1995 LDRD Final Report:
Electromagnetic Simulation of Electronic Packaging Designs
(95-ERP-003)

John A. Swegle
Lawrence Livermore National Laboratory
Livermore, CA 94551

RECEIVED
JUN 27 1996
OSTI

May 1, 1996

Work performed under the auspices of the U.S. Department of Energy by the
Lawrence Livermore National Laboratory under Contract W-7405-ENG-48.

MASTER

DISCLAIMER

This document was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor the University of California nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial products, process, or service by trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or the University of California. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or the University of California, and shall not be used for advertising or product endorsement purposes.

This report has been reproduced
directly from the best available copy.

Available to DOE and DOE contractors from the
Office of Scientific and Technical Information
P.O. Box 62, Oak Ridge, TN 37831
Prices available from (615) 576-8401, FTS 626-8401

Available to the public from the
National Technical Information Service
U.S. Department of Commerce
5285 Port Royal Rd.,
Springfield, VA 22161

1995 LDRD Final Report:

**Electromagnetic Simulation of Electronic Packaging
Designs**

(95-ERP-003)

John A. Swegle

**Lawrence Livermore National Laboratory
Livermore, CA 94551**

1. Introduction

The increasing demand by military and commercial consumers for speed, compactness, and miniaturization in electronic products is leading to major technical advances in microelectronics technology. Increases in computer clock speeds and device packaging density (both on-chip and in multi-chip systems) have dramatically decreased the cost and increased the performance of mass-produced electronic devices during the last decade. For this trend to continue, however, a host of new technical challenges must be overcome; among the most important is the understanding and amelioration of electromagnetic effects. Short rise-time clock pulses in digital systems lead to radiation that can couple signals between chips in a high-density packaging technology such as the multi-chip module (MCM). In addition to interchip coupling, increasing crosstalk effects are observed as more circuits are packed onto individual chips. Coupling between interconnects causes spurious "on" and "off" states in logic elements, resulting in unreliable hardware. Circuits that work in isolation can become unreliable when located in proximity to other chips or interconnects due to time-dependent electromagnetic effects, which greatly complicates the design of complex systems.

Simulation tools presently being used in industry for packaging design are based on quasi-static analysis of lumped-circuit elements. The characteristics of a typical high-speed electronics packaging analysis problem, however, suggest the use of time-domain electromagnetic simulation. Though not yet applied to MCM design, 3D time-domain EM simulation is a proven technique that has been used successfully for an increasing number of applications such as radar cross-section calculations, evaluation of electromagnetic interference effects, phased array antenna design, and for design of individual components of microwave integrated circuits such as isolation vias. LLNL has been involved in the development of these time-domain electromagnetic codes for a number of years and is considered a leader in the field.

The primary focus of the project summarized in this report has been to evaluate the performance of the 3D, time-domain electromagnetic code DSI3D in the simulation of structures used in microwave microelectronics circuits. We've adopted two test cases, coaxial and stripline transmission lines, for which well-known results are available so that results obtained with DSI3D could be easily and accurately checked. Our goals have been three-fold:

1. To develop specialized mode-launching capabilities for single-mode signals typically found in test geometries and the diagnostics necessary to evaluate the performance of the code in modeling the propagation of those signals.
2. To analyze the effect of different zoning schemes on the accuracy with which the code models the propagation of signals through the geometries

by checking against known analytic results and calculations performed with other codes.

3. To examine the effect of code modifications aimed at enhancing the accuracy of the simulations.

The calculated transmission line impedance was chosen as the primary means of evaluating code performance. Since the lowest-order propagating modes for the test cases were transverse electromagnetic (TEM) modes, the computation of impedance was reasonably straightforward. Both time- and frequency-domain values (the latter obtained from the code output by post-processing with a discrete Fourier transform) were obtained and compared.

Section 2 contains a short introduction to the DSI3D code, Section 3 provides a discussion of the general features of electromagnetic waves in axially-symmetric transmission lines and reviews the calculation of impedance in transmission lines. Following that, Sections 4 and 5 are devoted to a description of the simulations of coaxial and stripline transmission lines, respectively. Finally, Section 6 presents an overall summary and conclusions.

2. The DSI3D Code

Here we present a brief introduction to the 3D electromagnetics code, DSI3D of Niel Madsen, where DSI stands for "discrete surface integration." Our introduction closely follows that of Ref. 1. DSI3D is a simulation code designed to compute electromagnetic fields in three dimensions using both structured and unstructured meshes (the latter being a mesh that cannot map to an orthogonal logical mesh described by integer indices i , j , and k). The added power of using unstructured meshes stems from the fact that such meshes can be made to conform to irregular shapes, thus allowing one to avoid either incurring "stair-stepping" errors at irregularly shaped boundaries or being forced to construct very highly resolved meshes to prevent the occurrence of such errors. The code solves Ampere's and Faraday's laws:

$$\frac{\partial \mathbf{D}}{\partial t} = \nabla \times \mathbf{H} \quad (2.1)$$

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E} \quad , \quad (2.2)$$

with $\mathbf{D}$, $\mathbf{E}$, $\mathbf{H}$, and $\mathbf{B}$ related by the constitutive relations

$$\mathbf{D} = \epsilon \mathbf{E} \quad , \quad (2.3)$$

$$\mathbf{B} = \mu \mathbf{H} \quad . \quad (2.4)$$

In general, the permittivity and permeability, ϵ and μ , can be variable, although in the work described in this report, they were invariant.

The DSI method of solution for Eqs. (2.1) and (2.2) requires the use of both primary and dual grids. Here, we assume that the primary grid is composed of convex polyhedra. The dual grid is completely derivable from the primary. Away from the exterior surfaces of the primary grid, the nodes of the dual grid are located at the barycenter of each cell of the primary grid, defined as the average of the coordinates of the nodes of that cell. Two nodes of the dual grid will be connected by a dual edge if and only if the two cells of the primary grid in which they lie share a common face. For cells of the primary grid that have a face lying on the boundary of the simulation region, a dual edge is formed by connecting the dual node at the barycenter of the cell with a second dual node formed at the barycenter of the face of the primary cell lying on the boundary. Figure 2.1 shows a single cell of the dual grid formed within an eight-cell section of a primary grid composed of hexahedral cells. In this particular example, the dual cell is itself a hexahedron, although in general the dual cell shapes can be quite irregular.

Fig. 2.1 An eight-cell section of the primary grid consisting of hexahedral cells and containing, at its center, a single cell of the dual grid.

DSI solves for field variables on the edges of the primary and dual grids. Electric field variables are projected onto the edges of the primary grid, forming $\mathbf{E} \cdot \mathbf{s}$ with $\mathbf{s}$ a primary cell edge vector. Magnetic fields are projected onto the edges of the dual grid to form $\mathbf{H} \cdot \mathbf{s}^*$, where $\mathbf{s}^*$ is a dual cell edge vector. Full magnetic and electric displacement field vectors, $\mathbf{B}$ and $\mathbf{D}$, are associated with the faces of the primary and dual grids, respectively. These associations are illustrated in Fig. 2.2.

Fig. 2.2 Associations between the primary and dual grids and the electric and magnetic field vectors.

To understand the DSI algorithm, assume that time is discretized with time step Δt . The time dependence of a field quantity is designated with a superscript, so that, for example, $E^k = E(t^k) = E(k\Delta t)$. In the leapfrog time integration scheme of DSI, magnetic fields are updated at half-integer time steps, $t^{k+1/2}$, while electric fields are updated at t^k . The area-normal vector to a primary mesh cell face is defined to be $N = \int n \, dS$, where n is a unit normal to the cell face. It can be shown that N is independent of the shape of the surface, so that piecewise planar approximating surfaces are sufficient for the code computations. Integrating Eq. (2.2) over a cell face in the primary mesh, F , yields

$$\int_F \left(\frac{\partial B^k}{\partial t} \cdot n \right) dS \equiv \frac{dB^k}{dt} \cdot N_F = - \int_F (\nabla \times E^k) \cdot n dS = \oint_F E^k \cdot dl \quad (2.5)$$

This equation permits the computation of the component of the magnetic field normal to the cell face F by summing the projections of the electric field around the perimeter of the face.

To compute a full vector value for the time derivative of B^k at F , consider Fig. 2.3 on the following page. As shown in the figure, the face F is shared by 2 primary grid cells, since F is interior to the simulation region (on the other hand, in the special case that F were to lie on the boundary of the simulation region, F would be shared by only 1 primary grid cell). In general, the face F has P primary edges and nodes. We denote by F_{ij} the face of the cell i (where i equals 1 or 2 in the case shown) sharing edge j (which runs from 1 to 5 in the example) with F . At each of the nodes of F , we compute the three components of the

Fig. 2.3 Primary grid faces used in the computation of the time-advanced magnetic field vector.

magnetic field associated with that node, $\mathbf{B}_{ij}^k$, by solving the 3x3 system of equations:

$$\frac{d\mathbf{B}_{ij}^k}{dt} \cdot \mathbf{N}_F = - \oint_F \mathbf{E}^k \cdot d\mathbf{l}$$

$$\frac{d\mathbf{B}_{ij}^k}{dt} \cdot \mathbf{N}_{F_{ij}} = - \oint_{F_{ij}} \mathbf{E}^k \cdot d\mathbf{l}$$

(2.6)

$$\frac{d\mathbf{B}_{ij}^k}{dt} \cdot \mathbf{N}_{F_{mj}} = - \oint_{F_{mj}} \mathbf{E}^k \cdot d\mathbf{l}$$

where $m = (i \bmod P) + 1$. Having determined the 10 (for this particular case) vector values $\mathbf{B}_{ij}^k$, the vector value $\mathbf{B}_F^k$ for the face F is calculated by interpolation:

$$\frac{dB_F^k}{dt} = \frac{\sum_{j=1}^2 \sum_{i=1}^P |w_{ij}| \frac{dB_{ij}^k}{dt}}{\sum_{j=1}^2 \sum_{i=1}^P |w_{ij}|}, \quad (2.7)$$

where the weight

$$w_{ij} = N_F \cdot (N_{F_{ij}} \times N_{F_{mj}}) \quad (2.8)$$

is the volume of the j^{th} local coordinate system at node i of face F , or, alternatively, the determinant of the set of Eqs. (2.6).

