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Modeling of a Sinusoidal Lobed Injector: Vorticity & Concentration Fields for a Cold Flow

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Prepared by
Sandia National Laboratories
Albuquerque, New Mexico 87185 and Livermore, California 94550
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under Contract DE-AC04-94AL85000

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Modeling of a Sinusoidal Lobed Injector: Vorticity & Concentration Fields for a Cold Flow

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Abstract

In this report, we present a simple and somewhat preliminary numerical model of a sinusoidal lobed injector. The lobed (corrugated) injector is being considered by several investigators as a potentially efficient device to mix fuel and air for combustion purposes. In this configuration, air flows parallel to the troughs and valleys of corrugations which grow in amplitude in the stream-wise direction. These ramped corrugations produce stream-wise vortices which enhance the downstream mixing. For the lobed injector, the corrugations are actually double walled which allows one to inject fuel through the space between them into the flow downstream of the ramp. The simulation model presented herein is based on a vorticity formulation of the Navier-Stokes equations and is solved using an unsteady viscous vortex method. In order to demonstrate the utility of this method we have simulated the three-dimensional cold mixing process for injection of methane gas into air. The vorticity and fuel concentration field downstream of the injector are simulated for two different injector geometries. We observe from these two simulations that variation of the amplitude of the corrugations can be used to achieve considerably different mixing patterns downstream of the injector.

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Acknowledgment

The author would like to thank Bill Ashurst, Senior Member of the Technical Staff at Sandia National Laboratories, Livermore, CA and Ann Karagozian who is a professor in the Mechanical, Aerospace, and Nuclear Engineering Department at UCLA for their introduction to this very interesting problem.

This work was partially supported by DOE through the Office of Basic Energy Sciences, Division of Chemical Sciences.

Nomenclature

$A_{k,l}$	=	Influence coefficient matrix for lateral direction
a	=	Vortex core radius
a_f	=	Core radius of fuel blob
a_0	=	Initial vortex core radius
$B_{k,l}$	=	Influence coefficient matrix for vertical direction
h	=	Injector gap height
$\hat{i}$	=	Unit vector in x direction
j	=	Unit vector in y direction
k	=	Index
l	=	Index
m_f	=	Mass of fuel per unit volume
$\dot{m}_\lambda$	=	Mass flow rate of fuel per wavelength
$\hat{n}$	=	Unit vector normal to surface
$\hat{n}_k$	=	Unit vector normal to surface for k^{th} collocation point
N	=	Number of source or target points
r	=	Radius in x, y plane
Sc	=	Schmidt number
s	=	Distance along corrugation surface
Δs	=	Incremental distance along corrugation surface
s_λ	=	Total length along surface of corrugation per wavelength
t	=	Time
t_0	=	Initial time
u	=	Lateral velocity
$\hat{u}_k$	=	Velocity vector at k^{th} control point
u_s	=	Lateral surface velocity
v	=	Vertical velocity
x	=	Lateral direction
x'	=	$x - \xi$
y	=	Vertical direction
y'	=	$y - \eta$

Nomenclature (continued)

y_k	=	y coordinate to point on corrugation at k^{th} collocation point
y_m	=	Amplitude of corrugation
y_s	=	y coordinate to point on corrugation
z	=	Stream-wise direction
z_m	=	Length of corrugated ramp
W_∞	=	Free-stream velocity
Γ	=	Circulation strength
Γ_h	=	Circulation produced by one-half wavelength of corrugation
Γ_l	=	Circulation of l^{th} vortex
η	=	Vertical location of vorticity source
λ	=	Corrugation wavelength
ξ	=	Lateral location of vorticity source
ν	=	Kinematic viscosity
ρ_f	=	Fuel density
Υ	=	Strength of fuel blob
χ_f	=	Volume fraction of fuel
ω	=	Vorticity
ω_z	=	Vorticity in z direction

