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The High Energy Behavior of the Forward Scattering Parameters—An Amplitude Analysis Update

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Abstract

Utilizing the most recent experimental data, we reanalyze high energy $\bar{p}p$ and pp data, using the asymptotic amplitude analysis, under the assumption that we have reached 'asymptopia'. This analysis gives strong evidence for a $\log(s/s_0)$ dependence at *current* energies and *not* $\log^2(s/s_0)$, and also demonstrates that odderons are *not* necessary to explain the experimental data.

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†Deceased.

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I Dedication

This work is dedicated to the memory of Bernard Margolis, Rutherford Professor of Physics at McGill University, Montreal, Canada, who died shortly after this paper was presented in June, 1995. He was a magnificent physicist and a more magnificent friend. He will be sorely missed!

II Asymptotic Amplitude Analysis

In spite of the fact that there are excellent arguments [1] that the energy region in which present experiments are conducted—even at the Tevatron Collider—is too low to be considered asymptotic, we will consider here the consequences of assuming the *opposite*. This allows us to test specific hypotheses using a well-defined phenomenological analysis. We caution the reader that we *don't believe* we are in 'asymptopia' and thus *don't believe* the analysis is applicable as a true asymptotic analysis. We *do believe* that present day energies are too low to make a truly asymptotic analysis. Nonetheless, we feel that such an analysis is valuable as a guideline to what is and is not happening at present energies.

We apply a "standard" asymptotic analytic amplitude analysis procedure[1] to the now-available data on σ_{tot} , the total cross section and ρ , the ratio of the real to the imaginary portion of the forward scattering amplitude, in the energy region $\sqrt{s} = 5$ to 1800 GeV, including the new CDF cross sections. The data are parameterized in terms of even and odd analytic amplitudes. Consistent with all asymptotic theorems, this allows use of even amplitudes varying as fast as $\log^2(s/s_0)$ and odd amplitudes (the 'Odderon' family) that do *not* vanish as $s \rightarrow \infty$.

We show here only the large s limit of the even and odd amplitudes that are used[1]. We make five fits to the data:

- (i) Fit 1: $\log^2(s/s_0)$ energy dependence for the cross section, with no Odderon amplitude,
- (ii) Fit 2: $\log^2(s/s_0)$ energy dependence for the cross section, with an Odderon amplitude whose cross sectional dependence is $\log s$, the most rapid behavior allowed by asymptotic theorems,
- (iii) Fit 3: $\log^2(s/s_0)$ energy dependence for the cross section, with an Odderon amplitude whose cross sectional dependence is constant,
- (iv) Fit 4: $\log(s/s_0)$ energy dependence for the cross section, with no Odderon amplitude,
- (v) Fit 5: $\log(s/s_0)$ energy dependence for the cross section, with an Odderon amplitude whose cross sectional dependence is constant, the most rapid behavior allowed by asymptotic theorems for this choice of even amplitude.

In all cases, an odd amplitude which vanishes with increasing energy is also employed, as well as an even amplitude that mimics Regge behavior.

II.1 $\log^2(s)$ Energy Behavior

We introduce f_+ and f_- , the even and odd (under crossing) analytic amplitudes at $t = 0$, and define the $\bar{p}p$ and pp forward scattering amplitudes by $f_{\bar{p}p} = f_+ + f_-$ and $f_{pp} = f_+ - f_-$, giving total cross sections σ_{tot} and the ρ -values

$$\sigma_{\bar{p}p} = \frac{4\pi}{p} \text{Im } f_{\bar{p}p}, \quad \sigma_{pp} = \frac{4\pi}{p} \text{Im } f_{pp}, \quad \rho_{\bar{p}p} = \frac{\text{Re } f_{\bar{p}p}}{\text{Im } f_{\bar{p}p}}, \quad \text{and} \quad \rho_{pp} = \frac{\text{Re } f_{pp}}{\text{Im } f_{pp}}. \quad (1)$$

We parameterize the 'conventional' even and odd amplitudes f_+ and f_- by :

$$\frac{4\pi}{p} f_+ = i \left(A + \beta \left[\log \left(\frac{s}{s_0} \right) - i \frac{\pi}{2} \right]^2 + c s^{\mu-1} e^{i\pi(1-\mu)/2} \right) \quad (2)$$

$$\frac{4\pi}{p} f_- = -D s^{\alpha-1} e^{i\pi(1-\alpha)/2}. \quad (3)$$

The parameter α in Eq (3) turns out to be about 0.5, and thus this odd amplitude vanishes as $s \rightarrow \infty$.