The full $\mathbf{B}$ vector is advanced in time using a time-centered leapfrog algorithm:

$$\mathbf{B}^{k+1/2} = \mathbf{B}^{k-1/2} + \frac{d\mathbf{B}^k}{dt} \Delta t \quad (2.9)$$

with Δt the time step. The projection of the time-advanced magnetic field onto the dual edge $\mathbf{s}^*$ is then easily accomplished:

$$(\mathbf{H} \cdot \mathbf{s}^*)^{k+1/2} = \frac{1}{\mu} \mathbf{B}^{k+1/2} \cdot \mathbf{s}^* \quad (2.10)$$

Advancing the electric field in time proceeds on the dual grid, as opposed to the primary grid for magnetic field advances. In the dual mesh, the area-normal vector is defined similarly: $\mathbf{N}^* = \int \mathbf{n}^* dS^*$, where $\mathbf{n}^*$ is the unit dual surface normal. Proceeding analogously to the magnetic field case, one associates an electric displacement vector with each corner of the dual face F^* using area normal vectors from the faces of the cells adjoining F^* that share common edges. Therefore, the displacement vectors are determined from the set of equations :

$$\frac{dD_{ij}^{k+1/2}}{dt} \cdot N_{F^*}^* = - \oint_{F^*} H^{k+1/2} \cdot dl^* \quad (2.11)$$

$$\frac{dD_{ij}^{k+1/2}}{dt} \cdot N_{F_{ij}}^* = - \oint_{F_{ij}} H^{k+1/2} \cdot dl^*$$

$$\frac{dD_{ij}^{k+1/2}}{dt} \cdot N_{F_{mj}}^* = - \oint_{F_{mj}} H^{k+1/2} \cdot dl^*$$

These values are interpolated as in Eq. (2.7) to arrive at a value of the displacement vector on the dual face F^* . Then, this value is advanced in time using a similar time-centered leapfrog scheme:

$$D^{k+1} = D^k + \frac{dD^{k+1/2}}{dt} \Delta t \quad (2.12)$$

Finally, a time-advanced value of the electric field projection onto the primary edge s is obtained by

$$(E \cdot s)^{k+1} = \frac{1}{\epsilon} D^{k+1} \cdot s \quad (2.13)$$

This essentially completes the DSI3D time-advance algorithm. Additional discussion can be found in Ref. 1.

3. Electromagnetic Waves in Axially-Symmetric Transmission Lines

3.1. General Features

The systems modeled in the course of this study have the character of axially-symmetric waveguides or transmission lines. To understand the general features of the electromagnetic modes in such systems, consider a general axially invariant waveguide of general cross section as shown in Fig. 3.1. The axis lies along z . In this geometry, absent free charges giving rise to currents and space charge, it is convenient to write the wave equation obeyed by both E and B as

$$\left[\nabla_{\perp}^2 + \left(\frac{\partial^2}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right) \right] \begin{Bmatrix} E \\ B \end{Bmatrix} = 0 \quad (3.1)$$

Fig. 3.1 A waveguide section with an arbitrary, axially-invariant cross section.

where

$$\nabla_{\perp}^2 = \nabla^2 - \frac{\partial^2}{\partial z^2}$$

takes account of variations of the fields in the plane transverse to the axis. Further, following Ref. 2, we can decompose the electric and magnetic fields into components along and transverse to the z-axis; i.e.,

$$\mathbf{E} = \mathbf{E}_{\perp} + \hat{z}E_z \quad (3.2)$$

With this decomposition, Maxwell's equations can be written in the following form:

$$\begin{aligned} \frac{\partial \mathbf{E}_{\perp}}{\partial z} - \hat{z} \times \frac{\partial \mathbf{B}_{\perp}}{\partial t} &= \nabla_{\perp} E_z & \hat{z} \cdot (\nabla_{\perp} \times \mathbf{E}_{\perp}) &= -\frac{\partial B_z}{\partial t} \\ \frac{\partial \mathbf{B}_{\perp}}{\partial z} + \frac{1}{c^2} \hat{z} \times \frac{\partial \mathbf{E}_{\perp}}{\partial t} &= \nabla_{\perp} B_z & \hat{z} \cdot (\nabla_{\perp} \times \mathbf{B}_{\perp}) &= \frac{1}{c^2} \frac{\partial E_z}{\partial t} \\ \nabla_{\perp} \cdot \mathbf{E}_{\perp} &= -\frac{\partial E_z}{\partial z} & \nabla_{\perp} \cdot \mathbf{B}_{\perp} &= -\frac{\partial B_z}{\partial z} \end{aligned} \quad (3.3)$$

In the event that there are two or more separated conductors running parallel to the axis of the system, as is the case for a coaxial, cylindrical transmission line for example, a simple solution for this system results when $E_z = B_z = 0$. This condition implies that

$$\nabla_{\perp} \times \mathbf{E}_{\perp} = 0 \quad \text{and} \quad \nabla_{\perp} \cdot \mathbf{E}_{\perp} = 0 \quad (3.4)$$

The solutions to this pair of equations, the transverse electromagnetic (TEM) modes, are quasi-electrostatic in character. They also have the character of free-space waves in that $\mathbf{E}$ and $\mathbf{B}$ are related by

$$\mathbf{B}_{\text{TEM}} = \pm \frac{1}{c} \hat{\mathbf{z}} \times \mathbf{E}_{\text{TEM}} , \quad (3.5)$$

where both fields are transverse. Further, the transverse spatial dependence separates from the z and t dependence of the fields. They can thus be written in the form

$$\mathbf{E}_{\text{TEM}}(\mathbf{x}_{\perp}, z, t) = \mathbf{E}_{\perp}(\mathbf{x}_{\perp}) \left[g^{-}(z - ct) + g^{+}(z + ct) \right] ; \quad (3.6)$$

a similar form applies for the magnetic field. The exact form of the transverse portion of the TEM electric and magnetic fields, $\mathbf{E}_{\perp}$ and $\mathbf{B}_{\perp}$, must be determined by solving the two-dimensional (in the transverse plane) Laplace's equation for the boundary conditions of the particular geometry being investigated.

When conditions (3.4) do not apply, our freedom in separating the spatial and temporal dependences of the fields is somewhat more restricted. In this case, though, we can assume that the fields have a complex exponential dependence on z and t :

$$\mathbf{E} = \mathbf{E}_{\perp}^{\pm}(\mathbf{x}_{\perp}) \exp[i(kz \pm \omega t)] . \quad (3.7)$$

Signals of arbitrary shape can be constructed from Fourier sums or integrals of modes with different frequency and wavenumber. Now, assuming that the walls of the waveguide are perfect conductors, the tangential component of $\mathbf{E}$ and the normal component of $\mathbf{B}$ must vanish at that surface, S . Therefore, for our axially-invariant system, the condition on the tangential component of $\mathbf{E}$ is

$$E_z = 0 \text{ on } S , \quad (3.8)$$

while the condition on the normal component of $\mathbf{B}$ becomes, according to the second equation on the left of (3.3),

$$\frac{\partial B_z}{\partial \mathbf{n}} = 0 \text{ at } S, \quad (3.9)$$

where the derivative in (3.9) is taken normal to the surface. These differing boundary conditions on $\mathbf{E}$ and $\mathbf{B}$ allow us to further decompose the solutions to the wave equation governing the fields into two separate sets of modes:

- *Transverse magnetic (TM) modes*

with $B_z = 0$ everywhere, and $E_z = 0$ on S .

- *Transverse electric (TE) modes*

with $E_z = 0$ everywhere, and $\frac{\partial B_z}{\partial n} = 0$ on S .

For the TE and TM modes, it can be shown that the eigenmodes of Maxwell's equations, subject to the boundary conditions (3.8) and (3.9), obey a dispersion relation between frequency ω and the wavenumber k of the general form

$$\omega^2 = \omega_{co}^2 + k^2 c^2, \quad (3.10)$$

where the cutoff frequency, ω_{co} , depends on the geometry of the waveguide. There will be a hierarchy of cutoff frequencies, varying according to whether the modes are TM or TE and increasing in value as the structure of the mode in the complex plane becomes more complex.

The dispersion relation given by (3.10) implies that the phase velocity for the TE and TM modes,

$$v_{ph} = \frac{\omega}{k}, \quad (3.11)$$

will vary with k . Therefore, a pulse composed of a sum (or integral) of waves with different wavenumbers will not maintain its shape as it propagates along the transmission line; it will spread as the different component waves in the sum separate from one another. The TEM waves, on the other hand, are dispersionless. If we make the specific choice that their z and t dependence is given by a complex exponential, as in (3.7), their frequency and wavenumber are related by the free-space dispersion relation

$$\omega = kc. \quad (3.12)$$

Thus, the TEM modes, consistent with the general form given in (3.6), will not disperse as they propagate. Instead, their axial and temporal pulse shape will be maintained during propagation along the transmission line.

