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Dense Inclined Flows: Theory and Experiments

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Contractor: Cornell University, Ithaca, NY 14853

Contract Participants: James T. Jenkins, Michel Y. Louge

DOE Project Manager: Sean Plasynski

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Summary of accomplishments

Rapid, gravity-driven flows of granular materials down inclines pose a challenge to our understanding. Even in situations in which the flow is steady and two-dimensional, the details of how momentum and energy are balanced within the flow and at the bottom boundary are not well understood. Thus we have undertaken a research program integrating theory, computer simulation, and experiment that will focus on dense entry flows down inclines. The effort involves the development of theory informed by the results of simultaneous computer simulations and the construction, instrumentation, and use of an experimental facility in which the variables necessary to assess the success or failure of the theory can be measured.

In the present reporting period, we have completed the experiments with the bumpy base consisting of random two-dimensional packings of 1mm glass spheres; we have derived boundary conditions for a bumpy frictionless boundary for other than small slip velocities; and we have made numerical studies of hydraulic equations using a simple Lagrangian code that we have developed. We are now in the process of writing the final report and a journal article summarizing our findings.

1) Experiments with Vitamin-E Pills

We have also interpreted measurements of the impact properties for the vitamin-E pills employed in the rotating drum experiment at Lovelace. The pills consist of a thin gelatin shell trapping medium chain triglyceride, a liquid of viscosity in the range $0.0214 \leq \mu \leq 0.0232 \text{ kg/m}\cdot\text{sec}$ and density $0.89 \leq \rho \leq 0.93 \text{ g/cc}$. Unlike solid spheres, the pills do not entrain the entire liquid mass at a unique value of the spin. Instead, the solid shell transmits the angular impulse through successive layers of the viscous liquid. To characterize the dynamics of the impact, we solved the Navier-Stokes equation

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assuming that the liquid is composed of concentric spherical layers of constant spin. The resulting governing equation is

$$\frac{\partial u_\phi}{\partial t} = \nu \left[\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial u_\phi}{\partial r} \right) - 2 \frac{u_\phi}{r^2} \right], \quad (1)$$

where $u_\phi = \omega(r) r \sin \theta$ is the velocity parallel to the equator, $\nu = \mu/\rho$ is the kinematic viscosity of the fluid, θ is the latitude measured from the pole and $\omega(r)$ is the angular velocity of the liquid layer of radius r . The shear exerted by the fluid at a point of the solid shell is

$$\tau_{r\phi} = -\mu r \frac{\partial}{\partial r} \left(\frac{u_\phi}{r} \right)_{r=R}. \quad (2)$$

And the corresponding torque on the entire solid shell is

$$T = -\frac{8\pi}{3} \mu R^4 \left(\frac{\partial \omega}{\partial r} \right)_{r=R}, \quad (3)$$

where R is the radius of the liquid sphere. In this problem, it is convenient to follow the equatorial velocity $u = \omega r$. Using the dimensionless variables denoted by $\dagger$, $t^\dagger \equiv \nu t/R^2$, $\omega^\dagger \equiv \omega R^2/\nu$, $u^\dagger \equiv uR/\nu$, and $r^\dagger \equiv r/R$, the fluid momentum Eq.(1) becomes

$$\frac{\partial u^\dagger}{\partial t^\dagger} = \frac{1}{r^{\dagger 2}} \frac{\partial}{\partial r^\dagger} \left(r^{\dagger 2} \frac{\partial u^\dagger}{\partial r^\dagger} \right) - 2 \frac{u^\dagger}{r^{\dagger 2}}, \quad (4)$$

subject to the boundary conditions

$$\frac{\partial (u^\dagger/r^\dagger)}{\partial r^\dagger} = 0 \text{ @ } r^\dagger = 0; \quad (5)$$

$$\frac{u^\dagger}{r^\dagger} = \Omega^\dagger \text{ @ } r^\dagger = 1, \quad (6)$$

where $\Omega^\dagger$ is the dimensionless angular velocity of the solid shell around the fluid. The condition (5) expresses symmetry of the spin at the center. Because in this experiment the spheres have negligible spin at impact, the initial condition is

$$u^\dagger(r^\dagger) = 0 \text{ @ } t^\dagger = 0, \quad (7)$$

everywhere in the fluid.

