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**De Broglie Wavelets versus Schrödinger Wave Functions.
A Ribbon Model Approach to Quantum Theory
and the Mechanisms of Quantum Interference**

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Abstract

As an alternative to better physical explanations of the mechanisms of quantum interference and the origins of uncertainty broadening, a linear hopping model is proposed with "color-varying" dynamics to reflect fast exchange between time-reversed states. Intricate relations between this model, particle-wave dualism, and relativity are discussed. The wave function is shown to possess dual characteristics of a stable, localized "soliton-like" de Broglie wavelet and a delocalized, interfering Schrödinger carrier wave function.

1. Introduction

For almost a century quantum mechanics has been the cornerstone for theories of microscopic phenomena occurred in physical, chemical and biological systems. Despite its great success, the quantum behaviour in the microscopic world has long puzzled scientists ever since Einstein [1] introduced the concept of "photons" to the wave nature of electromagnetic waves, and later by De Broglie who introduced "matter wave" to particles [2]. The debates about the dualism of particle and wave, the logic of existing quantum theories, the origins of quantum interference, and interpretation of quantum measurements have never waned [3-7]. In this work a simple ribbon model with hopping, "color-varying" dynamics is presented to explain the mechanisms of quantum interference, as an alternative to the orthodox but often abstract approaches of Schrödinger wave mechanics [8] or Heisenberg matrix theory [9]. Quantum interference and uncertainty broadening in the microscopic world will be shown as results of fast exchange between internal degrees of freedom of time-reversed states, a consequence of intricate relations in Einstein's relativity. It will be shown using this model that the distribution function, or the wave function, possesses both characteristics of a "soliton-like" de Broglie wavelet and a Schrödinger-type traveling wave function. The shape-preserving, localized de Broglie wave-packet moves at a velocity v and exhibits a ballistic classical trajectory. In addition to such a localized de Broglie wavelet, there is another Schrödinger-type delocalized wave carrier moving at a phase velocity shown to be c^2/v . These results agree with earlier beliefs of Einstein who introduced photons as localized lumps of electromagnetic energy, and de Broglie who thought electrons as singular localized wavelets guided by a "matter wave" with the same phase. The phenomena of quantum interference and uncertainty broadening will be explained using this ribbon model as manifestation of fast

mixing between the internal degrees of freedom which are the time-reversed states. Illustrations will be made for uncertainty broadening and quantum interference effects to demonstrate why such phenomena are less likely to occur in the macroscopic world. Dual characteristics of the de Broglie wavelets and the Schrödinger guide-wave functions will also be investigated.

2. Linear ribbon model

We will consider here the hopping mechanism of "color-varying" particles along a linear ribbon. The "color" change represents oscillation between the two internal degrees of freedom which will be shown to be related to the time-reversal symmetry. The rate equation for the distribution function $f_{n,\mu}(t)$ is defined by

$$\frac{d}{dt} f_{n,\mu} = -\mu \frac{W}{2} (f_{n-1,\mu} - f_{n,\mu}) + \mu \frac{W}{2} (f_{n+1,\mu} - f_{n,\mu}) + \mu U_c f_{n,-\mu}, \quad (1)$$

where μ (1 or -1) is the "color" index for the two internal degrees of freedom, and n is the index for the site position. As illustrated in fig. 1, the top particle in a ribbon with $\mu = 1$ can only hop to the nearest right and the bottom particle with $\mu = -1$ can only hop to the nearest left with the same rate constant W . U_c is the exchange rate between particles at the same site but a different μ . The time dependence of the position and the "color" of particles can be determined from eq. (1) which can be solved by Fourier transform using $F_k(t) = \sum_n f_{n,1}(t) \exp(-i2\pi kn/N)$ and $G_k(t) = \sum_n f_{n,-1}(t) \exp(-i2\pi kn/N)$. Assuming a sufficiently long chain, so that $2\pi k/NR$ (R is the lattice length) can be replaced by a continuous variable κ , the solutions for $F_\kappa(t)$ and $G_\kappa(t)$ are given by