3.2. Defining Current and Voltage

Computing the current and voltage in a transmission line requires some definitions and assumptions. First, to look at the current, we refer to the differential form of Ampere's law,

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{j} + \frac{1}{c^2} \frac{\partial \mathbf{E}}{\partial t} . \quad (3.13)$$

Here, we have included the current density to take account of surface currents in the walls of the waveguide or transmission line. As in the previous section, we take the axis of the transmission line to lie along z . Integrating (3.13) over the cross-sectional area of the line, A , yields

$$\int_A \nabla \times \mathbf{B} \cdot d\mathbf{S} = \oint_C \mathbf{B} \cdot d\boldsymbol{\ell} = \mu_0 I + \frac{1}{c^2} \frac{\partial}{\partial t} \int_A \mathbf{E} \cdot d\mathbf{S} . \quad (3.14)$$

In the area integrals in this equation, the surface A is in the plane perpendicular to the axis, while $d\mathbf{S}$ lies along the axis (i.e., along z), while the contour in the line integral, P , lies on the perimeter of A . Therefore, in a transmission line, the current is accurately computed for TE and TEM modes, which have no axial electric field component, from

$$I = \frac{1}{\mu_0} \oint_C \mathbf{B} \cdot d\boldsymbol{\ell} = \oint_C \mathbf{H} \cdot d\boldsymbol{\ell} . \quad (3.15)$$

For TM modes, on the other hand, there is an ambiguity. In this case, one either applies (3.15) as a definition and lumps the time derivative of the electric flux into the current, or one computes the time derivative of the flux separately from some knowledge of the transverse mode structure of E_z and applies it as a correction on the right side of the equation. In our code tests, we dealt with systems with center conductors that would support TEM modes, so we could typically apply (3.15) to these modes with no ambiguity.

Similarly, ambiguities can arise in defining a voltage. To clarify, we start with the formulation of the electric field in terms of the scalar and vector potentials, ϕ and A :

$$\mathbf{E} = -\nabla\phi - \frac{\partial \mathbf{A}}{\partial t} . \quad (3.16)$$

Here, the gradient of ϕ represents the curl-free, or conservative, portion of the field. Accordingly, a path integral of this portion of the electric field will depend only on the endpoints and not on the path itself. Taking the curl of (3.16), bearing in mind that

$$\mathbf{B} = \nabla \times \mathbf{A} , \quad (3.17)$$

yields the differential form of Faraday's law:

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} . \quad (3.18)$$

If we integrate over a cross-sectional surface A again and let the outer and inner boundaries of the surface lie just inside the perfect conductors, C_1 and C_2 , where there are no fields, we get

$$\int_S \nabla \times \mathbf{E} \cdot d\mathbf{S} = \int_{C_2} \mathbf{E} \cdot d\boldsymbol{\ell} \text{ (path 1)} + \int_{C_1} \mathbf{E} \cdot d\boldsymbol{\ell} \text{ (path 2)} = -\frac{\partial}{\partial t} \int_A \mathbf{B} \cdot d\mathbf{S} . \quad (3.19)$$

Now, the right side of (3.19) vanishes for TM and TEM waves, so that the line integrals on the left side of the equation must cancel. Therefore,

$$\int_{C_2} \mathbf{E} \cdot d\boldsymbol{\ell} \text{ (path 1)} = -\int_{C_1} \mathbf{E} \cdot d\boldsymbol{\ell} \text{ (path 2)} = \int_{C_1} \mathbf{E} \cdot d\boldsymbol{\ell} \text{ (path 2)} , \quad (3.20)$$

so that the path integrals are independent of the path chosen. Thus, we can compute the voltage by taking the integral along any convenient path between the two conductors. Taking account of (3.16), with the integral involving the vector potential vanishing, we write

$$V \equiv \phi(C_2) - \phi(C_1) = -\int_{C_1}^{C_2} \mathbf{E} \cdot d\boldsymbol{\ell} . \quad (3.21)$$

As was the case with the computation of the current, (3.21) is strictly accurate only for certain modes, in this case the TM and TEM modes. Otherwise, for TE modes, one must either apply (3.21) by definition or compute a correction using (3.16) and (3.19) as well as a knowledge of the transverse structure of the modes in question.

3.3. Defining the Impedance

In loose terms, we could define the impedance Z as the ratio of the voltage to the current. Some clarification is in order, though, if the impedance is to be computed accurately and unambiguously using values obtained from the code. On a computational mesh, the voltage is computed at some distance z along the

axis as some function of time by taking the line integral of the electric field as prescribed in (3.21). In addition, we define the Fourier transform of the voltage

$$\tilde{V}(z, \omega) = \int_{-\infty}^{\infty} V(z, t) e^{i\omega t} dt . \quad (3.22)$$

Similarly, we define the current via Eq. (3.15) and its Fourier transform as in (3.22). On a computational mesh with uniform spacing along the axis, however, the magnetic fields will be computed not at z , but one-half zone away, at $z + \Delta z/2$. Thus, we would compute

$$\tilde{I}(z + \Delta z/2, \omega) = \int_{-\infty}^{\infty} I(z + \Delta z/2, t) e^{i\omega t} dt . \quad (3.23)$$

Now, in the case of a TEM mode, which travels along the axis at the speed of light $c = (\mu\epsilon)^{-1/2}$, we can time-shift to get $I(z)$, under the assumption that numerical dispersion is negligible over this small distance:

$$I(z, t) = I(z + \Delta z/2, t + \Delta z/2c) . \quad (3.24)$$

Inserting (3.24) in (3.23) and changing variables to $t' = t + \Delta z/2c$, we get

$$\tilde{I}(z, \omega) = \tilde{I}(z + \Delta z/2, \omega) e^{i\omega \Delta z/2c} . \quad (3.25a)$$

Alternatively, one could compute the current at $z - \Delta z/2$ and reverse the time shift to get

$$\tilde{I}(z, \omega) = \tilde{I}(z - \Delta z/2, \omega) e^{-i\omega \Delta z/2c} . \quad (3.25b)$$

Now, we consider the calculation of the impedance in the frequency and time domains. In the frequency domain, we define the impedance by

$$\tilde{Z}(z, \omega) \equiv \frac{\tilde{V}(z, \omega)}{\tilde{I}(z, \omega)} . \quad (3.26)$$

If we directly transform the values of the current computed at $z \pm \Delta z/2$, though, what we have in hand is $\tilde{I}(z \pm \Delta z/2, \omega)$. Using these values, we get

$$\frac{\tilde{V}(z, \omega)}{\tilde{I}(z \pm \Delta z/2, \omega)} = \frac{\tilde{V}(z, \omega)}{\tilde{I}(z, \omega) e^{\pm i\omega \Delta z/2c}} = \tilde{Z}(z, \omega) e^{\mp i\omega \Delta z/2c} . \quad (3.27)$$

Thus,

$$\tilde{Z}(z, \omega) = \frac{\tilde{V}(z, \omega)}{\tilde{I}(z \pm \Delta z/2, \omega)} e^{\pm i\omega \Delta z/2c} , \quad (3.28)$$

and if we are only interested in the magnitude of the impedance, this computation is good enough, since

$$|\tilde{Z}(z, \omega)| = \left| \frac{\tilde{V}(z, \omega)}{\tilde{I}(z \pm \Delta z/2, \omega)} \right| . \quad (3.29)$$

On the other hand, if we attempt to find $\tilde{I}(z, \omega)$ by averaging, e.g.,

$$\tilde{I}_{\text{avg}} \equiv \frac{1}{2} [\tilde{I}(z + \Delta z/2, \omega) + \tilde{I}(z - \Delta z/2, \omega)] , \quad (3.30)$$

we get, from (3.25a) and (3.25b),

$$\tilde{I}_{\text{avg}} = \tilde{I}(z, \omega) \cos\left(\frac{\omega \Delta z}{2c}\right) \quad (3.31)$$

and

$$\frac{\tilde{V}(z, \omega)}{\tilde{I}_{\text{avg}}} = \frac{1}{\cos\left(\frac{\omega \Delta z}{2c}\right)} \tilde{Z}(z, \omega) . \quad (3.32)$$

This latter relation is approximately accurate, providing

$$\omega \ll \frac{2c}{\Delta z} , \quad (3.33)$$

but becomes increasingly inaccurate for larger frequencies.

The small time shift in measuring the current a half zone width away from the voltage measurement affects the time-domain impedance similarly. Ideally, for a TEM wave traveling only in the +z direction, $V(z, t)$ and $I(z, t)$ will, according to (3.6), have the same z and t dependence, $g(z-ct)$. Therefore, the impedance should be a constant dependent only on the geometry of the line whether we compute it in the frequency or the time domain:

$$Z(z,t) \equiv \frac{V(z,t)}{I(z,t)} = \tilde{Z}(z,\omega) = Z_0 . \quad (3.34)$$

Once again, though, on the computational mesh, with the code, we compute not $I(z,t)$, but $I(z \pm \Delta z/2, t)$. Time shifting, we get

$$I(z \pm \Delta z/2, t) = I(z, t \mp \Delta z/2c) . \quad (3.35)$$

Now, if we attempt to ignore the time shift by making the approximation

$$Z \approx \frac{V(z,t)}{I(z \pm \Delta z/2, t)} , \quad (3.36)$$

the error can be significant. Consider, as an example, the impedance estimate using (3.36) for an ideal Gaussian pulse TEM wave defined by

$$V(\tau = t - z/c) = V_0 \exp \left[-\frac{(\tau - t_{\text{peak}})^2}{t_{\text{width}}^2} \right] \quad (3.37)$$

$$I(\tau = t - z/c) = \frac{V_0}{Z_0} \exp \left[-\frac{(\tau - t_{\text{peak}} - \Delta t)^2}{t_{\text{width}}^2} \right]$$

If the shift, $\Delta t = \Delta z/2c$, were zero, the impedance of the wave would be Z_0 . In Fig. 3.2, we show how the impedance varies with time at fixed z for $t_{\text{peak}} = 600$ ps, $t_{\text{width}} = 150$ ps, and $\Delta t = 0, 1, 2, 3$, and 4 ps. We can see that even though the time shift is, at most, a few percent of the width of the Gaussian, the impedance shifts are substantial. In Fig. 3.3, we compute the current by averaging two values of the current with $\Delta t = 0, \pm 1, \pm 2, \pm 3$, and ± 4 ps. Because the averaged value of the current is essentially second order in the shift when one expands about the maximum of the Gaussian, the impedances calculated this way are quite accurate around the peak. A blow-up of the area around the peak shown in Fig. 3.3b indicates the increasing softening of the edges of the impedance curve outside the peak region as the time shift grows in magnitude.

