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Democratic Reinforcement: Learning via Self-Organization

Dimitris Stassinopoulos* and Per Bak† ¶

**Center for Complex Systems, Florida Atlantic University*
Boca Raton, Florida 33431, USA
†Department of Physics, Brookhaven National Laboratory
Upton, New York 11973, USA

ABSTRACT

The problem of learning in the absence of external intelligence is discussed in the context of a simple model. The model consists of a set of randomly connected, or layered integrate-and fire neurons. Inputs to and outputs from the environment are connected randomly to subsets of neurons. The connections between firing neurons are strengthened or weakened according to whether the action is successful or not. The model departs from the traditional gradient-descent based approaches to learning by operating at a highly susceptible "critical" state, with low activity and sparse connections between firing neurons. Quantitative studies on the performance of our model in a simple association task show that by tuning our system close to this critical state we can obtain dramatic gains in performance.

I. Introduction

One of the most remarkable properties of biological neural networks is their ability to learn via *self-organization*. Simply put, this means that animals acquire experience and make sense of their environment without the aid of a "teacher" or some other form of external intelligence. To any non-expert that has ever seen a toddler acquiring a new skill with virtually no guidance or an animal adapting to a novel situation the statement would seem obvious. Yet for all its simplicity and common sense this idea has long remained on the fringe of the experimental and theoretical research concerning brain function, in all likelihood because of the severity of the contraints it imposes on brain modelling.

The term self-organization has been used in many different disciplines such as physics, chemistry, biology, and psychology, and often to convey different underlying mechanisms. Here, however, we will discuss self-organization solely in the context of learning[1].

The oldest and perhaps still most dominant approach to understanding the brain is what we call the "engineered brain" paradigm. According to this paradigm, brain

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function emerges because nature, in the role of the engineer, has created all the necessary mechanisms by establishing an intricate web that brings billions of pieces together. But how can evolution achieve such an engineering feat? We do not deny the role of evolution in many aspects of brain function – the very fact that our brain is different than a lobster's brain has to be attributed to evolution. Nevertheless, it cannot possibly account for the brain's ability to deal with unforseen situations, specific to an individual's experience, or for novel ones that evolution had never had the opportunity to confront.

In providing an alternative to this view, the field of artificial neural networks (ANN) has been instrumental (For reviews see Hertz *et al*[2] and Haykin[3]). The major contribution of ANN was that it demonstrated how non-trivial tasks can be achieved with networks composed of many simple computing elements. It also offered the first evidence that principles for brain function can be captured with models that have simple structure. Despite all the important insights ANN offered, however, they have not eliminated the need for an external intelligence. In the widely used *supervised learning* paradigm this takes the form of a "teacher" providing the system with a detailed scheme for the update of the synaptic weights based on knowledge of the goal to be achieved. Furthermore, most models for learning use gradient-based update rules, such as back-propagation, which are biologically implausible because they impose strong constraints on the architecture and they require computation that cannot be performed by the neural network itself. Thus, again the network is formed by design rather than by self-organization.

The issues of self-organization have been addressed in the context of reinforcement learning models[2,3]. These models are more realistic in the sense that there is no teacher explaining how to modify the synaptic weights, but only a "critic" telling the system whether its performance is successful or not. Most reinforcement learning models, however, still rely on back-propagation[4], or some other overseeing agency possessing prior knowledge of the problem, for the update of the synaptic weights. There is one exception, however. Barto[5], in one of the first variants of his *Associative Reward-Penalty*[6] (A_{R-P}) algorithm, discusses the idea of "self-interested" elements which do not have access to information other than a feedback signal from the environment broadcast simultaneously to all elements. We very much agree with his view that the difficulties in solving the problem of learning under the severe constraints imposed by self-organization are fundamental.