$$\begin{aligned}
F_{\kappa}(t) &= [\cos(\Omega_{\kappa}t) - i \sin\theta_{\kappa} \sin(\Omega_{\kappa}t)] F_{\kappa}(0) \\
&\quad + \cos\theta_{\kappa} \sin(\Omega_{\kappa}t) G_{\kappa}(0) \\
G_{\kappa}(t) &= -\cos\theta_{\kappa} \sin(\Omega_{\kappa}t) F_{\kappa}(0) \\
&\quad + [\cos(\Omega_{\kappa}t) + i \sin\theta_{\kappa} \sin(\Omega_{\kappa}t)] G_{\kappa}(0)
\end{aligned} \tag{2}$$

$$\Omega_{\kappa} = \sqrt{U_c^2 + W_{\kappa}^2} = \sqrt{U_c^2 + W^2 R^2 \kappa^2}$$

$$\theta_{\kappa} = \tan^{-1}(WR\kappa/U_c) .$$

By defining the vector α_{κ} as $(0, \Omega_{\kappa}\cos\theta_{\kappa}, -\Omega_{\kappa}\sin\theta_{\kappa})$, the above equation can also be expressed as

$$\begin{pmatrix} F_{\kappa}(t) \\ G_{\kappa}(t) \end{pmatrix} = e^{it\alpha_{\kappa} \cdot \sigma} \begin{pmatrix} F_{\kappa}(0) \\ G_{\kappa}(0) \end{pmatrix} , \tag{3}$$

where σ 's are the Pauli matrices. One can define here a 2×2 density matrix $\rho_{\kappa}(t)$ in terms of F_{κ} , F_{κ}^* , G_{κ} and G_{κ}^* such as

$$\rho_{\kappa}(t) = \begin{pmatrix} |F_{\kappa}(t)|^2 & F_{\kappa}(t) G_{\kappa}^*(t) \\ F_{\kappa}^*(t) G_{\kappa}(t) & |G_{\kappa}(t)|^2 \end{pmatrix} . \tag{4}$$

It can be shown that the evolution of $\rho_{\kappa}(t)$ is given by

$$\rho_{\kappa}(t) = U_{\kappa}(t) \rho_{\kappa}(0) U_{\kappa}^{\dagger}(t) \tag{5}$$

$$U_{\kappa}(t) = e^{it\alpha_{\kappa} \cdot \sigma} .$$

The above relation indicates that one may define here a Hamiltonian H as $-\hbar\alpha_k \cdot \sigma$ if one interprets the unitary operator $U_k(t)$ as $\exp(-iHt/\hbar)$. The Hamiltonian can be diagonalized by V which is $\exp[i(\pi/4 - \theta_k/2)\sigma_x]$ such that $VHV^+ = -\hbar\Omega_k\sigma_z$ and $VU_k(t)V^+ = \exp(i\Omega_k t\sigma_z)$. Thus, eq. (5) can also be expressed as

$$\rho_k(t) = V^+ e^{i\Omega_k t\sigma_z} V \rho_k(0) V^+ e^{-i\Omega_k t\sigma_z} V. \quad (6)$$

In the weak exchange coupling case with $U_c^2 \ll W^2 R^2 \kappa^2$, $\theta_k \approx \pi/2$ and $V \approx I$, and from eqs. (4) and (6) it can be shown that $F_k(t)$ and $G_k(t)$ are decoupled, and $\rho_k(t) \approx \exp(iWR\kappa t\sigma_z)\rho_k(0)\exp(-iWR\kappa t\sigma_z)$. On the other hand, in the strong exchange limit with $U_c^2 \gg W^2 R^2 \kappa^2$, $\theta_k \approx 0$ and $V \approx \exp(i\sigma_x\pi/4)$, one has $\rho_k(t) \approx \exp(iU_c t\sigma_y)\rho_k(0)\exp(-iU_c t\sigma_y)$. In this limit $F_k(t)$ and $G_k(t)$ are coupled. Because $|F_k(t)|^2 + |G_k(t)|^2$ is a conserved quantity as a result of the unitary property of U_k , there is interference between $F_k(t)$ and $G_k(t)$ even though either $F_k(0)$ or $G_k(0)$ may be zero initially.

