TOLERANCE DESIGN BASED ON VARIATION TRANSFER FUNCTION

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ABSTRACT

Tolerance design presented in this paper minimizes the expected quality loss due to system performance variations from the target value and the cost for controlling the tolerances of components and subsystems. The method is based on loss function, variation transfer function (VTF) and design of experiments methodology. The VTF developed in this paper makes it possible to assess the variance of system performances for any tolerance levels setting of the components or subsystems based on the data from only one tolerance levels setting with on more experiments or simulations to be conducted. The VTF transfers the variations in the parameters of the components or the subsystems to the variations in the quality characteristic of the system. An optimization model is presented for tolerance design and a method is given to find the coefficients for the VTF. An example is given to demonstrate the method.

Key Words: Design of Experiments, Taguchi, Tolerance Design.

1. INTRODUCTION

Significant progress has been made toward quality and productivity improvement over the past few decades. These developments include the exciting and interesting reforms in quality design methodology as well as philosophy. Japan has shown the world that improving quality leads to improved productivity at no impact on cost. In America, the traditional issues of quality control are concerned with the downstream side of a production process or on-line activities, focusing on control charts and inspection schemes. The emphasis now is on moving upstream to the prevention or the activities during the design and development stages of products. The technique of robust design as originally developed by Taguchi can build quality into products and make the products insensitive to manufacturing and environmental variations, deteriorations over time [1-13]. The approach has been practiced for static and dynamic systems [4]. Philosophically, robust design is an economical way to improve quality since it has no or little impact on cost. The method has three basic steps: (1). System Design, (2). Parameter Design and (3). Tolerance Design.

System design deals with the innovation of a workable prototype model of the system. Much of the previous and the current design effort in the United States is concentrated on this step. Parameter design can achieve significant improvement in quality and cost. Experiments and simulations are performed to search for the optimal level settings for design factors so that the system performance is less sensitive to undesired variations. To further improve quality, components or subsystems can be upgraded to balance quality loss due to variations and cost for controlling tolerances. To do this, Taguchi employs an equation to find the relationship between the variations of the system performance and the variations of the components or the subsystems with no consideration of interactions between the effects of the components [1]. The equation is intuitively appealing but no theoretical basis is given. In practice, the interactions between the effects of the components can not be ignored for many systems. This paper will present the theoretical basis of the variation transfer function (VTF) that can be used to transfer the variances of the components to the variance of the system. Based on quality loss function (QLF) and the VTF, economic analysis will be performed for tolerance design for the components or the subsystems to minimize the total loss to the customers and the producers.

The philosophy of robust design originates from the definition of quality in terms of the loss imparted to the society from the time that a product is shipped. Viewing quality from a societal perspective is unique since it deals with customers, manufacturers, and thus with the whole society. Stemming from this viewpoint, a quality loss function (QLF) is developed as a measure of quality in robust design. For the cases where

a symmetrical quadratic QLF is applicable, it is given by $L(y) = K(y-T)^2$ where y is the quality characteristic, T is the target value, K transfers the unit of y square to the monetary unit [5]. Due to the manufacturing or other variations, the parameters of components may have variations from target values. These variations will be reflected to the system performance and that causes the deviation from the target value. A quality loss is incurred, because the system fails to precisely implement the desired task. So the characteristic is a random variable denoted by Y . As a result, the expected quality loss can be assessed by $E[L(Y)] = K\sigma_y^2$, with the assumption that the mean has been adjusted to the target, where σ_y^2 is the variance of the quality characteristic [5]. The model and the approach to implementing tolerance design will be discussed based on in the following sections.

