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Multiresolution Reproducing Kernel Particle Methods in Acoustic Problems

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ABSTRACT

In the analysis of complex phenomena of acoustic systems, the computational modeling requires special attention for a realistic representation of the physics. As a powerful tool, the finite element method has been widely used in the study of complex systems. In order to capture the important physical phenomena, p-finite elements and/or hp-finite elements are employed. The reproducing kernel particle methods(RKPM) are emerging as an effective alternative due to the elimination of a mesh, and the ability to analyze a specific frequency range. Additionally, a wavelet particle method based on the multiresolution analysis encountered in signal processing has been developed. The interpolation functions consist of spline functions with built-in window. A variation in the size of the window implies a geometrical refinement, and allows the filtering of the desired frequency range. Preliminary analysis of the wave equation shows the effectiveness of this approach. The frequency/wave number relationship of the continuum case can be closely simulated by using the reproducing kernel particle methods. A similar methodology is also developed for the Timoshenko beam.

INTRODUCTION

Multiple scale methods, which are based on reproducing kernel and wavelet analysis, are developed. These permit the response of a system to be separated into different scales. These scales can either be the wave numbers corresponding to spatial variables or the frequencies corresponding to temporal variables; each scale response can be examined separately. This complete characterization of the unknown response is performed through the integral window transform. A space-

scale and time-frequency localization process is achieved by dilating the flexible multiple scale window function.

This flexible space-scale window function is constructed to resemble the well-known smooth particle hydrodynamics, and the unstructured adaptive finite element methods. However, the multiple scale adaptive refinements are performed simply by inserting nodes into the highest wavelet scale solution region, and at the same time narrowing the window function. Hence, hp-like adaptive refinements can be performed without a mesh.

These developed methods are applied to structural acoustics and structural dynamics with particular reference to fluid-structure interaction. These methods provide the capability to perform efficient and accurate computations of the structural response which combine both the low and medium frequency parts of the spectrum. This new development enables engineers not only to bring more detail into their structural system models, but also enhances their computer simulations of many classes of multiple scale structural analysis. It improves the accuracy, efficiency, and reliability of dynamic analysis. This is of great importance because the noise prediction of a complex structure relies heavily on the accuracy of the numerical procedures which are used to analyze them.

In this development, a microscope is constructed using a flexible space-wave number and time-frequency localized window function which *translates and dilates* in space and time to cover the entire domain of interest. This microscope can magnify, examine, and record the image of the various scales and frequencies of the response locally within the support of the window function. The degree of magnification depends on

Figure 1. The exponential convergence behavior of the multiple scale RKPM

the power of the microscope. This complete characterization of the unknown response is performed through the integral window transform. This localization process can be achieved by dilating the flexible multiple-scale window function. The *zoom in* and *zoom out* capability of the window function is especially useful in examining complex flow phenomena, such as flow induced vibration, dynamic stability of flow-structure interaction, turbulence structures, and high frequency structural dynamics response. We make use of the multiple scale decomposition of the response to study the dynamic behavior of acoustic fluid-structure systems.

MULTIRESOLUTION ANALYSIS

This section is devoted to a review of multiresolution analysis [see e.g. (2, 3, 8, 12, 16) and references therein]. The framework of a multi-resolution analysis consists of a sequence of nested closed subspaces, V_m , such that:

$$\{0\} = V_{-\infty} \subset \dots \subset V_1 \subset V_0 \subset V_{-1} \subset \dots \subset V_{\infty} = L^2(R). \quad (1)$$

The basis functions which generate these subspaces are scaling functions. Scaling functions are a family of functions which are generated by the translation and dilation of a single function $f(x)$

$$\phi_{mn}(x) = a^{-m/2} \phi(a^{-m}x - nb), \quad (2)$$

where 'a' and nb are the dilation and translation parameters, respectively.

Through the implementation of this type of window function and the exploitation of the Fourier transform, it is possible to develop a new type of shape functions that is closely related to finite elements. These new shape functions can still be used in the standard Galerkin formulation. The derivative of the shape function, and thus reproducing kernel, can be obtained by direct differentiation without a finite element mapping. The two parameters in the scaling function provide the ability to translate and dilate the window function.

