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FUZZY ATTRIBUTES FOR KNOWLEDGE REPRESENTATION
AND ACQUISITION

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Fuzzy attributes for knowledge representation and acquisition

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ABSTRACT

Repertory grids and other matrix-like structures can be used to represent knowledge and elicit knowledge from experts. A grid or matrix is a representation of a knowledge domain where the elements in the domain appear along the horizontal axis and constructs or attributes of the elements appear along the vertical axis. Each construct is rated for its presence in a given element or how much a construct applies to an element. Analysis of these ratings can determine similarities and differences between the elements.

Traditionally, constructs are bipolar entities where a rating falls on a range from one pole to the other. For example, temperature may be represented by the bipolar construct hot-cold and a range of 1 to 5 in which 1 represents hot and 5 represents cold. Ratings of 2, 3, and 4 lie in-between hot and cold. Additionally, all constructs in a grid have the same range of values and the range is arbitrarily chosen.

This paper presents a method for translating grid ratings in to fuzzy membership values. The fuzzy membership values become the values for describing and analyzing the associations between elements. Thus, constructs no longer need to use the same scaling range and no longer need to be bipolar. A construct of an element now becomes a true attribute of an element. An attribute can be rated in its own range and with its own unit of measurement. In the previous example, the bipolar construct hot-cold becomes simply, temperature measured in degrees. Experts or users need no longer translate to an artificial rating range.

Keywords: knowledge representation, fuzzy attributes, repertory grid

1 BACKGROUND

1.1 Introduction

Declarative knowledge is characterized by a set of facts. An object or entity is described by listing its characteristics or attributes. Objects that are similar may share the same set of attributes. Similar objects and their corresponding attributes can be grouped together into a structure like a grid or matrix. The rows of the

matrix are the attributes and the columns are the instances or examples of the object. When used for expert knowledge representation, an expert fill out the cells of the matrix. The expert "rates" each attribute for its presence in each of the examples. In this way, a matrix can describe the spectrum of an object.

1.2 Repertory grid background

The matrix is a simple description and perhaps a loose interpretation of a repertory grid. The repertory grid was developed by Kelly for personal construct psychology.⁶ Since then, the repertory grid has been used for representing and eliciting expert knowledge for designing expert systems.⁴ The attributes in a repertory grid are bipolar constructs and the examples are referred to as elements. A rating scale, such as 1 to 5, is selected and each bipolar construct is rated for each element. A 1 represents the left-hand pole of the construct and 5 the right-hand pole. Using distance and clustering algorithms, a grid can be analyzed to suggest refinement and further elicitation.⁴

Gaines and Shaw have applied fuzzy logic to repertory grids.³ The grid and its scaling system remains the same, but each of the two poles for a construct becomes a fuzzy set. Figure 1 shows an example construct turned into fuzzy sets. An assigned rating for a construct in an element is converted to a fuzzy membership value in each of the two fuzzy sets. Fuzzy membership values are between 0.0 and 1.0 inclusive, where 0.0 means no membership and 1.0 means complete membership. For example, a rating of 1 (on a 1 to 5 scale) has a membership value of 1.0 in the left-hand pole fuzzy set and a membership value of 0.0 in the right-hand pole fuzzy set as shown in Figure 1. The Lukasiewicz fuzzy implication operator is then used to derive implications between constructs.³

Figure 1: Two fuzzy sets for the bipolar construct hot-cold.

1.3 Modifying the repertory grid

The matrix used in this research is a modified version of the repertory grid Gaines and Shaw used in their fuzzy logic research.³ This matrix is more usable and a more general purpose representation structure than the repertory grid. The modifications also make it easier to create a fuzzy implication and are described in the following sections.

Attributes take the place of bipolar constructs. An attribute is a single entity. It has no poles or explicit extremes. The uniform rating system for constructs is no longer used. Each attribute has its own measure, thus each has its own units, scale, and range. The elements remain the same, but are referred to as instances or examples. Like the repertory grid, the attributes are along the rows of the matrix and the instances are along the columns.