Recently, we have proposed *Democratic Reinforcement* (*DR*) as a new approach to the long-standing issues of learning via self-organization[7]. A similar approach was originally used to solve a non-trivial tracking problem by a continuous modification of its synaptic weights[8]. An evaluative feedback is sent democratically to all neurons simultaneously. The reinforcement rule operates in two modes: a "learning" mode when the evaluative feedback is positive, and an "exploration" mode when the evaluative feedback is negative. The reinforcement rule depends on the firing states of the presynaptic and postsynaptic neurons only. The novel feature of our model is that the threshold for firing is regulated in order to keep the output activity minimal. This sets the system up at or near a critical state, which turns out to be crucial for the performance of the network.

To the best of our knowledge, the A_{R-P} and the DR represent the only attempts to address the problem of learning via self-organization. However, the two algorithms are fundamentally different. While the A_{R-P} is a gradient-descent algorithm, DR solves problems by operating at or near a highly-susceptible "critical" state in which the system becomes very sensitive to modification of the synaptic weights. To make our point more concrete we shall discuss the two algorithms in the context of a specific association task.

II. The Model

The problem we are addressing can be summarized as follows: How can many "agents", be it neurons or some other kind of computing element, operating under local rules and receiving input only from a small fraction of other agents, cooperatively perform macroscopic tasks imposed on them by their environment? For the agents to perform a task, they are given the freedom to tune some local parameters, such as their synaptic weights, to appropriate values. The severity of the problem arises from the requirements: i) that there can be no external intelligence with prior knowledge of the problem to instruct the agents on how to tune their parameters, only some form of overall evaluative feedback that tells the agents when some parameters essential for "survival" fall out of bounds, and ii) that the system is robust and versatile, allowing for solutions that are not task or architecture-specific. From an agent's point of view any effective adjustment can be made only when the adjustment has a detectable impact on the overall, collective behavior of the system, allowing for a robust feedback. This, however, is a non-trivial requirement. Most neural networks are rather insensitive to small changes in their parameters. Then we are left with a situation where no agent can "learn" from its actions because there is no way for it to know whether it should get "credit" or "blame" in the final evaluation. This is known as the *credit-assignment problem*[9].

The above problem remains largely unresolved. Of course, real biological networks serve as examples par-excellence that solutions to this problem exist, but how do we mimic this in a simple model?

Our model imposes no constraints on the architecture whatsoever. In the most general case neurons are connected randomly to each other via unidirectional connections[7]. Each neural unit receives input and sends output to a small fraction of the total number of units. This ensures that the majority of units in our system are hidden, i.e., units that interact with the environment indirectly through other units. The system interacts with the outer world in three ways (Fig. 1); via i) its input units which receive input from the outside and thus provide the system with information about the state of the world; ii) its output units which allow the system to act on the environment; and iii) a binary yes/no feedback that is broadcast to all units and indicates whether the action to the environment was successful or not.

Although from a conceptual point of view the random architecture is the most appealing, we found the layered architecture of Fig.1 to work better[10]. Here each unit is connected to its three nearest neighbors in the next layer. The units can be either in a firing state, $n_i = 1$, or in a quiescent state, $n_i = 0$. In standard fashion, each unit integrates the input of its presynaptic units, $h_i = \sum_j J_{ij} n_j$. The unit fires if the input exceeds a threshold T. Input patterns, indicated in Fig. 1 by dark disks, are presented to the system by setting the corresponding units into a firing state. The system acts on the environment via its output sites, shaded disks in Fig. 1. The feedback, r, broadcast by the environment, takes two values; positive, r_0, if the action was evaluated as successful and negative, $-r_0$, if not. Both the synaptic modification and the regulation of the threshold depend on the evaluative feedback, r.

The update rule affects only connections between firing neurons, $J_{ij} \rightarrow J_{ij} + [rJ_{ij} + h_{ij}]n_i^{\prime} n_j$, where $n_i^{\prime}$ denotes the state of the i'th neuron at the next time step and h_{ij} is a random noise between $-h_0$ and h_0. The outgoing weights are normalized, $J_{ij} \rightarrow J_{ij} / \sum_i J_{ij}$. The rule differs from standard gradient-descent based update rules in one crucial aspect: When $r > 0$ the system operates in a "learning" mode in which connections are being strengthened and the performance improves but when $r < 0$ the system operates in an "exploratory" mode in which strong connections are being weakened and weaker connections are being strengthened. Typically, during this phase the performance deteriorates. In contrast, standard reinforcement schemes, such as A_{R-P}, rely on an improvement of the performance both for positive and negative r and perform the exploration stochastically.

