CROSS-FLOW VERSUS COUNTER-CURRENT FLOW
PACKED-BED SCRUBBERS: A MATHEMATICAL ANALYSIS

Vasilis M. Fthenakis

February 1996

Prepared for presentation at the 1996 American Institute
of Chemical Engineers (AIChE) Spring National Meeting
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ABSTRACT

Little is known about the mass transfer properties of packing media exposed to a crossflow of gas and liquid, whereas there is abundant information related to counter-current scrubbers. This paper presents a theoretical analysis of mass transfer and hydrodynamics in cross-flow packed-bed scrubbers and compares those with information available for counter-current towers, so that the first can be evaluated and/or designed based on data derived for the second. Mathematical models of mass transfer in cross-flow and counter-current packed-bed scrubbers are presented. From those, one can predict the removal effectiveness of a crossflow scrubber from the number of transfer units (NTU) calculated for a similar counterflow operation; alternatively, when the removal effectiveness in counterflow is known, one can predict the corresponding NTU in crossflow.
1. INTRODUCTION

Packed towers were used in liquid/gas mass transfer unit operations since early 1900s. The theoretical basis of packed tower design was developed through the 1920’s and 1930’s (e.g., Lewis and Whitman, 1924; Chilton and Colburn, 1935; Colburn, 1939), culminating in 1940 when Sherwood and Holloway published two papers which summarized the state-of-the art and laid the foundation for much of the future work in the field (Little, 1990). All these studies, were based on counter-current configurations where the liquid and gaseous streams enter the column from opposite ends. Crossflow packed-bed scrubbers differ from counterflow scrubbers in that water is sprayed at right angles, instead of the opposite direction, on the gas stream. For a given feedwater concentration and flow rates of liquid and gas streams, counterflow allows for greater concentration gradients between the liquid and gas phases and, subsequently, it gives greater solute removal than crossflow. A crossflow design, however, may prove to be more economical for large liquid flow rates because it causes less pressure drop than a counterflow one. Also, crossflow scrubbers have the advantage of low profile and space savings.

Little is known about the mass transfer properties of packing media exposed to a crossflow of gas and liquid, whereas there is abundant information related to
counter-current scrubbers. The objective of this paper is to analyze the mass
transfer and hydrodynamics in cross-flow packed-bed scrubbers and compare
those with information available for counter-current towers, so that the first can
be evaluated and/or designed based on data derived for the second. The only
studies found in the literature regarding cross-flow packed-bed units, are those
of Thibodeaux et al. (1977), Little (1990) and Little and Selleck (1991) which
apply to stripping towers. In this article we expand Little’s work to absorption
units and recirculating liquid flow.

2. A MODEL OF MASS TRANSFER IN CROSS-FLOW PACKED-BED
SCRUBBERS

The elements of a crossflow packed-bed scrubber are shown schematically
in Figure 1. The liquid introduced to the top of the packed bed falls by gravity
through a depth Z, and the air is drawn through a distance X at right angles to
the water stream. The width across the face of the tower is T. In absorption
units, the concentration in the gas phase leaving the unit at X near the top (z=0)
is lower than the concentration near the bottom of the unit (z=Z). It is assumed
that the operation is isothermal, the drag of the air stream on the water does not
displace the water to the side of the tower, and no dispersion occurs in either the
air or water streams (plug flow).
With reference to Figure 1, mass balances taken about the control element combined with the commonly used expression of mass transfer based on the overall mass transfer coefficient $K_{\text{os}}$ gives

$$K_{\text{os}}a(y^*-y) = -G' \frac{dy}{dx} = L' \frac{dc}{dz}$$

(1)

in which $y = \text{solute mass fraction in the gas-phase}$; $c = \text{solute mass fraction concentration in the liquid-phase}$; $y^* = \text{solute mass fraction in the gas-phase at equilibrium with that in the bulk of the gas-phase}$; $a = \text{gas-liquid interfacial area per unit of bed volume}$, $L' = \text{liquid mass flux}$, and $G' = \text{gas mass flux}$. $y^*$ and $c$ for the very dilute mixtures in this application, are related by the Henry's law,
\[ y^* = mc, \quad \text{where } m \text{ (or } H\text{) is a Henry's law constant.} \quad (2) \]

When Equation 2 is combined with Equation 1 and the dimensionless distances \( J = z/Z \) and \( \chi = x/X \) are introduced, the following are obtained:

\[
\frac{dc}{dJ} = N \frac{G}{L} (y - mc) \quad (3)
\]

\[
\frac{dy}{d\chi} = -N (y - mc) \quad (4)
\]

where \( N = (K_{oo} a Z)/G' \) is the number of transfer units; \( L \) and \( G \) are the mass flow rates of the liquid and gas streams, respectively. The boundary conditions for these equations are:

- \( c = c_{in} \) along the top of the bed at \( J = 0 \), and
- \( y = y_{in} \) along the face of the bed at \( x = 0 \).