Asymptotic theorems by Eden and Kinoshita[1] prove that the *difference* of cross sections can not grow faster than $\log^{\gamma/2}(s)$, when the cross section grows as $\log^{\gamma}(s)$. Thus, odd amplitudes which do *not* vanish as $s \rightarrow \infty$ for this case are :

$$\frac{4\pi}{p} f_-^{(0)} = -\epsilon^{(0)}, \quad \frac{4\pi}{p} f_-^{(1)} = - \left[\log \left(\frac{s}{s_0} \right) - i \frac{\pi}{2} \right] \epsilon^{(1)}, \quad \text{and}, \quad \frac{4\pi}{p} f_-^{(2)} = - \left[\log \left(\frac{s}{s_0} \right) - i \frac{\pi}{2} \right]^2 \epsilon^{(2)}.$$

The complete odd amplitude is formed by adding any one (or none) of the $f_-^{(i)}$ to the conventional odd amplitude f_- of Eq (3). We then fit the experimental ρ and σ_{tot} data, for both pp and $\bar{p}p$, for energies between 5 and 1800 GeV, to obtain the real constants $A, \beta, s_0, c, \mu, D, \alpha, \epsilon^{(i)}$. The data used below 500 GeV are listed in [1], and the high energy points are from UA1, UA4, E710 and CDF[1]. We emphasize that what we really fit for the UA4 and CDF cross sections is the measured experimental quantity $\sigma_{tot} \times (1 + \rho^2)$, which is appropriate for experiments that measure a 'luminosity-free' cross section, whereas for UA1 and the 1020 GeV point of E710, we fit the experimental quantity $\sigma_{tot} \times \sqrt{1 + \rho^2}$, which was their experimentally measured quantity (they measured a 'luminosity-dependent' cross section).

II.2 Fitted Results for $\log^2(s)$ Behavior

- (i) Fit 1—This fit uses no Odderons in the odd amplitude and uses the even amplitude of Eq (2). The $\chi^2/\text{d.f.}$ ($\chi^2/\text{degree of freedom}$) for the fit was 1.94, a rather large number. The fitted constants are shown in Table 1, Fit 1—the computed curves are shown in Fig. 1 (for σ_{tot}) and Fig. 2 (for ρ). The most obvious features of the fit are

- (a) the predicted value of the total cross section is much too high to fit the experimental values (E710 and CDF) at 1800 GeV,
- (b) it predicts much too high a ρ -value at 546 GeV.

We conclude that a simple $\log^2(s)$ fit does not fit the data.

(iii) Fit 2— We fit the data with an additional degree of freedom, by adding Odderon 2 to f_- of Eq (3), along with the even amplitude of Eq (2). The parameters are summarized as Fit 2, in Table 1. Again, we conclude that this combination doesn't fit the data, since the high energy cross section predicted at 1800 GeV is much too high. Although the ρ -value predicted at 540 GeV is slightly lower, the ρ values predicted are still too high.

(iv) Fit 3—The odd amplitude added to the conventional f_- of Eq (3) was Odderon 1. The parameters are given as Fit 3 in Table 1. Again, the fit suffers from the same defect as the Odderon 2 fit, giving much too high a total cross section at 1800 GeV, as well as predicting a UA4/2 ρ -value which was much too high.

The addition of Odderon 0 can have no effect on the cross section. Since it turns out to have a negligible effect on ρ , we will not consider it further.