In addition to the synaptic update rules, r regulates the threshold, T. The objective is that the output activity is kept to a minimum. This is essential to ensure that the system attributes credit and blame to the minimum possible number of active units, in order to keep the network intact for other problems. In our first versions of the DR algorithm (Ref. 7, 8) the output activity was regulated to a small but arbitrarily chosen level of activity. Choosing a value for this parameter, however, assumes prior knowledge of the task to be completed which runs counter to the self-organization philosophy. More recently[11] we have introduced a threshold mechanism that depends solely on r, $T \rightarrow T + \delta(r)$, where δ assumes a positive value, δ_+, if $r > 0$, and a negative value, δ_-, if $r < 0$ ($|\delta_-| >> \delta_+$).

The typical criterion for success, $r > 0$, is that the selected output sites are active and for failure, $r < 0$, that at least one of the selected output sites is inactive. On first thought, that might sound like nonsense: the system can trivially obtain a positive feedback and thus get its reward by lowering its threshold, for instance, and keeping all of its output units active! However, the solution that the system opts for is the one where the selected output sites are active and all the rest are inactive. But even if we accept that such a rule makes computational sense what sense does it make in terms of biology? It is true that in some simple situations we may view this reward/penalty coming from the environment. This was the case in Ref. 7 where we used the analogy of a monkey that presses one or more buttons. In such a situation the environment indeed provides food as long as the selected buttons are among the ones pressed by the monkey. However, the monkey can not be considered successful merely because now and then it happens to press the right buttons. It is important to reduce the incorrect actions. In that respect it makes more sense to view $r > 0$ not as an external reward but rather as the default mode of operation; an innate tendency of the system to minimize its efforts, while still having success. In contrast, one might view the $r < 0$ signal as an external wake-up call announcing that something is wrong, for instance when some parameter that is crucial to survival exceeds a certain value. The preference for passivity is sharply interrupted when $r < 0$. There is no symmetry between $r < 0$ and $r > 0$.

How does the system solve problems? How does it successfully attribute credit and blame where it is due? Our studies indicate that this involves a build-up process in which the synaptic "landscape" reaches a near critical, highly susceptible state in which small changes in the synaptic weights can have a big effect on the collective activity. In such a state the system can establish efficiently causal relationships between changes in the synapses and the output. To achieve such a critical state the system: i) assigns credit and blame only to connections between active neurons, ii) keeps the activity low by means of the global regulation of the threshold and the local learning rules. By combination of these two mechanisms the system attributes credit and blame selectively by driving the system to the interface between success and failure.

III. Self-organization in a Simple Association Task

In an association task we ask the system to generate a certain input/output pattern. The insets in Figs. 2a, b offer examples of a simple association task. The system accomplishes the task by "carving" paths between the input sites and the output sites. In previous work[7,8,12] we have investigated the performance of DR in a variety of situations: multiple input/output patterns, recovery from "damage", tracking, conditioning, and so on. Here we will be concerned with the question of degradation of performance as the size of the association task grows.

We consider an $L_1 \times L_2$ layered network (Fig. 2a, inset). The number of layers in this networks is kept fixed, $L_1 = 16$, while the lateral dimension is varied, $L_2 = 16, 32, 64, \ldots$ The number, c, of input and output sites in the input/output pattern is varied accordingly, $c = L_2/16$. Here the input is confined to the top row and the output to the bottom row. To minimize crossover between paths we keep the input and output sites in pairs, well separated from each other. More precisely, each of the c columns of a given network contains a single input and a single desired output site.