In the special case of TEM waves, one can nevertheless take advantage of the fact that the z and t dependence for the current and voltage will be the same. Further, because these waves are dispersionless (except perhaps for numerical dispersion, which we will ignore at this point), the peak value of the current measured as a function of time at two different locations will be the same, even

Fig. 3.2 Impedance estimate as a function of the shift Δt based on Eq. (3.36) for an ideal 50- Ω line carrying a Gaussian pulse described by Eq. (3.37) with the parameters $t_{\text{peak}} = 600$ ps and $t_{\text{width}} = 150$ ps.

Fig. 3.3 An approximation to the impedance of an ideal 50- Ω line carrying a Gaussian pulse based on averaging currents measured with shifts of $\Delta t = 0, \pm 1, \pm 2, \pm 3$, and ± 4 ns; (b) is a blow-up of (a) in the vicinity of the peak impedance.

though the time at which the peak occurs will be shifted between the two locations. Therefore, in this special case, it will be sufficient to compare peak values of V and I , even if they are measured by computing these peaks at nearby, but not coincident, points to find the impedance.

Finding the impedance associated with transmission line waves composed of some sum of TE and TM modes, however, is not as straightforward. The phase of each mode advances as

$$\phi_{\pm} = kz \pm \omega t = k(z \pm v_{ph}t) , \quad (3.38)$$

and, unfortunately, the phase velocity varies with the wavenumber. This introduces a number of complicating factors. First, unless one is dealing with a single mode at a single frequency and single wavenumber, dispersion of the wave packet will affect the error introduced by spatial shifts in the data. Second, it will be more difficult to correct for the spatial shifts in the data by time shifting because, in the case of a multimode wave packet, there will be a different phase velocity for each mode to be applied in calculating the time shift; even in the case of a single mode, the necessary time shift will have to be computed with a phase velocity in mind that varies with the wavenumber. Finally, phase shifts induced by inductive and capacitive effects will make the time-domain impedance less meaningful, necessitating the use of a frequency-domain – and frequency-dependent – impedance found by Fourier-transforming the current and voltage.

4. Simulations of a Coaxial Line

4.1. Fields, Impedance, and Non-TEM Modes

An ideal coaxial cylindrical transmission line consists of coaxial inner and outer conductors with circular cross sections of radii r_i and r_o . Here, the outer line will be assumed to be grounded at zero potential, while the inner line will be held at a positive potential V_0 . The azimuthally-invariant TEM waves within this structure can be cast in the form shown in Eq. (3.6), where the transverse structure of the electric field is governed by the equation

$$\frac{1}{r} \frac{d}{dr}(rE_r) = 0 , \quad (4.1)$$

which has the solution $E_r = E_0/r$. The field magnitude E_0 can be related to V_0 by integrating the solution to (4.1), using (3.21):

$$V_0 = E_0 \ln\left(\frac{r_o}{r_i}\right) . \quad (4.2)$$

The magnetic field between the conductors, which is azimuthal, can be gotten from (3.5):

$$B_\theta = \frac{E_0}{cr} = \frac{V_0}{\ln(r_0/r_1)} \frac{1}{cr} . \quad (4.3)$$

Integrating this expression according to (3.15) yields the current in the line:

$$I = \frac{1}{\mu_0} \oint \mathbf{B} \cdot d\boldsymbol{\ell} = 2\pi \left(\frac{\varepsilon}{\mu_0} \right)^{1/2} \frac{V_0}{\ln(r_0/r_1)} , \quad (4.4)$$

where $\varepsilon = \varepsilon_r \varepsilon_0$, with ε_0 the free-space permittivity, and μ_0 is the free-space permeability, which is used everywhere in this report. Now, the TEM impedance is the ratio of the voltage V_0 to the current I :

$$Z_{\text{TEM}} = \frac{1}{2\pi} \left(\frac{\mu_0}{\varepsilon} \right)^{1/2} \ln \left(\frac{r_0}{r_1} \right) . \quad (4.5)$$

In our work, transmission lines with an impedance of 50 ohms were modeled, in which case $r_0/r_1 = 2.30$ when the relative impedance is unity.

Because of their simple dispersion relation, Eq. (3.12), the TEM waves travel nondispersively down the line at the speed of light and can be excited with spectral content for all frequencies down to zero. In fact, they propagate with an arbitrary dependence on the combination $z \pm ct$, as indicated in (3.6). As the frequency content of the excitation for the line increases, though, the chances that higher frequency TE and TM waves will be excited on the line increase. This can happen under two conditions:

1. There are frequency components of the excitation above the lowest, or cutoff, frequencies for these modes, ω_{co} in (3.10); and
2. The symmetry of the excitation is such that when it is spatially decomposed in a sum over all possible modes, it has some component in TE or TM modes above cutoff.

The cutoff frequencies for the TE and TM modes in a cylindrical coaxial line can be computed by solving the wave equations, (3.1), for fields of the form given in (3.7):

$$\mathbf{E}_\perp = \mathbf{E}_\perp^-(x_\perp) \exp[i(kz - \omega t)] = E_n(r) \exp[i(n\theta - kz - \omega t)] . \quad (4.6)$$

Individual components of the field must obey (3.3) and the relevant boundary conditions for the TE and TM modes. The dispersion relations are derived from the boundary conditions, and the expressions for the cutoff frequency are obtained by taking $k = 0$. From Ref. 3, ω_{co} for the cylindrical coaxial line obeys

$$\text{TE: } J'_n\left(\Omega_{co} \frac{r_o}{r_i}\right) N'_n(\Omega_{co}) - J'_n(\Omega_{co}) N'_n\left(\Omega_{co} \frac{r_o}{r_i}\right) = 0 \quad (4.7a)$$

$$\text{TM: } J_n\left(\Omega_{co} \frac{r_o}{r_i}\right) N_n(\Omega_{co}) - J_n(\Omega_{co}) N_n\left(\Omega_{co} \frac{r_o}{r_i}\right) = 0 , \quad (4.7b)$$

where

$$\omega_{co} = \left(\frac{c}{r_i}\right) \Omega_{co} . \quad (4.8)$$

The dimensionless cutoff frequencies for the lowest-order TE and TM modes are shown in Fig. 4.1. Note that for 50-ohm lines with $r_o/r_i = 2.30$, $\Omega_{co} = 2.5$ for the lowest-order TE mode and 2.4 for lowest-order TM mode. The actual cutoff frequency depends also on the inner radial dimension of the transmission line. In the cases considered here, r_i was taken to be 7 mm, with r_o therefore equal to 16 mm. For these dimensions, the cutoff frequencies of the lowest modes are

$$\begin{aligned} \omega_{co} &= 1.07 \times 10^{11} \text{ rad/s (TE)} \\ &= 1.03 \times 10^{11} \text{ rad/s (TM)} . \end{aligned}$$

Dividing by 2π , the cutoff frequencies, measured in Hertz, are 17.0 and 16.4 GHz for the TE and TM modes. One must bear in mind that waveforms with frequency content above these limits can potentially excite non-TEM components in the line.

Fig. 4.1 The dimensionless cutoff frequencies for the lowest-order TE (dotted) and TM (solid) modes in a coaxial cylindrical line as a function of the ratio of the outer to the inner radius.

4.2. Simulations

Meshes for all problems considered here were generated with the INGRID mesh generator⁴. Simulations were performed in two ways, one using the full cross section of the coax and a version of the code maintained by Scott Brandon, and the second using one section of the coax and a version of the code written and maintained by J. Brian Grant. The full cross section of the coax was modeled with the mesh shown in Fig. 4.2 in an end-on view. The mesh shown in the figure was broken into 4 radial sections and 8 azimuthal sections; the dark region at the inside of the mesh is a perspective view of the inner surface of the coax. Inner and outer radii of 0.7 and 1.6 cm were chosen, which would yield an impedance of 49.58 ohms if the inner and outer surfaces of the coax were perfectly cylindrical. In fact, the surfaces are faceted, as shown in the figure, with the corners of the cross section lying on the circular cylinders of radius 0.7 and 1.6 cm. The length of the computational region along the axis was 40 cm, which was usually divided into 200 zones, so that $\Delta z = 0.2$ cm.

A driving function was written to model the $1/r$ dependence of the TEM component of the radial electric field at a driving plane situated at $z = 0$. The time dependence of the driving function was a Gaussian of the form given in Eq.

Fig. 4.2 End-on view of a 4x8 (r- θ) mesh modeling a coaxial, cylindrical waveguide. The dark region at the interior of the cross section is a perspective view of the inner surface of the coax looking along the axis.

(3.37). Typically, the driving pulse had unit amplitude and the parameters $t_{\text{peak}} = 0.6$ ns and $t_{\text{width}} = 0.15$ ns; the pulse existed for $0 < t < t_{\text{max}} = 1.2$ ns, and was zero otherwise. Thus, the pulse extended over 36 cm in the axial direction, and the width of 0.15 ns was resolved over 22.5 zones in the axial direction.

A voltage diagnostic was developed to integrate the radial electric field along the mesh edges shown in Fig. 4.2 from the inner to the outer radius of the waveguide. Similarly, the code had a diagnostic that computed $\mathbf{H} \cdot d\mathbf{S}$ along an edge of the dual mesh. By summing these individual contributions, one could obtain the line integral of the magnetic field about the center conductor, from which one could compute the current in the line using Eq. (3.15) and the constitutive relation in Eq. (2.4). It must be noted that the current was therefore calculated at a location displaced along the axis of the coax by one-half of a cell, $\Delta z/2$. Therefore, the time displacement of the current relative to the voltage when computed in this fashion was, as given in Eq. (3.37), $\Delta t = \Delta z/2c = 3.33$ ps. The time-lag of the current is shown in Fig. 4.3, where the current (dashed line) has been scaled upward by a factor of approximately 50 to make it comparable in magnitude to the voltage. If one divides the voltage by the current, the resulting impedance shown in Fig. 4.4 has the same type of temporal behavior indicated in Fig. 3.2. If we assume that the numerical dispersion of the line is small, then the waveform should propagate along the line with little distortion, since there is no

Fig. 4.3 An illustration of the time-lag of the current (dashed line), measured with respect to the voltage (solid line), in the code calculations. For convenience, the current has been scaled to have approximately the same magnitude as the voltage.

dispersion for TEM waves on an ideal line. Therefore, we can compute the impedance of the modeled coax by taking the ratio of the peak voltage to the peak current. The results are shown in Table 4.1. Somewhat surprisingly, the impedance appears to increase with the number of radial zones from values lying below that of the ideal cylindrical line to values lying above. This issue will be addressed in the summary.

	n_θ				
n_r	8	12	16	20	24
2		51.4	50.0	49.6	49.6
3	54.7	51.7	50.4	49.8	49.8
4	54.9	51.9	50.5	50.1	50.1
5	55.0	52.0	50.6	50.2	50.2

Table 4.1
Cylindrical coaxial transmission line impedances computed with DSI3D

Fig. 4.4 The impedance Z computed by taking the ratio of the voltage and the time-delayed current. The waveform of the voltage pulse is shown for reference.

