Stability of Strange Quark Stars with Nuclear Crusts against Radial Oscillations*

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STABILITY OF STRANGE QUARK STARS WITH NUCLEAR CRUSTS AGAINST RADIAL OSCILLATIONS

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ABSTRACT

This paper investigates the stability of the complete sequence of strange-matter stars with nuclear crusts against radial pulsations (acoustical modes). It is found that a broad class of white dwarf-like strange stars is stable against such pulsations. The same holds for the much denser strange stars, which are the strange counterparts of neutron stars. All stars possessing still higher central densities (e.g., charm stars) turn out to be unstable.

1. Introduction

Elsewhere in this volume¹, the properties of the complete sequence of strange-matter stars with nuclear crusts are reviewed. It consists of massive strange stars, which constitute the strange counterparts of neutron stars, and strange white dwarfs, whose bulk properties are similar to those of ordinary white dwarfs. Here we investigate the stability of the complete sequence against radial pulsations (acoustical modes). (For more details, see Refs. ²,³,⁴)

2. Mass-Radius Relationship

For later purpose, we show in Fig. 1 the mass-radius relationship of the complete sequence of non-rotating strange-matter stars whose inner crust density is equal to neutron drip⁵,⁶. The central density along this curve decreases monotonically from ‘E’, which corresponds to the minimum-mass star of the charm-quark star sequence to ‘A’ where the strange core has shrunk to zero. Stars located between ‘A’ and ‘C’ (the lightest star of the sequence) are referred to as strange dwarfs⁷. ‘B’ refers to the maximum-mass strange dwarf.

3. Stability Analysis

The oscillatory motion of a star in its nth normal mode (n = 0 is the fundamental
mode) is expressed in terms of an amplitude $u_n(r)$ given by $^5,^6$

$$\delta r(r, t) = e^{\nu} u_n(r) e^{i\omega_n t} / r^2 ,$$

(1)

where $\delta r(r, t)$ denotes small (Lagrangian) displacements. The underlying metric is of the form

$$ds^2 = e^{2\nu(r)} dt^2 - e^{2\lambda(r)} dr^2 - r^2 \left( d\theta^2 + \sin^2 \theta \, d\phi^2 \right) .$$

(2)

The quantity $\omega_n^2(t)$ is the star’s oscillation frequency, which is given as the solution of the following (Sturm-Liouville type) eigenequation,

$$\frac{d}{dr} \left( \Pi(r) \frac{d u_n(r)}{dr} \right) + \left( Q(r) + \omega_n^2 W(r) \right) u_n(r) = 0 .$$

(3)

Stable (unstable) radial oscillations are associated with $\omega_n^2 > 0 \ (\omega_n^2 < 0)$. The functions $\Pi(r)$, $Q(r)$, and $W(r)$ are expressed in terms of the equilibrium configurations of the star, and are given by

$$\Pi = e^{(\lambda + 3\nu) r^{-2}} \Gamma P ,$$

(4)

$$Q = -4 e^{(\lambda + 3\nu) r^{-3}} \frac{dP}{dr} - 8\pi e^{3(\lambda + \nu) r^{-2}} P (\epsilon + P)$$

$$+ e^{(\lambda + 3\nu) r^{-2}} (\epsilon + P)^{-1} \left( \frac{dP}{dr} \right)^2 ,$$

(5)

$$W = e^{(3\lambda + \nu) r^{-2}} (\epsilon + P) .$$

(6)
Figure 2: Oscillation frequencies, \( \omega_n^2 \), of the lowest four \( (n = 0, 1, 2, \text{ and } 3) \) normal radial modes of strange stars as a function of central star density. The quantity \( \Phi \) is defined by \( \Phi \equiv \text{sign}(a) \log [1 + \text{abs}(a)] \), where \( a \equiv (\omega_n/\text{sec}^{-1})^2 \). Stars with \( \Phi < 0 \) are unstable against oscillations. The dotted curve shows qualitatively the behavior of mass as a function of density. ‘D’ and ‘E’ refer to the same stars as in Fig. 1.

The quantities \( \epsilon \) and \( P \) in Eqs. (4)-(6) denote energy density and pressure of the star. The pressure gradient, \( dP/dr \), is obtained from the Oppenheimer-Volkoff equations. The symbol \( \Gamma \) denotes the varying adiabatic index at constant entropy, given by \( \Gamma = [(\epsilon + P)/P]dP/d\epsilon \). Finally, the boundary conditions for Eq. (3) are \( u_n \sim r^3 \) at star’s origin \( (r = 0) \), and \( du_n/dr = 0 \) at star’s surface \( (r = R) \).

The four lowest-lying eigenfrequencies of massive strange stars and strange dwarfs (inner crust density equal to neutron drip) are shown in Fig. 2. A qualitative comparison with the mass-central density relationship (dashed curve)\(^3\) shows that these equilibrium configurations possess a characteristic mode of vibration of zero frequency \( (\omega_0^2 = 0) \) when and only when \( dM/d\epsilon = 0 \), that is, only when the star’s mass attains an extremum, as one expects from the theorem in Ref. \(^7\).

An enlargement of the low-density portion of Fig. 2 is shown in Fig. 3. It reveals that, in the direction of increasing central densities, the lowest-lying eigenmode passes through zero at the heaviest strange dwarf, ‘B’, becomes zero again for the lightest strange dwarf, ‘C’, and turns positive at the higher densities again. Since \( \omega_0^2 < 0 \) corresponds to unstable radial oscillations, it follows from Figs. 2 and 4 that no quark-matter star can exist stably that is either denser than ‘B’ or more compact than the densest strange star, ‘D’. Specifically this rules out the possible existence of the even denser charm stars, which are located beyond ‘E’.

So far our discussion was restricted to eigenfrequencies of strange dwarfs with inner crust density equal to neutron drip (the maximum possible value). Figure 4 shows the behavior of the eigenfrequencies of such stars for \( \epsilon_{\text{crust}} < \epsilon_{\text{drip}} \). One sees
Figure 3: Enlargement of the low-density portion of Fig. 2. The dotted curve shows qualitatively the dependence of mass on density; 'C' and 'B' denote maximum- and minimum-mass strange dwarfs. The integers denote the first four eigenmodes. The labels 'A', 'B', and 'C' refer to the same stars as in Fig. 1.

Figure 4: Oscillation frequencies of the fundamental mode of oscillation, $\omega_0^2$, versus central star density. The inner crust densities are $\epsilon_{\text{crust}} = 10^9$ g/cm$^3$ (curve a), $10^{10}$ (b), $10^{11}$ (c), and $4.3 \times 10^{11}$ (d) $^3$. 
that strange dwarfs of sequences with $\epsilon_{\text{crust}} \leq 10^9 \text{ g/cm}^3$ down to the termination point are all stable against radial oscillations, which is due to the fact that such sequences terminate before they reach the maximum-mass strange dwarf peak where $\omega^2$ would become negative (sequences ‘b’ to ‘d’).

4. Summary

Hydrostatic equilibrium sequences of strange matter stars with nuclear crusts extend from white-dwarf-like strange stars (strange dwarfs) to dense strange stars with properties similar to those of ordinary neutron stars. We find that most stars along such sequences are stable against radial oscillations and thus could exist stably in the universe if Bodmer’s and Witten’s strange matter hypothesis is true. The only exceptions are stars that are either denser than strange stars (i.e. charm stars) or strange dwarfs with central densities that place them into the mass-radius region of unstable white dwarfs.

5. Acknowledgement

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ABSTRACT

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Figure 1: Mass versus radius of strange-star configurations with nuclear crust (cf. 1,3). The labels are explained in the text.

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