Each attribute is divided into a number of overlapping fuzzy sets or fuzzy membership functions. Recall in Figure 1, each pole of a bipolar construct is converted into a fuzzy set. Since (in a repertory grid) all constructs

have the same rating system, the same two fuzzy sets describe all constructs. For the matrix, each attribute may have its own set of fuzzy sets and always more than just two sets. This provides for more detail. As an example, Figure 2 shows the fuzzy membership functions for the attribute temperature. This attribute is a matrix version of the bipolar construct hot-cold shown in Figure 1. There are three fuzzy membership functions for temperature, which are Low, Medium, and High. The range of the attribute temperature is 60 to 100 degrees and is along the x axis. The degree of membership is along the y axis.

Figure 2: Fuzzy membership functions for the attribute temperature.

From a user or expert's perspective, an actual temperature for an example is entered into the matrix. Once the matrix is filled out of all its attributes and examples, the corresponding fuzzy values can be calculated. For example, an entry of 65 degrees has a membership value of 0.4 in the fuzzy set Low and a membership value of 0.6 in the fuzzy set Medium. However, calculation of the fuzzy values is transparent to the expert. This is done by the underlying program.

Whereas analysis of a repertory grid looks for single implications between "like" fuzzy sets describing constructs, analysis routines for the matrix reveals fuzzy rules composed of multiple attributes. That is, repertory grid analysis only looks for implications between one left-handed pole of a construct and a left-hand pole of another construct and likewise for right-hand poles. It does not look for implications between a left-hand pole of a construct and a right-hand pole of another construct (or visa-versa). This makes sense due to the nature of a bipolar construct. The poles of the construct oppose one another. Alternatively, since an attribute range does not necessarily represent opposing ends of a spectrum, it makes sense to look for rules between the different combinations of the different fuzzy sets describing an attribute. Also, with the added knowledge of which attributes are inputs to a control system and which is output, rules with logically AND'd inputs can be revealed.

1.4 Advantages to the matrix form

There are several advantages to the matrix form over repertory grids. Many of the advantages deal with cognitive psychology or user-interface aspects of the matrix. These are human-computer interaction issues that directly effect the expert or other user who fills the matrix with expert knowledge. Differences between the matrix and repertory grid make it easier and more straightforward for an expert to deal with the matrix.

Values for attributes are entered into the matrix in their original form. They need not be converted to another form such as an artificial and arbitrary rating scale. Absence of conversion means one less step for the expert. This makes the elicitation and use of the attributes more natural. An attribute's original form consisting of its scale, range, and unit of measure is something with an expert should be intimately familiar. Attributes are also more natural in the sense that an expert deals with "one" entity and not an entity described by opposing poles or adversary-like extremes.

Since attribute values are now acquired in their original form, it is possible to seek knowledge from other

"experts." These other experts include, but are not limited to, indexed knowledge bases (like databases) and on-line information archives. Sources of this type may possess historical knowledge recorded from past human experts. On-line sources could streamline and increase knowledge acquisition.

Rules that encompass most or many of the attributes may lead to a clearer picture of the knowledge being represented. This seems especially true for control system type knowledge. The point of the rules is to express relationships between attributes and the sum of the relationships may express an overall behavior. If the rules are all single implications (one attribute implies another attribute) and there are many attributes, this may lead to a fragmented view with no apparent overall behavior. The matrix and its analysis strives for revealing multiple-antecedent implications as shown in Equation 1 to suggest overall behavior.

$$(a \wedge b) \rightarrow c \quad (1)$$

2 HOW IT WORKS

2.1 Fuzzy control system framework

Analysis of the matrix is based on the way fuzzy control systems work. In a fuzzy control system there are fuzzy sets for the inputs and output of the system and rules to map the inputs to output. Input values are converted to fuzzy input values and subsequently mapped to a fuzzy output depending on which rules apply. The fuzzy output value is then defuzzified to obtain an actual output value for the control system.

Each example within the matrix has values for inputs and an associated output. It is the rules of the system that need to be revealed. In this way the problem is similar to a fuzzy control system problem, except for solving for a different result. Rules are obtained by inspecting all the combinatorically possible rules and calculating which ones best fit the attribute examples.