To solve the partial differential equation (PDE) (4), it is convenient to carry out the change of variables

$$w \equiv \frac{u^\dagger}{r^\dagger} - \Omega^\dagger \quad (8)$$

and thus obtain the following PDE:

$$\frac{\partial w}{\partial t^\dagger} = \frac{\partial^2 w}{\partial r^{\dagger 2}} + \frac{4}{r^\dagger} \frac{\partial w}{\partial r^\dagger} - \dot{\Omega}^\dagger, \quad (9)$$

subject to the homogeneous boundary conditions

$$\frac{\partial w}{\partial r^\dagger} = 0 \text{ @ } r^\dagger = 0, \quad (10)$$

$$w = 0 \text{ @ } r^\dagger = 1; \quad (11)$$

and the initial condition

$$w(r^\dagger) = 0 \text{ @ } t^\dagger = 0. \quad (12)$$

The overdot in Eq. (9) represents the derivative with respect to $t^\dagger$. We find the eigenfunctions of the homogenous PDE (4) using the method of separation of variables, assuming that the solution has the form

$$u^\dagger = R(r^\dagger) T(t^\dagger). \quad (13)$$

Substituting this expression in the PDE yields the ordinary differential equations (ODEs)

$$\frac{\dot{T}}{T} = \frac{1}{r^{\dagger 2} R} (r^{\dagger 2} R')' - \frac{2}{r^{\dagger 2}} = -\lambda^2, \quad (14)$$

where the primes denote differentiation with respect to $r^\dagger$ and λ^2 is a positive constant.

From Eq. (14), we find that the eigenfunctions are spherical Bessel functions of fractional order, namely

$$R(r^\dagger) = j_1(\lambda r^\dagger) \text{ or } y_1(\lambda r^\dagger). \quad (15)$$

Application of the boundary condition (5) dismisses $y_1(\lambda r^\dagger)$, which is infinite at $r^\dagger=0$.

The homogeneous counterpart of boundary condition (6) yields an equation satisfied by all eigenvalues,

$$j_1(\lambda_i) = 0, \quad (16)$$

where

$$j_1(z) \equiv \frac{\sin z}{z^2} - \frac{\cos z}{z}. \quad (17)$$

Thus, Eq. (16) is equivalent to $\tan(\lambda_i) = \lambda_i$. Its solutions are tabulated in the Handbook of Mathematical Functions. The first three are

$$\lambda_1 = 4.493409; \lambda_2 = 7.725252; \lambda_3 = 10.904122; \text{ etc...}$$

These solutions imply that the variable w satisfying the homogeneous problem of Eqs. (9) to (11) with $\dot{\Omega}^\dagger = 0$ may be written

$$w = W(r^\dagger) T(t^\dagger), \quad (18)$$

with the eigenfunctions

$$W_i(r^\dagger) = \frac{j_1(\lambda_i r^\dagger)}{r^\dagger}. \quad (19)$$

Because the ordinary differential equation (ODE) (14) is of the Sturm-Liouville type, the eigenfunctions W_i satisfy the orthogonality

$$\int_0^1 W_i W_j r^{\dagger 4} dr^\dagger = \delta_{ij} \int_0^1 W_i^2 r^{\dagger 4} dr^\dagger, \quad (20)$$

where δ_{ij} is the Kronecker delta symbol. Integration of the right-hand-side of this Eq. is achieved by manipulating the ODE for W_i and integrating by parts. We obtain

$$\int_0^1 W_i^2 r^{\dagger 4} dr^\dagger = j_2^2(\lambda_i)/2, \quad (21)$$

where $j_2(z)$ is a spherical Bessel function of fractional order,

$$j_2(z) = \left(\frac{3}{z^3} - \frac{1}{z}\right) \sin z - \frac{3}{z^2} \cos z.$$

Using Eq. (16), we find

$$\int_0^1 W_i^2 r^{\dagger 4} dr^\dagger = \frac{\sin^2(\lambda_i)}{2\lambda_i^2}. \quad (22)$$

