis to shear center axis.
2Based on transformation to an equivalent steel cross section.
and the geometry of the connection between the plate girders and piers shown in [1 and 3].

6. COMPARISON WITH MEASURED DYNAMIC PROPERTIES AND PROPERTIES IDENTIFIED WITH DETAILED FINITE ELEMENT MODELS

Table II compares the resonant frequencies identified by the simplified beam model with those measured on the I-40 Bridge and those calculated with the detailed finite element model BR3W discussed in [3]. In Table II it can be seen that the percent difference between resonant frequencies calculated with the beam model and the measured resonant frequencies is less than 15% for the first flexural mode and the first two torsional modes. However, for higher frequency modes the error in the resonant frequencies becomes significantly larger. The boundary conditions specified in the beam model were intended to simulate those in the detailed finite element model BR3W. When compared with the resonant frequencies calculated with this detailed finite element model, the beam model again shows similar comparisons as were observed with the experimental data.

| TABLE II |
| Comparison of Simplified-Beam-Model Analytical Modal Analysis Results with Experimental Modal Analysis Results and Detailed Finite Element Analytical Modal Analysis Results |

<table>
<thead>
<tr>
<th>Resonant Frequency (Hz)</th>
<th>Exp.</th>
<th>Beam Model</th>
<th>BR3W</th>
<th>% Diff. Exp. vs Beam Model</th>
<th>% Diff. BR3W vs Beam Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mode 1 (Bending)</td>
<td>2.48</td>
<td>2.84</td>
<td>2.59</td>
<td>14.5</td>
<td>9.65</td>
</tr>
<tr>
<td>Mode 2 (Torsion)</td>
<td>2.96</td>
<td>2.63</td>
<td>2.78</td>
<td>11.1</td>
<td>5.40</td>
</tr>
<tr>
<td>Mode 3 (Bending)</td>
<td>3.50</td>
<td>4.36</td>
<td>3.71</td>
<td>24.6</td>
<td>17.5</td>
</tr>
<tr>
<td>Mode 4 (Bending)</td>
<td>4.08</td>
<td>5.53</td>
<td>4.32</td>
<td>35.5</td>
<td>28.0</td>
</tr>
<tr>
<td>Mode 5 (Torsion)</td>
<td>4.17</td>
<td>4.06</td>
<td>3.96</td>
<td>2.64</td>
<td>2.52</td>
</tr>
<tr>
<td>Mode 6 (Torsion)</td>
<td>4.63</td>
<td>5.59</td>
<td>4.50</td>
<td>20.7</td>
<td>24.2</td>
</tr>
<tr>
<td>Ave. % Diff.</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>18.2</td>
<td>14.5</td>
</tr>
</tbody>
</table>

The first flexural and torsional mode shapes calculated with the simplified beam model are shown in Figs. 5 and 6. In these figures very stiff beam elements with negligible mass have been added perpendicular to the beam elements representing the bridge so that torsional modes can be visualized. A qualitative comparison can be made visually by comparing these modes to those obtained experimentally from forced vibration tests, Figs. 7 and 8, and with those obtained from the detailed finite element model BR3W shown in Figs. 9 and 10. In all cases it appears that the simplified model is accurately predicting the experimental results and the results obtained with the detailed finite element model.
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REFERENCES