Translation is required to move the window function around the domain since the window functions themselves have a compact support. The *ability to translate replaces the need to define elements*. The *dilation parameter is used to provide refinement*. This dilation parameter also controls the convergence rate of the multiple scale RKPM. As shown in Fig. 1, the rate of convergence of the L_2 norm (and H_1 norm) of a smooth Laplacian solution varies from 2 to 16 (and 1 to 15) for a Gaussian window by simply changing the *refinement* or dilation parameter of a single window function. This combination of translation and dilation produces a *meshless p-like adaptive variable-node multiple scale RKPM*.

As the basic characteristics of the multiresolution analysis dictates, an orthonormal wavelet basis exists for a collection of closed subspaces presented in Eqn. (1):

$$V_{n+1} = V_n + W_n. \quad (3)$$

where W_n is the wavelet basis which complements V_n , $m \in \mathbb{Z}$ where

$$Z = \{\dots, -1, 0, 1, \dots\}. \quad (4)$$

A *Wavelet* generates the spaces, W_n , in the same manner as a scaling function $f(x)$ generates the spaces V_n . From Eqn. (3), this family of subspaces of $L^2(R)$ yields a direct-sum decomposition of $L^2(R)$ such that

$$L^2(R) = \dots + (V_{-2} - V_{-1}) + (V_{-1} - V_0) + (V_0 - V_1) + \dots = \dots + W_{-1} + W_0 + W_1 + \dots \quad (5)$$

A wavelet y in $L^2(R)$ is called an orthogonal wavelet if $\{\psi_{nk}\}$ satisfies

$$\langle \psi_{nk}, \psi_{kl} \rangle = \delta_{nk} \delta_{kl} \quad (6)$$

Figure 2. RKPM yields to the resolution limit. $k=p/Dx$

where $\langle \cdot \rangle$ is the inner product operator and δ_{mk} is the Kronecker delta. If y is an orthogonal wavelet, then the direct sum in Eqn. (5) becomes an orthogonal sum, so that the reconstruction formula is given by

$$u = \sum_{m,n \in \mathbb{Z}} \langle u, \psi_{mn} \rangle \psi_{mn}(x) \quad (7)$$

In a classical Fourier analysis, in which the sine and cosine waves do not have a compact support, such modifications require the computation of a new set of Fourier coefficients which can be very costly. Presently, this is a common practice in the numerical study of the computed physical response.

In wavelet analysis, unlike the classical Fourier analysis, only a few wavelet coefficients need to be computed because of the space-scale localization process. In three dimensions, this localization process will involve a space-wave number and time-frequency localization window.

In the present study, the concept of multi-resolution analysis to separate the numerical noise from the high scale components (frequencies and wave numbers) of the compressible flow-structure phenomena.

MULTIPLE SCALE RKPM

The reproducing kernel particle method [5-9] is given by

$$u^h(x) = \int_{\Omega} C_A(x; x-y) \phi\left(\frac{x-y}{a}\right) u(y) dy \quad (8)$$

where $C_A(x; x-y)$ is the boundary correction term, its dependence on the dilation parameter is implied by and the

subscript 'a.' With the aid of the boundary correction term, the reproduction of a given function can be treated as a finite convolution of the function, $u(y)$, and the kernel function $\phi(\frac{x}{a})$. From the Fourier transform analysis, we note that this kernel function serves as a low-pass filter in the reconstruction procedure. The multiple scale RKPM is defined by a family of kernel functions so that the wavelets corresponding to these kernel functions are defined by:

$$\psi_{m+1}(x) = \phi_m(x) - \phi_{m+1}(x) \quad (9)$$

Therefore, a multiple scale decomposition of a given response is given as

$$\begin{aligned} u^h(x) &= u_0(x) && \text{finest scale} \\ &= w_1(x) + u_1(x) && \text{two-level decomposition} \\ &= w_1(x) + w_2(x) + u_2(x) && \text{three-level decomposition} \\ &= \sum_{i=1}^n w_i(x) + u_n(x) && \text{n-level decomposition} \end{aligned} \quad (10)$$

where

$$u_n(x) = \int_{\Omega} C_{2^n}(\mathbf{x}; \mathbf{x}-\mathbf{y}) \phi_n(\mathbf{x}-\mathbf{y}) u(\mathbf{y}) d\mathbf{y} \quad (11)$$