The solution to the complete, inhomogeneous PDE (9) is found by the method of variation of parameter. We assume a series solution of the form

$$w(r^\dagger, t^\dagger) = \sum_{i=1}^{\infty} b_i(t^\dagger) W_i(r^\dagger), \quad (23)$$

where b_i is a function of time. To establish an ODE for each b_i , we differentiate Eq. (23) with respect to time, apply orthogonality (20), substitute derivatives of w with respect to radius for the time derivative using PDE (9), and integrate twice by parts from $r^\dagger=0$ to 1. To calculate the corresponding initial conditions, we apply orthogonality to (23) at time zero. We find

$$\dot{b}_i + \lambda_i^2 b_i = \dot{\Omega}^\dagger \frac{2}{\sin(\lambda_i)}, \quad (24)$$

subject to the initial condition $b_i(0) = 0$. Then, the spin of spherical fluid layers is

$$\omega^\dagger(r^\dagger, t^\dagger) = \Omega^\dagger + \sum_{i=1}^{\infty} b_i(t^\dagger) \frac{j_1(\lambda_i r^\dagger)}{r^\dagger}. \quad (25)$$

From Eq. (3), the dimensionless torque exerted by the fluid on the shell is

$$T^\dagger \equiv \frac{3}{8\pi} \frac{\rho}{\mu^2 R} T = - \sum_{i=1}^{\infty} b_i(t^\dagger) \sin(\lambda_i) . \quad (26)$$

We now write a balance of linear momentum for the solid shell during impact,

$$I_s \dot{\Omega} = F(t) R_s + T , \quad (27)$$

where I_s and R_s are the moment of inertia and outer radius of the shell, respectively, and F is the tangential force exerted by the other colliding body upon the solid shell. Because during impact, the appropriate time scale is the total duration t_c of the collision, we define $t' \equiv t/t_c$ and $\Omega' \equiv \Omega t_c$ and write ODE (24) as

$$A \frac{db_i}{dt'} + \lambda_i^2 b_i = \frac{2A^2}{\sin(\lambda_i)} \frac{d\Omega'}{dt'} , \quad (28)$$

where

$$A \equiv \frac{R^2}{v t_c} \quad (29)$$

is the dimensionless number defining the impact regime. In the limit of vanishing A , the fluid reacts immediately to the spin of the shell and the pill behaves as a rigid, inhomogeneous sphere. In the limit of infinite A , the fluid does not participate in the dynamics of the spin. Although the fluid may avert collapse of the shell upon collision, its spin is not appreciably affected by the angular impulse of the shell in that limit.

To proceed further, we assume that the time-history of the tangential force is sinusoidal i.e.,

$$F = \frac{\pi}{2} \frac{J}{t_c} \sin(\pi \frac{t}{t_c}) H(t) , \quad (30)$$

where

$$J = \int_0^{t_c} F dt \quad (31)$$

is the tangential impulse. The Heavide's function $H(t)$ equals zero for $t < 0$ and one otherwise. In this case, ODE (27) may be written

$$\frac{d\Omega'}{dt'} = B \sin(\pi t') H(t') - C \sum_{i=1}^{\infty} b_i \sin(\lambda_i) , \quad (32)$$

where

$$B \equiv \frac{\pi}{2} R_s \frac{J t_c}{I_s}; \quad (33)$$

$$C \equiv \frac{8\pi}{3} \frac{\mu^2 R t_c^2}{\rho I_s}. \quad (34)$$

We solve the simultaneous ODEs (28) and (32) in the Laplace domain. From ODE (28), we derive

$$\tilde{b}_i = \frac{2A}{\sin(\lambda_i)} \frac{p\tilde{\Omega}}{p + \lambda_i^2/A}, \quad (35)$$

where $p = j2\pi f$ is the Laplace parameter with $j^2 = -1$, and $\tilde{\Omega}$ and $\tilde{b}_i$ denote the Laplace transforms of Ω' and b_i , respectively. Similarly, we find from PDE (32)

$$p\tilde{\Omega} = B \frac{\pi}{\pi^2 + p^2} - C \sum_{i=1}^{\infty} \tilde{b}_i \sin(\lambda_i). \quad (36)$$

using the approximation of the tangential force in Eq. (30). Eliminating $\tilde{b}_i$ between Eqs. (35) and (36), we obtain

$$\tilde{\Omega} = B \frac{\pi}{p(\pi^2 + p^2)} \frac{1}{1 + 2AC \sum_{i=1}^{\infty} \left(\frac{1}{p + \lambda_i^2/A} \right)} \quad (37)$$