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1 INTRODUCTION

In this report we present a simple and somewhat preliminary numerical model of a sinusoidal lobed injector. Lobed mixers have been used in jet engine exhaust systems as a means to reduce noise [1] and for mixing enhancement in chemical lasers [2] and combustors. Computational and experimental studies have been conducted by Elliott et. al. [3] on such mixers. In the mixer configuration, air flows parallel to the troughs and valleys of corrugations which grow in amplitude in the stream-wise direction. These ramped corrugations produce stream-wise vortices which enhance the downstream mixing. Lobed injectors which are potentially useful for lowering NO_x have been studied by Smith et. al. [4]. Lobed injectors differ from lobed mixers in that the corrugations are actually double walled which allows one to inject fuel through the space between them into the flow downstream of the ramp.

The geometry of the lobed injector is shown in Figure 1. In this configuration, fuel flows between a double wall corrugation and begins to mix with the external flow field at the mixer exit plane. The corrugated ramp is characterized by the three length scales λ , y_m , and z_m which are the corrugation wave length, the corrugation exit plane amplitude, and the ramp length respectively. In the present studies, the waveform of the corrugation is taken to be a simple sin wave although virtually any waveform may be analyzed using the present methodology.

For the present studies, the upstream flow field is considered to be moving at a uniform velocity W_∞ prior to encountering the corrugated ramp. The corrugated ramp generates stream-wise vorticity in the x, y plane in the z direction. The amount of stream-wise vorticity which is generated can be calculated by satisfying the boundary conditions which require that the corrugations are not penetrated by the flow. For cases where the velocity on the top of the splitter plate is different from that on the bottom, vorticity whose direction is in the x, y plane will also be generated. Treatment of such cases is beyond the scope of the present study.

Figure 1. Sinusoidal Lobed Injector Geometry

2 METHODOLOGY

Our general approach is to first convert the steady three-dimensional flow field into an unsteady two-dimensional one by assuming that the z coordinate can be replaced by tW_∞ in the governing equations. This transformation is best visualized by considering an observer moving in the z direction with a velocity W_∞ . As the corrugated ramp is encountered, its surfaces appear to be expanding into the flow in the y direction with a velocity equal to $y_s W_\infty / z_m$ where y_s is the y coordinate for a given point on the corrugation. In order to satisfy the condition that the flow cannot penetrate the corrugated surface, vorticity must be generated on the surface. As one moves at the velocity W_∞ downstream of the injector exit plane, the vortical flow field evolves under the combined action of diffusion and convection of the vorticity field.

2.1 Generation of Stream-wise Vorticity

If we assume that the interaction of the flow with the corrugated ramp is of sufficiently short duration, the vorticity generated by the ramp will not have had time to diffuse very far away from the surface by the time the flow reaches the exit plane. Thus, we will assume that the stream-wise vorticity at the exit plane is contained in a thin layer which is congruent with the sinusoidal corrugation. The net distribution of vorticity generated by both the upper and lower sides of the corrugation is obtained by using relationships between the vorticity field and velocity field along with satisfaction of the velocity boundary condition.

For a set of stream-wise vortices with circulation Γ which repeat in the x direction with a periodicity of λ , we can write the following equations for the velocity components in the x, y plane:

$$\begin{aligned} u(x, y) &= \frac{\Gamma}{2\lambda} \frac{\sinh(2\pi y'/\lambda)}{\cosh(2\pi y'/\lambda) - \cos(2\pi x'/\lambda) + 1 - \cos(2\pi a/\lambda)}, \\ v(x, y) &= \frac{\Gamma}{2\lambda} \frac{\sin(2\pi x'/\lambda)}{\cosh(2\pi y'/\lambda) - \cos(2\pi x'/\lambda) + 1 - \cos(2\pi a/\lambda)}, \end{aligned} \quad (1)$$

where $x' = x - \xi$ and $y' = y - \eta$. Here, the vorticity source is located at ξ, η while the evaluation or target point is located at x, y . It should be noted that Equation 1 is a regularized version of the formulation given by Lamb [5] for a set of discrete vortices. The regularization is achieved with the addition of the term in the denominator $1 - \cos(2\pi a/\lambda)$. Here a represents a core radius. The vorticity field for this set of periodically placed vortices is given by:

$$\omega = \frac{\pi\Gamma}{\lambda^2} \frac{[1 - \cos(2\pi a/\lambda)] [\cosh(2\pi y'/\lambda) \cos(2\pi x'/\lambda)]}{[\cosh(2\pi y'/\lambda) - \cos(2\pi x'/\lambda) + 1 - \cos(2\pi a/\lambda)]^2}. \quad (2)$$