$$w_n(x) = \int_{\Omega} \psi_n(\mathbf{x}-\mathbf{y}) u(\mathbf{y}) d\mathbf{y} \quad (12)$$

with

$$\begin{aligned} \psi_n(\mathbf{x}-\mathbf{y}) &= C_{2^{n+1}}(\mathbf{x}; \mathbf{x}-\mathbf{y}) \phi_{n+1}(\mathbf{x}-\mathbf{y}) \\ &\quad - C_{2^n}(\mathbf{x}; \mathbf{x}-\mathbf{y}) \phi_n(\mathbf{x}-\mathbf{y}) \end{aligned} \quad (13)$$

Similar to the multi resolution analysis, the multiple scale RKPM always starts from the finest scale and the different levels of the response can be derived by the decomposition algorithm in Eqn (10). It is emphasized that all computations are performed in the *space-time domain*. However, because of the multiple scale characteristics of the *new time-frequency and space-wave number basis* for representing functions, the physical interpretation of the computed results are apparent.

CHOOSING THE OPTIMAL DILATION PARAMETER

Aliasing Control approach

A practical approach for choosing the dilation parameter 'a' for the reproducing kernel window is to determine the interaction between the size of the given window and the sampling rate of the given nodal system in the Fourier transform domain. The Fourier transform of the reproducing kernel window function $f(x)$ is also a window $\hat{f}(x)$ in the transform domain. A large window (in function domain) filters out the fine scales (small wave numbers or high frequencies) which can be caused by sampling aliasing; consequently, oscillation in the solution may occur. A small window (in function domain), on the contrary, may introduce aliasing, but will cover wider scale/frequency bands.

hp-like adaptivity

The exponential convergence behavior of the multiple scale RKPM, depicted in Fig. 1, indicates that the RKPM has a *similar* feature of the adaptive p-finite element methods [see e.g. (1, 13, 15, 18) and references therein]. However, the high convergence rate, or equivalently, the high value of p, can be achieved simply by increasing the size of the refinement parameter of a *single* window function. This avoids the restrictive requirement of the traditional p-finite element methods. There is no apprehension of the compatibility (or continuity) along the element boundaries of the hp-finite element mesh, since the RKPM requires only a set of nodes and the global sampling shape functions can be C^∞ .

The multiresolution concept can further be used to construct hp-adaptive algorithms. The multiple scale adaptive refinements are performed simply by inserting nodes into the highest wavelet scale solution region, and at the same time, narrowing the size of the window function to pick up the fine-scale structure of the response. This zooming in process, together with the addition of nodes, will result in an hp-like adaptive refinement algorithm. Hence, hp-like adaptive refinements can be performed without a mesh. Examples in Fig. 2 show that RKPM almost reaches the resolution limit; $k=p/Dx$ and the ratio of the harmonic wave number, n, and the number of nodes (particles), p, approaches one.

Based on the multiple scale decomposition and the highest wavelet scale response, a convergence parameter or an error estimation indicator can be determined to locate the adaptivity regions. The adaptive refinement is accomplished by a *single p-order* (or C^∞) flexible space-scale window

function that translates and dilates covering all the nodes in the computational domain.

DISPERSION RELATIONSHIP

Wave Equation

In one dimensional space, the wave equation is given as

$$\frac{1}{c^2} \ddot{u} = u_{,xx} \quad x \in [0, L] \quad (14)$$

where c is the wave speed, a superposed dot indicates the temporal derivative, and a subscript comma denotes the spatial derivative. In computational analysis, appropriate approximation functions are employed to represent the solution

$$u^h(x, t) = \sum_{i=1}^{NP} N_i(x) u_i(t) \quad (15)$$

where a superscript "h" indicates the approximate solution, NP refers to the total number of grid points (as in the FEM) or particles (as in the RKPM). N_i are the approximation functions, and u_i denotes the approximate solution at each grid point or particle.