With the expressions derived in eqs. (3)-(6), we can now explore the physical meanings of this model. Following de Broglie's concept [2], the Planck relation for photons can be extended to other massive particles such that energy $E = \hbar\Omega$ and momentum $p = \hbar\kappa$. One can readily see from eq. (2) that $E^2 = p^2 W^2 R^2 + \hbar^2 U_c^2$. By comparing this result with Einstein's mass-energy relation [10] of $E^2 = p^2 c^2 + m_0^2 c^4$ for a free massive particle, one has $WR = c$ and $U_c = m_0 c^2/\hbar$. For an electron, one may define the natural length unit R as the classical radius of an electron r_e ($r_e = \alpha\hbar/m_e c \approx 2.82 \times 10^{-13}$ cm) and the natural time unit W_e^{-1} as c/r_e (about 9.41×10^{-24} s), where r_e is related to the Compton wavelength $\hbar/m_e c$ by the

fine-structure constant $\alpha = 1/137.04$. It seems that the lattice unit R can be arbitrarily small so long as the assumption of $\kappa R \ll 1$ is met and the time unit W^{-1} is defined as R/c . For other particles with a mass m_0 one may define the Compton wavelength $\hbar/m_0c$ as the natural length unit R .

3. De Broglie wavelets and Schrödinger wave functions

We will first consider the strong exchange, nonrelativistic limit with $U_c^2 \gg W^2 R^2 \kappa^2$. From eq. (2) one has $\theta_\kappa \approx 0$. By inverse Fourier transform of eqs. (2) or (3) and with the unitary transform $U_\kappa(t) = \exp(iU_c t \sigma_y)$, it can be shown

$$f_\mu(x, t) \approx f_{\mu,+}(x, t) + f_{\mu,-}(x, t) , \quad (7.a)$$

where

$$\begin{aligned} f_{1,\pm}(x, t) &= \frac{1}{2\pi} \int d\kappa e^{i\kappa x + i\Omega_\kappa t} F_\kappa(0) , \\ f_{-1,\pm}(x, t) &= \frac{1}{2\pi} \int d\kappa e^{i\kappa x + i\Omega_\kappa t} G_\kappa(0) . \end{aligned} \quad (7.b)$$

Let us assume that particles are initially localized within a width L around the origin, and also assume that both $F_\kappa(0)$ and $G_\kappa(0)$ are peaked at κ_0 , corresponding to a speed $\hbar\kappa_0/m$. For example, if an initial Gaussian distribution is assumed then $F_\kappa(0) = (2\pi L^2)^{-1/2} \exp[-(\kappa - \kappa_0)^2 L^2]$. One can expand Ω_κ of eq. (2) around κ_0 , and to the first order approximation one has $\Omega_\kappa \approx \Omega_{\kappa_0} + \Omega'_{\kappa_0}(\kappa - \kappa_0)$, where Ω'_{κ_0} is the first derivative of Ω_κ at κ_0 and $\Omega'_{\kappa_0} = W^2 R^2 \kappa_0 / \Omega_{\kappa_0}$. Using $WR = c$ and $U_c = m_0 c^2 / \hbar$ as discussed earlier, and with the Planck-Einstein's relation of $E = \hbar\Omega_{\kappa_0} = mc^2 = m_0 c^2 / (1 - v^2/c^2)^{1/2}$ and $p_0 = \hbar\kappa_0 = mv = m_0 v / (1 - v^2/c^2)^{1/2}$, one can show $\Omega'_{\kappa_0} = v$ and $\Omega_{\kappa_0} / \kappa_0 = c^2 / v$. Eq. (7b) can be reduced to

$$\begin{aligned}
f_{1,+}(x,t) &\approx e^{i\kappa_0 x - i\Omega_{\kappa_0} t} f_1(x-vt, 0) \\
f_{-1,-}(x,t) &\approx e^{i\kappa_0 x - i\Omega_{\kappa_0} t} f_{-1}(x-vt, 0) .
\end{aligned}
\tag{8}$$

