Threshold Detection of Radar Signals off the Sea Surface in Non-Gaussian Clutter and Deterministic Interference

II: Statistical Analysis of ROI Surface Data

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Lawrence Livermore National Laboratory

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Abstract:

The purpose of this report is to motivate and outline a program of data analysis, for data obtained from radar returns from ocean surfaces perturbed by internal waves and wind-wave interactions. The ultimate aims of this analysis are to provide the appropriate statistics of the signals returned from these ocean surfaces for: (1) use in implementing and evaluating optimum and near-optimum signal processing procedures for detecting and evaluating (i.e., measuring) these internal wave effects and, (2) to provide quantitative physical insight into both the surface scatter and subsurface mechanisms which determine the received radar signals.

Here the focus is initially on the needed statistics of the radar returns. These are primarily: (i) the (instantaneous) amplitude and envelope probability densities, (pdf's) and distributions (PDF'S) of the returns and, (ii) analogous statistics for the intensities (associated with the pixel data). Also required are: (iii) space-time covariance data of the returns, for further improvement of detection capabilities. Preliminary evidence and earlier experiments suggest that these data [(i), (ii)] are nongaussian and strongly so at times. This in turn, if not properly taken into account, can greatly degrade signal detection in the usual weak-signal régimes [1],[2].

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1. Introduction

The experimental data we are concerned with here are obtained from monostatic radar returns from wind-wave surfaces, under which the medium produces various internal waves. These in turn manifest themselves as modulations of the wave surface, which are scanned by the radar in question. With small grazing angles ($\Psi \leq 20^\circ$) and short pulses we may expect that the scatter return off such surfaces will exhibit a nongaussian structure, particularly with bursts of multiple scatter and occasional breaking waves. As we have already remarked in Section 3 of [1], where the affect of these mechanisms are briefly described, the resulting random bursts (vis-à-vis an essentially gaussian background of comparatively small, uncorrelated scatter components) suggest the physically derived Class A, 3-component nongaussian noise model (Sec. 3.1 [1]), briefly summarized below (Sec. 3ff.). In any case, whatever the model — physical or mathematically ad hoc—our purpose here is to outline or otherwise describe:

- 1) The types of (clutter) noise data - instantaneous amplitudes, envelopes, intensities - needed for adaptive (near-) optimum detection of weak signals in such noise environments;
- 2) How much data are needed and applied in the detection process;
- 3) The constraints upon, and tests for, the validation of the experimental data itself;
- 4) Related topics, concerning sample size, stationarity, etc.

The cumulative intent of the above is to provide a guide to the data analyses required for full exploitation of the locally optimum detection algorithms indicated by theory for optimum performance, cf. Sections 2, 4 of [1].

Our topics in the following include:

- i) Optimum threshold detection algorithms in the coherent reception régimes*;

* We shall extend these results in a subsequent study [3], now underway, to the important cases of incoherent reception for independent and correlated noise samples.

- ii) Needed statistics for their implementation;
- iii) Class A noise models: first-order pdf's;
- iv) Other, empirical noise models;
- v) Various procedures for analyzing the input data.

As we have stated above, our primary purpose in this Report is to provide a guide to statistical data analysis needed for ROI implementation.

2. Canonical Optimum Threshold Algorithms —The Coherent Régime [1]:

The canonical threshold algorithm for optimum detection, namely, the locally optimum Bayer detector (LOBD), here for coherent reception, directly identifies the various statistical data needed for: 1) implementation of the algorithm and, 2) prediction of its performance. In practice, of course, we seek to approximate this theoretical ideal as closely as possible, consistent with existing constraints of time and cost.