The wave characteristics of Eq.(14) are studied by considering its homogeneous form. A general harmonic wave solution is given as

$$u = u_0 e^{i(\omega t - kx)} \quad i = \sqrt{-1} \quad (16)$$

where u_0 is the amplitude, ω is the circular frequency, and k is the wave number. The discrete form of Eq.(16) for the Ith grid point or particle becomes

$$u_I(t) = u_0 e^{i(\omega t - kx_I)} \quad (17)$$

The characteristic equation is obtained by focusing on the Ith grid point/particle

$$\begin{aligned} & -\frac{\omega^2}{c^2} \{ (1 - \alpha) M_{II} \\ & + \alpha \left[\sum_{m=1}^{n_L} M_{I, I-m} e^{ik \sum_{j=1}^m \Delta x_{I-j}} + M_{II} + \sum_{m=1}^{n_R} M_{I, I+m} e^{-ik \sum_{j=0}^{m-1} \Delta x_{I+j}} \right] \\ & + \sum_{m=1}^{n_L} K_{I, I-m} e^{ik \sum_{j=1}^m \Delta x_{I-j}} + K_{II} + \sum_{m=1}^{n_R} K_{I, I+m} e^{-ik \sum_{j=0}^{m-1} \Delta x_{I+j}} \} = 0 \end{aligned} \quad (18)$$

where Δx_i is the spacing between the Ith and (I+1)th mesh point/particle locations; n_R and n_L are the number of points to the right and to the left of the Ith location which fall under the window function and $\alpha \in [0, 1]$ which determines the amount of lumped mass, M_{II}^* , versus the consistent mass, M_{II} .

In a standard finite element analysis with linear Lagrange interpolants, the Ith stiffness term is coupled to (I-1)th and (I+1)th terms only, hence, $n = 1$. By employing linear

approximation functions and a lumped mass, the wave characteristic Eq.(18) can be rewritten as:

$$-\frac{\omega^2}{c^2} + \frac{4}{h^2} \sin^2\left(\frac{1}{2}kh\right) = 0 \quad (19)$$

Figure 3 Dispersion Relationship for wave equation with varying 'a' (consistent mass)

In reproducing kernel particle method, the shape function are defined in terms of window functions

$$N_l(x) = \left[c_1(x) + c_2(x) \frac{x - x_l}{a} \right] \frac{1}{a} \phi\left(\frac{x - x_l}{a}\right) \Delta x_l \quad (20)$$

where $f(x)$ is a window function, 'a' is the dilation parameter which determines the width of the window, and the function c_1 and c_2 are the boundary correction functions.

For uniform spacing between the particles, the general matrix form of the discretized wave equation with RKPM is introduced

$$-\frac{\omega^2}{c^2} \left\{ (1 - \alpha) M_{11}^l + \alpha \sum_{m=n}^n M_{1,1+m} \cos(mkh) \right\} + \sum_{m=n}^n K_{1,1+m} \cos(mkh) = 0 \quad (21)$$

The frequency equation for the continuum wave equation can be shown to be

$$\frac{\omega^2}{c^2} - k^2 = 0 \quad (22)$$

This equation indicates that there is no dispersion. For $\omega = 0$, which is representative of the rigid body motion, the corresponding wave number is zero. The spurious mechanisms associated with the existing discretization methods can be studied by comparing their characteristic equation to Eq. (22).

The comparison of the RKPM with varying 'a' and the analytic dispersion (frequency vs. wave number) relation is depicted in Fig. 3. As can be seen, by increasing dilation parameter the dispersion relation for RKPM has more agreement with the analytic one.

A Multiple Scale RKPM Timoshenko Beam

The coupled equations for the Timoshenko beam can be expressed in matrix operator form as:

$$\begin{bmatrix} \rho A \frac{\partial^2}{\partial t^2} - \kappa GA \frac{\partial^2}{\partial x^2} & \kappa GA \frac{\partial}{\partial x} \\ \kappa GA \frac{\partial}{\partial x} & -\rho I \frac{\partial^2}{\partial t^2} + EI \frac{\partial^2}{\partial x^2} - \kappa GA \end{bmatrix} \quad (23)$$