2. OPTIMIZATION MODEL FOR TOLERANCE DESIGN

The objective of tolerance design is to balance quality loss due to variations from the target and quality cost for controlling tolerances (precision or allowances). In other words, tolerance design is performed to minimize the total loss that consists of quality loss due to variations from the target and the cost for controlling tolerances. There are conventional approaches for tolerance design [6], one of which is Taylor series method. Assume that a system or a product has n components. The quality characteristic of the system is a function of these components and is given by

$$Y = f(X_1, X_2, \dots, X_n) \quad (1)$$

where $X_1, X_2, \dots, X_n$ are the parameters of the components that are random variables. Let μ_i, σ_i^2 be the mean and the variance of X_i ($i=1,2,\dots,n$). By using Taylor series approximation on eq. (1) at the point $(\mu_1, \mu_2, \dots, \mu_n)$, the variance of Y can be found to be

$$\sigma_y^2 \approx \sum_{i=1}^n \left[\frac{\partial f(\mu_1, \mu_2, \dots, \mu_n)}{\partial x_i} \right]^2 \sigma_i^2 \quad (2)$$

with the assumption that $X_1, X_2, \dots, X_n$ are independent random variables. This conventional method has a good mathematical explanation. But the disadvantage is that an analytical model and computation of derivatives are needed. Moreover, it may lead to a less accurate result.

A system especially an electronic system may have many components and subsystems. The variabilities in the characteristics of components and subsystems will adversely affect the quality of the system. Reducing variations can reduce the loss to the customer but it may increase the cost to the producer. On the other hand, larger tolerances or larger allowable variations will bring in a larger loss to the customer. To balance this, an optimization model can be formulated. The objective function consists of two portions. The first part is a quality loss due to variations or $K\sigma_y^2$, which is an increasing function of the allowable tolerance limits for components and subsystems. The second part is the cost for controlling tolerances. For a system having n components, the tolerance control cost for the i th component is a function of its tolerance level or the standard deviation σ_i , and the cost is modeled by $c_i/\sigma_i^{\alpha_i}$ where c_i and α_i are positive constants. As a result, the optimization model for tolerance design can be formulated as

$$\begin{aligned} \text{Minimize } & \left\{ K\sigma_y^2 + \sum_{i=1}^n \frac{c_i}{\sigma_i^{\alpha_i}} \right\} \\ \text{Subject to } & (\sigma_1, \sigma_2, \dots, \sigma_n) > 0 \end{aligned} \quad (3)$$

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σ_y^2 can be expressed as a function of $\alpha_1, \alpha_2, \dots, \alpha_n$ or $\sigma_y^2 = VTF(\alpha_1, \alpha_2, \dots, \alpha_n)$ where VTF is the variations transfer function that can be used to estimate the variance of the system for any given set of tolerance levels or ($\alpha_1, \alpha_2, \dots, \alpha_n$) of components/subsystems. (3) is a nonlinear programming (NLP) problem. The NLP algorithms may be used to solve the problem. However, due to the structure of the model that consists of posynomials, geometric programming may also be used. In addition, sophisticated NLP algorithms may not always be needed, depending on the nature of the practical problem, such as the example given in Section 6. If the system model is given as (1), then (2) could be used as a VTF. If an analytical model is not given, experimental design and simulation can be used to estimate the parameters for VTF, which will be addressed later.

3. SIMULATION OF NOISE FACTORS

Manufacturing or other variations may cause the parameters to deviate from the nominal values. These effects are called noise factors. Monte Carlo methods may be utilized to simulate these noise factors, but it is not efficient for the cumbersome computation. In this section, the simulation of noise factors will be discussed. It is well known that the responses on three levels of a factor can evaluate the linear and the quadratic effects of the factor [7,18]. Since the discussion is limited to the linear and quadratic effects, three levels should be assigned to each factor to simulate noises associated with this factor. For the n -component system, let's consider the noise factor associated with the i th component. The parameter of this component has a mean μ_i and a standard deviation σ_i . For symmetrically-distributed noise factor, three levels can be assigned to this component:

$$\begin{cases} \text{the 1st level} = \mu_i - h\sigma_i \\ \text{the 2nd level} = \mu_i \\ \text{the 3rd level} = \mu_i + h\sigma_i \end{cases} \quad (4)$$