From Eq. (2.6) of [1] we write directly for the LOBD in the coherent cases:

$$\text{LOBD: } g^{(21)}(z)_{\text{coh}}^* = \log \mu_{21} + \hat{B}_{\text{coh}}^{(21)*} - \overline{S_D^{(21)}} \sum_{j=m,n}^J l(z_j), (\mu_{21} = p_2 / p_1) \quad (2.1)$$

where (cf. Sec. 2 of [1]) $z_j = z_{mn}$, ($m = 1, \dots, M$; $n = 1, \dots, N$) are the received (normalized) data samples, and where the bias term in (2.1) is specifically:

$$\hat{B}_{\text{coh}}^{(21)*} \doteq -L^{(2)} \langle \theta^{(1)} \rangle \cdot \overline{S_D^{(21)}}, \quad \overline{S_D^{(21)}} \ll 2 \langle \theta^{(1)} \rangle \quad (2.2)$$

$$\overline{S_D^{(21)2}} \equiv \left\langle a_{\text{o-coh}}^{(21)2} \right\rangle_{\min}^* \quad (2.2a)$$

under the often reasonable approximation of local stationarity and homogeneity over the $J (=MN)$ space-time samples* (pixels/sec • seconds of observation, cf. Sec 4.1, [1]). As before, l and $L^{(2)}$ are determined from the accompanying non-gaussian noise, including a purely random (additive) noise component, through the relations:

$$I. \quad l(z) = \frac{d}{dz} \log w_1(z|H_0); \quad L^{(2)} \equiv \langle l^2 \rangle_{H_0} = \int_{-\infty}^{\infty} l(z)^2 w_1(z|H_0) dz = \int_{-\infty}^{\infty} \left(\frac{w_1'}{w_1} \right)^2 w_1 dz. \quad (2.3)$$

In the gauss noise cases (2.3) is readily obtained, since

$$w_1(z|H_0)_{\text{gauss}} = \frac{1}{\sqrt{2\pi}} e^{-z^2/2}, \text{ and } \therefore l(z)_{\text{gauss}} = -z; \quad L_{\text{gauss}}^{(2)} = 1. \quad (2.3a)$$

Thus, items I, (2.2) (2.3) above are our first set of needed algorithm statistics, to be applied to the LOBD (2.1). This means that, fundamentally, to begin with we need the first-order noise pdf $w_1(z|H_0)$, cf. (2.3), which is generally nongaussian.

For implementation of the LOBD (2.1) we need two additional statistical quantities here, viz:

$$1. \quad \underline{\text{The Minimum Detectable Signal, } \overline{S_D^{(21)^2}}, \text{ cf. (2.2a);}} \quad (2.4a)$$

$$2. \quad \underline{\text{The bias } \hat{B}_{j-\text{coh}}^{(21)*}, \text{ Eq. (2.2), or equivalently } \langle \theta^{(1)} \rangle.} \quad (2.4b)$$

To achieve the quantification of (2.4a,b) we proceed as follows in these coherent régimes. We begin with the prediction of performance and use (4.1) of [1], namely:

$$\left[\sigma_{j-\text{coh}}^{(21)*} \right]^2 \doteq L^{(2)} \overline{JS_D^{(21)^2}} \quad (2.5)$$

* It is recognized that this is frequently an approximation to the actual state of the data, hence the need for statistical testing for stationarity and homogeneity (s.+h.), noted below, cf. Sec. 4 III. It may be necessary to split the data set into a number of (s.+h.) subsets, for example, and use (2.1), (2.2) for each set.

in (4.1) and (4.6), Sec. 2 of [1]. Thus, by choosing an acceptable false alarm probability $\alpha_F^* (= \beta_1^{2*})$, (4.6), [1], and an acceptable (conditional) probability p_D^* of correctly detecting the presence of the desired signal, we can determine the resulting minimum detectable signal, $\overline{S_D^{(21)^2}}$ here, from (4.9), [1], viz:

$$\overline{S_D^{(21)^2}} \equiv \left\langle a_{o-coh}^{(21)^2} \right\rangle^* = \frac{2}{L^{(2)}_J} \left[\Theta^{-1}(1 - 2\alpha_F^*) + \Theta^{-1}(2p_D^* - 1) \right]^2, \quad \begin{pmatrix} \alpha_F^* > \frac{1}{2} \\ p_D^* > \frac{1}{2} \end{pmatrix}, \quad (2.6)$$