We conclude that an even amplitude varying as $\log^2(s/s_0)$ *does not fit* the cross section data. We see that the experimental cross section does not rise as rapidly as $\log^2(s/s_0)$, in the present-day energy region. The addition of an Odderon term does not change this conclusion.

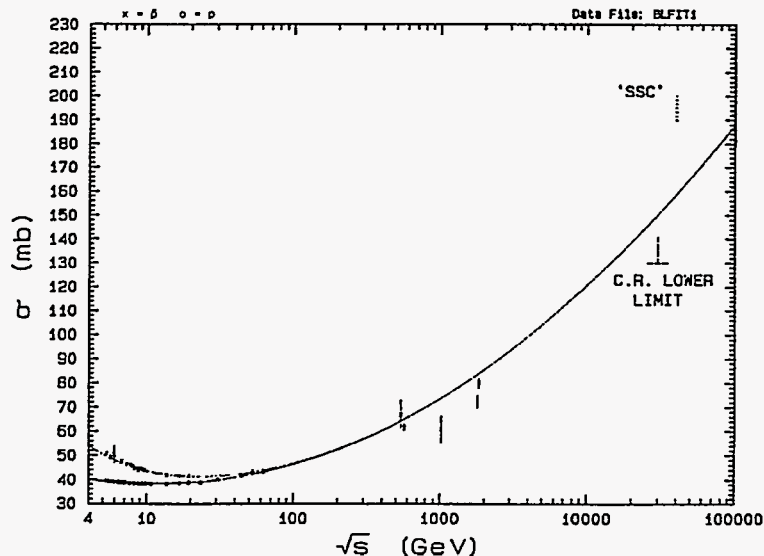


Figure 1: The total cross section σ_{tot} , in mb, for $\bar{p}p$ and pp scattering *vs.* the energy, \sqrt{s} , in GeV, for Fit 1, described in Table I. The fit was made with a $\log^2(s)$ energy variation, and no Odderon. The crosses are for the $\bar{p}p$ experimental data and the circles indicate pp data. The dot-dashed curves are for $\bar{p}p$, and the solid curves for pp . The pp cosmic-ray lower limit[1] is appended to the curve, but is *not* used in the fit.

II.3 $\log(s)$ Energy Behavior

Since the experimental cross section in the energy region 5-1800 GeV did not vary as fast as $\log^2(s/s_0)$, we now consider an asymptotic variation that goes as $\log(s/s_0)$. We

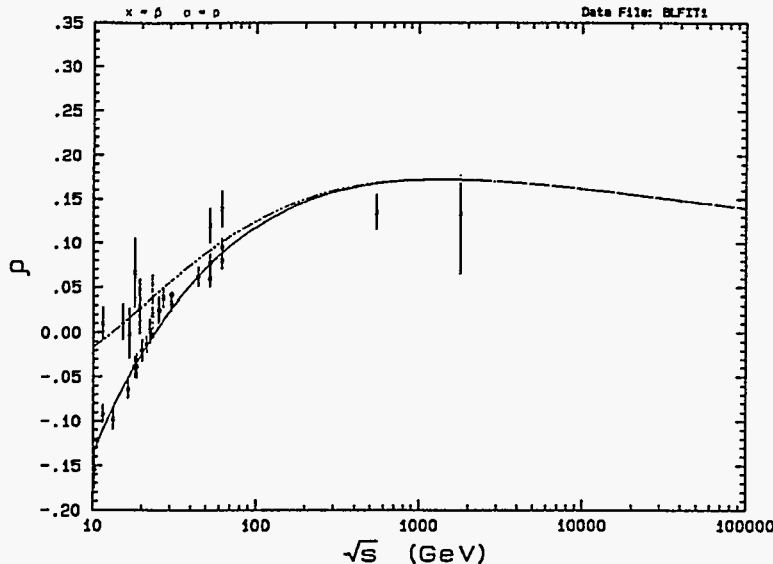


Figure 2: The ρ -value for $\bar{p}p$ and pp scattering *vs.* the energy, \sqrt{s} , in GeV, for Fit 1, described in Table I. The fit was made with a $\log^2(s)$ energy variation, and no Odderon. The crosses are for the $\bar{p}p$ experimental data and the circles indicate pp data. The dot-dashed curves are for $\bar{p}p$, and the solid curves for pp .

