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Order in Very Cold Confined Plasmas

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The study of the structure and dynamic properties of classical systems of charged particles confined by external forces, and cooled to very low internal energies, is the subject of this talk. An infinite system of identical charged particles has been known for some time to form a body-centered cubic lattice and is a simple classical prototype for condensed matter. Recent technical developments in storage rings, ion traps, and laser cooling of ions, have made it possible to produce such systems in the laboratory, though somewhat modified because of their finite size. I would like to discuss what one may expect in such systems [1] and also show some examples of experiments.

If we approximate the potential of an ion trap with an isotropic harmonic force

$$F = -Kr$$

then the Hamiltonian for this collection of ions is the same as that for J. J. Thomson's 'plum pudding' model of the atom, where electrons were thought of as discrete negative charges imbedded in a larger, positive, uniformly charged sphere. The harmonic force macroscopically is canceled by the average space-charge forces of the plasma, and this fixes the overall radius of the distribution. What remains, are the residual two-body Coulomb interactions that keep the particles within the volume as nearly equidistant as possible in order to minimize the potential energy. The configurations obtained for the minimum energy of small ionic systems [2] in isotropic confinement are shown in figure 1. Indeed this is an 'Exotic Atom' and fits well into the subject of this symposium honoring the 60th birthday of Professor Toshi Yamazaki.

The configurations even show a shell structure -- though not simply related to the atomic shell structure that we know. Up to 12 ions the minimum energy configuration consists of ions in a shell, equidistant from the origin, but the 13th ion sits in the center. This may be understood qualitatively in terms of the radius of the object becoming larger than the spacing between the ions on the shell when the number of ions exceeds 4π . Above 13, further ions go into the outer shell but also gradually more ions join the one in the center, until for 60 ions there are 48 in the outer shell and 12 in the inner. The latter form the same regular icosahedron as did 12 ions by themselves, and the radius of the outer shell of 48 being twice that of the inner; above 60 ions a third shell starts to form.

In the limit of many ions a multiple shell structure forms [2] as is shown for 20000 ions in figure 2. The pattern of ions on each shell consists of approximate equilateral triangles -- with some defects -- and the surface density (and thus the size of the triangles) is the same on each shell. Thus the ions between different shells cannot interlock in a simple fashion since the angular separations between ions is different. There is a correlation between shells in that the directions in which lines of particles orient themselves seem to persist from one shell to the next.

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For ionic systems that are at a finite temperature above the minimum potential energy the shells become more diffuse, eventually disappear, and then the radius of the charge distribution begins to grow. This is illustrated in figure 3 where Γ is a dimensionless quantity in units of the Coulomb energy between two ions at the Seitz-Wigner radius that is a measure of the interparticle spacing a_{sw}

$$\Gamma = \frac{q^2/a_{sw}}{kT}, \text{ where } a_{sw} = 1/\rho^{1/3}, \text{ with } \rho \text{ being the density.}$$

Since the lowest state of an infinite system is body-centered cubic, for a sufficiently large spherical cloud the interior should show cubic structure while on the surface the shell structure seen here is likely to dominate. It is not well understood where this transition should occur -- it may happen for clouds larger than 20000 ions where simulations have not yet been carried to the point of reaching the minimum-energy configuration.

If the confining force is anisotropic, which often happens in ion traps, ($F_z \neq F_r$, where the notation is for cylindrical coordinates) then the outer shells becomes spheroidal and the inner shells are still equally spaced. An example is shown in figure 4.

How the lowest energy state of a system of ions accommodates itself to anisotropic confinement shows a systematic pattern that is reminiscent of phase transitions. Obviously, for F_z sufficiently weaker than F_r , the ions will stretch out along the z axis. As the force along the z-direction is slowly increased, the configuration will suddenly become two-dimensional, and at a further value of this force it becomes three-dimensional. This is illustrated in figure 5. At the other extreme, when $F_z \gg F_r$, the configuration of ions again becomes a two-dimensional disk. These dimensional transitions have all the characteristics of phase transitions in the configurations [3]. Of course, there are other more complex transitions in the ion configurations as well, but these have not been studied so systematically.

The limiting case, when $F_z = 0$, is that of a beam -- the configuration becomes cylindrical and the pattern of ions is determined by the linear density of ions, the number per unit length [1,4]. With a sufficiently large number of ions there is again a set of cylindrical shells, shown in figure 6, with the same triangular order on the shells as is seen in the spherical and spheroidal case.

Next consider some of the dynamic properties of these systems. For a spherical cloud, for instance, the simplest normal modes correspond to distortions of the cloud from its equilibrium shape [5]. Perturbations from spherical harmonics cause hydrodynamic oscillations in shape which are quite reminiscent of the giant resonances of nuclei. The simplest such mode is the monopole mode, a simple volume oscillation of the spherical cloud. This seems to proceed without any damping. For the higher multipoles, which are volume-conserving shape oscillations, however, there is appreciable damping of the normal modes as shown in figure 7. The expansion of the hydrodynamic modes into the exact eigen-modes of the N-particle system in figure 8 shows this damping. The hydrodynamic monopole mode has the frequency of the plasma frequency ω_p , and is the only multipole mode that is an exact eigenmode of these systems.