or

$$\overline{S_D^{(21)}} = \left(\frac{2}{L^{(2)}_J} \right)^{1/2} \left\{ \Theta^{-1}(1 - 2\alpha_F^*) + \Theta^{-1}(2p_D^* - 1) \right\} (> 0), \quad (2.6a)$$

with $\Theta(x) \equiv (2/\sqrt{\pi}) \int_0^x e^{-t^2} dt \equiv \text{erf}.x$, as before. Equations (2.6), (2.6a) allow us to write equivalently for (2.5):

$$\begin{aligned} \left(\sigma_{J-coh}^{(21)*} \right)^2 &\doteq 2 \left\{ \Theta^{-1}(1 - 2\alpha_F^*) + \Theta^{-1}(2p_D^* - 1) \right\}^2, \\ &= L^{(2)}_J \left[\left\langle \theta^{(2)} \right\rangle - \left\langle \theta^{(1)} \right\rangle \right]^2. \end{aligned} \quad (2.7)$$

When $\langle \theta^{(1)} \rangle$ cannot be separately measured, we cannot obtain the bias $\hat{B}_{j-coh}^{(21)*}$ explicitly. We can, however, bypass the evaluation of this bias term, as we shall see below. We begin by noting that (cf. (2.12), [1])

$$\alpha_F^* \equiv \frac{1}{2} \left\{ 1 - \Theta \left[\frac{\sigma_{J-coh}^{(21)*}}{2\sqrt{2}} + \frac{\log(K/\mu_{21})}{\sqrt{2}\sigma_{J-coh}^{(21)*}} \right] \right\}, \quad (2.8)$$

where $\sigma_{J-coh}^{(21)*} = \sigma_{J-coh}^{(21)*}(p_D^*, a_F^*)_{J-coh}$, cf. (2.5) – (2.6a). Thus, p_D^* and α_F^* being preselected, $\sigma_{J-coh}^{(21)*}$ is, of course, initially determined, cf. (2.7), as well as $\overline{S_D^{(21)}}$ subsequently via (2.6a). Furthermore, (2.8) specifies the threshold ratio (K/μ_{21}) .

To proceed further, however, we must return to the threshold evaluation of α_F^* using the fact that $g^{(21)}(z)_{\text{coh}}^*$, (2.1), is asymptotically normal (AN), to get alternatively to (2.8),

$$\alpha_F^* \equiv \frac{1}{2} \left\{ 1 + \Theta \left[\frac{\left\langle g^{(21)*} \right\rangle_{H_0} - \log K}{\sqrt{2} \sigma^{(21)*}} \right] \right\}, \quad (2.9)$$

with

$$\left\langle g_{J-\text{coh}}^{(21)*} \right\rangle_{H_0} = \log \mu_{21} + \hat{B}_{j-\text{coh}}^{(21)*}, \quad (2.9a)$$

which now explicitly introduces the bias term in the false alarm probability α_F^* . Accordingly, (2.9a) in (2.9) yields directly

$$\alpha_F^* \equiv \frac{1}{2} \left\{ 1 - \Theta \left[\frac{\log(K / \mu_{21}) - \hat{B}_{J-\text{coh}}^{(21)*}}{\sqrt{2} \sigma_{J-\text{coh}}^{(21)*}} \right] \right\}. \quad (2.10)$$

The equivalent decision situation here is (cf. (4.13) of [1]) with (2.1) now modified to:

Decide H_2 : "desired signal is present", if:

$$\hat{g}_{J-\text{coh}}^{(21)*} \equiv \overline{S_D^{(21)}}^2 \sum_j l(z_j) = g_{J-\text{coh}}^{(21)*} - \log \mu_{21} - \hat{B}_{j-\text{coh}}^{(21)*} \geq \log \left(\frac{K}{\mu_{21}} \right) - \hat{B}_J^{(21)*} \left(\equiv \log \hat{K}^{(21)} \right); \quad (2.11a)$$

$$\text{Decide } H_1: \text{ "desired signal not present", if: } \hat{g}_{J-\text{coh}}^{(21)*} < \log \hat{K}^{(21)}. \quad (2.11b)$$