The error in computing the impedance of the coax in this fashion comes essentially from two factors: the "model error" resulting from the approximation of the circular cylinders by faceted cylinders, and the computational error of the code in computing the fields on the mesh. Deriving an exact analytical result for the impedance of the faceted model used by the code in order to determine the model error is quite difficult. We made an attempt to estimate the bounds on the impedance for the faceted model by assuming that the exact value should lie between the minimum and maximum values for circular cross section transmission lines obtained as follows. From Eq. (4.5), the impedance is a function of the ratio of the inner and outer radii of the coax. We can circumscribe an outer cylinder about the outer surface of the faceted-face model of Fig. 4.2 that contacts the face at its corners and has radius r_o , and we can inscribe another cylinder within that surface that strikes the faceted face at points of tangency to the cylinder and has radius $r_{o,min} = r_o \cos(\pi/n_\theta)$, where n_θ is the number of azimuthal sections in the faceted face. Similarly, we can circumscribe and inscribe such cylinders on the inner surface of the faceted-face model, obtaining radii r_i and $r_{i,min} = r_i \cos(\pi/n_\theta)$. From these, we can compute upper and lower bounds on the impedance:

$$Z_{\text{upper}} = \frac{1}{2\pi} \left(\frac{\mu_0}{\varepsilon} \right)^{1/2} \ell n \left(\frac{r_o}{r_{i,\min}} \right) \quad (4.9)$$

$$Z_{\text{lower}} = \frac{1}{2\pi} \left(\frac{\mu_0}{\varepsilon} \right)^{1/2} \ell n \left(\frac{r_{o,\min}}{r_i} \right)$$

The difference of these two values is

$$\Delta Z = Z_{\text{upper}} - Z_{\text{lower}} = -\frac{1}{\pi} \left(\frac{\mu_0}{\varepsilon} \right)^{1/2} \ell n \left[\cos \left(\frac{\pi}{n_\theta} \right) \right], \quad (4.10a)$$

which for large values of the azimuthal number obeys

$$\Delta Z \approx \left(\frac{\mu_0}{\varepsilon} \right)^{1/2} \frac{\pi}{2n_\theta^2}. \quad (4.10b)$$

The computed values of Z_{upper} and Z_{lower} are given in Table 4.2. We can see that the computed impedances of Table 4.1 always lie well within the bounds given in Table 4.2, so that our estimate of the upper and lower bounds on the model error is too coarse.

n_θ	Z_{upper}	Z_{lower}
8	59.07	40.08
12	53.74	45.42
16	51.91	47.25
20	51.06	48.09
24	50.61	48.55

Table 4.2

Upper and lower bounds on the impedance of the coax model used in the DSI3D computations.

To check the results of the code, the commercial, frequency-domain code MAXWELL EMINENCE was used to compute the impedance of the same model of the coax. This finite-element code provides the TEM impedance of the line directly as one of its standard output values. After several iterations to reduce the error in the calculations, the results obtained are summarized in Table 4.3. In contrast to the time-domain results obtained with DSI3D, these values

consistently lie below the analytic result for an ideal circular-cylindrical coax, 49.58 ohms, although the values are approaching this impedance uniformly with increasing azimuthal resolution of the model; by the time $n_\theta=24$, the error is minuscule. It would appear, if these results are at least qualitatively accurate, that the impedance of the model configuration increases, albeit by a small amount, toward the ideal circular-cylindrical value as the number of azimuthal sections increases. It is noted, however, that the increase from 8 to 24 sections is only about 1%.

n_θ	Z_{MAXWELL}
4	44.56
8	48.98
12	49.41
16	49.49
20	49.52
24	49.54

Table 4.3
Impedances computed with MAXWELL EMINENCE.

As a further check, Grant's version of the DSI code was used to compute the impedance. For the purposes of this study, Grant's version differed in two essential respects: (1) it had the capability to take advantage of the symmetry of the INGRID mesh to compute the impedance using only a single azimuthal section of the model, as shown in Fig. 4.5, and (2) it computed the line integrals of Eqs. (2.6) and (2.11) somewhat differently than Brandon's version, breaking the path integrals along each edge into two segments, dividing the edge at the midpoint, rather than using only one segment along the edge. Using only a single section of the whole coax cross section, it was simplest to subdivide the section in what amounts to the x-direction in the figure. Thus, this set of simulations was characterized by numbers of zones in the azimuthal-, radial-, and x-directions ($n_\theta \times n_r \times n_x$). With this greater resolution, it was hoped that it would be possible to obtain more accurate values for the impedance of the faceted-face model approximations to the ideal cylindrical coax. The results obtained from these simulations are summarized in Table 4.4. The values obtained with this version of the code are quite different in a couple of respects from those computed earlier, as can be seen from the table. First, the impedances increase from below toward the value obtained with MAXWELL EMINENCE as the resolution of the mesh becomes greater. Second, except for a few exceptions, the impedances approach the frequency-domain value quite uniformly as the both the azimuthal and radial resolution is increased, in contrast to the trend with the other version

Fig. 4.5 A single section of an eight-sided INGRID model of the coax as used in simulations with Grant's version of DSI.

of the code differ from the frequency-domain results by 0.5-1%. Closer analysis of the results of the calculations indicate that the difference in the impedance for zoning schemes with the same number of azimuthal and radial zones is a result of differences in the computed current; i.e., the voltages for runs with different values of n_x typically differ by very little or not at all.

4.3. Summary and Conclusions

The version of DSI3D originally used with the full model geometry of the coax was most probably the least accurate of the methods used here, which also included the frequency-domain code MAXWELL EMINENCE and Grant's version of DSI. Several comments are in order here, though. First, with fewer zones in the cross-sectional plane, it featured less resolution than the runs done with Grant's version. Further, Grant's version, by using two segments along each side of a cell to compute the line integrals, as opposed to one in Brandon's version, was inherently more accurate, roughly by a factor of two for each of the three dimensions, or 8 in total ($2 \times 2 \times 2$). It should be noted, also, that while Grant's version was capable of analyzing the full model geometry, it was only used on a section of the model. For analysis of the coax in isolation as a single component, this is acceptable; however, when the coax is embedded in a larger system, the full coax must be used in order to model the full system. In such a circumstance, one also cannot assume that the excitation will be almost purely TEM, as was the case here. In fact, in a larger system with asymmetric excitations of the coax, one would expect that TE and/or TM components could

$n_\theta = 8$	n_x			
n_r	2	4	6	8
2	46.87			
3	47.55			
4	47.68	48.18		
5	47.89	48.33		
6	47.85		48.55	
7	47.89	48.46	48.61	48.66
8		48.46	48.65	48.70
9				48.73
$n_\theta = 12$				
5	49.12			
6				
7			49.2	
$n_\theta = 16$				
5		49.2		
6				
7		49.3		
$n_\theta = 20$				
5		49.30		
6				
7		49.39	49.41	
$n_\theta = 24$				
5	49.50	49.27		

Table 4.4
Impedances computed using sections of the coax.

be launched along the device if the frequency content of the excitation extended above the lower cutoffs that can be computed from Eqs. (4.7) and (4.8). The computation of the impedances, or, more generally, transfer matrices for the device, would become more complex, as discussed in Secs. 3.2 and 3.3. With consistency in the definitions of voltages and currents, though, the impedances and matrix elements can be determined.

Despite its apparent accuracy, the frequency-domain code has its shortcomings. Most notably, as a frequency-domain code, it computes the impedance for a single assumed frequency. Because the TEM waves in an ideal coax are dispersionless (i.e., the phase velocity of the waves along the line is a constant, independent of the frequency or wavenumber), this limitation is inconsequential; whatever the waveform of the exciting pulse, it will be

replicated at the output end of the line. Larger configurations containing a coax are unlikely to be dispersionless, however, which would make the behavior of such configurations frequency-dependent. Alternatively, if the center conductor were removed from the transmission line, only dispersive TE and TM modes could exist on the line, so that a signal with arbitrary frequency content would be distorted as it traveled along the line. In either case, the frequency-domain code remains applicable to situations with arbitrary temporal excitations only if enough runs can be made to effectively allow one to invert the Fourier transform in going from the frequency to the time domain.

The apparently anomalous behavior of DSI3D illustrated in Table 4.1 is initially puzzling, but nevertheless plausible if we reason simply as follows. The analytic expression for the radial electric field varies as $1/r$. Now imagine that along the radially-directed edge of each zone in the computational mesh, the code computes a value of the electric field for that edge that is equal to the value of the field at the small-radius end of the edge. Under this circumstance, the code would consistently overestimate the value of the electric field along that edge. In taking the line integral along the edges to compute the voltage, then, the code would sum each of those edge fields multiplied by the edge length in a way that makes the code-computed voltage larger than the analytic value. As the resolution improved, though, the edges would get shorter and the amount by which the code overestimated the fields would decrease, so that the computed voltage and impedance would decrease. In the limit of an infinite number of edges with infinitely small lengths, the impedance would eventually converge downward to the analytic value. It is reasonable to believe that this is how the code produces the behavior seen in the calculations.