The average solute mass transfer \( y_{out} \) in the gas stream leaving the packed-bed is

\[
y_{out} = \int_0^1 y(J, 1) dJ \quad (5)
\]
in which \( y(J,1) \) is the concentration in the gas stream leaving the packed bed at \( \chi = 1 \).

The following exact analytical solution of equations 3 and 4 subject to the boundary conditions above, was obtained by means of Laplace transforms (Fthenakis, 1995):

\[
\frac{\overline{y}_{out} - mc_{in}}{y_{in} - mc_{in}} = e^{-B-N} \phi_3(2,2,B,NB) \tag{6}
\]

where \( B=N/A, \quad N=\frac{KoGaZ}{G} \quad \text{and} \quad A=\frac{L}{mG} \) is the Absorption Factor.

The hypergeometric function \( \Phi_3 \) can be expanded in terms of a double sum series as:

\[
\frac{\overline{y}_{out} - mc_{in}}{y_{in} - mc_{in}} = e^{-B-N} \sum_{i=0}^{\infty} \sum_{k=0}^{\infty} \frac{1+i}{(1+i+k)! k!} (B)^i (NB)^k \tag{7}
\]

which converges rapidly for values of \( N \) and \( A \) typically encountered in scrubbing operation.

For once-through liquid flow, \( C_{in}=0 \), and Equation 7 readily gives the removal efficiency of a cross flow scrubber. For recirculating liquid flow, \( C_{in}, y_{in} \) and \( y_{out} \) are related via mass balance equations,

\[
C_{in} = \frac{G (y_{in} - y_{out})}{LB_f} (1-Bf),
\]
where $B_f$ is the ratio of the liquid blowdown flow-rate over the inlet liquid flow rate, and the removal efficiency of the scrubber is given by the equation:

$$\text{Removal} = 100 \left(1 - \frac{\frac{y_{out}}{y_{in}}}{1 + \frac{k_2 + m(k_1 - k_1 k_2)}{1+m(k_1 - k_1 k_2)}}\right) = 100 \left(1 - \frac{y_{out}}{y_{in}}\right),$$

where

$$k_1 = \frac{G}{B_f L},$$

and

$$k_2 = e^{-B-N} \sum_{i=0}^{\infty} \sum_{k=0}^{\infty} \frac{1+i}{(1+i+k)!k!} (B)^i (NB)^k.$$

A counter-current flow equation developed under the same assumptions, (Fthenakis, 1995), is given below:

$$\text{Removal} = 100 \left(1 - \frac{\frac{y_{out}}{y_{in}}}{1 + \frac{k_2 + m(k_1 - k_1 k_2)}{1+m(k_1 - k_1 k_2)}}\right) = 100 \left(1 - \frac{y_{out}}{y_{in}}\right),$$

where

$$k_1 = \frac{G}{B_f L},$$

and

$$k_2 = \frac{1-(1/A)}{e^\lambda - (1/A)}$$

and

$$\lambda = N \left(1 - \frac{1}{A}\right).$$
Values obtained from Equations 8 and 9, for \( c_0 = 0 \) (liquid free of solute), are plotted in Figures 2 and 3 for \( A=1.2 \) and \( A=2 \) correspondingly. As an example, if \( N = 4, \) and \( A = 2, \) then a 96% removal of solute is predicted for crossflow and 99% for counterflow; for same \( N \) and \( A=1.2, \) 83% removal is predicted for crossflow, and 88% for counterflow. This illustrates the gain in mass transfer efficiency that is achieved with a counterflow design.

<table>
<thead>
<tr>
<th>A</th>
<th>N</th>
<th>Counter Flow</th>
<th>Cross Flow</th>
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<tbody>
<tr>
<td>1.2</td>
<td>2</td>
<td>74.69</td>
<td>68.70</td>
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<tr>
<td></td>
<td>3</td>
<td>83.14</td>
<td>75.60</td>
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<tr>
<td></td>
<td>4</td>
<td>88.03</td>
<td>79.80</td>
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<tr>
<td></td>
<td>5</td>
<td>91.16</td>
<td>82.70</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>93.30</td>
<td>84.80</td>
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<td></td>
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<td>94.83</td>
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<tr>
<td></td>
<td>8</td>
<td>95.95</td>
<td>87.70</td>
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<tr>
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<td>2</td>
<td>92.74</td>
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</tr>
<tr>
<td></td>
<td>3</td>
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<td>8</td>
<td>99.98</td>
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</table>
The above equations allow for a mathematical comparison of cross-flow and counter-current packed-bed scrubbers. One can predict removal effectiveness for a given number of transfer units (N) and initial conditions. Alternatively, when the efficiency removal is known, an iterative numerical procedure can be used to solve Equation 8 or 9 for N. The packing depth Z can then be determined by multiplying N by the height of a transfer unit HTU.