It is interesting to note that the “core function” represented by Equation 2 behaves in a similar fashion to a Gaussian core given by:

$$\frac{\omega\lambda^2}{\Gamma} = \frac{1}{\pi} \left(\frac{\lambda}{a}\right)^2 e^{-\left(\frac{r}{a}\right)^2}, \quad (3)$$

as demonstrated by the plots in Figure 2. Here, $r^2 = x'^2 + y'^2$.

Figure 2. Comparison of Core Function with Gaussian Core

In order to enforce the velocity boundary condition on the corrugation (no flow through the surface) we first note that the surface appears to be moving in a vertical direction with a velocity $v_s = y_s W_\infty / z_m$. The horizontal velocity of the surface u_s is equal to zero. We next discretize the corrugation as shown in Figure 3 such that there are N control points or targets and N vortices or sources in the first quarter wavelength. Images or reflections of the vortex system in the first quarter wavelength appear in the second, third, and fourth quarter of the wavelength. The velocity at a control point k can be written as:

$$\hat{u}_k = [A_{k,l} \hat{i} + B_{k,l} \hat{j}] \{\Gamma_l\}, \quad (4)$$

where the coefficient matrices $A_{k,l}$ and $B_{k,l}$ are obtained by evaluating Equation 1 for the l^{th} source at the k^{th} control point with proper consideration of images and reflections.

Figure 3. Discretization of the Corrugation

We next form a set of N linear equations in the N unknown values of Γ_l by equating the normal surface velocity to the fluid velocity normal to the surface. The unit vector which is normal to the surface at the collocation point k can be written as:

$$\hat{n}_k = -\left(\frac{\partial y}{\partial s}\right)_k \hat{i} + \left(\frac{\partial x}{\partial s}\right)_k \hat{j}, \quad (5)$$

where s is the distance along the surface. The resulting linear equation set is given by:

$$\left[-\left(\frac{\partial y}{\partial s}\right)_k A_{k,l} + \left(\frac{\partial x}{\partial s}\right)_k B_{k,l} \right] \{\Gamma_l\} = \left(\frac{\partial x}{\partial s}\right)_k \left(\frac{y_k W_\infty}{z_m} \right), \quad (6)$$

where y_k is the y coordinate at collocation point k . We note that Equation 6 applies to general corrugation wave forms, not just the sinusoidal form being studied here.

Solution of Equation 6 for the N values of Γ_l yields the net vorticity distribution on a sheet which is congruent with the corrugation at the exit plane. No attempt has been made to satisfy the no-slip condition at the surface. Satisfaction of the no-slip condition can be achieved by splitting the single sheet into two sheets in such a way so as to produce the correct tangential surface velocity between the two sheets. An alternate approach developed by Kempka et. al. [6] is to write a system of equations which could be used to replace Equation 6 and which would satisfy the no-slip condition directly. For the present work, only the net vorticity has been calculated.