One motivation for this analysis is to demonstrate in a convincing manner the difference between DR and A_{R-P}. We were not so interested in the absolute performances of the two algorithms, which tend to be sensitive to the tuning of the various parameters, but rather to the scaling of the performance with the size of the network. For our simulations with A_{R-P} the same layered architecture was chosen (Fig. 1) but the number of layers was set to $L_1 = 4$ (more layers would degrade the performance of A_{R-P} too much). The lateral dimension is varied, $L_2 = 4, 8, 12, \ldots$ The input/output patterns were chosen with similar considerations as in the DR case (see insets in Figs. 2a,b for a comparison). Figures 2a and 2b show examples of the performance, P, for DR and A_{R-P} as a function of time. P is defined as the temporal average of the activity at the selected output sites minus the activity at the rest of the output sites. Appropriate normalization assures that best performance ($P = 1$) corresponds to persistent firing at the selected output sites only, whereas worst performance ($P = -1$) corresponds to persistent firing everywhere except at the selected sites.

The DR is characterized by intervals of rapid improvement in performance, interrupted by sudden dips. This behaviour is a signature of the dual mode of operation of the algorithm: i) the learning or exploitation mode in which the system strengthens connections and "weeds out" irrelevant paths, and ii) the exploration mode in which the system tends to spread the activity in an attempt to explore new possibilities with subsequent decrease in the performance P. The A_{R-P} performance versus time, although also highly irregular, seems to have a very different structure. It is dominated by very fast fluctuations at the smallest time scale (not seen here due to averaging of P). At longer time scales it seems to be dominated by long periods during which the system seems trapped at a certain level of performance. Once the system escapes this barrier the transition to a new performance level appears to be very fast. We would like to point out that at the individual level, and with the limited information available to it, each neuron *always* opts for the change that it expects will increase the collective performance. In its decisions, however, it cannot take into account the positive or negative contributions of the other neurons. Therefore, it is only in a statistical sense that the system senses the gradient towards a better performance and can tune its synapses accordingly. The stochastic nature of the A_{R-P} can also be witnessed in Fig. 3, ($\triangle$). Here we depict the time to completion of the task, $\bar{t_s}$ (averaged over many runs obtained with different initialization) as a function of the number, c, of input/output pairs. In a first order approximation, it seems that $\bar{t_s}$ scales exponentially with c, $\bar{t_s} \sim e^{\alpha c}$, with $\alpha \simeq 1.6$.

DR (Fig. 3, $\bigcirc$) has a significantly better scaling behavior. When plotted in a log-log plot (inset of Fig. 3) $\bar{t_s}$ might follow a power law, $\bar{t_s} \sim c^{\gamma}$, which subsequently breaks down around $c = 8$. If this is true it would not be inconsistent with our suggestion that the algorithm operates near a "critical", highly susceptible regime. Although evidence of such a critical regime have previously been seen in the dynamics of our system[7], this scaling behavior offers the first quantitative evidence. Clearly, this initial data seems to be amenable to more than one interpretation, therefore it is imperative that the direct consequences of our critical-state hypothesis are tested further.

The convergence towards the critical state is accomplished by ensuring that the patterns of activity for different input signals do not overlap, on one hand, while, on the other hand, not being too sparse to connect inputs with desired outputs. In "sand" models of self-organized criticality,[13] overlap of events ("avalanches") is avoided by keeping the input rate low. Here, criticality is achieved by keeping the output low.

It turns out that one can improve the efficiency, and carry the system closer to the critical point by further ensuring that changes in the activity, due to threshold modulation do not overlap in time, while not happening too infrequently with respect to the synaptic modification rate. We do so by allowing a variable rate δ_+ for increasing the threshold T. More precisely the rise of the threshold is governed by $\delta_+(t, r)$, $\delta_+ \to a\delta_+$, where $a \gtrsim 1$. Notice that now δ_+ is time dependent in the sense that, while $r > 0$, it is constantly increased and r dependent in the sense that it is reset to a small value whenever r becomes negative, $\delta_+ \to a^{-L_1}\delta_+$. The rate of decrease of T, δ_- is kept constant as before. The modified algorithm leads to a significant improvement of the performance (Fig. 3, (□)). Furthermore, the new curve, $\bar{t}_s(c)$, gives a stronger indication for the existence of a power law with exponent $\gamma \simeq 1.3$. The modification was chosen for its simplicity rather than its performance and, based on our experience, it appears to be straightforward to obtain further improvements.