Unfortunately, we do not know how to invert this transform analytically. However, in the limit where $2AC$ is a small parameter, we approximate the second ratio in Eq. (37) and find

$$\tilde{\Omega} \approx B \frac{\pi}{p(\pi^2 + p^2)} - \frac{2ABC\pi}{p(\pi^2 + p^2)} \sum_{i=1}^{\infty} \left(\frac{1}{p + \lambda_i^2/A} \right), \quad (38)$$

which can be readily inverted to yield the time-history of the spin of the shell during impact:

$$\Omega' = \frac{B}{\pi} [1 - \cos(\pi t')] H(t') - 2ABC \pi \sum_{i=1}^{\infty} f_i(t') H(t'), \quad (39)$$

where

$$f_i(t') = \frac{\pi^2(1 - \exp[-\frac{\lambda_i^2}{A}t']) + \frac{\lambda_i^2}{A}[1 - \cos(\pi t')] - \pi \frac{\lambda_i^2}{A} \sin(\pi t')}{\pi^2 \frac{\lambda_i^2}{A} (\pi^2 + \frac{\lambda_i^4}{A^2})}. \quad (40)$$

Thus, in dimensional form at the end of impact, we obtain

$$\Omega(t=t_c) = \frac{R_s J}{I_s} (1 - A^2 C \sum_{i=1}^{\infty} \frac{2\lambda_i^4 + \pi^2 A^2 (1 - \exp[-\lambda_i^2/A])}{(\lambda_i^4 + \pi^2 A^2) \lambda_i^2}). \quad (41)$$

In this Eq., the sum is a function of A alone,

$$f(A) \equiv \sum_{i=1}^{\infty} \frac{2\lambda_i^4 + \pi^2 A^2 (1 - \exp[-\lambda_i^2/A])}{(\lambda_i^4 + \pi^2 A^2) \lambda_i^2}, \quad (42)$$

and the product $A^2 C$ represent inertial properties of the pill,

$$A^2 C = \frac{8\pi \rho R^5}{3 I_s} = 5 \frac{\rho}{\rho_s} \frac{R^5}{R_s^5 - R^5}, \quad (43)$$

where ρ_s is the material density of the shell. To a good approximation, the function can be fit using

$$f(A) \approx \frac{0.1996}{[1 + (0.09475A)^{1.2643}]^{0.3744}}. \quad (44)$$

We estimate the magnitude of the collision time t_c by recording the impulse exerted by the impact of a pill on the flat face of a 3mm cylindrical steel post connected to a piezoelectric load cell. A typical impact time-history is shown in Fig. 1.

Fig. 1. Impact of a 4mm Vitamin-E pill onto a 3mm stainless steel post. The collision time is approximately 55μsec.

We invoke the Hertzian theory to infer the corresponding time for the collision of two pills. Assuming that the stiffness of the steel is much larger than that of the pill, the collision time of two pills is longer by a factor of $2^{1/5}$ than the same parameter for impact with a rigid half-space. Thus, we adopt $t_c \sim 63\mu\text{sec}$. For the Vitamin-E pills, we have $\rho_s \sim 1\text{g/cc}$,

$\rho \sim 0.91\text{g/cc}$, $R_s \sim 1.975\text{mm}$, $R \sim 1.841\text{mm}$, and $\nu \sim 24.5 \cdot 10^{-6} \text{ m}^2/\text{sec}$. Then, we find $A \sim 2300$, $f(A) \sim 0.016$, and $A^2C \sim 9.6$.

Interpretation of the collision experiments requires the knowledge of the particle spin following impact. Because we did not measure spin in these experiments, we inferred it from the fluid theory outlined above and from the conservation of linear and angular momenta in the collision. We now review these equations and modify them with the expressions for the fluid spin derived earlier.