where h is a constant. To perform experiments or simulations, we assign the component with the above levels to an orthogonal array (either full factorial or partial factorial). The component has the same number of times to take each value of the three levels in the orthogonal array. Thus, it can be reasonably implied that the parameter of the component has an equal probability $1/3$ to take each of the three levels. Perhaps inspired by making the mean and the variance of this discrete distribution equal to those of the true distribution, Taguchi suggests $h = \sqrt{3/2}$ [11,19]. D'Errico and Zaino [10] propose other selections of the noise levels to give a better approximation to the true distribution, such as $h = \sqrt{3}$ rather than $\sqrt{3/2}$ assuming that the component has a probability $1/6$ to take the 1st level and the 3rd level, a probability $4/6$ to take the 2nd level. Under this assumption, this discrete distribution can match the first five moments of the true distribution. However, the implementation of D'Errico and Zaino's recommendation for design and analysis seems very complicated because one must use unequal-weighted data to compute the sum of squares and perform analysis.

4. VTF FOR LINEAR EFFECT MODEL

In Taguchi's parameter design, experiments are first performed on low-priced components with large variations to determine the robust parameter level setting. If the parameter design can not meet the quality requirement, one must upgrade the components to balance quality and cost. VTF makes it possible to estimate the variances of Y for other tolerance level settings of the components with no more experiments to be conducted. For simplicity, a two-component system will be discussed that has components A and B . The same results can be obtained for a system composed of three or more components. The model may be known or unknown. Assume μ_A, μ_B are the mean or the nominal value of A or B respectively, and σ_A^2, σ_B^2 are the variances.

For a linear model, the components have the significant linear effects on Y . Since the levels of the noise factors with A and B are fixed in terms of (4), the model is called fixed model [7,18]. Assume the model is given as follows for A at the i th level, B at the j th level, for the k th repetition:

$$Y_{ijk} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + e_{ijk} \quad (i, j = 1, 2, 3, k = 1, \dots, n) \quad (5)$$

where α_i is the effect of A at the i th level, and it may be represented by $k_A(A_i - \mu_A)$; β_j is the effect of B at the j th level, and it may be represented by $k_B(B_j - \mu_B)$; $(\alpha\beta)_{ij}$ is the interaction between the effect of A at the i th level and the effect of B at the j th level, it can be represented by $\gamma\alpha_i\beta_j$ [7]; e_{ijk} is the error term, which is normally distributed with a mean zero and a variance σ_e^2 ; k_A, k_B and γ are constants; A_i and B_j are the parameters of

A at the i th level, B at the j th level. The total sum of squares for Y can be partitioned as the sums of the squares for effects and interactions [7,18]:

$$SS_T = SS_A + SS_B + SS_{AB} + SS_e \quad (6)$$

Since the model is a fixed model, the expected mean square (EMS) for each term on the right-hand side of (6) is given by substituting A_i and B_j in terms of (4) into the following equations:

$$EMS_A = \sigma_e^2 + \frac{3n \sum_{i=1}^3 [k_A(A_i - \mu_A)]^2}{(3-1)} = \sigma_e^2 + C_A \sigma_A^2 \quad (7)$$

$$EMS_B = \sigma_e^2 + \frac{3n \sum_{j=1}^3 [k_B(B_j - \mu_B)]^2}{(3-1)} = \sigma_e^2 + C_B \sigma_B^2 \quad (8)$$

$$EMS_{AB} = \sigma_e^2 + \frac{n \sum_{i=1}^3 \sum_{j=1}^3 [\gamma k_A k_B (A_i - \mu_A)(B_j - \mu_B)]^2}{(3-1)(3-1)} = \sigma_e^2 + C_{AB} \sigma_A^2 \sigma_B^2 \quad (9)$$

$$EMS_e = \sigma_e^2 \quad (10)$$

where $C_A = 3nk_A^2 h^2$, $C_B = 3nk_B^2 h^2$, $C_{AB} = n\gamma^2 k_A^2 k_B^2 h^4$ are parameters not related to the variances. Since the three levels of the components are set to simulate the noise factors, MS_T is an unbiased estimator of the variance of Y , namely $\sigma_y^2 = EMS_T$. Taking the expected value of eq. (6), we have

$$(9n-1)\sigma_y^2 = 2C_A \sigma_A^2 + 2C_B \sigma_B^2 + 4C_{AB} \sigma_A^2 \sigma_B^2 + 9(n-1)\sigma_e^2 \quad (11)$$