IV. Conclusions

In this paper we have been concerned with the issues of self-organization which must play a central role in brain function. We propose a new mechanism through which efficient self-organized learning takes place. The central element of is a build-up process that allows the system to operate at a "critical" state, characterized by high sensitivity to small modifications of the synaptic weights and low output activity. The combination of those features allows the system to establish strong cause-effect relationships that allow the coexistence of many input/output patterns.

We suggest that the mechanisms that enables self-organizion in our model might also underlie real brain function, so *DR* can serve as an excellent testbed for further exploration of the consequences of such a hypothesis. It might be worthwhile exploring whether some signature of the critical state described in our model can also be observed in actual experiments.

References

1. A qualification is necessary here to avoid confusion of our use of the term "self-organization" as this used in *unsupervised learning.* In this paradigm the system modifies its synaptic ways with no feedback or any supervision whatsoever from the environment. Our view is that the consideration of environmental constraints is essential in biological learning and intelligent behavior in general, therefore, unsupervised learning has been entirely omitted from our discussion.

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3. S. Haykin, *Neural Networks, a Comprehensive Foundation* (Macmillan, New York, 1994).

4. Based on the partial information provided by the critic a target pattern is determined and the output-weight errors computed. The rest of the weights can then be updated by back-propagating this error-signal through the network (see Hertz *et al* in Ref. 2).

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10. This is primarily due to computer time limitations since the layered architecture gives substantially better performance[7] but also due to our interest in making comparisons with the A_{R-P} algorithm which is implemented in feedforaward architectures.

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Figure captions

Figure 1. Block diagram of the model, here shown for the layered architecture. The input sites that receive signals from the environment are shown as dark discs. The output sites are shown as shaded disks. Periodic boundary conditions are assumed for the layers.

Figure 2. a) *DR*: Performance vs. time, for a 16×64 system and, for the input/output pattern shown in the inset. Light sites denote quiescent units and dark sites denote firing ones. The firing sites connect the input and output sites by effectively forming wires. The parameters of the algorithm have been set to: $r_0 = 0.1, \delta_+ = 0.01/16, \delta_- = -0.05/16$, and $h_0 = 0.01$. The performance is obtained by averaging over 50 time steps. b) A_{R-P}: Performance vs. time (measured in 'trials'), for a 4×16 system, and for the input/output pattern shown in the inset. The connections between input and output sites are more complicated. The central element of the algorithm is the update rule for the synaptic weights, $J_{ij} \rightarrow J_{ij} + \eta(r[n_i - < n_i >] + \lambda(1-r)[-n_i - < n_i >])n_j$, where η is the 'learning' coefficient, λ is the 'penalty learning rate factor', $< n_i >= \tanh(\beta \sum_j J_{ij} n_j)$ is the average firing state, and r is the evalutation feedback (for details see Hertz *et al* in Ref. 2). The parameters have been set to: $\eta = 0.5, \lambda = 0.001$, and $\beta = 0.5$. The performance is obtained by averaging over 100 trials. *Insets:* Typical successful ($P = 1$) activity patterns for DR and A_{R-P}.

Figure 3. Average time elapsed, $\bar{t_s}$, to completion of an association task vs. number of input/output pairs, c. ($\triangle$) A_{R-P}: systems of size $L_1 \times L_2$, $L_1 = 4$ and $L_2 = 4, 8, 12$, and 16 have been considered. For each case twenty runs were performed (with the exception of the 4×16 system for which we conducted five runs only, due to computing time limitations) for the same association task but with different initialization; ($\bigcirc$) DR: systems of size $L_1 \times L_2$, $L_1 = 16$ and $L_2 = 16, 32, 64, 128$, and 192 have been considered. For the same association task fifty runs, with differing initializations, were considered; ($\square$) DR with variable δ_+ ($a = 1.05$): systems of size $L_1 \times L_2$, $L_1 = 16$ and $L_2 = 16, 32, 64, 128$, and 256 have been considered. For the same association task, fifty runs with differing initializations, were considered. Vertical bars denote standard deviation. *Inset:* Same data in log-log plot. The solid lines represent least-squares fits. ($\triangle$): $\sim e^{1.6c}$; ($\square$): $\sim c^{1.3}$.