We consider two colliding spherical pills of masses m with centers located at $\mathbf{r}_1$ and $\mathbf{r}_2$. We define the unit normal along the line joining the centers of the two spheres $\mathbf{n} \equiv (\mathbf{r}_1 - \mathbf{r}_2) / |\mathbf{r}_1 - \mathbf{r}_2|$. During the collision, pill 2 exerts an impulse $\mathbf{J}$ onto pill 1. Prior to the collision, the pills have translational velocities $\mathbf{c}_1$ and $\mathbf{c}_2$ and angular velocities $\boldsymbol{\omega}_1$ and $\boldsymbol{\omega}_2$; the corresponding post-collision velocities are denoted by primes. The velocities before and after the collision are related by

$$m (\mathbf{c}_1' - \mathbf{c}_1) = -m (\mathbf{c}_2' - \mathbf{c}_2) = \mathbf{J} , \quad (45)$$

and

$$\boldsymbol{\omega}_1' - \boldsymbol{\omega}_1 = \boldsymbol{\omega}_2' - \boldsymbol{\omega}_2 = -\frac{R_s}{I_s} \lambda \mathbf{n} \times \mathbf{J} , \quad (46)$$

where $\lambda \equiv [1 - A^2 C f(A)]$ from Eq. (41). Thus knowledge of the impulse and λ is sufficient to infer the post-collision velocities from the values of these quantities before the collision. In order to determine the impulse, we calculate the relative velocity $\mathbf{g}$ of the point of contact,

$$\mathbf{g} = (\mathbf{c}_1 - \mathbf{c}_2) - R_s (\boldsymbol{\omega}_1 + \boldsymbol{\omega}_2) \times \mathbf{n} . \quad (47)$$

The incident angle γ between $\mathbf{g}$ and $\mathbf{n}$ characterizes the impact geometry,

$$\cot \gamma \equiv \mathbf{g} \cdot \mathbf{n} / |\mathbf{g} \times \mathbf{n}| . \quad (48)$$

Because impacts occur when $\mathbf{g} \cdot \mathbf{n} \leq 0$, this angle lies in the range $\pi/2 \leq \gamma \leq \pi$. Using Eqs. (45) and (46), the contact velocities before and after the collision are related by

$$\mathbf{g}' - \mathbf{g} = \frac{2}{m} \mathbf{J} + 2\lambda \frac{R_s^2}{I_s} (\mathbf{n} \times \mathbf{J}) \times \mathbf{J} = \left(\frac{2}{m}\right) [(1+\eta) \mathbf{J} - \eta \mathbf{n} (\mathbf{n} \cdot \mathbf{J})] , \quad (49)$$

where

$$\eta \equiv \lambda \frac{m R_s^2}{I_s} = \frac{5}{2} \lambda \frac{1 + \left(\frac{R}{R_s}\right)^3 \left(\frac{\rho}{\rho_s} - 1\right)}{1 - \left(\frac{R}{R_s}\right)^5} . \quad (50)$$

In the case of two rigid homogeneous spheres, $\lambda=1$ and $\eta=5/2$. For the Vitamin-E pills, $\lambda \sim 0.85$ and $\eta \sim 6.16$.

It remains to model the impact. Because the fluid is incompressible, in the normal direction we do not expect the pills to exhibit a markedly different behavior than rigid spheres. Then the usual coefficient of restitution e characterizes the incomplete restitution of the normal component of $\mathbf{g}$,

$$\mathbf{n} \cdot \mathbf{g}' = -e \mathbf{n} \cdot \mathbf{g} , \quad (51)$$

where $0 \leq e \leq 1$.

Collisions with incident angles near $\pi/2$ involve gross sliding. For these, we assume that sliding is resisted by Coulomb friction and that the tangential and normal components of the impulse $\mathbf{J}$ are related by the coefficient of friction μ ,

$$\mathbf{n} \times \mathbf{J} = \mu (\mathbf{n} \cdot \mathbf{J}) , \quad (52)$$

where $\mu \geq 0$. For greater values of the incident angle, the impact no longer involves gross sliding as parts of the contact patch are brought to rest. When γ exceeds the limiting angle γ_0 , we replace Eq. (52) by

$$\mathbf{n} \times \mathbf{g}' = -\beta_0 \mathbf{n} \times \mathbf{g} , \quad (53)$$

where $\gamma_0 \equiv \pi - \arctan[(1+e)\mu(1+\eta)/(1+\beta_0)]$ and β_0 is the tangential coefficient of restitution with $0 \leq \beta_0 \leq 1$. For simplicity, we then categorize the collision as “sticking” and assume that the entire contact point is brought to rest during impact. For sticking collisions the definition of β_0 in Eq. (53) implies that some of the elastic strain energy stored in the solid during impact is recoverable through tangential compliance, so the tangential velocity of the point of contact may be reversed. In this simple model, Eqs. (52) and (53) are mutually exclusive i.e., the point of contact is either slipping (Eq. 52) or sticking (Eq. 53).