2.2 Wake Evolution

2.2.1 Vorticity Field

Downstream of the exit plane of the corrugated injector, the stream-wise vorticity begins to evolve through the combined action of convection and diffusion. We model the flow with a number of overlapping vortex blobs with core radii a and circulation strength Γ_l . We again assume that the velocity in the stream-wise or z direction is a constant equal to W_∞ which allows us to observe the flow as an unsteady one whose evolution is given by the two-dimensional vorticity formulation of the Navier-Stokes equation:

$$\frac{\partial \omega_z}{\partial t} + (\hat{u} \cdot \nabla) \omega_z = \nu \nabla^2 \omega_z. \quad (7)$$

In order to satisfy Equation 7, we use a time-splitting scheme in which we first solve for the evolution due to convection and then for the evolution due to diffusion. In other words,

$$\frac{\partial \omega_z}{\partial t} = \left(\frac{\partial \omega_z}{\partial t} \right)_c + \left(\frac{\partial \omega_z}{\partial t} \right)_d, \quad (8)$$

where

$$\left(\frac{\partial \omega_z}{\partial t} \right)_c + (\hat{u} \cdot \nabla) \omega_z = 0, \quad (9)$$

and

$$\left(\frac{\partial \omega_z}{\partial t} \right)_d = \nu \nabla^2 \omega_z. \quad (10)$$

With regard to convection of the vorticity field, we note that Equation 9 may be rewritten for each of the vortex blobs as:

$$\frac{D\Gamma_l}{Dt} = 0. \quad (11)$$

Equation 11 simply states that as the vortex blob moves at the local fluid velocity its circulation strength must remain constant. The local fluid velocity is obtained by summing the contribution at a point (as calculated from Equation 1) for each of the vortex blobs in the flow.

There are several methods by which the viscous diffusion process given by Equation 10 may be modeled when using Lagrangian vortex elements. These methods include the Gaussian random walk method [7], the diffusing core method [8], the particle strength exchange (PSE) method [9], and the diffusion velocity method originally proposed by Ogami and Akamatsu [10] and extended by Kempka and Strickland [11].

In the present work we will use the diffusing core method due to its ease of use. In this method, we use an exact solution for Equation 10 for blobs with a Gaussian vorticity distribution. The diffusion process evolves by expansion of the core radius according to:

$$a = \sqrt{a_0^2 + 4\nu(t - t_0)}, \quad (12)$$

where a_0 is the core radius at time t_0 . Since the diffusion equation is linear, each vortex blob may be diffused independently and the new vorticity field obtained from superposition of these blobs will exactly satisfy the diffusion equation for the field as a whole.

Contrary to some previous opinions, the diffusing core method is a valid method for solving the diffusion step. It should be noted however, that one should not allow the blobs to grow too large because this introduces errors in convecting the vorticity at points which are far removed from the center of the blob. The diffusion velocity method, which also solves the diffusion step exactly, produces both a translation and an expansion of the blob but the expansion is typically much smaller than that associated with the diffusing core method. We also note that in the present case, the cores only approximate that of a Gaussian as has been indicated by Figure 2.

2.2.2 Concentration Field

In order to simulate the concentration field we model the fuel by blobs which are analogous to the vortex blobs. For this particular problem, we can assume that the centers of the vortex blobs coincide with the center of the fuel blobs since they both originate along the corrugation at the injector exit plane. The centers of these fuel blobs are convected at the local fluid velocity. The diffusion of the fuel is governed by an equation similar to Equation 10:

$$\frac{\partial m_f}{\partial t} = \frac{\nu}{Sc} \nabla^2 m_f, \quad (13)$$

where m_f is the mass of fuel per unit volume of space and Sc is the schmidt number associated with the diffusion of the fuel into the air. By dividing both sides of Equation 13 by the density of the fuel ρ_f we obtain the evolution equation for the volume of fuel per volume of space or simply the volume fraction of fuel χ_f :

$$\frac{\partial \chi_f}{\partial t} = \frac{\nu}{Sc} \nabla^2 \chi_f. \quad (14)$$