3. DETERMINING HTU IN CROSS-FLOW

The resistance to mass transfer between the gas and the liquid phases is described by the mass transfer coefficient $K_{\text{oo}}$ in the mass conservation equations. Frequently, data on mass transfer resistance are presented in the form of a height of a transfer unit (HTU), where $\text{HTU} \equiv H_{\text{oo}} = \frac{G''}{K_{\text{oo}} a}$, and $G''$ is the gas mass flux. HTU multiplied with the number of the transfer units (N) gives the overall height of a separation column.

Several correlations exist for calculating the gas-film and liquid-film mass transfer coefficients for different packing types; these have been summarized elsewhere (Fthenakis, 1996). All correlations, however, have been derived via counter-current experiments. Some comparisons of HTU in counter-current and cross flows were given by Little (1990). He measured crossflow mass transfer
characteristics of two types of 2-inch polypropylene packing (Jaeger Tri-Packs™ (spherical shapes) and 2-inch Norton Super Intalox™ (saddles).\(^1\) For spherical packing, Little’s experiments showed that both the gas-side and the liquid-side mass transfer characteristics of the spherical packing used in his tests were the same in crossflow and counterflow. The observed liquid-side resistances (or \(H_L\)) were within ±20% of those predicted by Norman’s and Onda’s equations\(^2\). The observed gas-side \(H_G\) was about 50% less than those predicted by Onda’s gas-side correlation, but it fell within ±25% of the values predicted by Sherwood and Pigford gas-side power regression equation. This finding indicates that Onda’s correlation may overestimate \(H_G\) in the considered unit by about 50%, but this needs further investigation. For the Norton Intalox saddle packing type and air and water flow rates in the range of 0.4-2.5 kg/m\(^2\)s and 1.3-10.2 kg/m\(^2\)s respectively, Little (1990) reported that the liquid-side \(H_L\) did not appreciably change between crossflow and counterflow, whereas the gas-side \(H_G\) was about 25% greater than that predicted by Onda’s gas-side correlation. If, as mentioned above, Onda’s correlation overestimates \(H_G\), then the difference between cross-flow and counter-current-flow could be even greater (e.g., about 75%).

From the above studies it appears that for spherical packing the mass transfer coefficients and HTUs are essentially the same for crossflow and

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\(^1\) The packed bed was 3.6 m high, 0.8 wide and 1.2 m long, and air and water flow rates were in the range of 0.4-2.5 kg/m\(^2\)s and 1.3-10.2 kg/m\(^2\)s respectively

\(^2\) Both Onda’s and Norman’s equations were derived from counterflow experiments.
counterflow, but for saddles, the HTU in crossflow can be approximately 50% higher than that in counterflow, for same packing and flow rates. Packings such as Cascade Mini-Rings which were developed to handle high flow rates in counterflow operation and thus have a preferential flow direction (vertical) would most probably behave in a very different manner under crossflow.

Also adjustments based on the different cross sections of the gas and the liquid flows are due. According to the two-film theory, a convenient expression of the counterflow mass transfer coefficient, is

\[
\text{HTU} \equiv H_{OG} = H_G + H_L/A
\]  \hspace{1cm} (10)

in which \(H_{OG} = G'/(K_{OG}a)\), \(H_L = L'/(k_La)\), and \(H_G = G'/(k_Ga)\) are the overall-, liquid- and gas-side heights of a transfer unit, and \(k_L\) and \(k_G\) the liquid- and gas-side mass transfer coefficients, respectively. Equation 10 can be adjusted for crossflow by including the ratio of the cross-sections to the gas and liquid streams \((Z/X)\), thus

\[
H_{OG} = H_G + \left( \frac{X}{Z} \frac{H_L}{A} \right)
\]  \hspace{1cm} (11)
4. EMPIRICAL CONSIDERATIONS

4.1 Backmixing

The models described above correspond to ideal mixing conditions (e.g., plug flow). In real applications, however, a major factor that can cause different efficiencies in crossflow and counterflow units is backmixing which decreases the performance of a scrubber (i.e., increases the required number of transfer units for a certain separation efficiency). Counter-current packed-bed scrubbers are designed to operate under conditions of limited backmixing (i.e., near plug flow), whereas crossflow scrubbers operating at large gas flow rates may deviate substantially from plug flow behavior. Experimental studies show that the mass transfer characteristics of spheres are the same in both counterflow and crossflow devices, whereas those of preferential flow packings (e.g., saddles, rings) can be different in the two configurations.