There are, however, modes that depend on interparticle forces in an ordered system and are not hydrodynamic. An example is a torsional mode of the cloud where the restoring force depends entirely on the inter-ionic forces. These modes have much lower frequencies than the multipole modes mentioned above and also tend to be damped into complex eigenmodes for finite systems.

Experimentally, considerable progress has been made in observing this form of ordering in ion traps. In particular, multiple shell structures have been seen in Penning traps in spheroidal geometry [6] -- and the cylindrical structure was seen in a torroidal 'ring trap' [7] where the formation of shells and the structure within was beautifully confirmed. Typically, the spacing between ions is of the order of tens of microns. For ions moving in an accelerator storage ring crystallization has not yet been observed and the experimental problems are formidable. Laser cooling has allowed ions to be 'cooled' so that their longitudinal velocities are uniform and the 'longitudinal temperature' is in the mK regime [8]. But as ions are bent around the ring they are inevitably sheared and this may be a major source of perturbation. A cooling technique that would force particles to move with constant angular velocity around the ring would eliminate most of this problem -- but this is not what laser-cooling techniques do. The question of equilibration between the easily-cooled longitudinal motion and the transverse components of the velocities also needs further study.

Finally, the question arises of where the limit is to these classical forms of order⁹. The quantum-mechanical ground state of these systems is not too far from what has already been observed -- it should be reached in the mK regime. This limit is also close to the point where laser-cooling can no longer be effective because the individual ions cannot absorb momentum from photons. At this point, with masses on the order of typical ionic masses (e.g. $^{24}\text{Mg}^+$) the wave functions of the ions in a typical confining field will be localized to about 1% of the inter particle spacings. The systems are likely to remain classical even in their quantum-mechanical ground states. This could be quite different with **much** stronger confining fields, or with a cold system of electrons.

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Figure Captions

- Fig. 1 Minimum energy configurations for up to 16 charged particles in an isotropic harmonic well. Note that 8 particles do not form a cube, but two squares twisted by 45° .
- Fig. 2 The configuration for 20,000 ions in an isotropic harmonic well cooled to a minimum in potential energy. The top figure shows only the ions on the surface of the spherical cloud -- note the pattern of equilateral triangles that also forms the structure of the interior shells. The lower part of the figure shows the radial density of the ions -- the outer shell is the sharpest and the interior ones have gradually increasing widths -- nevertheless all 18 shells appear to be present.
- Fig. 3 Radial density for 1000 ions under isotropic confinement at different temperatures (characterized by the dimensionless parameter Γ defined in the text. Successive curves from the bottom up are for $\Gamma > 10,000$ (effectively the minimum energy configuration), then $\Gamma = 100$, 10, and 1. The successive curves are each displaced horizontally and vertically, for better viewing. The top curve represents the distribution expected for a cold charged fluid under the same confinement.
- Fig. 4 20000 ions under prolate anisotropic confinement. The lower part of the figure shows the ion positions projected onto a plane using the absolute z coordinate and the cylindrical radius to show the pattern for all ions. Note that successive interior shells are approximately at equal perpendicular distances from their nearest neighbors, giving sharp points at the ends. Lines that are drawn in are equally spaced from the outer ellipse.
- Fig. 5 Dimensional changes in 70 ions as the anisotropy of the focusing field changes. The harmonic focusing constant K_z is given assuming that the radial focusing constant $K_r = 1$. For weak z -focusing, shown on the upper left, the ions all sit on the axis. For increasing K_z the ions pop out near the center in a planar zig-zag pattern that increases in size from upper right to lower left. Note that the radial scale is exaggerated by a factor of 10. At a yet higher value of K_z the pattern then twists into three dimensions. These dimensional phase transitions are a general characteristic of these systems.
- Fig. 6 Radial distribution of ions under cylindrical confinement. The parameter λ is the number of particles per unit length (in units of a_{sw}). Note that for small λ the particles remain on axis, for $\lambda > 0.7$ they expand into a zig-zag pattern, and then into various helices and other structures on the surface of a cylinder. As the cylinder grows eventually a new string fits onto the axis -- and multiple shells develop. The triangular pattern on each cylinder's surface is very similar to that shown in figure 2.
- Fig. 7 Damping of two normal modes of a 1000 ion sphere of ions, initially very cold. Time is measured in the units of the period of one particle's oscillation in the harmonic field. Note the absence of damping in the monopole mode.

Fig. 8 Expansion of the hydrodynamic quadrupole mode in terms of the 3000 true eigenmodes of a 1000 ion system and a 100 ion system. The frequency is given in units of the plasma frequency of the clouds.

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Fig. 1

Fig. 2

Fig. 3

Fig. 4

70 PARTICLES

Fig. 5

Fig. 6

Fig. 7

Fig. 8