Now, with the definition (2.11a) of the new threshold $\log \hat{K}^{(21)}$ and new test statistic $\hat{g}^{(21)*}$, we see that (2.10) becomes

$$\alpha_F^* \equiv \frac{1}{2} \left\{ 1 - \Theta \left[\frac{\log \hat{K}^{(21)}}{\sqrt{2} \sigma_{J-\text{coh}}^{(21)*}} \right] \right\}. \quad (2.12a)$$

Since α_F^* is preset and is already determined by (2.9), Eq. (2.12a) specifies the new threshold, e.g.,

$$\log \hat{K}^{(21)} = \sqrt{2} \sigma_{J-\text{coh}}^{(21)*} \Theta^{-1}(1 - 2\alpha_F^*). \quad (2.12b)$$

In this fashion we have eliminated the need to evaluate the bias term $\hat{B}_{J-\text{coh}}^{(21)*}$ in (2.1), explicitly[†], and hence $\langle \theta^{(1)} \rangle$ in (2.2) as well. We can still carry out the decision process (2.11a,b) with the now modified algorithm (2.11a), (vs. (2.1)), as well as obtain the minimum detectable signal $\overline{s_D^{(21)}}^2$ (2.6), and the optimum signal detection parameter $\sigma_{J-\text{coh}}^{(21)*}$ here, cf. (2.5). All this is achieved because we have preset the probability controls α_F^*, p_D^* . Of course, our preset values of α_F^*, p_D^* will not assure, with the same p_D^* , the successful detection of signals less than the minimum detectable signal determined by (2.6). Setting smaller values of p_D^* with a given α_F^* will yield smaller minimum detectable signals. Moreover, setting α_F^* and/or $p_D^* = 1/2$ is equivalent to "flipping" an (unbiased) coin, with the unconditional outcome weighted by the a prior probabilities p_2, p_1 .

3. Statistical-Physical Noise Models:

From the analysis in Section 2 above, it is clear that the central statistical information required in developing and testing our threshold detection algorithms [cf. (2.1) here] is the first order pdf $w_1(z|H_0)$, vide (2.3), of the non-deterministic part of the clutter noise, including all additive gaussian noise components associated with the scattered radar return, any ambient sources, and inherent system noise.

[†] However, the proper bias is still necessary in p_D^* in order to assure consistency of the test and its asymptotic optimality.

As explained in [1], Section 3, a candidate model here is Class A noise [2],[4], which in its usual form (with additive, zero-mean gauss noise) is a 3-parameter gauss mixture model. Reverberation (acoustic) and clutter (EM) are typical sources of such noise, since they consist of sums of independent scattered “replicas” of the incidental signal. The associated pdf (2.3) here is analytically described below. The observed data $\{z_{j=mn}\}$, for which we construct an appropriate Class A model, may take several forms, depending on the manner of its initial processing before application to an LOBD like (2.1) above. We have specifically:

(i). z_j = narrow-band, instantaneous amplitudes

$$= z_c^{(m)}(t_n) \cos \omega_0 t_n + z_q^{(m)}(t_n) \sin \omega_0 t_n, \quad (3.1a)$$

$z_c, z_q \equiv$ in-phase, and quadrature components ;

(ii). z_j = instantaneous envelopes,

$$= E^{(m)}(t_n); \text{ or} \quad (3.1b)$$

$$E_n^{(m)} \equiv \left(z_{cn}^{(m)^2} + z_{qn}^{(m)^2} \right)^{1/2};$$

(iii). z_j = instantaneous intensities

$$= I_n^m = E_n^{(m)^2} = I^{(m)}(t_n), \text{ etc.} \quad (3.1c)$$

For example, the data shown in Figure 3.3 of [1] are partially time-averaged pixel intensities, at distinct periods of time (n) and spatial location (m). Original, unaveraged data are given by (i), or by the instantaneous envelopes (after initial detection), before subsequent optimal processing according to the algorithm (2.1) for test (2.11), for example. Similar remarks may also be applied to the instantaneous intensities (iii), (3.1c).