Let p be the percent contribution ratio. Thus,

$$\begin{aligned} p_A &= \frac{2C_A \sigma_A^2}{(9n-1)\sigma_y^2}, \text{ estimated by } \frac{2MS_A - MS_e}{SS_T} \\ p_B &= \frac{2C_B \sigma_B^2}{(9n-1)\sigma_y^2}, \text{ estimated by } \frac{2(MS_B - MS_e)}{SS_T} \\ p_{AB} &= \frac{4C_{AB} \sigma_A^2 \sigma_B^2}{(9n-1)\sigma_y^2}, \text{ estimated by } \frac{4MS_{AB} - MS_e}{SS_T} \\ p_e &= 9(n-1)\sigma_e^2 / (9n-1)\sigma_y^2, \text{ estimated by } 1 - p_A - p_B - p_{AB} \end{aligned}$$

For a known or unknown system model, we can estimate these percent contribution ratios and σ_y^2 by statistics MS_A, MS_B, MS_{AB} and MS_e by performing experiments and analysis of variance (ANOVA) on one tolerance-level setting (perhaps the low-cost tolerance-level setting as suggested by Taguchi) of A and B . For another set of tolerance levels of A and B , or σ_A^2 and σ_B^2 , we can find the variance of Y or σ_y^2 without any more experiment or simulation to be conducted, using the following VTF, which can be derived from eq. (11):

$$\sigma_y^2 = \sigma_e^2 \left[p_A \left(\frac{\sigma_A^2}{\sigma_A^2} \right) + p_B \left(\frac{\sigma_B^2}{\sigma_B^2} \right) + p_{AB} \left(\frac{\sigma_A^2 \sigma_B^2}{\sigma_A^2 \sigma_B^2} \right) + p_e \right] \quad (12)$$

Let $\omega_A = p_A \sigma_A^2 / \sigma_A^2$, $\omega_B = p_B \sigma_B^2 / \sigma_B^2$, $\omega_{AB} = p_{AB} \sigma_A^2 \sigma_B^2 / \sigma_A^2 \sigma_B^2$, $\omega_e = p_e \sigma_e^2$. VTF can be given by

$$\sigma_y^2 = \omega_A \sigma_A^2 + \omega_B \sigma_B^2 + \omega_{AB} \sigma_A^2 \sigma_B^2 + \omega_e \quad (13)$$

For the system with three or more components, a VTF can be derived in a similar way.

5. VTF FOR NONLINEAR EFFECT MODEL

The problem arising in practice may have significant nonlinear effects. Therefore, nonlinear models must be considered. The discussion is limited to the cases where only linear and quadratic effects are significant, the method is applicable for the model with cubic or higher-order effects, as long as we set appropriate levels to simulate the noises associated with the components and choose appropriate factorial coefficients. We still use a two-component system for discussion. Assume that the levels for the simulation of noises are set as eq. (4) with $h = \sqrt{3/2}$. Thus, $a=b=3$. For A at the i th noise level, B at the j th noise level and the k th repetition, the response of the system for the quadratic model is given by

$$\begin{aligned} Y_{ijk} &= m + k_{Ai} A_i + k_{Aj} A_j^2 + k_{Bi} B_i + k_{Bj} B_j^2 + k_{ABij} A_i B_j \\ &\quad + k_{AAij} A_i^2 B_j^2 + k_{BBij} A_i^2 B_j^2 + k_{AABij} A_i^3 B_j^2 + \dots \end{aligned} \quad (14)$$

Where m and $k_{Ai}, \dots, k_{AABij}$ are parameters depending on the particular model. These parameters may not be necessarily known. If the model is unknown, experiments or simulations can be used to collect the response data. The total sum of squares for Y can be partitioned as SS_A, SS_B and SS_{AB} , which can be further decomposed into the sums of squares for the linear and the quadratic effects as well as the interactions given below:

$$SS_T = SS_{A1} + SS_{A2} + SS_{B1} + SS_{B2} + SS_{AB1} + SS_{AB2} + SS_{A1B1} + SS_{A1B2} + SS_{A2B1} + SS_{A2B2} \quad (15)$$