substitute for the even amplitude in Eq (2) a new amplitude f_+ varying as $\log(s/s_0)$, $\frac{4\pi}{p} f_+ = i \left(A + \beta \left[\log \left(\frac{s}{s_0} \right) - i \frac{\pi}{2} \right] + c s^{\mu-1} e^{i\pi(1-\mu)/2} \right)$. We use the conventional odd amplitude of Eq (3), along with no Odderon or Odderon 1, in Fits 4 and 5, respectively. We make the important observation that since the energy variation of the cross section is now only $\log(s)$, Odderon 2 is *not* allowed by the asymptotic theorems.

II.4 Fitted Results for $\log(s)$ Behavior

- (i) Fit 4—The data are fitted with a $\log(s/s_0)$ cross section energy behavior, with no Odderon. The results are detailed in Table 1, and plotted in Fig. 3 and Fig. 4. The fit is quite satisfactory, giving a $\chi^2/\text{d.f.}$ of 1.22, fitting reasonably well to all cross section data over the entire range of energy. Most importantly, it now fits the UA4/2 ρ -value at 546 GeV, as well as the E710 ρ -value at 1800 GeV.
- (ii) Fit 5— The data are fitted with a $\log(s)$ cross section energy behavior, along with Odderon 1. The results are given in Table 2. This fit (as is Fit 4) is quite satisfactory, giving a $\chi^2/\text{d.f.}$ of 1.24. Indeed, it is almost indistinguishable from fit 4.

We find that the experimental cross sections and ρ -values in the energy domain 5–1800 GeV can be reproduced using a $\log(s/s_0)$ energy variation. Further, the introduction of an Odderon amplitude is not needed to explain the experimental data. Also, using the new CDF cross sections does not change these conclusions.

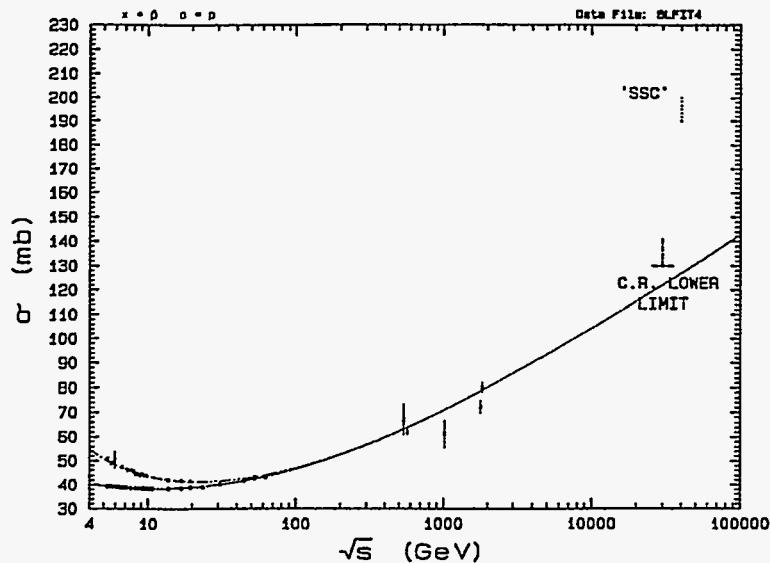


Figure 3: The total cross section σ_{tot} , in mb, for $\bar{p}p$ and pp scattering *vs.* the energy, \sqrt{s} , in GeV, for Fit 4, described in Table I. The fit was made with a $\log(s)$ energy variation, and no Odderon. The crosses are for the $\bar{p}p$ experimental data and the circles indicate pp data. The dot-dashed curves are for $\bar{p}p$, and the solid curves for pp . The pp cosmic-ray lower limit[1] is appended to the curve, but is *not* used in the fit.