The desired relationship between frequency and wave number is obtained by setting the determinant of the Fourier matrix operator to zero. Thus, the continuum frequency equation derived from Eq. (23) is:

$$a \eta^4 + b \eta^2 + c = 0 \quad (24)$$

where η is wave number and

$$a = \frac{EI}{\rho A}$$

$$b = -\frac{I}{A} \left(1 + \frac{E}{\kappa G} \right) \omega^2$$

$$c = \frac{\rho I}{GA \kappa} \omega^4 - \omega^2 \quad (25)$$

solving for η gives:

$$\eta = \pm \left\{ \left[\frac{\rho}{G \kappa} + \frac{\rho}{E} \right] \omega^2 \pm \sqrt{\frac{\rho A}{EI} \omega^2 + \frac{1}{4} \left[\frac{\rho}{G \kappa} - \frac{\rho}{E} \right]^2 \omega^4} \right\}^{\frac{1}{2}} \quad (26)$$

The semi-discrete RKPM eqs. for a linear Timoshenko beam, formulated for the case of equal length, $h = \Delta x$, between two nodes at an interior node, 'j', are

$$\begin{aligned} \rho A m_j^w \ddot{w}_j = & -\kappa GA (k_{j,j-n_w}^w w_{j-n_w} + \dots + k_{j,j-1}^w w_{j-1} \\ & + k_{j,j}^w w_j + k_{j,j+1}^w w_{j+1} + \dots + k_{j,j+n_w}^w w_{j+n_w}) + \kappa GA (k_{j,j-n_w}^{\theta} \theta_{j-n_w} \\ & + \dots + k_{j,j-1}^{\theta} \theta_{j-1} + k_{j,j}^{\theta} \theta_j + \dots + k_{j,j+n_w}^{\theta} \theta_{j+n_w}) \end{aligned} \quad (27)$$

$$\begin{aligned} \rho A m_j^{\theta} \ddot{\theta}_j = & -EI (k_{j,j-n_{\theta}}^{\theta} \theta_{j-n_{\theta}} + \dots + k_{j,j-1}^{\theta} \theta_{j-1} + k_{j,j}^{\theta} \theta_j \\ & + k_{j,j+1}^{\theta} \theta_{j+1} + \dots + k_{j,j+n_{\theta}}^{\theta} \theta_{j+n_{\theta}}) + \kappa GA (k_{j,j-n_w}^w w_{j-n_w} + \dots \\ & + k_{j,j-1}^w w_{j-1} + k_{j,j}^w w_j + \dots + k_{j,j+n_w}^w w_{j+n_w}) - \kappa GA (k_{j,j-n_{\theta}}^{\theta} \theta_{j-n_{\theta}} \\ & + \dots + k_{j,j-1}^{\theta} \theta_{j-1} + k_{j,j}^{\theta} \theta_j + k_{j,j+1}^{\theta} \theta_{j+1} + \dots + k_{j,j+n_{\theta}}^{\theta} \theta_{j+n_{\theta}}) \end{aligned} \quad (28)$$

where m_j^w and m_j^{θ} are evaluated as lumped masses and $k_{j,j}^w$, $k_{j,j}^{\theta}$, $k_{j,j+1}^w$ and $k_{j,j+1}^{\theta}$ are the non-zero coefficients of the stiffness matrices K_{AB}^w , K_{AB}^{θ} , $K_{AB}^{w\theta}$ and $K_{AB}^{\theta w}$, respectively, and n_w , n_{θ} , $n_{w\theta}$ and $n_{\theta w}$ are dependent on the number of nodes covered under the support of the window functions with rotation and transverse displacement dilation parameters, a_{θ} and a_w .

The discrete Fourier matrix operators corresponding to the RKPM formulation can be shown to be

$$\begin{bmatrix} \rho A m_j^w \omega^2 - \kappa G A \alpha^w & i \kappa G A \alpha^{w\theta} \\ i \kappa G A \alpha^{\theta w} & \rho I m_j^\theta \omega^2 - E I \alpha^\theta - \kappa G A \alpha^{\theta\theta} \end{bmatrix} \quad (29)$$

where $m_j^w = \int_0^L N_A^w dx$; $m_j^\theta = \int_0^L N_A^\theta dx$, and

$$\alpha^\theta = \sum_{m=-n_\theta}^{n_\theta} k_{j,j+m}^\theta \cos m\eta h$$

$$\alpha^w = \sum_{m=-n_w}^{n_w} k_{j,j+m}^w \cos m\eta h$$

$$\alpha^{w\theta} = \sum_{m=-n_{w\theta}}^{n_{w\theta}} k_{j,j+m}^{w\theta} \sin m\eta h$$

$$\alpha^{\theta w} = \sum_{m=-n_{\theta w}}^{n_{\theta w}} k_{j,j+m}^{\theta w} \sin m\eta h$$

$$\alpha^{\theta\theta} = \sum_{m=-n_{\theta\theta}}^{n_{\theta\theta}} k_{j,j+m}^{\theta\theta} \cos m\eta h. \quad (30)$$