For the present purposes we find it convenient to use fuel volume fraction blobs in place of fuel mass blobs. From the analogy between χ_f and ω we can deduce the following influence equation for a single volume fraction blob:

$$\chi_f = \frac{\pi \Upsilon}{\lambda^2} \frac{[1 - \cos(2\pi a_f/\lambda)] [\cosh(2\pi y'/\lambda) \cos(2\pi x'/\lambda)]}{[\cosh(2\pi y'/\lambda) - \cos(2\pi x'/\lambda) + 1 - \cos(2\pi a_f/\lambda)]^2}, \quad (15)$$

where Υ is the blob strength and a_f is the blob radius. The strength Υ is obtained from:

$$\Upsilon = \int \chi_f dA, \quad (16)$$

where the integration is over the area which contains the blob. In the present case, the quantity Υ is conserved since there is no destruction of fuel and the flow is incompressible. The strength Υ can easily be obtained from conditions at the exit plane where $\chi_f = 1.0$ in the injector gap and zero elsewhere. Denoting the gap width by h and the increment along the corrugation which is associated with the blob as Δs then $\Upsilon = h\Delta s$.

We again use the expanding core method to satisfy the diffusion equation represented by Equation 14 with the approximation that the initial core radius is $0.5h$. Therefore, the evolution equation for the volume fraction blob radius a_f is given by:

$$a_f = 0.5 \sqrt{h^2 + 16 \frac{\nu}{Sc} (t - t_0)}. \quad (17)$$

The total mass flow rate $\dot{m}_\lambda$ through the injector per wavelength is equal to $\rho_f h s_\lambda W_\infty$. Here, the arc length s_λ of a sinusoidal corrugation per wavelength is given approximately by:

$$\frac{s_\lambda}{\lambda} = 1.000 + 1.418 \frac{y_m}{\lambda} - 0.679 \left(\frac{y_m}{\lambda} \right)^2 + 0.206 \left(\frac{y_m}{\lambda} \right)^3. \quad (18)$$

3 EXAMPLE SIMULATIONS

3.1 Calculation of Half-Wavelength Circulation

It is advantageous to calculate the magnitude of circulation which is produced by the injector over each half wavelength. In general, the vorticity which is produced by the sinusoidal injector is positive (counter clockwise when looking upstream) for $-0.25 \leq x/\lambda \leq 0.25$, negative for $0.25 \leq x/\lambda \leq 0.75$, and so on. As we shall see later, the vorticity produced from these half wavelength segments tends to roll up into a single vortex under certain conditions.

For those cases, the net effect of the injector is to produce sets of counter rotating vortices which are separated by a distance $x = 0.5\lambda$.

In Figure 4 we show the results of several calculations which yield the magnitude of the circulation Γ_h produced by the injector over a half wavelength. The circulation Γ_h which has been scaled by λ , z_m , and W_∞ as indicated, can be shown to be only a function of y_m/λ by careful examination of the governing equations.

Figure 4. Half-Wavelength Circulation Generated by Injector

When plotted using a log-log scale, the non-dimensional half-wavelength circulation can be seen to be roughly proportional to the square of the corrugation height normalized by the wavelength. However, a better fit, which is shown in Figure 4, is given by the following third order polynomial:

$$\frac{\Gamma_h z_m}{\lambda W_\infty \lambda} = 0.394 \frac{y_m}{\lambda} + 2.077 \left(\frac{y_m}{\lambda} \right)^2 + 0.125 \left(\frac{y_m}{\lambda} \right)^3. \quad (19)$$

3.2 Distribution of Circulation at Exit Plane

We have found that the distribution of circulation at the exit plane may also be cast in terms of a similarity solution, although this is not apparent from the governing equations. In Figure 5 we plot the x gradient of Γ which has been normalized by λ and Γ_h versus x/λ . We have made a number of calculations which are typified by the ones shown in Figure 5 for three different values of y_m/λ . These numerical results show that the plot is basically independent of y_m/λ . We also note that the plot is independent of the parameter z_m/λ since the magnitude of both Γ_h and $d\Gamma/dx$ scale on z_m/λ . Furthermore, the numerically calculated data can be well represented by a simple cosine curve given by:

$$\frac{\lambda}{\Gamma_h} \frac{d\Gamma}{dx} = \pi \cos(2\pi x/\lambda). \quad (20)$$