A convenient way to interpret the data is to follow Maw, Barber and Fawcett (1976,1981) and produce a plot of $\Psi_2 \equiv -(\mathbf{g}' \cdot \mathbf{t})/(\mathbf{g} \cdot \mathbf{n})$ versus $\Psi_1 \equiv -(\mathbf{g} \cdot \mathbf{t})/(\mathbf{g} \cdot \mathbf{n})$, where $\mathbf{t}$ is a unit vector located in the collision plane $(\mathbf{g}, \mathbf{n})$ and tangent to both spheres. In collisions of homogeneous spheres that involve gross sliding,

$$\Psi_2 = \Psi_1 - (1+e)\mu(1+\eta) \text{sign}(\mathbf{g} \cdot \mathbf{t}); \quad (54)$$

and in collisions that do not,

$$\Psi_2 = -\beta_0 \Psi_1 . \quad (55)$$

For positive values of $\mathbf{g} \cdot \mathbf{t}$, Ψ_1 represents the magnitude of the tangent of the incident angle. Similarly, the ratio (Ψ_2/e) is the tangent of the recoil angle γ' between $\mathbf{n}$ and $\mathbf{g}'$.

In the experiments, we measure the unit vectors $\mathbf{n}$ and $\mathbf{t}$ and the linear velocities of the center of mass of the pills before and after impact. From Eq. (45), these measurements provide both components of the impulse. Because our apparatus releases particles without spin, the relative velocity $\mathbf{g}$ of the contact point is known directly through $\mathbf{g} = \mathbf{c}_1 - \mathbf{c}_2$. Complete interpretation of the data requires a prediction of the spin after impact. For rigid spheres, the prediction is straightforward and requires only conservation of angular momentum at the contact point. For liquid-filled shells, our theory predicts the degree of involvement of the fluid; it leads to Eq. (49), which deduces both components of $\mathbf{g}'$ and thus Ψ_2 from measured quantities,

$$\Psi_2 = -\frac{\mathbf{g} \cdot \mathbf{t}}{\mathbf{g} \cdot \mathbf{n}} - \frac{1}{m} (1+\eta) \frac{\mathbf{J} \cdot \mathbf{t}}{\mathbf{g} \cdot \mathbf{n}}. \quad (56)$$

Fig. 2. Measured relative velocities at contact. The line is a least-squares fit to the data using $e=0.89 \pm 0.07$.

Results of the binary impacts of Vitamin-E pills are shown in Figs. 2 and 3. The normal coefficient of restitution is obtained from a least-squares fit of $\mathbf{g}' \cdot \mathbf{n}$ versus $\mathbf{g} \cdot \mathbf{n}$ (Fig. 2). We find $e=0.89 \pm 0.07$. From a plot of Ψ_2 versus Ψ_1 (Fig. 3), we find that, within experimental scatter, the pills always “stick” upon contact with negligible tangential restitution i.e., $\beta_0 \sim 0$. We attribute this absence of apparent sliding to the large value of η exhibited by the pills. Because for the pills $\eta \sim 6.2$ is much greater than the corresponding

value $\eta \sim 2.5$ for rigid spheres, the pills have a limiting angle γ_0 much closer to $\pi/2$. Therefore, unless the friction coefficient of the shell surface is very large or the incident angle is near grazing, the pills cannot slide upon impact.

Another peculiar characteristic of the pills is that the apparent spin of the shell decreases following impact, as momentum slowly diffuses in the liquid. A consequence is that the pills exhibit "memory" of previous impacts that complicate theoretical interpretation of their granular dynamics.

Fig. 3. Values of Ψ_2 inferred from Eq. (56) versus Ψ_1 measured in the experiments. The dashed line is a model that assumes a slipping contact (Eq. 54). Clearly, the data do not follow the trends of this Eq. We attribute the experimental scatter to variability in the diameter and mass of the shells.

2) Next Research

In the next reporting period, we plan to write the final report.

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