The frequency equation can be obtained by setting the determinant of Eq. (30) equal to zero. Thus

$$a_r \omega^4 + b_r \omega^2 + c_r = 0 \quad (31)$$

where

$$\begin{aligned} a_r &= \rho^2 A I m_j^w m_j^\theta \\ b_r &= -[\rho A m_j^w (E I \alpha^\theta + \kappa G A \alpha^{\theta\theta}) + \kappa G A \alpha^w \rho I m_j^\theta] \\ c_r &= [\kappa G A \alpha^w (E I \alpha^\theta + \kappa G A \alpha^{\theta\theta}) + (\kappa G A)^2 \alpha^{w\theta} \alpha^{\theta w}] \end{aligned} \quad (32)$$

It is noted that by replacing m_j^w and m_j^θ with $\tilde{m}^w$ and $\tilde{m}^\theta$, respectively, the consistent mass or mixed mass matrices formulation can also be obtained. The parameters, $\tilde{m}^w$ and $\tilde{m}^\theta$, are given as

$$\begin{aligned} \tilde{m}^w &= [(1-r) m_j^w + r \alpha^{ww}] \\ \tilde{m}^\theta &= [(1-r) m_j^\theta + r \alpha^{\theta\theta}] \end{aligned} \quad (33)$$

where

$$\alpha^{ww} = \sum_{m=-n_{ww}}^{n_{ww}} k_{j,j+m}^{ww} \cos m\eta h. \quad (34)$$

$\tilde{m}^w$ and $\tilde{m}^\theta$ give a lumped mass formulation when $r = 0$, and a consistent mass matrix formulation when $r = 1$.

Note that Eq. (31) is not only a function of the wave number, η , and frequency, ω , it is also a function of the *two dilation parameters for the transverse displacement and rotation*, a_w and a_θ , respectively. The optimal values for the dilation parameters can be selected such that the discrete frequency relation Eq (31) is the same as that of the continuum frequency equation. In other words, by performing a dispersion analysis of the RKPM Timoshenko beam an optimal ratio of the dilation parameters for the transverse displacement and rotation can be determined. This optimal ratio dictates that the interpolation functions for the transverse displacement and rotation are constructed with a correct match between the bending and transverse shear frequencies (wave numbers or scales).

By substituting the exact wave number into both the discrete frequency relation Eq. (31) and the continuum frequency equation, the optimal values of dilation parameters a_w and a_θ can be found so that the frequency error is minimized. The comparison of the RKPM and the analytic dispersion relation for the Timoshenko beam is depicted in Fig. 4.

Figure 4 Dispersion relation with varying energy ratio for a_θ

NUMERICAL RESULTS

Radiation for an Arch

Problem statement

$$\nabla^2 \phi + \kappa^2 \phi = 0 \quad \text{in } \Omega \quad (35)$$

with boundary conditions

$$\phi(1) = \begin{cases} 1 & -\alpha \leq \theta \leq \alpha \quad (\alpha=30^\circ) \\ 0 & \text{otherwise} \end{cases} \quad (36)$$

and Sommerfeld condition at the infinity

$$\lim_{r \rightarrow \infty} \Gamma(\phi_r - i\kappa\phi) = 0 \quad (37)$$

In this example, Sommerfeld condition is replaced by a 1st order DtN mapping on $r=2$, which is given as

Original Mesh (684 nodes=19x36)

Figure 5(a) an hp-like adaptivity in acoustic problem. (Mesh and Solution)

Figure 5(b) an hp-like adaptivity in acoustic problem. (Decomposition)

Adaptive Refinement 2 (1805 nodes)

Figure 5(c) an hp-like adaptivity in acoustic problem (Adaptivity)

In this example, Sommerfeld condition is replaced by a 1st order DtN mapping on $r=2$, which is given as

$$B_1 \phi|_{r=2} = 0 \quad (38)$$

where

$$B_1 = \frac{\partial}{\partial r} - ir + \frac{1}{2r} \quad (39)$$

The meshless hp-adaptivity is given in Fig. 5. As can be seen in the figure, only a set of nodes is required for adaptivity, and there is no apprehension of compatibility problem.