The above expression for the wave function $f_{1,+}(x,t)$ clearly possesses both characteristics: a "soliton-like" de Broglie wavelet represented by $f_1(x-vt,0)$; and a Schrödinger-type travelling plane wave represented by $\exp(i\kappa_0 x - i\Omega_{\kappa_0} t)$. The shape-preserving, localized wave-packet of $f_1(x-vt,0)$ moving along the x axis at a velocity v exhibits a particle-like classical trajectory. The "soliton-like" solution has been discussed by Vigier for a non-linear Schrödinger equation [11] which is different from our model proposed here. In addition to such a localized de Broglie wavelet, there is another delocalized Schrödinger-type wave carrier, represented by $\exp(i\kappa_0 x - i\Omega_{\kappa_0} t)$. It travels also along the x axis but with a phase velocity given by $\Omega_{\kappa_0}/\kappa_0$ or c^2/v as shown earlier. These results agree with earlier beliefs of Einstein who thought photons as localized lumps of electromagnetic energy, and de Broglie who thought massive particles as singular localized wavelets guided by a "matter wave" with the same phase. Similar to $f_{1,+}(x,t)$, $f_{-1,+}(x,t)$ also represents a Schrödinger-type delocalized wave carrier with a localized de Broglie wavelet $f_{-1}(x-vt,0)$ travelling along the x axis. Two other components $f_{1,-}(x,t)$ and $f_{-1,-}(x,t)$, however, representing a wave form travelling along the $-x$ axis and have a time-reversed property as an anti-particle [12].

In the earlier analysis, the Taylor's expansion of Ω_κ at κ_0 was carried only up to the first order in κ . By including the second derivative of Ω_κ at κ_0 which is $(W R U_c)^2 / (\Omega_{\kappa_0})^3 = \hbar(1-v^2/c^2)^{3/2}/m_0 = \hbar/m_0$, one has $\Omega_\kappa \approx m_0 c^2/\hbar + v(\kappa - \kappa_0) + \hbar(\kappa - \kappa_0)^2/2m_0$. Eq. (7b) can now be reduced to

$$\begin{aligned}
f_{1,\pm}(x,t) &\approx \phi_{\pm}(x,t) e^{\mp im_0 c^2 t/\hbar} f_{\pm}(x \mp vt, 0) \\
f_{-1,\pm}(x,t) &\approx \phi_{\pm}(x,t) e^{\mp im_0 c^2 t/\hbar} f_{\mp 1}(x \mp vt, 0) \\
\phi_{\pm}(x,t) &\equiv e^{i(\kappa_0 x \mp t \hbar^2 \kappa_0^2 / 2m_0)} .
\end{aligned} \tag{9}$$

By separating the rapidly oscillating term of $\exp(\mp im_0 c^2 t/\hbar)$ due to the rest mass energy, one can show that the remaining carrier wave function $\phi_{\pm}(x,t)$ is a solution of the following equation

$$\begin{aligned}
i\hbar \frac{\partial}{\partial t} \phi_{\mu,+}(x,t) &\approx -\frac{\hbar^2}{2m_0} \frac{\partial^2}{\partial x^2} \phi_{\mu,+}(x,t) \\
-i\hbar \frac{\partial}{\partial t} \phi_{\mu,-}(x,t) &\approx -\frac{\hbar^2}{2m_0} \frac{\partial^2}{\partial x^2} \phi_{\mu,-}(x,t) .
\end{aligned} \tag{10}$$

Thus, $\phi_{\mu,+}(x,t)$ is the solution of the ordinary Schrödinger equation but $\phi_{\mu,-}(x,t)$ is the solution of the time-reversed Schrödinger equation. It can be shown that $f_{\mu}(x,0)$ in the rest frame satisfies the following equation

$$\frac{\partial^2}{\partial x^2} f_{\mu}(x,0) + \frac{m_0^2 c^2}{\hbar^2} f_{\mu}(x,0) = 0 . \tag{11}$$

In a three-dimensional space $\mathbf{x}$, one has

$$\nabla^2 f_\mu(\mathbf{x}, 0) + \frac{m_0^2 c^2}{\hbar^2} f_\mu(\mathbf{x}, 0) = 0 . \quad (12)$$

The solution of eq. (12) can be expressed as a linear combination of a radial function, given by the spherical Bessel function $j_\ell(m_0 c |\mathbf{x}| / \hbar)$ and the spherical Neumann function $n_\ell(m_0 c |\mathbf{x}| / \hbar)$, with an angular function given by the spherical harmonic function $Y_{\ell,m}(\theta, \phi)$. These spherical Bessel and Neumann functions are related to the ordinary Bessel function by $j_\ell(r) = (\pi/2r)^{1/2} J_{\ell+1/2}(r)$ and $n_\ell(r) = (1)^{\ell+1} (\pi/2r)^{1/2} J_{-\ell-1/2}(r)$. The size of the wavelet is related to the Compton wavelength of the particle $\hbar/m_0 c$, and its internal structure is given by the solution of the above Helmholtz equation.