Before the system can be used, fuzzy membership functions must be created for each of the input and output attributes. An actual attribute value will then be automatically be converted to a fuzzy value in its corresponding fuzzy set. The number of fuzzy membership functions in each input and output determine the number of rules possible. For example, if the rules are the type shown in Equation 1 then a and b are input attributes and c is the output attribute. Suppose that attributes a and b each have three membership functions and attribute c has four membership functions. If this was a control system, the two inputs a and b would be mapped to the output c using nine rules ($3 \times 3 = 9$). Each combination of the inputs can only imply one output value out of the four. However, when trying to reveal a rule, all the combinations of inputs and output must be considered. For this example, there are 36 rules to consider.

2.2 Deriving the fuzzy relation R

In fuzzy control systems a fuzzy relation R is used to calculate a fuzzy output value given a fuzzy input value and a fuzzy rule. A rule maps inputs to outputs. Since each input and output is constructed of many fuzzy sets, there are many possible rules. A corresponding fuzzy relation R must be derived for each rule. Fuzzy relations are often expressed in matrix form.

Continuing with the previous fuzzy membership functions example, Figure 3 shows the fuzzy membership functions for the inputs A and B . Three fuzzy sets or membership functions which are **Low**, **Medium**, and **High** make up the inputs A and B . The membership functions are "triangle" or "tent" functions because of their shape. Some of them (**Low** and **High**) are "half tents." Figure 4 shows the fuzzy membership functions for an

output C . Four membership functions, **Poor**, **Fair**, **Good**, and **Excellent** make up the output C . These output membership functions are also constructed of half-tent and tent functions.

Figure 3: Fuzzy membership functions for inputs A and B .

Figure 4: Fuzzy membership functions for output C .

The half-tent function is the minimal description of a fuzzy set or fuzzy membership function in this system. Recall, that given three membership functions for each A and B , and four membership functions for C , there are 36 possible combinations of inputs to outputs. Taking into account the half-tent function increases the number of combinations. Each A and B , has four half-tent membership functions and C has six half-tent membership functions. This can be seen in Figures 3 and 4. The number of possible combinations of inputs to output and thus possible rules is 96.

A fuzzy relation R is derived with a fuzzy implication or "ply"⁸ operator. There are several different ply operators. The ply operator used here is based on the Lukasiewicz ply operator¹ which is shown in Equation 2. Yager⁸ and Bandler and Kohout¹ study some characteristics and properties of different ply operators. The Lukasiewicz ply operator is modified to handle two inputs instead of one. The modification is shown in Equation 3. Logical basis for the modification is demonstrated in the truth tables for the left-hand side and right-hand side of Equation 3. The left-hand side of Equation 3 is equivalent to Equation 4. The truth table for Equation 4 is shown in Table 1. The truth table for the right-hand side of Equation 3 is shown in Table 2. Crisp values (0 and 1) are used in the truth tables.

$$x \rightarrow z = \min(1, 1 - x + z) \quad (2)$$

$$(x \wedge y) \rightarrow z = \min(1, 1 - (\min(x, y)) + z) \quad (3)$$

$$\neg x \vee \neg y \vee z \quad (4)$$

Suppose $f(a)$, $f(b)$, and $f(c)$ are half-tent membership functions which represent a particular rule. The ply operator to derive the fuzzy relation R then becomes Equation 5. Each half tent is broken into ten intervals giving 11 discrete points for each a , b , and c . There are 121 rows in the matrix derived from the AND'ing of a with b and 11 columns in the matrix from c . This leads to an R matrix of 1331 elements.

$$(f(a) \wedge f(b)) \rightarrow f(c) = \min(1, 1 - (\min(f(a), f(b))) + c) \quad (5)$$

x	$\neg x$	y	$\neg y$	z	$\neg x \vee z$	$\neg y \vee z$	$\neg x \vee z \vee \neg y \vee z$
1	0	1	0	1	1	1	1
1	0	1	0	0	0	0	0
1	0	0	1	1	1	1	1
1	0	0	1	0	0	1	1
0	1	1	0	1	1	1	1
0	1	1	0	0	1	0	1
0	1	0	1	1	1	1	1
0	1	0	1	0	1	1	1

Table 1: Truth table for Equation 4.

x	y	z	$\min(x, y)$	$1 - \min(x, y)$	$1 - (\min(x, y)) + z$	$\min(1, 1 - (\min(x, y)) + z)$
1	1	1	1	0	1	1
1	1	0	1	0	0	0
1	0	1	0	1	2	1
1	0	0	0	1	1	1
0	1	1	0	1	2	1
0	1	0	0	1	1	1
0	0	1	0	1	2	1
0	0	0	0	1	1	1

Table 2: Truth table for right-hand side of Equation 3.