5.0 Simulations of a Stripline Transmission Line

5.1 Analytical and Frequency-Domain Impedance Calculations

The stripline transmission line has the cross-sectional configuration shown in Fig. 5.1, with a thin, electrically conducting strip running along the center of a rectangular waveguide. In general, the region between the strip and the waveguide walls can be filled by a dielectric of relative permittivity ϵ_r , so that the overall permittivity of the dielectric is $\epsilon = \epsilon_r \epsilon_0$.

The TEM impedance of a stripline with a finite-thickness center conductor can be found by using the Schwarz-Christoffel transformation. The general solution obtained this way is expressed in terms of hypergeometric functions dependent on four parameters⁵. When the side walls of the rectangular waveguide are long compared to the distance between the wall and the center conductor, though, the expressions for the impedance can be simplified. Further, if the thickness of the center conductor, t , is less than $b/3$, the impedance takes the approximate form⁶

$$Z = \frac{30\pi}{\epsilon_r^{1/2} \left[\frac{w/b}{1-t/b} + \frac{2C_f(t/b)}{\pi C_f(0)} \ln \left(1 + \coth \frac{\pi g}{b} \right) \right]} \text{ ohms ,} \quad (5.1)$$

where

$$C_f(t/b) = \frac{\epsilon}{\pi} \left[\left(\frac{b}{b-t} \right) \ln \left(\frac{2b-t}{t} \right) + \ln \frac{t(2b-t)}{(b-t)^2} \right]$$

and

$$C_f(0) = \frac{2\epsilon \ln 2}{\pi} .$$

Note here that the dimensions w , t , and g are all normalized to b .

Fig. 5.1 Cross section of a stripline transmission line. The perfectly conducting walls of the waveguide are separated by b in the vertical direction and by $2g+w$ in the horizontal. The perfectly conducting center strip has a thickness t and width w and is spaced symmetrically from the walls at the center of the rectangular waveguide.

In the limit of an infinitely thin center conductor – i.e., $t \rightarrow 0$ – the direct computation of the impedance using the Schwarz-Christoffel transformation is not as difficult⁷. In order to find the impedance in this case, one first finds the modulus k in the relation

$$\frac{K(k)}{K(k')} = \frac{2}{b} (2g + w) , \quad (5.2)$$

where $K(k)$ is the complete elliptic integral of the first kind⁸ and

$$k' = (1 - k^2)^{1/2} . \quad (5.3)$$

With k (and k') in hand, one defines

$$\xi = K(k') \frac{g}{b} , \quad (5.4)$$

with which one computes

$$\lambda^2 = 1 - (k')^2 \left(\frac{\text{sn} \xi}{\text{cn} \xi} \right)^4 , \quad (5.5)$$

where sn and cn are Jacobian elliptic functions. Similarly to Eq. (5.3),

$$\lambda' = (1 - \lambda^2)^{1/2} , \quad (5.6)$$

and with these two parameters, one can find the capacitance per unit length L of the line:

$$\frac{C_s}{L} = 2\epsilon \frac{K(\lambda)}{K(\lambda')} . \quad (5.7)$$

Finally, the impedance is⁹

$$Z_0 = \frac{L\epsilon_r^{1/2}}{cC_s} , \quad (5.8)$$

where c is the speed of light in vacuum.

The dimensions and relative permittivity of the lines examined here were the following:

$$\begin{aligned} w &= 0.102 \text{ cm} \\ b &= 0.152 \text{ cm} \\ g &= 0.152 \text{ cm} \\ t &= 0.006 \text{ cm} \\ \epsilon_r &= 2.94 \end{aligned} . \quad (5.9)$$

Because of the thinness of the center conductor, simulations were performed with both non-zero and zero thickness center conductors. In the limit of vanishing t , the right side of Eq. (5.2), $2(2g+w)/b = 5.34$. This value is quite large, implying that k in that equation is quite close to unity. As a consequence, a

number of approximations can be made, as described in Ref. 7, which result in an expression for the impedance that is identical to the expression derived from Eq. (5.1) in the limit that t goes to zero:

$$Z_0 = \lim_{t \rightarrow 0} Z = \frac{30\pi}{\epsilon_r^{1/2} \left[\frac{w}{b} + \frac{2}{\pi} \ln \left(1 + \coth \frac{\pi g}{b} \right) \right]} . \quad (5.10)$$

Using the values of the parameters from Eq. (5.9) yields impedances of 49.7 and 49.4 ohms according to Eqs. (5.8), and the preceding expressions, and (5.10), respectively. The difference between the two values amounts to 0.8%.

Impedance calculations using the commercial frequency-domain code HFSS were also provided¹⁰ for an almost identical configuration, except that the dimensions were specified in inches as

$$\begin{aligned} w &= 0.0396 \text{ in} \\ b &= 0.060 \text{ in} \\ 2g + w &= 0.160 \text{ in} . \end{aligned}$$

To three significant figures, these dimensions are identical to those in Eq. (5.9), except for w , which converts to 0.101 cm. HFSS computes a complex frequency-domain impedance in three forms, Z_{pi} , Z_{pv} , and Z_{vi} . With the voltage and current, V and I , complex phasor quantities in the frequency domain computed essentially by taking the integrals in Eqs. (3.12) and (3.15), and the power P defined in terms of an area integral of the Poynting flux,

$$P = \int_A \mathbf{E} \times \mathbf{H}^* \cdot d\mathbf{S} , \quad (5.11)$$

the three forms of the impedance are defined as follows:

$$Z_{pi} = \frac{P}{(I \cdot I^*)} \quad (5.12a)$$

$$Z_{pv} = \frac{(V \cdot V^*)}{P} \quad (5.12b)$$

$$Z_{vi} = (Z_{pv} Z_{pi})^{1/2} = \left(\frac{V \cdot V^*}{I \cdot I^*} \right)^{1/2} = \frac{|V|}{|I|} . \quad (5.12c)$$

The values obtained from HFSS for these three forms of the impedance were the following:

$$Z_{pi} = 51.1 \text{ ohms}$$

$$Z_{pv} = 48.7 \text{ ohms}$$

$$Z_{vi} = 49.9 \text{ ohms.}$$

For comparison, if w is set to 0.101, Eqs. (5.8) and (5.10) yield impedances of 50.0 and 49.7 ohms, respectively.

5.2 Simulations

The excitation function for a TEM wave on the stripline was obtained by modifying that for the coax. Each of the shaded rectangular regions at the end of the line shown in Fig. 5.2 was treated as a small angular segment of the cross section of a coax with radii of the order of ten times the distance between the conductors. The radial electric fields of the driving functions for the two coaxes

Fig. 5.2 Excitation of the electric fields in the stripline was accomplished by approximating the vertical fields in the shaded regions by the radial fields in the small angular segments of the two coaxial lines shown.

were essentially vertical and directed oppositely in each shaded region. Although the resulting field distribution pattern in the plane of the excitation was not that of the TEM eigenmode, the fields rapidly redistributed themselves into the desired eigenmode after propagating a short distance along the line. The downstream cross-sectional distribution of the magnitude of the electric field is shown in Fig. 5.3, where one can see that the fields are most intense in the region in which they were initially excited, which is where the transverse distance between conductors was smallest. Measurements of the axial components of

Fig. 5.3 The magnitude of the electric field in the cross-sectional plane of the stripline transmission line. Red regions correspond to the largest magnitudes. Essentially no axial fields were measured, so that the fields were those for a TEM mode.

the fields showed them to be essentially nonexistent. The temporal behavior of the TEM pulses launched on the lines was, as in the case of the coaxial lines, Gaussian. With unit amplitude, the pulses typically had temporal parameters of $t_{\text{peak}} = 40$ ps and $t_{\text{width}} = 10$ ps for $0 < t < t_{\text{max}} = 80$ ps.

The stripline mesh was uniform in the axial, or x-, direction. Lines with a length of 0.445 cm were simulated using 35 zones along the axis, so that the mesh spacing in this direction was $\Delta x = 0.0127$ cm. Given that the speed of light along the line was $c' = c/\epsilon_r^{1/2} = 1.75 \times 10^{10}$ cm/s, the 10-ps rise of the Gaussian pulse was resolved over $c't_{\text{width}}/\Delta x \approx 14$ cells of the mesh. In the cross-sectional plane of the line, three different configurations were used to model the conducting strip at the center of the waveguide:

- A rectangular conductor of nonzero thickness;
- A conductor of nonzero thickness and beveled edges, giving the strip the appearance of a flattened hexagon, as illustrated in Fig. 5.4; and
- A conducting strip of zero thickness, which necessitated the development of a boundary condition for internal, perfectly conducting surfaces by Scott Brandon. The problem with the first model is that the strip is quite thin, so that one must either use a minimum of $b/t = 0.152/0.006 \approx 25$ zones in the y-direction to resolve the strip, or one must use some sort of variable thickness zoning scheme to reduce the number of zones. In this latter case, though, the thinness of the center strip in combination with the Courant condition for stability that $\Delta t < \Delta y/c'$ will perhaps create longer simulation runs than desired due to the shortness of the time step. The use of bevelled edges was an attempt to nevertheless resolve the thickness of the strip without requiring the use of thin zones, albeit at the expense of a more complicated mesh with nonrectangular

Fig. 5.4 Cross-sectional view of the beveled-edge model of the center conductor in the stripline transmission line. The horizontal width of the conductor from tip to tip (which is along the z -direction) is w , and the thickness at the center (which is along the y -direction) is t .

zones. Finally, the use of internal conducting surfaces, which models the strip as a conductor of zero thickness, was deemed to generally be the best solution, although one must accept the small error associated with the loss of the strip's thickness. An additional source of computational error associated with these models is mathematical singularities in the fields in the vicinity of the corners and sharp edges of these strip models. We'll return to this topic later.