Therefore, an individual particle is represented by a localized wavelet of a dimension about the Compton wavelength, as obtained in the solution of eqs. (11) or (12). A localized oscillating finite energy solution for a massless Klein-Gordon wave equation has been discussed by Barut [13] which is related but different from this work. For an ensemble of particles or wavelets with an initial distribution of κ_0 , $\phi_\mu(x, t)$ describes the nonlocalized Schrödinger-type carrier wave function where one obtains the probabilistic interpretation in the standard quantum theory. In a double-slit experiment by Tonomura et al. [14] single dots were observed as an individual particle hit a screen. In each single event, a dot corresponds to the impact of a "soliton-like" wavelet. As enough ensemble of these dots were collected in a memory device, the interference pattern emerged. Such an observation agrees with the probabilistic interpretation of the delocalized Schrödinger-type wave function $\phi_\mu(x, t)$, and the localized wavelet nature of an individual particle discussed here.

4. Uncertainty broadening

Now we will examine the phenomena of uncertainty broadening. The distinction between wavelets and carrier wave functions, as we have discussed earlier, will not be emphasized here. We will examine the probabilistic properties of the wave functions based on the ribbon model. Let us consider an ensemble of particles represented by a travelling Gaussian distribution at a nonrelativistic speed $v = \hbar\kappa_0/m_0$ such that $f_1(x, t=0) = 0$ and $f_{-1}(x, t=0) = \exp(-(x-vt)^2/4L^2)/(2\pi L^2)^{1/4}$, using the normalized condition $\int dx |f_1(x, t=0)|^2 = 1$. Here L is assumed to be much greater than the Compton wavelength in this nonrelativistic, fast exchange regime. By Fourier transform, one has $F_\kappa(0) = (2\pi L^2)^{1/4} \exp(-(\kappa-\kappa_0)^2 L^2)$ and $G_\kappa(0) = 0$. It can be shown from eq. (2) or (4)

$$\begin{aligned}
 & |f_1(x, t) + i f_{-1}(x, t)|^2 + |f_1(x, t) - i f_{-1}(x, t)|^2 \\
 &= \frac{1}{\sqrt{2\pi L^2 \left[1 + \frac{t^2 \hbar^2}{4m_0^2 L^4}\right]}} \exp\left\{-\frac{(x-vt)^2}{2L^2 \left[1 + \frac{t^2 \hbar^2}{4m_0^2 L^4}\right]}\right\} .
 \end{aligned} \tag{13}$$

The above equation indicates that the center of the packet travels at a speed of v . In addition, the time dependence of the population distribution clearly demonstrates spreading spatial uncertainty. The uncertainty of the particle's position is given by the second moment $\sigma^2(t) = L^2(1 + t^2 \hbar^2 / 4m_0^2 L^4)$ which grows in time. Such a property is exactly identical to the result of the Heisenberg uncertainty broadening.

In this model, the exchange interaction U_c causes fast exchange between f_1 and f_{-1} which represent time-reversed states of two degenerate components with same kinetic energy but opposite momentum. It is shown here that the uncertainty