Figure 5. Distribution of Circulation at Injector Exit Plane

3.3 Evolution of the Vorticity Field for Two Cases

We next present calculations for the evolution of the vorticity fields behind two injector geometries with different corrugation amplitudes y_m/λ but with fixed ramp slopes $y_m/z_m = 0.125$. The Reynold's number based on λ and W_∞ is equal to 10,000 for both cases. The initial thickness of the vorticity layer $h_0 = 2a_0$ was set equal to $0.04y_m$. In order to provide good resolution and a reasonable amount of core overlap, 400 vortices per wavelength were used. The gross evolution of the flow is relatively insensitive to significant variations in the number of vortices used in the simulation. In Figures 6 and 7 we present contour plots of the absolute value of the vorticity field. As noted previously, the vorticity is positive (counter-clockwise) on the half wavelength to the left of a positive lobe and negative (clockwise) on the half wavelength to the right of a positive lobe.

For the first case, the corrugation amplitude $y_m/\lambda = 0.25$. The evolution of the vorticity field is shown in Figure 6 at several locations downstream of the exit plane. At a downstream location of $z/\lambda = 5.0$ we notice that the half-wavelength vortex sheets are beginning to roll up at their ends and that the strength of the vorticity is the greatest in those regions. As we move further downstream, the two rolled up regions move closer to one another. Eventually, the two like-signed rolled-up-patches of vorticity coalesce into a single vortex. At this stage, the flow field may be described by pairs of counter-rotating vortices spaced at a distance of 0.5λ apart whose shapes are roughly circular.

For the second case, shown in Figure 7, the corrugation amplitude $y_m/\lambda = 1$. At a downstream location of $z/\lambda = 5$ we note the same tendency for the half-wavelength vortex sheets to roll up at their ends but as we move further downstream the two rolled up regions do not move closer to one another in any significant way. At a downstream location of $z/\lambda = 25$ we note that the sheet in the center region appears to be breaking down and that vorticity may, in fact, be moving from the central region to the vortices at $y = \pm y_m$. Thus we observe a very different evolution for this case as compared to the first case.

Figure 6. Vorticity Field Behind Injector ($y_m/\lambda = 0.25$, $z/\lambda \leq 10$)

Figure 6. (cont.) Vorticity Field Behind Injector ($y_m/\lambda = 0.25$, $10 \leq z/\lambda \leq 25$)

Figure 7. Vorticity Field Behind Injector ($y_m/\lambda = 1.0$, $z/\lambda \leq 10$)

Figure 7. (cont.) Vorticity Field Behind Injector ($y_m/\lambda = 1.0, 10 \leq z/\lambda \leq 25$)

3.4 Evolution of the Concentration Field for Two Cases

We next present calculations for the evolution of the concentration fields behind the two injector geometries with different corrugation amplitudes y_m/λ but with fixed ramp slopes $y_m/z_m = 0.125$. Again, the Reynold's number based on λ and W_∞ is equal to 10,000 for both cases. We use a Schmidt number of 0.84 which is valid for dilute mixtures of methane in air at standard atmospheric conditions [12]. The initial thickness of the fuel layer h was set equal to $0.04y_m$.

In Figures 8 and 9 we present contour plots of the fuel concentration field χ_f which is in terms of volume fraction. While we have not actually introduced any combustion model into the present code, we can make several observations about the potential combustion process by noting that the flammability limits are between 4.36% and 15.53% volume fraction [13] when burning methane in air. For stoichiometric conditions, the percent volume fraction of fuel is 9.47%.

By comparing Figures 6 and 7 with Figures 8 and 9 we observe that the character of the vorticity and concentration fields are similar. However, there are some fundamental differences, even if the Schmidt number were to be set equal to 1.0. For instance, the diffusion process will eventually annihilate the vorticity downstream of the injector due to the cancelation of oppositely signed vorticity which has diffused together. On the other hand, there is no destruction of fuel as a result of its diffusion into the air.