2.3 Composite function

A composite function is used to solve fuzzy relational equations. Equation 6 shows that the composition of S with R is equal to T . When S and R are represented in matrix form the composite function is similar to the product operation of matrices.⁷ For fuzzy relational equations the product operation in the composite function is replaced with a fuzzy operation. There are many different composite functions, but the two most common are the max-min and the max-prod composite functions.⁵

$$S \circ R = T \quad (6)$$

The max-min composite functions is used in this work. Given two inputs and an output to the control system, the composite function looks like Equation 7. a' and b' are the input attributes and each represent fuzzy values in specific half-tent membership functions. R is the fuzzy relation matrix corresponding to a rule which maps the input half-tent membership functions to an output half-tent membership function. In this case, the result of the composite function, $f'(c)$, is the output half-tent membership function.

$$(a' \wedge b') \circ R = f'(c) \quad (7)$$

The min part of the max-min composite function takes place between the value of $(a' \wedge b')$ and every element in the R matrix, proceeding column by column within each row. This results in a matrix the size of the R matrix. The max part of the max-min composite function takes place between all values in each column of the result matrix. The final result is a matrix the size of one row of the former matrix and this describes a half-tent membership function for the output of the fuzzy control system.

2.4 Truth of a rule

The composite function results in an output membership function and this, when taken in context with the input functions used to calculate the output function, reveals a rule. The rule should reflect the attribute example, but how true is the rule? A belief in the rule is established with two different figures. The difference between the actual c (c_{act}) from the attribute example and a calculated c (c_{calc}) from the output membership function result is calculated. A level of truth based on the degree of membership in each of the membership functions a rule describes is also calculated.

A value for c_{calc} is derived using a sort of "defuzzification" process. Defuzzification is the process of converting an output fuzzy set or membership function into a crisp value.² Typically, defuzzification is accomplished with one of several methods. The two most common methods are the composite maximum and the centroid method.² For this system, the output membership function is defuzzified by selecting the interval where c_{act} occurs in its membership function. If the output membership function resulting from the composite function is a close match, then c_{act} and c_{calc} will also be a close match.

The degree of membership that a , b , c_{calc} , and c_{act} have in their respective membership functions is averaged together to obtain a level of truth. This value, along with the difference between c_{act} and c_{calc} , is used to measure belief in the rules revealed. A rule with a low difference between c_{act} and c_{calc} and a high degree of membership is believed to best describe the attribute example from which it was derived.

3 TESTING AND RESULTS

3.1 Introduction

Initial testing of the system was done with only a few attribute examples at a time. Each attribute example illustrated a specific rule. Membership functions for the inputs and output were similar to those shown in Figures 3 and 4 with actual values ranging from 0 to 1. Later, more exhaustive test sets were run which contained large numbers of attribute examples. The large test sets varied the inputs and output values over an increment to generate all possible combinations.

The initial testing was used to try out different composite functions and ways of measuring the belief in the rules revealed. The more exhaustive test sets helped show reliability across the changing input/output values and membership functions. They also showed that all possible rules were being revealed. Since the input/output values (from one example to the next) changed with small increments, there was a "concentration" of revealed rules. That is, multiple attribute examples revealed the same rule. This might be a useful measure when dealing with similar knowledge acquired from different experts or as a means of measuring the "accumulation" or "emphasis" of a rule.

3.2 Oil and gas example

Once some confidence was demonstrated in the system, a "real-world" example was constructed. This example deals with domain knowledge from the application of oil and gas well logs. Briefly, data from well logs is gathered from tools lowered down the borehole of oil and gas wells. Various measurements, depending on the type of tool, are taken at each foot in a specified range of depth in the well. The measurements describe characteristics and properties of the geology immediately surrounding the borehole of the well.