broadening is a result of f_1 and f_2 components moving opposite to each other. The increasing uncertainty broadening prescribed in eq. (13) could be detectable if the observation time is larger than or comparable to $L^2 m_0 / \hbar$. For free electrons initially localized within an angstrom. i.e., $L \sim 10^{-8}$ cm, delocalization starts after 0.1 fs. For mercury atoms initially localized within 10^{-6} cm, it would take 10^{-7} s for the packet to double its size. As a contrast, for particles as big and heavy as a golf ball it may have to take 10^{20} years to have significant increase in spatial uncertainty. A daily life object in the macroscopic world typically has a very large dimension L and a heavy mass m_0 , thus $\sigma^2(t) = L^2(1 + t^2 \hbar^2 / 4L^4 m_0^2) \approx L^2$. These unfavorable factors coupled with large thermal fluctuations make quantum interference unnoticeable and uncertainty broadening insignificant. The travelling and spreading population distribution in eq. (13) is shown in fig. 2 to illustrate the uncertainty broadening caused by fast mixing and opposite motion of f_1 and f_2 components. The coordinate x is normalized to L , the initial width of delocalization. The time t is also normalized to T defined as $m_e / m_0 L^2$, where m_e / m_0 is the ratio of the electron mass to the mass of the particle in consideration. For a given L , a heavier object takes m_0 / m_e more time to delocalize. In contrast to the axiomatic but often obscure wave mechanics or matrix mechanics, this simple hopping model provides better physical insights into the difference between the classical behavior in the macroscopic world and the peculiar quantum interference in the microscopic world.

5. Quantum interference

To demonstrate quantum interference using this "color-varying" ribbon model, let us consider an ensemble of nontravelling particles represented by two Gaussians

peaked at $-a_0$ and a_0 , i.e., $f_1(x, t=0) \propto \exp(-(x+a_0)^2/4L^2) + \exp(-(x-a_0)^2/4L^2)$ and $f_1(x, t=0) = 0$. The delocalization width L is assumed to be much larger than the Compton wavelength in the nonrelativistic, fast exchange regime. It can be shown from eq. (2) that

$$|f_1(x, t) + if_{-1}(x, t)|^2 + |f_1(x, t) - if_{-1}(x, t)|^2 = \frac{e^{-\frac{x^2+a_0^2}{2\sigma^2(t)}}}{\sqrt{2\pi}\sigma(t) [1+e^{-a_0^2/2L^2}]} \cosh\left[\frac{xa_0}{\sigma^2(t)}\right] \left\{1 + \operatorname{sech}\left[\frac{xa_0}{\sigma^2(t)}\right] \cos\left[\frac{\hbar t x a_0}{2m_0 L^2 \sigma^2(t)}\right]\right\} \quad (14)$$

$$\sigma^2(t) = L^2 [1 + t^2 \hbar^2 / 4m_0^2 L^4] .$$

The above result illustrates quantum interference between two particle packets. It is a quantum analogy to the Young's double-slit experiment in optics, where L is the width for the slit opening and $2a_0$ is the distance between two slits. If particles are assumed to have a constant speed along the z axis (perpendicular to the slits which are aligned along the x axis), then the time variable t is directly related to the distance between the double slits to a screen. The above result indicates a spreading line shape with expanding $\sigma^2(t)$. In addition, the interference fringes emerge which are represented by the cosine term with a time dependent phase $\Phi(t)$ given by $\hbar a_0 x t / [2m_0(L^4 + t^2 \hbar^2 / 4m_0^2)]$. At a very short time $t \ll 2m_0 L^2 / \hbar$, the intensity of fringes is very weak and the phase $\Phi(t) \approx \hbar a_0 x t / 2m_0 L^4$ corresponds to a frequency $\omega = \hbar a_0 x / m_0 L^4$. As time evolves the packets spread out due to increasing second moment and the interference fringes start to emerge. At $t \gg 2m_0 L^2 / \hbar$, the oscillation in phase $\Phi(t) \approx 2a_0 x m_0 / \hbar t$ slows down and the intensity of fringes decreases again. As illustrated in figs. 3 and 4, the quantum interference effects resemble the optical interference in a double-slit experiment. Both surface and contour plots are

shown for the cases of $a_0/L = 10$ and 20 . In the beginning, two initial Gaussian wave forms expand in shape exhibiting uncertainty broadening. As the wave front of these two individual distributions cross each other, interference fringes emerge. The width between fringes, given by $2m_0(L^4 + t^2\hbar^2/4m_0^2)/\hbar a_0 t$, also changes in time.

At a very short t , the fringe intensity starts to grow and the width between fringes starts to narrow until $t = 2m_0L^2/\hbar$. At $t > 2m_0L^2/\hbar$, it starts to grow weaker in intensity and wider between fringes. For a smaller a_0/L , interference fringes emerge at a much later time. The width between fringes is narrower and the intensity of interference is weaker.