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Figure 1

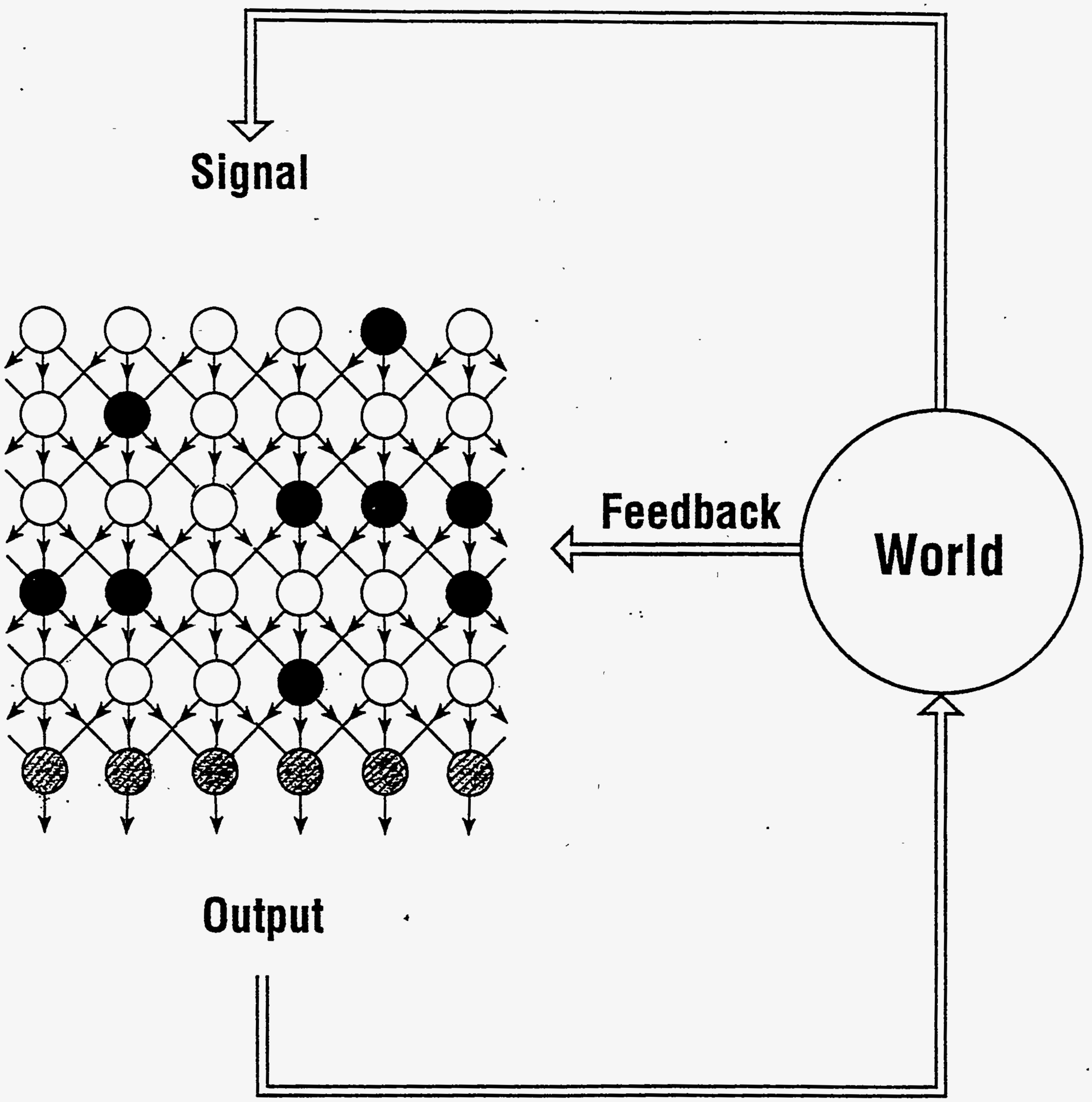
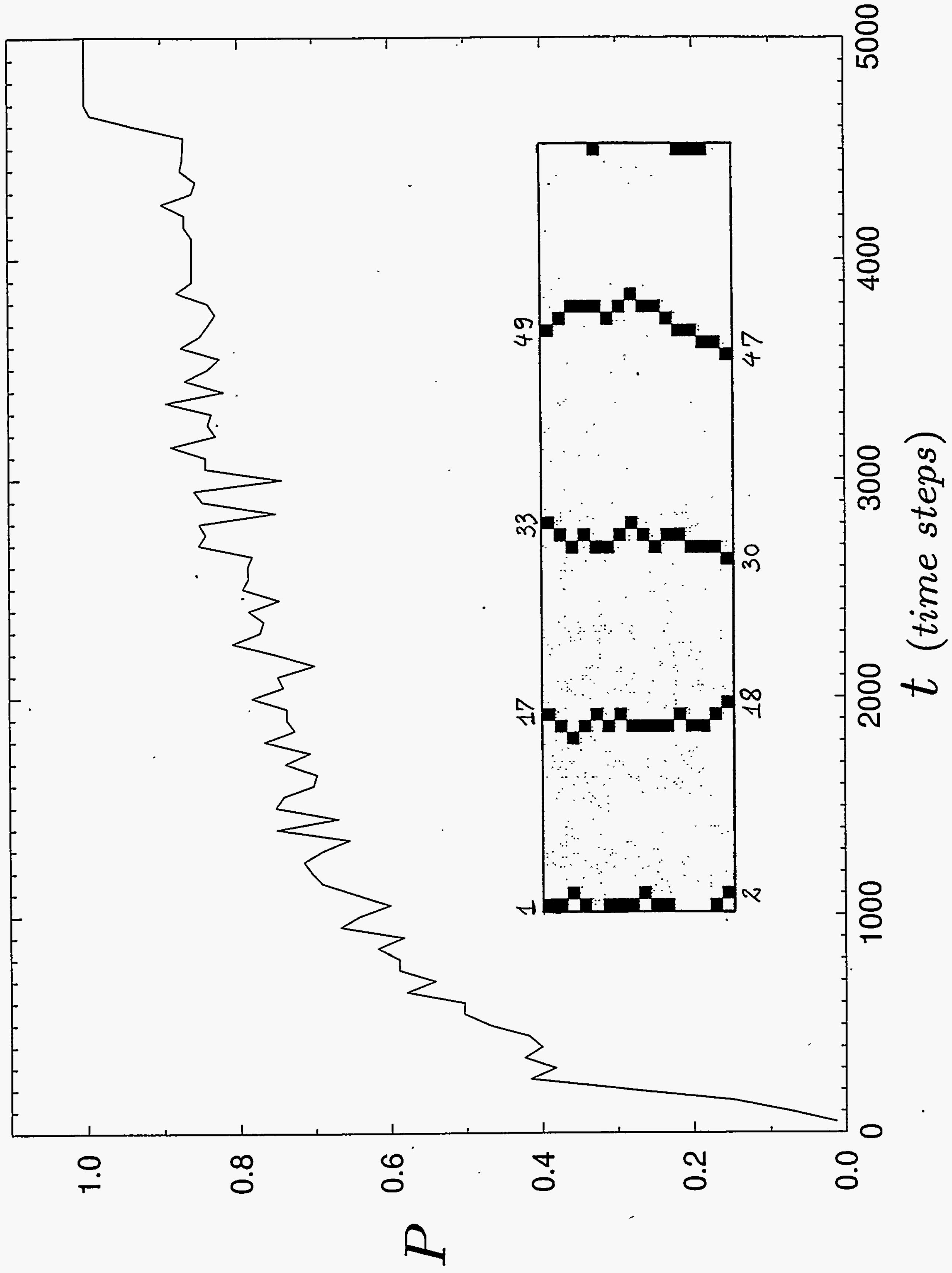