Three measurements or attributes are identified in some example well logs for use in testing the knowledge representation system. The attributes are porosity, water saturation, and oil in place. Porosity and water saturation are considered to be input attributes and oil in place the output attribute. Six different well logs are used, each containing 20 to 40 attribute examples. This is akin to the knowledge of six different experts. The membership functions for porosity, water saturation, and oil in place are shown in Figures 5, 6, and 7 respectively. The only difference between these membership functions and the examples shown in Figures 3 and 4 are the range and units of values.

Figure 5: Fuzzy membership functions for the input porosity.

Figure 6: Fuzzy membership functions for the input water saturation.

Figure 7: Fuzzy membership functions for the output oil in place.

The expectation is that as the water saturation increases relative to the porosity, the oil in place decreases. As the water saturation decreases relative to the porosity, the oil in place increases. This is reflected in the context

of the rules that are revealed. Table 3 shows some of the rules (divided in three sets) that are revealed by the six different well logs. In the first set of rules, the porosity remains **Low** while the water saturation increases from **Medium** to **High**. Thus, the oil in place decreases from **Fair** to **Poor**. A similar effect is seen in the second set of rules. The third set of rules shows the reverse relationship. While the water saturation remains **Medium**, the porosity increases from **Low** to **High**. Thus, the oil in place increases from **Poor** to **Good**.

If porosity is Low and water saturation is Medium then oil in place is Fair
If porosity is Low and water saturation is High then oil in place is Poor
If porosity is Medium and water saturation is Low then oil in place is Good
If porosity is Medium and water saturation is Medium then oil in place is Fair
If porosity is Medium and water saturation is High then oil in place is Poor
If porosity is Low and water saturation is Medium then oil in place is Poor
If porosity is Medium and water saturation is Medium then oil in place is Fair
If porosity is High and water saturation is Medium then oil in place is Good

Table 3: Rules revealed from the well log attribute examples.

4 CONCLUSIONS AND FUTURE WORK

This matrix form of knowledge representation has been shown to successfully reveal rules from expert knowledge in the form of fuzzy attribute examples. The rules are two-input/one-output control-system type rules. This increases the capacity of knowledge of other systems that only derived single-antecedent rules and may lead to the expression of overall behavior. The success of this work also suggests that to derive single-antecedent rules directly from a ply operator³ is more than the ply operator is designed to accomplish.

Although generating expert rules was an original goal of this work, there were several other benefits. Many of these benefits deal with the user-interface aspects of the matrix. As previously discussed, the matrix and its characteristics provide ease of use for an expert, a more natural form of declarative knowledge, and the ability to acquire knowledge from other expert sources including databases and knowledge archives. These benefits alone may make the matrix form (instead of a repertory grid) more appealing to an expert and/or knowledge engineer.

This work has also generated many questions, some of which may be answered in future work. Although the revealing of two-input/one-output control-system type rules is a breakthrough from past work, how realistic are two-input/one-output control systems? The increase of input-to-output attributes and/or the increase of fuzzy membership functions for each attribute leads to a combinatoric explosion of possible rules. Parallel processing could be an answer to this problem, but it would also be advantageous to figure out a way to intelligently prune the number of rule combinations that must be checked.

Another area yet to be explored deals with the resolution of conflicting knowledge. What happens when one or more experts have attribute examples in their matrices which conflict? Which piece of knowledge or expert should be believed and trusted? This problem appears to be similar to the problem of noisy data. A better or creative way of calculating the measure of belief or a measure of confidence might help solve this problem.

Still other work can be done in extending the type of knowledge the matrix represents and the type of knowledge revealed from the matrix. Fuzzy attributes allow knowledge with continuous, ordinal scales and ranges (like temperature) to be represented. By adding rough set capabilities, it would be possible to represent discrete, non-ordinal knowledge (like blood types). Right now, expert system type rules are revealed from the attribute or declarative knowledge in the matrix. Further work needs to be done to turn the rules or sets of rules revealed into methods or procedural knowledge. The procedural knowledge should express the overall or broad behavior

of a knowledge domain.

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