As an example, the computation of SS_{A1} is discussed. SS_{A1} is the sum of squares for the linear effect of A and it can be computed by the contrast $C_{A1} = L_{A1} Y$, where L_{A1} is the row vector of the coefficients of the contrast for the linear effect of A which can be found in [7] or [8]; Y is a vector of the response totals Y_{ij} for each set of the level combination of A and B (Similar notations will be used in the context and bold letters are vector quantities unless otherwise specified). Thus [7,18],

$$SS_{A1} = \frac{(L_{A1} Y)^2}{L_{A1} L_{A1}^T} = \frac{\left[\sum_{i=1}^n (Y_{A1i} - Y_{A2i})^2 \right]}{6n} \quad (16)$$

Taking the expectation of (16) and also using (14), (4) with $h=\sqrt{3/2}$, we have

$$EMS_{A1} = F_{A1} \sigma_A^2 + \sigma_e^2 \quad (17)$$

Similarly, we can derive the EMS for other effects or interactions:

$$EMS_{B1} = F_{B1} \sigma_B^2 + \sigma_e^2 \quad (18)$$

$$EMS_{Bq} = F_{Bq} \sigma_B^2 + \sigma_e^2 \quad (19)$$

$$EMS_{A1B1} = F_{A1B1} \sigma_A^2 \sigma_B^2 + \sigma_e^2 \quad (20)$$

$$EMS_{A1Bq} = F_{A1Bq} \sigma_A^2 \sigma_B^2 + \sigma_e^2 \quad (21)$$

$$EMS_{AqB1} = F_{AqB1} \sigma_A^2 \sigma_B^2 + \sigma_e^2 \quad (22)$$

$$EMS_{AqBq} = F_{AqBq} \sigma_A^2 \sigma_B^2 + \sigma_e^2 \quad (23)$$

where $F_{A1}, F_{Aq}, \dots, F_{AqBq}$ are coefficients not related to the variances or tolerance levels. For more mathematical details of these F coefficients, see also [1]. The corresponding ANOVA is given in Table 1.

To compute σ_y^2 , the variance of y for another set of tolerances σ_A' and σ_B' without any more experiment or simulation to be conducted, we can obtain the VTF by taking the expectation of eq. (15) and substituting the corresponding terms:

$$\sigma_y^2 = \sigma_e^2 \left[p_{A1} \left(\frac{\sigma_A'^2}{\sigma_A^2} \right) + p_{Aq} \left(\frac{\sigma_A'^2}{\sigma_A^2} \right) + p_{B1} \left(\frac{\sigma_B'^2}{\sigma_B^2} \right) + p_{Bq} \left(\frac{\sigma_B'^2}{\sigma_B^2} \right) + p_{A1B1} \left(\frac{\sigma_A'^2 \sigma_B'^2}{\sigma_A^2 \sigma_B^2} \right) + p_{A1Bq} \left(\frac{\sigma_A'^2 \sigma_B'^2}{\sigma_A^2 \sigma_B^2} \right) + p_{AqB1} \left(\frac{\sigma_A'^2 \sigma_B'^2}{\sigma_A^2 \sigma_B^2} \right) + p_{AqBq} \left(\frac{\sigma_A'^2 \sigma_B'^2}{\sigma_A^2 \sigma_B^2} \right) + p_e \right] \quad (24)$$

where $p_{A1}, \dots, p_{AqBq}$ are the percent contribution ratios (p coefficients), whose estimated values can be found using Table 1. Since the EMS of the insignificant term is σ_e^2 , it can be combined into the error term. If all interactions are insignificant, the percent contribution ratios for interactions are zeros. The VTF is reduced to the model with no interactions, which is used in Taguchi's tolerance design. Let $w_A = p_{A1} \sigma_A^2 / \sigma_A^2$, $w_B = p_{B1} \sigma_B^2 / \sigma_B^2$, $\dots$, $w_{AqBq} = p_{AqBq} \sigma_A^2 \sigma_B^2 / \sigma_A^2 \sigma_B^2$, $w_{AqBq} = p_{AqBq} \sigma_A^2 \sigma_B^2 / \sigma_A^2 \sigma_B^2$. The VTF can be rewritten based on these w coefficients:

$$\sigma_y^2 = w_{A1} \sigma_A'^2 + w_{Aq} \sigma_A'^4 + w_{B1} \sigma_B'^2 + w_{Bq} \sigma_B'^4 + w_{A1B1} \sigma_A'^2 \sigma_B'^2 + w_{A1Bq} \sigma_A'^2 \sigma_B'^4 + w_{AqB1} \sigma_A'^4 \sigma_B'^2 + w_{AqBq} \sigma_A'^4 \sigma_B'^4 + \sigma_e^2 \quad (25)$$

Table 1. ANOVA For 2-Component System

Source	df	SS	MS	EMS*	Estimated value of p	Estimated value of w
A_1	1	SS_{A1}	MS_{A1}	$\sigma_e^2 + F_{A1} \sigma_A^2$	$(MS_{A1} - MS_e) / SS_T$	$p_{A1} \sigma_A^2 / \sigma_A^2$
A_q	1	SS_{Aq}	MS_{Aq}	$\sigma_e^2 + F_{Aq} \sigma_A^4$	$(MS_{Aq} - MS_e) / SS_T$	$p_{Aq} \sigma_A^2 / \sigma_A^4$
B_1	1	SS_{B1}	MS_{B1}	$\sigma_e^2 + F_{B1} \sigma_B^2$	$(MS_{B1} - MS_e) / SS_T$	$p_{B1} \sigma_B^2 / \sigma_B^2$
B_q	1	SS_{Bq}	MS_{Bq}	$\sigma_e^2 + F_{Bq} \sigma_B^4$	$(MS_{Bq} - MS_e) / SS_T$	$p_{Bq} \sigma_B^2 / \sigma_B^4$
$A_1 \times B_1$	1	SS_{A1B1}	MS_{A1B1}	$\sigma_e^2 + F_{A1B1} \sigma_A^2 \sigma_B^2$	$(MS_{A1B1} - MS_e) / SS_T$	$p_{A1B1} \sigma_A^2 / \sigma_A^2 \sigma_B^2$
$A_1 \times B_q$	1	SS_{A1Bq}	MS_{A1Bq}	$\sigma_e^2 + F_{A1Bq} \sigma_A^2 \sigma_B^4$	$(MS_{A1Bq} - MS_e) / SS_T$	$p_{A1Bq} \sigma_A^2 / \sigma_A^2 \sigma_B^4$
$A_q \times B_1$	1	SS_{AqB1}	MS_{AqB1}	$\sigma_e^2 + F_{AqB1} \sigma_A^4 \sigma_B^2$	$(MS_{AqB1} - MS_e) / SS_T$	$p_{AqB1} \sigma_A^2 / \sigma_A^4 \sigma_B^2$
$A_q \times B_q$	1	SS_{AqBq}	MS_{AqBq}	$\sigma_e^2 + F_{AqBq} \sigma_A^4 \sigma_B^4$	$(MS_{AqBq} - MS_e) / SS_T$	$p_{AqBq} \sigma_A^2 / \sigma_A^4 \sigma_B^4$
Error	$9(n-1)$	SS_e	MS_e	σ_e^2	1-sum of the above	$p_e \sigma_e^2$
Total	$9n-1$	SS_T	MS_T^{**}	$\sigma_y^2^{**}$	1.00	

(* $F_{A1}, \dots, F_{AqBq}$ are coefficients not related to tolerance levels of A and B; ** Since the levels of the components are set to simulate the noise factors, then MS_T can be used as the estimator of σ_y^2)

Generally speaking, the method can be extended to the system which has three or more components. Consider a system that has components 1, 2, $\dots$, n and the interactions between three or more components are insignificant, the VTF can be derived as follows: when we consider the sum of squares for the effect of a certain component or an interaction between two components, the response under different levels of other components not included in this effect or interaction can be considered as repetitions. For instance, for the sum of squares for the interaction of the linear effect of component 1 and the linear effect of component 2, the responses under the different levels of the components 3, $\dots$, n are combined as a response total Y_{ij} . In this way, a VTF can be appropriately derived.