Figure 2a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18
33
30
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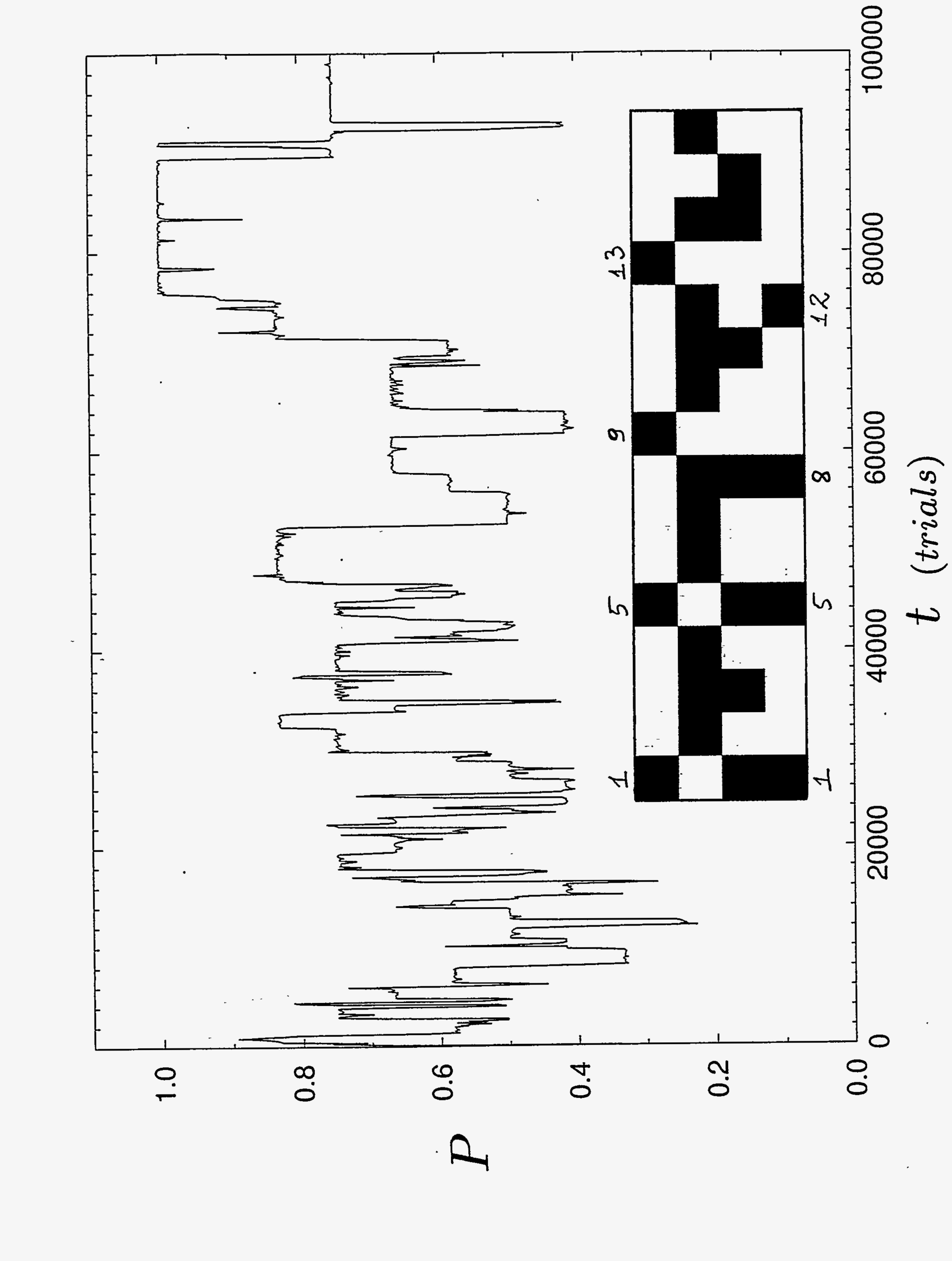
Figure 2b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