References

- [1] For a complete bibliography, see M. M. Block *et al.*, p. 373, Proceedings, XXIII International Symposium on Multiparticle Dynamics, Aspen, CO, 1994, World Scientific, M. M. Block and A. White, Editors.

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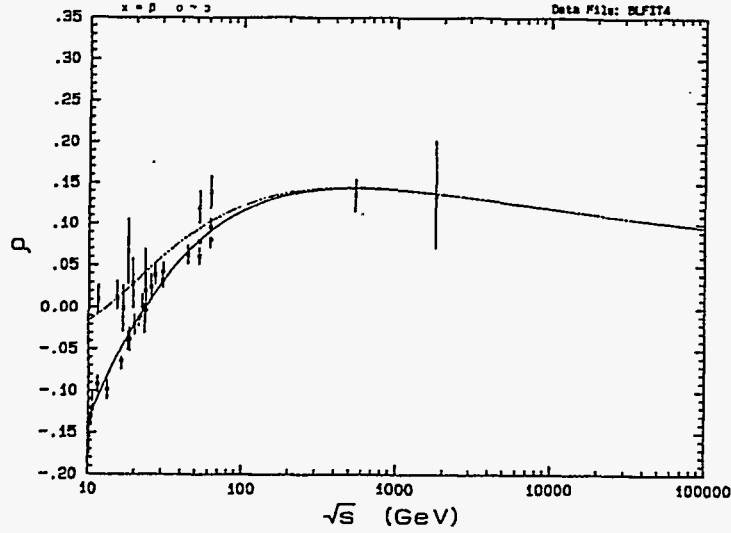


Figure 4: The ρ -value for $\bar{p}p$ and pp scattering vs. the energy, \sqrt{s} , in GeV, for Fit 4, described in Table I. The fit was made with a $\log(s)$ energy variation, and no Odderon. The crosses are for the $\bar{p}p$ experimental data and the circles indicate pp data. The dot-dashed curves are for $\bar{p}p$, and the solid curves for pp .

Parameters	$\sigma_{\text{tot}} \sim \log^2(s/s_0)$			$\sigma_{\text{tot}} \sim \log(s/s_0)$	
	Fit 1	Fit 2	Fit 3	Fit 4	Fit 5
A (mb)	$40.3 \pm .20$	$40.2 \pm .24$	$41.6 \pm .04$	-26.0 ± 12.7	-22.5 ± 11.7
β (mb)	$.47 \pm .02$	$.48 \pm .02$	$.57 \pm .01$	$9.6 \pm .9$	$9.4 \pm .8$
s_0 ((GeV) ²)	200 ± 20	207 ± 25	346 ± 11	500	500
D (mb(GeV) ^{2(1-α)})	-40.9 ± 1.9	-38.8 ± 2.1	-36.8 ± 1.6	-43.2 ± 2.1	-43.3 ± 2.0
α	$.46 \pm .02$	$.47 \pm .02$	$.49 \pm .02$	$.45 \pm .02$	$.45 \pm .02$
c (mb(GeV) ^{2(1-μ)})	30.9 ± 4.1	26.6 ± 4.3	5.9 ± 2.6	159 ± 17	154 ± 16
μ	.46	.49	.49	$.86 \pm .01$	$.86 \pm .01$
$\epsilon^{(2)}$ (mb)		$-.024 \pm .011$			
$\epsilon^{(1)}$ (mb)			$.035 \pm .040$		$-.016 \pm .043$
$\chi^2/\text{d.f.}$	1.94	1.83	2.58	1.22	1.22
d.f.	82	81	81	82	81

Table 1: Results of fits to total cross sections and ρ -values, including Odderons. Fit 1, Fit 2 and Fit 3 correspond to an asymptotic cross section variation of $\log^2(s/s_0)$, with no Odderon, Odderon 2 and Odderon 1, respectively, whereas Fit 4 and Fit 5 correspond to an energy dependence of $\log(s/s_0)$, with no Odderon and Odderon 1, respectively.