4.2 Liquid Hold-Up

The liquid holdup is an important characteristic of packed beds because it is related to the effective specific interfacial area, gas pressure drop and flooding (Norman, 1961). Pittaway and Thibodeaux (1980)\(^3\) reported that liquid holdup in crossflow is approximately the same as that predicted by correlations for counterflow operation of the same packing. Little (1990) also reported liquid

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\(^3\) They used a 1x1x1 ft cube packed with 0.5 inch ceramic Raschig rings.
holdup and dispersion approximately the same in crossflow and counterflow, for both spheres and saddles.

4.3 Gas Pressure Drop

The principle factors affecting the pressure drop in packed towers are the size and shape of the packing, the ratio of the tower diameter to packing diameter and the packing supports (Norman, 1961). In general, pressure drop data vary widely due to differences in packing density and manufacture. A generalized graphical correlation prepared by Eckert (1975) is considered sufficient for most purposes (Treybal, 1980). The relationship of the pressure drop to the air and liquid loading rates for a packed bed will be highly dependent on the geometry of the packing. For spherical packing not having a preferential flow direction, the generalized correlation developed by Eckert will generally suffice. However, many newer packing types exhibit pressure drops that behave differently than what the Eckert correlation predicts. For crossflow operation using spherical packing, pressure drop gradients (ΔP/z) are expected to be the same to that of the counterflow unit, under a low loading rate. However, in a cross-flow scrubber where the cross-section perpendicular to the gas flow is smaller than the area available to the liquid flow (i.e., height < length), the total pressure drop through the device (ΔP), will be substantially less than in counterflow. This allows crossflow operation at gas-to-liquid flow ratios higher than those causing flooding in a counterflow operation.
5. CONCLUSION

Design of a crossflow separation equipment is limited by a complete lack of design information and the unavailability of a crossflow process simulation model. Since much information exists on counter-current scrubbers, and their performance can be predicted by a number of commercial process simulators, an analytical correlation of the two configurations is very useful.

Equations 8 and 9 allow a mathematical comparison of cross-flow and counter-current packed-bed scrubbers. From those, one can predict the removal effectiveness of a crossflow scrubber from the number of transfer units (N) calculated for a similar counterflow operation. Alternatively, when the removal effectiveness in counterflow is known, an iterative procedure can be used to solve Equation 8 for N in crossflow. The packing depth Z can then be determined by multiplying N by the height of a transfer unit HTU. The later can be determined by the value given for counterflow, with a geometry adjustment according to Equation 11.

It is noted that equations 8 and 9 were developed under the assumption of plug flow, which is rarely the case in large scrubbers. Deviations from plug flow may be implicitly accounted for in the empirical mass transfer coefficients, if the later are derived from large-scale tests. A model of crossflow that explicitly
accounts for backmixing can be developed by implementing the solution of the mass balance equations 3 and 4 on the interactive control volumes of a compartmental backmixing model. This will be addressed in a future study.

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NOTATION

\[ \lambda = N(1-1/A) \]
\[ A = \text{Absorption factor (LP/mG)} \]
\[ a = \text{gas-liquid interfacial area per unit of bed volume} \]
\[ B = \text{N/A} \]
\[ B_r = \text{ratio of blowdown flow / scrubber inlet liquid flow} \]
\[ c = \text{solute mass fraction in the liquid-phase} \]
\[ G' = \text{gas mass flux} \]
\[ G = \text{gas mass flow rate} \]
\[ H = \text{Henry law coefficient} \]
\[ H_o = \text{gas-phase transfer height} \]
\[ H_l = \text{liquid phase transfer height} \]
\[ H_{O\infty}, \text{HTU} = \text{overall height of a transfer unit based on gas-phase resistance.} \]
\[ L' = \text{liquid mass flux} \]
\[ L = \text{liquid mass flow rate} \]
\[ m = \text{slope of the equilibrium curve} \]
\[ N, \text{NTU} = \text{number of transfer units} \]
\[ T = \text{third dimension of a cross flow scrubber} \]
\[ X = \text{horizontal dimension of a cross-flow scrubber} \]
\[ y = \text{solute mass fraction in the gas-phase} \]
\[ y^* = \text{solute mass fraction in the gas-phase at equilibrium with that in the bulk of the gas-phase} \]
\[ Z = \text{vertical dimension of a cross-flow scrubber} \]

Subscripts

\[ \text{in} = \text{scrubber inlet} \]
\[ \text{out} = \text{scrubber outlet} \]
4. LITERATURE CITED