Wave propagation in Bar

The governing equation for the one dimensional wave propagation in a bar of length L in the frequency domain is given as

$$\frac{d^2 \phi}{dx^2} + k^2 \phi = F(x) \quad \phi(0) = \phi_0 \quad \phi(L) = \phi_L \quad (40)$$

where k is the wave number.

Figure 6 Wave propagation in bar, Case (i)

Figure 7 Wave propagation in bar, Case (ii)

Two numerical examples are performed to compare the results of the RKPM with the available analytical solutions:
Case (i): Homogeneous differential equation with nonhomogenous boundary conditions, i.e.

$$F(x) = 0, \phi(0) = 1 \text{ and } \phi(L) = 0$$

It can be shown that the analytical solution for Case (i) is

$$\phi(x) = \frac{\sin(k(L-x))}{\sin(kL)} \quad (41)$$

Case (ii) : nonhomogeneous differential equation with homogenous boundary conditions, with $F(x) = 1$. The analytical solution for this case is given as

$$\phi(x) = \frac{1}{k^2} \left[1 - \cos(kx) + \frac{\cos(kL) - 1}{\sin(kL)} \sin(kx) \right] \quad (42)$$

Figure 8(a) Case (i)

Figure 8(b) Case (ii)

Figure 8 Convergence rate with different wave number, k

Figure 9(a) Transverse displacement

Figure 9(b) Rotation

In numerical examples, the bar length is taken as 10 units. The RKPM is used with 11, 21 and 41 particles to obtain a measure for the convergence rate. In order to show the significance of the dilation parameter, ' α ', several cases are considered by keeping the number of the particles constant and varying ' α '. The comparison of middle point displacement between the analytical and the RKPM solutions are depicted in Fig. 6 and Fig. 7. For both of the numerical examples introduced earlier, the RKPM solution yields excellent agreement for low wave numbers.

$$U = \frac{U_{\text{RKPM}}}{U_{\text{Analytic}}} \quad (43)$$

With increasing dilation parameter the RKPM solution correctly represents a bigger portion of the wave number spectrum. However, the difference between the analytical and the RKPM solutions start deteriorating beyond a value of 1.17 for the dilation parameter. This implies the existence of an optimum value of the dilation parameter.

By increasing the number of particles, the convergence of the RKPM solution to the analytical solution is determined. Figures 8 depict the rate of convergence for cases (i) and (ii).

For higher frequencies, the convergence rate is higher. Even with a relatively small number of particles a convergence rate of 4 indicates the effectiveness of the RKPM.

Timoshenko Beam

To present the locking phenomenon in the Timoshenko beam, we examine the static simply-supported steel beam with a rectangular cross-section and thickness 0.1 cm and carrying a uniform load. The analytical solution is given as:

$$\theta = \frac{1}{EI} \left(\frac{L^2}{8} x - \frac{x^3}{6} \right) \quad (44)$$

and

$$w = \frac{1}{2\kappa GA} \left(x^2 - \frac{L^2}{4} \right) - \frac{1}{EI} \left(\frac{x^4}{24} - \frac{L^2 x^2}{16} + \frac{5L^4}{384} \right) \quad (45)$$

The results shown in Figure 9 depict an excellent agreement between the RKPM solution and the analytic solution for both transverse displacement and rotation. In contrast to FEM, no locking is observed with full integration of RKPM.

7. CONCLUSIONS

Preliminary results demonstrate that the multiple scale methods developed here have great promise in dealing effectively with vibration control, noise control, structural dynamics in which the medium frequencies are important. Unlike the usual perturbation approaches, which are based on scaling of small parameters, multiple scale methods are able to handle a wider class of problems. The developed methods take advantage of a frequency/wave number shift and frequency/wave-number band approximations; however, they are developed in a space-time domain.

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