For the first case, the corrugation amplitude $y_m/\lambda = 0.25$. The evolution of the concentration field is shown in Figure 8 at several locations downstream of the exit plane. At a downstream location of $z/\lambda = 5.0$ we notice that the half-wavelength fuel-rich sheets are beginning to roll up at their ends due to the underlying vorticity field and that the fuel concentration is the greatest in those regions. As we move further downstream, the two rolled up regions move closer to one another. Eventually, the two like-signed rolled-up-patches of vorticity coalesce into a single vortex which contains the richest concentration of fuel. At this stage, the flow field may be described by pairs of fuel-rich counter-rotating vortices spaced at a distance of 0.5λ apart whose shapes are roughly circular. We note that a downstream location of $z/\lambda = 5.0$ the flow is well mixed and that a major portion of the mixed region is within the flammability limits.

For the second case, shown in Figure 9, the corrugation amplitude $y_m/\lambda = 1$. At a downstream location of $z/\lambda = 5$ we note the same tendency for the half-wavelength fuel-rich sheets to roll up at their ends but as we move further downstream the two rolled up regions do not move closer to one another in any significant way. At a downstream location of $z/\lambda = 25$ we note that the sheet in the center region appears to be breaking down and that the fuel-rich vortices at $y = \pm y_m$ are reasonably stable. We also observe that at $z/\lambda = 25$ and $y = \pm y_m$ there are fuel rich regions which are still outside the upper flammability limit. Thus we observe that the fuel is much less mixed in this case as compared to the first case.

Figure 8. Fuel Volume Fraction Behind Injector ($y_m/\lambda = 0.25$, $z/\lambda \leq 10$)

Figure 8. (cont.) Fuel Volume Fraction Behind Injector ($y_m/\lambda = 0.25$, $10 \leq z/\lambda \leq 25$)

Figure 9. Fuel Volume Fraction Behind Injector ($y_m/\lambda = 0.25, z/\lambda \leq 10$)

Figure 9. (cont.) Fuel Volume Fraction Behind Injector ($y_m/\lambda = 0.25, 10 \leq z/\lambda \leq 25$)

4 SUMMARY AND CONCLUSIONS

This model is capable of predicting the vorticity at the injector exit plane which is generated by the ramped corrugations of a lobed injector. In addition, it will predict the vorticity and concentration fields downstream of the injector exit plane. By simulating the flow over two different geometries in which the corrugation amplitude was varied, we have demonstrated that the character of the flow downstream of the injector exit plane is highly dependent on the geometry of the device.

In this report, we have presented a simple and somewhat preliminary model of a sinusoidal lobed injector. The following restrictions apply:

- The stream-wise velocities of the air flow above and below the lobed injector as well as that of the fuel are assumed to be equal.
- It is assumed that only minimal diffusion of vorticity occurs as the flow passes over the lobed injector.
- There is no introduction of vorticity in the x, y plane by the lobed injector surfaces.
- There is no turbulence in the free-stream flow.
- Fluid properties are assumed to be constant.
- No combustion occurs.

This simulation model may be enhanced by removing the present restrictions which are placed upon it. The following is an overview of possible enhancements:

- This method can be extended to include the production of a hot combustion product by generating product blobs in regions of the flow where combustion is occurring. These product blobs would provide dilatation sources in the flow due to the expansion of hot gases. This also requires negative fuel blobs to be superimposed on the field to account for the transformation of fuel into product. With this extension, the centers of the vorticity, fuel, and product blobs would not necessarily coincide. We also note that a combustion model of some form is necessary in order to predict the amount of product and its temperature. Variable fluid properties should also be considered due to their strong dependence on temperature.
- The modeling of diffusion of stream-wise vorticity and its subsequent convection away from the lobed injector surface could be modeled relatively easily. The diffusion and subsequent convection of vorticity whose direction is in the x, y plane away from the lobed injector surface would be much more difficult to model. The production of vorticity whose direction is in the x, y plane will cause one to have to go to a fully three-dimensional formulation.
- If the stream-wise velocities in the flow above and below the lobed injector are different, vorticity whose direction is in the x, y plane will be produced and the simulation will have to be three-dimensional.

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