6. Discussion and conclusions

In this work, we have presented a simple hopping and "color-varying" ribbon model to explain the difference between de Broglie wavelets and Schrödinger carrier wave functions, the origins of quantum interference, and the mechanisms for uncertainty broadening. As an alternative to the doctrinal but often obscure approaches of Schrödinger wave mechanics or Heisenberg matrix theory, better physical views are offered here to improve our understanding of peculiar quantum behavior in the microscopic world. The hopping and "color-varying" dynamics along a linear ribbon chain is simple. No imaginary factors are required in eq. (1), in contrast to the doctrine of Schrödinger's approach which demands we shall replace the Hamiltonian H by $i\hbar\partial/\partial t$ and the momentum p by $-i\hbar\partial/\partial x$. Quantum interference is explained in this work as a result of fast "color-varying" or exchange between internal degrees of freedom f_1 and f_{-1} which represent two degenerate components (time-reversed states) with same kinetic energy but opposite momentum and spin. The uncertainty broadening is shown to be the manifestation of such a fast

internal oscillation and opposite motion between f_1 and f_{-1} .

The ribbon model is connected to Einstein's relativity through de Broglie's hypothesis in extending the Planck's relation for a photon's energy and momentum to a massive particle. It was shown that F_κ and G_κ are interchangeable by a time-reversal operation. We have shown that the solution for eq. (1) possesses dual characteristics of a stable, localized "soliton-like" de Broglie wavelet and a delocalized, interfering Schrödinger carrier wave function. An individual particle is represented by a localized wavelet with a dimension related to the Compton wavelength. For a physical particle in a three-dimensional space, the wavelet solution for eq. (12) can be expressed in terms of the spherical Bessel, Neumann and harmonic functions. On the other hand, an ensemble of particles or wavelets with a distribution of κ_0 is described by $\phi_\mu(x,t)$. It represents the delocalized Schrödinger-type carrier wave function where the probabilistic interpretation in the standard quantum theory comes into play.

It is demonstrated in eq. (13) that the uncertainty broadening can be described by $\sigma^2(t) = L^2(1 + t^2\hbar^2/4L^4m_0^2)$. It is also shown in eq. (14) that quantum interference between two particle packets can occur at a small separation, very similar to the Young's double-slit interference experiment in optics. Because of the small Planck constant $\hbar$, quantum interference and uncertainty broadening are negligible unless the mass m_0 and the physical size of the particles are very small. Although quantum behavior emerges mostly in the microscopic world and classical mechanics dominates in the macroscopic world, a few large scale quantum phenomena do occur in the macroscopic world such as in superconductivity. The situation is different there. Because of the presence of sufficient numbers of electrons of time-reversed states

near the fermi surface, Cooper pairs can be formed via second-order electron-phonon interactions [15]. As discussed in our recent work [16], in a BCS superconductor [17] the exchange coupling U_c between the pairs is about 10^{11} s, the natural length unit R is about 10^{-8} cm, and the natural time unit W^{-1} is about 10^{-16} s. Such a weak exchange coupling and large values for R and W^{-1} , as compared to quantum electrodynamics in which $r_e \sim 10^{-13}$ cm and $W_e^{-1} \sim 10^{-23}$ s, it is not too surprised to see large scale quantum coherence exist between these Cooper pairs which lead to superconductivity in the macroscopic world.

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Figure captions

Fig. 1. A schematic diagram for the linear ribbon model with hopping and "color-varying" dynamics.

Fig. 2. The two-dimensional surface plot of the population versus x/L and t/T , showing uncertainty broadening. The coordinate x is normalized to L (in cm), the initial width of localization, and the time t is also normalized to T (in s) defined as $m_e/m_0 L^2$. The uncertainty broadening is demonstrated as a result of fast exchange interaction between f_1 and f_{-1} components and their opposite motion.

Fig. 3. The surface plot of the population distribution showing interference effects between two initially localized wave packets with $a_0/L = 10$. The quantum interference patterns resemble the optical interference by double slits.

Fig. 4. The contour plot of the population distribution showing quantum interference for the case of $a_0/L = 20$. In the beginning, two Gaussian wave forms first expand in shape exhibiting uncertainty broadening. As the wave fronts of two individual distributions cross each other, interference fringes emerge.

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