The signal structures corresponding to (3.1) take the form

$$(i). \quad q_j^{(2or1)} = \left[a_n^{(m)} s_n^{(m)} (t_n - \varepsilon) \right]_{2or1} = a_{on}^{(m)} \sqrt{2} \cos \left[\omega_0 (t - \varepsilon) - \phi_n^{(m)} \right]_{2or1} ; \quad (3.2a)$$

$$\left. \begin{aligned} \overline{s_j^2} &= 1 \\ a_{on}^{(m)} &= A_{on}^{(m)} / \sqrt{2\Psi_z} \\ s_n^{(m)} &= A_{on}^{(m)} s_n^{(m)} / \sqrt{2} \\ &= \sqrt{\Psi_z} a_{on}^{(m)} s_n^{(m)}; \end{aligned} \right\} \quad (3.2a1)$$

$$(ii). \quad \theta_j^{(2or1)} = \left[a_{on}^{(m)} \right]_{2or1} = E_{on}^{(m)} \Big|_{(2or1)} ; \quad (3.2b)$$

$$(iii). \quad \theta_j^{(2or1)} = \left[I_{on}^{(m)} \right]_{2or1} = \left(a_{on}^{(m)^2} \right) \Big|_{(2or1)} . \quad (3.2c)$$

Thus, according to the type of data [(i)–(iii)] of (3.1), or their corresponding locally averaged values, i.e., time averages of each pixel display, we have corresponding signal forms and noise statistics. Specifically, for our Class A models, we have

$$w_1 = w_1(z|H_0)_{A+G} = e^{-A} \sum_{m=0}^{\infty} \frac{A^m}{m! \sqrt{4\pi\hat{\sigma}_m^2}} e^{-z^2/4\hat{\sigma}_m^2} : [\text{instantaneous amplitudes: } z = z_{amp}], \quad (3.3a)$$

$$= e^{-A} \sum_{m=0}^{\infty} \frac{z A^m}{m! \hat{\sigma}_m^2} e^{-z^2/2\hat{\sigma}_m^2} : [\text{instantaneous envelopes: } z = z_{amp} \sqrt{2}], \quad (3.3b)$$

$$= e^{-A} \sum_{m=0}^{\infty} \frac{A^m}{m! \hat{\sigma}_m^2} e^{-z/\hat{\sigma}_m^2} : [\text{instantaneous intensity: } z = I = z_{amp}^2 / 2], \quad (3.3c)$$

in which

$$2\hat{\sigma}_m^2 = (m/A + \Gamma'/(1+\Gamma')). \quad (3.3d)$$

The quantities Γ' and A are defined here as

$$\Gamma' = \sigma_G^2 / \Omega_{2A} = \text{the "gaussian factor" = ratio of the average intensity of the gaussian background* to that of the nongaussian noise (clutter) component.} \quad (3.4a)$$

$$A = \text{"overlap index" = average no. of emission events (re-radiation "bursts") per second x the mean duration of a typical "burst".}$$

For noticeably nongaussian Class A noise, we have $A < O(0.5)$ and $\Gamma' \leq O(10^{-1})$.

Finally, other, ad hoc models, e.g., K-distributions [6], log-normal pdf's [7], etc., may be needed if the data show more complex mechanisms are at work statistically.

* We remark that for the single-scatter ($k = 1$), or "speckle" component of the clutter (cf. 3), Sec. 3.1, of [1], each pixel [$1.5m \times 44m$], vide Fig. 3.3, [1] namely each pulse return "footprint" of the radar, may support 0(10–20) "facets" $O(1.5m \times 1.5m)$, so that approximately by the Central Limit Theorem (CLT), [5], Sec. 7.7-3) this component is gaussian. It is included with, and is probably the main contributor to, the postulated gaussian background (G) in our Class A noise model above.