There was one significant difference in the treatment of the boundary conditions at the downstream, or far, end of the stripline transmission line, as opposed to the coaxial lines. In the case of the coax, the far end of the line was a perfect conductor, which caused the propagating pulse to reflect there. Nevertheless, the pulses were short enough compared to the total length of the line that all the required measurements of the voltage and current could be made before the reflected wave interfered with the incident wave. For the stripline, a simplified graded absorber was placed at the end of the line; this absorber reduced the magnitude of the reflected TEM signal by up to three orders of magnitude, so that the reflected signal could essentially be ignored (although such absorbers will not prevent the reflection of evanescent modes, in our case the launching plane was spaced a sufficient distance from the absorber to allow us to ignore this possibility). The use of this absorber was suggested by R. B. Grant, adopting a method described in Ref. 11. Basically, the method used here consists of adding a set of layers at the end of the line, each with a thickness of a single axial zone and graded electric and fictitious magnetic conductivities. Numbering these added layers by $i=1,2,3,\dots,N$, starting with the layer closest to the input end, the electric and magnetic conductivities have a quadratic increase with distance along the absorber:

$$\sigma_{e,i} = \left(\frac{i}{N}\right)^2 \sigma_{e,max} \quad (5.13a)$$

$$\sigma_{h,i} = \left(\frac{\mu_0}{\epsilon_r \epsilon_0}\right) \sigma_{e,i} \quad (5.13b)$$

These conductivities necessitate the modification of Maxwell's equations through the addition of current densities equal to $\sigma_e E$ and $\sigma_h H$ on the right sides of Eqs. (2.1) and (2.2), respectively. Figure 5.5 shows the degree to which the reflected fields were reduced for 20- and 40-layer absorbers as a function of $\sigma_{e,max}$. One can see that the ratio of the reflected field to the incident field, $R = |E_r| / |E_{inc}|$, diminishes exponentially with the maximum conductivity, at least until a minimum is reached in the case of 20 layers. The results shown there are for an infinitely thin center strip. In succeeding simulations, a maximum conductivity of $7 (\text{ohm} \cdot \text{m})^{-1}$ was typically used.

Although only a limited number of simulations were performed with the rectangular, nonzero thickness model of the strip, the mode pattern of Fig. 5.3 was determined using this model. In addition, a plot was made of the field intensities of a pulse traveling along the transmission line; this is shown in Fig. 5.6. There, the view is of the mode pattern in the x-z plane of the line.

Fig. 5.5 The ratio of the reflected to the incident electric fields at the end of a stripline transmission line terminated in a graded absorber as a function of the maximum electric conductivity in Eq. (5.3a).

Fig. 5.6 Color plot of the field strength (green is the strongest, dark blue the weakest) for a Gaussian pulse in the x-z plane of the stripline transmission line. This plane is at the midplane of the line in the y-direction.

Returning to the beveled-edge model of Fig. 5.4, one can see that the particular scheme shown consists of an 18×48 mesh in the y-z plane. In the y-direction, the dimensions of the waveguide, b and t , are as given in Eq. (5.9), while in the z-direction, the strip width w is the distance between the sharp edges of the strip. Along the outer edges of the strip at $z = \pm 0.203$ cm, the mesh spacing is uniform in y , at $\Delta y = 0.152/18 = 0.00844$ cm. Additional meshes in the same proportions, with the same width of the bevel, were also used: 6×16 , 12×32 , and 24×64 . The impedance computed for each mesh is shown in Fig. 5.7, where the horizontal axis is the ratio of the number of zones in the z-direction, N_z , normalized to the maximum number, $N_{z,\max} = 64$. The dotted line is the

Fig. 5.7 Impedance of the stripline as a function of the number of zones in the z-direction for the beveled-edge model of the strip. The dashed line is the impedance computed using the frequency-domain code MAXWELL EMINENCE.

impedance Z_{vi} (following the definition in Eq. (5.12)) computed with MAXWELL EMINENCE for a beveled-edge stripline with the same dimensions (for further comparison, $Z_{pi}=47.9$ ohms and $Z_{pv}=47.7$ ohms in that same calculation). Clearly, greater resolution in the mesh translates to greater accuracy in the computation of the impedance; however, even with $24 \times 64 = 1536$ zones in the y-z plane, the agreement between the frequency- and time-domain calculations is only 2.5%.

Computations with the zero-thickness model for the strip were performed using three different zoning schemes in the cross-sectional plane: uniform, nonuniform, and ratio zoning. The uniform zoning scheme essentially repeated the calculations for the beveled-edge model with the thickness of the strip reduced to zero. Results were obtained for meshes that were 6×16 , 12×32 , 18×48 , and 24×64 in the y-z plane. These are compared in Fig. 5.8 with an analytic impedance calculated from Eq. (5.10). Again, greater resolution in the cross-sectional plane produces a more accurate value of the impedance, which is increasing toward the analytic value as the resolution is increased. At the maximum resolution, the difference between the analytic and numerical values is about 2.4%, almost identical to the value for the beveled-edge model.

To attempt to gain greater resolution in the vicinity of the strip without generating an intractable number of zones overall in the cross-sectional plane, nonuniform zoning, with greater numbers of zones near the strip, was tried.

Fig. 5.8 Impedance of the stripline as a function of the number of zones in the z-direction for the zero-thickness model of the strip using uniform zoning. The dashed line is the impedance computed using the analytic result of Eq. (5.10).

To attempt to gain greater resolution in the vicinity of the strip without generating an intractable number of zones overall in the cross-sectional plane, nonuniform zoning, with greater numbers of zones near the strip, was tried. Two nonuniform zoning schemes that were used are shown in Fig. 5.9. In Fig. 5.9a, the idea was to concentrate the zones near the tips of the strip. Overall, there were 52×24 zones in the y-z plane, with 20 zones between $y = \pm 0.0253$, meaning $\Delta y = 0.00253$ in that region, and 20 zones between $z = 0.0255$ and $z = 0.0765$, meaning $\Delta z = 0.00255$ there. The impedance computed with this mesh was 48.5 ohms. In the 60×24 zoning scheme of Fig. 5.9b, high resolution was maintained over the whole strip region. The same vertical mesh size was employed, but 50 zones were placed between $z = \pm 0.0638$ cm, which yielded the same value for Δz . In this case, the impedance dropped slightly to 48.2 ohms. In either case, the computed impedance is larger and closer to the analytic value, and therefore more accurate, even though somewhat fewer zones have been used. The conclusion at this point was that the largest contribution to the computational error came from the vicinity of the strip, which, not surprisingly, is the region of greatest field stresses.

Nonuniform zoning had the disadvantage of employing adjacent zones that vary significantly in size. To get a smoother variation in zone size, it was decided to take advantage of the ratio zoning capability of the mesh generator, INGRID. This capability allows one to specify, along some axis, the number of

Fig. 5.9 Nonuniform zoning schemes used with the zero-thickness strip model.

zones and the ratio of the dimensions of adjacent zones along that axis; for example, along the z-axis, with R the ratio, the size of zone i, $(\Delta z)_i$, is related to that of the adjacent zones, i+1 and i-1, by $(\Delta z)_{i+1} = R(\Delta z)_i$ and $(\Delta z)_i = R(\Delta z)_{i-1}$.

Six different ratio zoning schemes were used in the simulations; they are described in Table 5.1, and the mesh in the y-z plane is shown in Fig. 5.10.

Simulation no.	N_g	$N_{w/2}$	$N_{b/2}$	R_y	R_z
1	12	4	6	1.2	1.1
2	12	4	6	1.3	1.2
3	24	8	12	1.2	1.1
4	20	12	12	1.2	1.1
5	20	12	12	1.1	1.05
6	24	8	12	1.2	1.1/1.3

Table 5.1

Description of the ratio zoning meshes used in the simulations. Here, N_g is the number of zones in the z-direction between the side of the waveguide and the edge of the strip (note that g is this distance in Fig. 5.1), $N_{w/2}$ is the number of zones in the z-direction over half of the strip, and $N_{b/2}$ is the number of zones in the y-direction between the strip and the top of the waveguide; R_y and R_z are the ratios of adjacent zones in the y- and z-directions, respectively.

Fig. 5.10 The ratio-zoned mesh in the y-z plane for simulation no. 1. The strip at the center of the mesh is highlighted, although in reality it has vanishing thickness in the y-, or vertical, direction.

In the sixth simulation, the different ratios in the y-direction refer to the ratios between the strip and the side of the waveguide ($R_y = 1.1$) and in the region containing the strip ($R_y = 1.3$). The impedances computed with these meshes are given in Table 5.2. These impedances are to be compared with the analytic result of 49.4 ohms. As one can see, the impedances computed with the

ratio-zoned mesh are more accurate than the other meshes, for similar numbers of zones. For example, simulations 1 and 2 were each performed with a 12×32 mesh in the y-z plane and had errors of 3.2% and 2.4%, respectively. This is to be compared with the 12×32 uniform mesh, for which the error was 5.7%.

Simulation no.	Z(ohms)
1	47.8
2	48.2
3	49.1
4	49.0
5	48.7
6	49.1

Table 5.2
Impedances computed with the meshes of Table 5.1.

Simulations 3 through 5 used 24×64 meshes and a minimum error of 0.6% was achieved, compared to the 2.0% error with the uniform mesh of the same size.

5.3 Analysis and Conclusions

Overall, with the calculation of impedance as a measure of code performance, one would conclude that this version of DSI3D was less accurate than the frequency-domain code, MAXWELL EMINENCE; however, MAXWELL EMINENCE does not have the ability to handle time domain problems without the performance of an inverse transform. Further, such codes must invert matrices of increasing size as the problem grows in complexity and volume, which limits their ultimate capability. With that in mind, we can compare the accuracy of the results obtained with DSI3D using the different meshing schemes.

Clearly, the performance is maximized by getting the greatest resolution in the area where the electric fields exhibit the most variation. Therefore, the nonuniform and ratio zoning schemes, which were used to concentrate the zones in the regions of most-rapid electric field variation, produced results of the greatest accuracy, as determined by comparison with analytic results. In addition, the ratio zoning schemes were the more accurate of the two, presumably due to the smoother variation of the zone dimensions across the mesh.

