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Transverse Instability Driven by Trapped Electrons

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Abstract

An instability of a positron beam can be driven by electrons trapped in the combined average electric field of the beam and the magnetic field of the dipoles. An estimate of the growth rate is given. The growth rate of the instability is smaller than the growth rate of the fast ion instability by a factor 50 for the PEP-II LER parameters.

Tendency to use long trains of bunches in storage rings intensified interest of stability of such trains. In the last year, two new instabilities were discovered: fast transverse instability, driven by one-turn ions for an electron storage ring^[1], and a transverse instability driven by interaction of a positron beam with photo-electrons (Ohmi-effect^[2]). Cornell instability has been also explained^[3] as instability driven by the interaction of a beam with electrons trapped in the combined magnetic field of dipoles and the electrostatic field of distributed ion pumps (DIPs) leaking into the beam pipe. All these instabilities are caused by the effective wake fields defined by the charge density of the beam environment rather than due to geometric or resistive wall wake fields.

Another instability of a positron beam that has features of all three instabilities mentioned above is considered in this paper: electrons produced mostly by photo-effect as for Ohmi-effect, they interact with the beam similarly to the interaction of ion instability, and they are trapped by combined electric and magnetic fields as

it takes place for the CESR instability. For the instability under consideration, the electric field is the average field of a positron beam rather than the field of DIPs, and electrons are produced by photo-effect on the residual gas.

It is worth noting that the kinematics of electrons used in the Ohmi-effect is based on the assumption that photo electrons are generated in straight sections between dipoles where the magnetic field is zero. However, the experimental observation in the Photon Factory indicates that at least partially the instability is caused by processes within dipoles.

A simple consideration of relevant effects and the rough estimate of the growth rate is presented here. Parameters of the low energy ring (LER) of PEP-II^[4] are used for illustrations.

The flux of synchrotron radiation (SR) in the high energy, high current machines like LER is substantial. For example, the LER beam produces SR power $P_0 = 32.6$ W/cm at a current $I = 2$ A, and an energy $E = 3$ GeV. In addition to the high P_0 , the cross-section of the photo-effect is maximum at low energies, close to the ionization potential of atoms, which enhances the flux of photo-electrons. Low energy electrons generated at the walls cannot, however, come close to the beam in dipoles because the Larmor radius in the magnetic fields of the bending dipoles is small. For example, it is equal to 4.6μ in the LER field $B_0 = 0.75$ T (bend radius $\rho = 13.5$ m). Nevertheless, photo-electrons can be generated directly in the residual gas with the rate dN_e/ds that is comparable to the rate dN_i/ds of ion production by a bunch in inelastic collisions. Estimating the ratio $\Delta = N_e/N_i$ for Thomas-Fermi atom with atomic number Z and for a short dipole $l_d < \sqrt{2b\rho}$ with bending radius ρ we get

$$\Delta = 0.12 \frac{P_0}{\hbar\omega_c} \frac{l_d}{I/e} Z^{7/9} \left(\frac{U_0}{\hbar\omega_c} \right)^{1/3} \left(\frac{a_0^2}{\sigma_{ion}} \right). \quad (1)$$

Here $l_d = 0.5$ m is dipole length, $\hbar\omega_c = 5$ keV is the critical energy of SR, $U_0 = 13.5$ eV, $a_0 = 0.5 \times 10^{-8}$ cm. The numbers here correspond to LER parameters giving $\Delta \simeq 0.45$. For the cross-section $\sigma_{ion} = 2$ Mbarn, the rate of inelastic collision is

$$\frac{dN_i}{ds} = 0.06 \frac{p}{T_{orr}} N_b \text{ cm}^{-1}, \quad (2)$$

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where p is pressure of the residual gas and N_b is the number of particles per bunch. Hence, the rate of photo-electron production is $dN_e/dz = 16$ of photo-electrons per cm. An additional number of electrons can be generated by the SR coming from upstream dipoles, and 10% of secondary SR scattered from the wall.

Photo electrons spiral along the magnetic field lines with large frequency: $\omega_L \sigma_l / c = 4.3$ for the LER bunch rms length $\sigma_l = 1$ cm. Therefore, the Larmor motion can be averaged out and a Larmor circle can be considered as a macroparticle.

The horizontal component of the average electric field E_x of a beam combined with the dipole magnetic field B causes longitudinal drift with a drift velocity of $v_d/c = E_x/B_0$. The field E_x is defined by the average potential of the beam

$$E_x = -\frac{\partial \bar{U}}{\partial x}, \quad \bar{U} = -\frac{Z_0 I}{4\pi} \ln \frac{b^2}{r^2}, \quad (3)$$

where $Z_0 = 120\pi$ Ohm, and $b = 3$ cm is the beam pipe radius, giving $v_d/c = 0.5 \times 10^{-4}$. A Larmor circle drifts by negligible 0.6×10^{-2} cm in the time between bunches, $s_b = 1.2$ m. Without a gap in the bunch train, the equilibrium electron density

$$n_{av} = \frac{l_d dN_e/dz}{(v_d/c)s_b} \quad (4)$$

would be large, $n_{av} = 3. \times 10^5$. However, electrons are cleared out by a gap, and only electrons accumulated in one revolution time have to be taken into account.