4. General Procedures:

At this point let us summarize the principal items of statistical necessity noted above in Sections 2 and 3, and indicate the accompanying tests required for "data validation". We need to obtain:

I. Required Statistical Data:

- (1). $w_1(z|H_0)|_{\text{amp}} = \text{1st order pdf of the normalized data } z (= Z/\sqrt{\Psi}) \text{ for:}$
 $= \text{instantaneous amplitudes: } Z(t) = I(t) \cos \omega_0 t + Q(t) \sin \omega_0 t$
 $= E(t) \cos [\omega_0 t + \phi(t)];$
 $E = \sqrt{I^2 + Q^2}; \phi = \tan^{-1}(Q/I);$
 $\omega_0 = 2\pi f_0 = \text{radar carrier (RF or IF) frequency;}$
- (2). $w_1(z|H_0)|_{\text{envel.}} = \text{1st-order pdf of the normalized envelope data,}$
 $z = E = E/\sqrt{2\Psi};$ (4.1)

$E = \text{instantaneous envelopes: } E(t) = \sqrt{I(t)^2 + Q(t)^2}, \text{ with}$
instantaneous phases: $\phi(t) = \tan^{-1}(Q/I) .$

- (3). $w_1(z|H_0)|_{\text{intensity}} = \text{1st-order pdf of the normalized intensity data } \hat{I}(t):$
 $\hat{I}(t) = z(t)^2 = Z(t)^2 / \Psi = E(t)^2 / 2 = E(t)^2 / 2\Psi = (I^2 + Q^2) / \Psi.$

Here, as earlier [1], we have

$$\overline{Z(t)} = 0; w(z|H_0)_{\text{amp}} = w(-z|H_0)_{\text{amp}} : \text{symmetry} \quad (4.2)$$

$$w(z|H_0)_{\text{inst.env.}} \geq 0 : \text{one-sided,}$$

and

$$\Psi = \text{noise intensity} = \overline{Z^2} = \overline{(I^2 + Q^2)} = \overline{E^2} / 2, \quad (4.3)$$

is a basic parameter of all these pdf's.

II. Class A Noise Models:

Here we have the three parameters:

$$\begin{aligned} (1) \quad \Psi_{A+G} (= \Psi_A + \Psi_G) &= \Omega_{2A} + \sigma_G^2; \\ (2) \quad \Gamma' &\equiv \sigma_G^2 / \Omega_{2A}; \\ (3) \quad A &\equiv \text{overlap index}, \end{aligned} \tag{4.4}$$

which are needed in (3.3) for the Class A model. Our task here is to:

- (i) Estimate the Class A parameters;
- (ii) Test how well $w_1(z|H_0)_{A+G}$, (3.3), fits the actual pdf $w_1(z|H_0)$ above.

III. Validation of the Data:

Various standard statistical tests should be applied to the data. These involve:

- (i). Tests of stationarity, i.e., homogeneity: are the data in a multiple element data sample from the same ensemble?
- (ii) Tests of "goodness of fit": how well do our various analytical models [Class A, log-normal [7], K-distributions [8], etc.] fit the data?
- (iii) Obtaining bounds, i.e., standard deviations ($\equiv \sqrt{\text{variance}}$) of parameter and pdf values.

Reference [8] provides an elementary starting point here for these tasks. Note, in addition, that one problem is obtaining sufficient accuracy (small σ 's) out on the "tails" of the distribution's (pdf's here), say at 4-6 standard deviations (σ 's). Also, one should start with "good" data, i.e., not obviously contaminated by surface ships. (Reference [9], and Dr. Axelrod (LLNL 510-422-0929), can offer additional advice on questions of statistical data analysis here).

IV. Related Statistics:

These are given by (2.3), viz:

$$l(z) = \frac{d}{dz} \log w_1(z|H_0) = w_1'(z|H_0) / w_1(z|H_0), \quad (4.5a)$$

$$L^{(2)} \equiv \int_{-\infty}^{\infty} l(z)^2 w_1(z|H_0) dz, \quad (4.5b)$$

and are needed in the implementation of the LOBD and for its expected performance, cf. (2.6), etc. For the Class A model one can readily apply (3.3) to (4.5a) to obtain analytic results, whose numerical values are then describable when the Class A parameters (4.4) have been obtained from the data at hand. (For examples, See [4], Sections 3, 4, and also Refs. [80]–[82] therein, for various additional approaches to estimating these parameters.)

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