To increase the accuracy of the calculations, two approaches were considered. The first, and conceptually the simplest, stemmed from the hypothesis that even though the strip was meant to have zero thickness, the dual-mesh feature of the code might make it appear as if the strip were wider and perhaps longer by some distance of the order of a zone width. Therefore, if the strip dimensions were modified by some amount proportional to the zone dimensions, a more accurate impedance might be obtained. A check of the effect of increasing the thickness of the strip by a zone width for the uniformly zoned

model showed that the impedances became too low in this instance; the 6×16 , 12×32 , and 24×64 meshes were dropped in impedance to 37.6, 39.6, and 40.5 ohms, respectively. Alternatively, we examined the effect of modifying the width of the strip while maintaining a thickness of zero. Figure 5.11 shows the impedances computed with DSI3D as a function of the thickness of the zone adjacent to the strip, Δz . Also in the figure, the upper solid curve is the analytic impedance from Eq. (5.10) using the dimensions in Eq. (5.9) with t set to 0. The lower solid curve is the impedance with w increased by $2\Delta z$ (i.e., the strip was increased in width by one zone on either side) while g was decreased by Δz (i.e., the overall width of the line was held constant). The dashed curve between the two was for an increase in the strip width by a half zone on either side, with a corresponding decrease in g . One can see that all of the computed impedances, denoted $Z_{\Delta z}$, lie between the two solid curves, and that most lie between the solid and the dashed curve. Therefore, any correction to the strip width would be less than a zone width. Figure 5.12 is a plot of the increase required in the strip width, δ ,

Fig. 5.11 Computed impedances (crosses) vs the width of the zone adjacent to the edge of the strip. The two solid curves are the analytic impedances with and without the increase of the strip width by a zone width on either side, while the dashed curve is an analytic impedance assuming the strip width is increased by half a zone width on either side.

normalized to Δz , in order to make the analytic impedance equal to the computed impedance; thus, in the analytic calculation, $w \rightarrow w + 2\delta$, while $g \rightarrow g - \delta$. We can see that there is quite a spread in the amount of correction required, relative to the width of the zone adjacent to the edge of the strip. Closer examination indicates that the two largest relative corrections are for the nonuniform zoning schemes of Fig. 5.9, while the relatively small relative corrections for the three largest values of Δz were for the uniform zoning scheme. It appears that a correction of about $0.3\Delta z$ would improve the result in most circumstances, but the choice of that value is justifiable only empirically, and not entirely at that.

Greater accuracy should have been achievable on more rigorous grounds by modifying the basis functions used in the performance of the line integrals on the right sides of Eqs. (2.6) and (2.11). These line integrals are performed around a contour consisting of edges in the mesh; the integrals consist of a sum of the products of the edge field, assumed to be a constant along the edge, and the edge length. This approximation breaks down in the vicinity of the edge of the strip,

Fig. 5.12 Strip width corrections, relative to the width of the zone adjacent to the edge of the strip, required to make the analytic impedance agree with the code impedance as a function of the zone width.

where the fields diverge as $1/r^{1/2}$, where r is the distance from the edge in the cross-sectional plane¹². This problem has been addressed in FDTD simulations,

for example, in Refs. 13 and 14. An implementation of such a method would have required the identification of the "special" edges at which such a treatment would have been required, and a determination of the exact treatment to be applied (by comparison with the divergence at the edge of a thin strip, fields at the edge of a 90-degree corner diverge as $1/r^{1/3}$, so that some account would have to have been taken of the angular separation of the two conducting planes coming together to form the edge). J. B. Grant made an effort to implement such a special treatment in his version of the code in order to determine if improved accuracy resulted; unfortunately, he was unable to realize a stable version of his treatment during the conduct of this project.

6.0 Final Summary and Conclusions

Here, two implementations of the three-dimensional DSI algorithm in codes by Scott Brandon and J. B. Grant were used to examine the launching, propagation, and diagnosis of signals on coaxial and stripline transmission lines. The accuracy of the code was determined for a wide variety of zoning schemes developed with the INGRID mesh generator. Code results were compared with analytic calculations and computations made with commercially available frequency-domain codes. Errors of a few percent were commonly seen in the time-domain calculations presented here, and errors of less than one percent were obtained with sufficient resolution in the mesh.

Several approaches were taken to reduce the errors in the simulation of stripline transmission lines. Apparently, the sharp edges in the line, particularly near the edges of the strip, were a major source of computational error. Increases in the mesh resolution near those edges reduced the computational error, with smoothly-varying ratio zoning doing the best job. The sensitivity of the analytic impedance calculation was also examined to determine if empirical adjustments to the strip width would improve the errors in the computation. Apparently, the impedances computed with the ratio-zoned meshes could be brought into better agreement with the analytic results if the strip were widened on either side by two- to four-tenths of a zone in the analytic calculation. Presumably, the converse of this would be that narrowing the strip in the computational mesh by a similar amount would bring the computed impedance into closer agreement with the analytic result for the unaltered line parameters. Nevertheless, it must be noted that this empirical correction does not hold for all of the simulations; for example, simulations with nonuniform zoning and a relatively small absolute error in the computed impedance required a correction of almost a full zone width in the analytic impedance to bring the two values into agreement. Corrections in the code based on the use of nonlinear basis functions in order to more accurately compute the fields near the sharp edges of the strip would have been more rigorously satisfying. Unfortunately, in the limited time of this study, J. B. Grant was unable to obtain a stable realization of such a scheme. Nevertheless, this technique has been applied in two-dimensional FDTD codes, and it should, in principle, be applicable to this code.

Finally, it is noted that changes in programmatic direction within the Defense and Nuclear Technology Directorate have led to the termination of this particular project, which was aimed at the electromagnetic analysis of signal propagation and interference in microwave microelectronic circuits. Nevertheless, work on electromagnetic analysis and codes continues within the Engineering Directorate. There were, at the time of this project's termination, at least two outstanding issues surrounding the version of the code used in this study. First, in the latter stages of this investigation, the algorithm underlying the code was shown by Brandon and Rambo to be unstable in so-called chevron grids, in which the edges of the zones have a chevron or sawtooth structure¹⁵. Since the completion of this project, the instability has been addressed in two ways. In one approach, the depth of the chevron structures can be modified to reduce the instability's growth. Niel Madsen has developed a grid-smoothing technique that accomplishes this. Alternatively, one can add dissipation to the time integration scheme so that the resulting damping counters the growth of the instability. Doug Riley at Sandia National Laboratories in Albuquerque has added an adjustable dissipative correction that allows one to add just enough damping to prevent unstable growth. In principle, this dissipation could perhaps be applied locally to prevent damping out the electromagnetic signal in regions where there is no instability.

The second issue was the difficulty in obtaining outgoing boundary conditions at the end of the bounded transmission systems considered here. The special challenge of such systems is that the guided waves they support are incident on the end of the line at a wide range of angles, depending on the frequency content of the wave. Of particular difficulty is the case of single-frequency waves near cutoff, in which event the wave reaches the end of the guide essentially at grazing incidence. The simple graded-layer absorbers considered here provided acceptable reductions in the amplitude of waves reflected from the end of the line for the special case of normally-incident TEM pulses. For more complex modal content, tensor conductivities would have been required, and, even in this case, the method would not have worked for evanescent waves. Evidently the Lindman-Higdon condition has been applied on orthogonal grids to obtain good results except at frequencies very close to cutoff. This method, or some variant, may be generalized to non-orthogonal grids in the future to provide the necessary boundary condition.

Acknowledgments

I would like to acknowledge in particular the contributions of Scott Brandon in the conduct of this work. In addition, Brian Poole was very generous of his time in helping me use the MAXWELL EMINENCE code, Grant Cook provided a number of valuable clarifications and suggestions in the use of DSI3D, and David Steich provided comments on current treatments of the chevron-grid instability and outgoing boundary conditions.

References

1. N. K. Madsen, *J. Comp. Phys.* **119**, 34 (1995).
2. J. D. Jackson, *Classical Electrodynamics* (John Wiley and Sons, New York, 1975), 2nd ed., pp. 339-342.
3. S. Ramo, J. R. Whinnery, and T. Van Duzer, *Fields and Waves in Communication Electronics* (John Wiley and Sons, New York, 1965), pp. 446-447.
4. M. A. Christon, D. Dovey, *INGRID: A 3-D Mesh Generator for Modeling Nonlinear Systems, User Manual*, UCRL-MA-109790 (1992).
5. T.-S. Chen, *IRE Trans. on Microwave Theory and Tech.* **MTT-8**, 510 (1960).
6. Also given in B. C. Wadell, *Transmission Line Design Handbook* (Artech House, Norwood, MA, 1991), p. 136.
7. J. C. Tippet and D. C. Chang, *IEEE Trans. Microwave Theory and Tech.* **MTT-24**, 602 (1976).
8. M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions* (National Bureau of Standards, Washington, DC, 1964), p. 590.
9. P. Silvester, *Proc. IEE* **115**, 43 (1968).
10. K. Virga, Hughes Aircraft, personal communication.
11. J.-P. Berenger, *J. Comp. Phys.* **114**, 185 (1994).
12. See, for example, J. D. Jackson, *op. cit.*, p. 77; or D. S. Jones, *The Theory of Electromagnetism* (Pergamon, Oxford, 1964), p. 566.
13. G. Mur, *IEEE Trans. on Microwave Theory and Tech.* **MTT-29**, 1073 (1981).
14. J. Van Hese and D. De Zutter, *IEEE Trans. on Microwave Theory and Tech.* **40**, 547 (1992).
15. S. T. Brandon and P. W. Rambo, *IEEE Conference Record -- Abstracts, 1995 IEEE Int. Conf. on Plasma Sci*, no. 95CH35796, June 508, 1995, Madison, WI, p. 202.