The Larmor circles are trapped in the vertical motion by the average field E_x . The frequency Ω of vertical oscillations depends on the distance $r^2 = x^2 + y^2$ of an electron from the beam. For small $y \ll r$,

$$\Omega^2 \simeq \frac{eZ_0 I}{2\pi} \frac{1}{mx^2}. \quad (5)$$

The frequency Ω is about the same as the frequency of ion oscillations at small amplitudes: the factor mx^2 at $x \simeq 1$ cm is comparable with $M\sigma_x\sigma_y$ for ions with mass M .

The oscillation are unstable for small x . This can be proved in a standard way considering a transform from the coordinates $y, \dot{y}$ to coordinates $\bar{y}, \dot{\bar{y}}$ over a period s_b/c between bunches:

$$\bar{y} = \dot{y} - \Lambda y, \quad \dot{\bar{y}} = y + \frac{s_b}{c} \dot{y}, \quad (6)$$

where

$$\Lambda = \frac{|e|IZ_0}{2\pi mc^2} \frac{s_b}{r^2}. \quad (7)$$

The motion is unstable for

$$\frac{x}{s_b} > \sqrt{\frac{|e|IZ_0}{8\pi mc^2}}, \quad (8)$$

or $x > 1$ cm for LER parameters. At smaller x a Larmor circle is unstable similar to the well known overshoot of the betatron oscillations. The maximum depth of the potential well Eq. (3) for stable electrons is $eU_m = 130$ eV at $x = 1$ cm, $b = 3$ cm, and $I = 2$ A.

The n -th bunch in a train with the vertical offset of the bunch centroid $y_n \ll x$ produces photo electrons with the same offset and changes the momentum of trapped electrons, produced by previous bunches, by

$$\Delta P = -\partial V_n / \partial Y = \frac{Z_0 I}{2\pi} \frac{(Y - y_n)}{x^2}. \quad (9)$$

Here y_n is the centroid of the n -th bunch taken at the moment $t_n = s + ns_b/c$, when it passes the location s of the ring.

Trapped electrons with the density $f_n(y, t)$ affect the position of the centroids of the following bunches:

$$\ddot{y}_m(t) + \omega_b^2 y_m(t) = -\frac{r_0}{\gamma} \sum_n^m \frac{\partial u_n}{\partial y}, \quad (10)$$

where

$$u_n = \frac{1}{2b} \frac{\partial N_e}{\partial z} \int \frac{d\vec{r}' f_n(y', t)}{|\vec{r} - \vec{r}'|}. \quad (11)$$

The system of coupled equation for the centroid of the k -th bunch situated at the distance $z = ks_b$ from the head of the train, $y_k(t) = y(t, z)$, and the position

$Y_l(t, s)$ of the centroid of a group of photo electrons, generated at the location s of the ring by the l -th bunch, are the same as for ion instability^[1]:

$$Y_l''(t, s) = -\Omega^2[Y_l(t, s) - y(t, t - z)], \quad (12)$$

$$y''(t, z) + \omega_b^2 y(t, z) = -\frac{\pi r_0}{bx\gamma} \frac{dN_c}{ds} \sum_{l=1}^k [y(t, z) - Y_l(t, t - z)]. \quad (13)$$

The only difference is that the product of transverse rms bunch size $\sigma_x \sigma_y$ is replaced by bx , where $x \simeq 1$ cm is the minimum distance of stable trapped electrons from the beam. All results of the ion instability can be retained with corresponding replacement. In particular, the amplitude of vertical oscillations of the n -th bunch in a train of n_b bunches is

$$y_n(t) = a_0 e^{(n/n_b)\sqrt{s/s_b}} \quad (14)$$

where

$$\frac{1}{s_c} = \frac{(\Omega s_b n_b)^2}{2\Omega\omega_b} \left(\frac{dN_i}{ds} \right) \left(\frac{r_0}{\gamma s_b bx} \right). \quad (15)$$

The growth rate is reduced in comparison with the ion instability by a factor $\sqrt{bx/\sigma_x \sigma_y} \simeq 50$ times for PEP-II $\sigma_x \sigma_y = 10^{-3}$ cm².

Frequency spread of trapped electrons due to the nonlinearity of the potential well further reduces the growth rate of the instability similarly to the reduction of the growth rate of the ion instability^[5]. The quasi-exponential growth Eq. (14) slows down and the growth become linear in time when amplitudes of trapped electrons become large, $Y_l > \sigma_y$ ^[6].

This regime can be studied describing trapped electrons by a distribution function ρ_0 with temperature $T \simeq U_m$,

$$\rho_0 = \sqrt{\frac{m\Omega^2}{2\pi T}} e^{-\frac{m\omega^2 y^2}{2T}}. \quad (16)$$

A small perturbation f due to the interaction of the electrons with an offset bunch,

$\rho = \rho_0 + f$, can be found from the linearized Vlasov equation

$$\frac{\partial f}{\partial t} + \{H_0, f\} = -\{V, \rho_0\}. \quad (17)$$

The perturbation of the density $f(y, t) = \int dp f(p, y)$ at the location s in the ring due to the kick of the n -th bunch is equal to

$$f_n(y, t) = -\frac{\Omega s_b}{c} \frac{2y y_n}{x^2} \rho_0(y) \sin \Omega(t - t_n) \quad (18)$$

where $y_n = y_n(t_n)$ is the position of the centroid of the n -th beam taken at $t_n(z) = (z + ns_b)/c$. f_n defines, see Eq. (11), a kick to the next bunch giving coupling between bunches.

In summary, the growth rate of the instability due to electrons trapped by combined E and B fields for the PEP-II LER parameters is smaller than the damping time of the transverse feedback system and is not dangerous for the PEP-II LER parameters.

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