6. EXAMPLE

An electronic device consists of three components, R, L and C. The quality characteristic of the system is the current and given by:

$$Y = \frac{110}{\sqrt{R^2 + (120 \times L - \frac{1}{120 \times C})^2}} \quad (26)$$

The target value of Y is 2.00. Assume the symmetric quadratic QLF is appropriate and given by

$$Loss = 250(Y - 2.00)^2 \quad (27)$$

In this example, we will select tolerances for R, L, C based on QLF, VTF and DOE methodology. Anyone who is interested in Taguchi's parameter design is referred to [1], [9].

The tolerance of R or C can be chosen from three grades with the associated relative cost and the tolerance of L can be chosen from two grades with the associated relative cost (see Table 2). Our objective is to select the best tolerances for R, L and C to minimize the quality loss due to variations and the cost due to tolerances.

Suppose that the robust nominal values of R, L and C after parameter design using the lowest-priced components, namely grade C for R, grade B for L and grade C for C, are given as: $R=40$, $L=0.17$ and $C=100 \times 10^{-6}$. We are investigating the linear and the quadratic effects of the components and their interactions. Three noise levels for the components can be used according to (4) with $h=\sqrt{3/2}$. Thus, the noise levels associated with R, L and C are given in Table 3. The three-way array for simulation data of the response Y for different noise level combinations using full factorial is given by Table 4 (Generally, the data may be available from parameter design if the parameter design is followed by the tolerance design). For these responses, we can perform ANOVA to find significant effects and interactions (Table 5). The insignificant term can be pooled into errors, since its expected mean sum of squares (EMS) equals to the variance of the error. As an example of the computation, SS_{A1B1} is given by

$$SS_{A1B1} = \frac{(2 \sum C_{11} Y_{11})^2}{4 \sum C_{11}^2} = \frac{(6.61 - 6.27 - 5.5 + 5.5)^2}{3(1^2 + (-1)^2 + (-1)^2 + 1^2)} = 0.00013$$

The VTF for σ_y^2 given another set of σ_R , σ_L and σ_C can be given by

$$\sigma_y^2 = \sigma_e^2 \left\{ 0.274 \left(\frac{\sigma_R^2}{\sigma_R^2} \right) + 0.0001 \left(\frac{\sigma_L^2}{\sigma_L^2} \right) + 0.6164 \left(\frac{\sigma_C^2}{\sigma_C^2} \right) + 0.0003 \left(\frac{\sigma_R^2}{\sigma_R^2} \right) + 0.0003 \left(\frac{\sigma_L^2}{\sigma_L^2} \right) + 0.0003 \left(\frac{\sigma_C^2}{\sigma_C^2} \right) + 0.0003 \left(\frac{\sigma_R^2 \sigma_L^2}{\sigma_R^2 \sigma_L^2} \right) + 0.0003 \left(\frac{\sigma_R^2 \sigma_C^2}{\sigma_R^2 \sigma_C^2} \right) + 0.0003 \left(\frac{\sigma_L^2 \sigma_C^2}{\sigma_L^2 \sigma_C^2} \right) + 0.0003 \right\} \quad (28)$$

where σ_R , σ_L and σ_C are tolerances of the lowest-grade components or $\sigma_R=2.666$, $\sigma_L=11.33 \times 10^{-6}$, $\sigma_C=6.67 \times 10^{-6}$. Since ANOVA in Table 5 is obtained by simulating the noises associated with the components, σ_y^2 can be estimated by MS_T . Using this VTF, we can compute the estimated values of the variance of Y for different σ_R , σ_L , σ_C without conducting more experiments. The total loss or the relative cost of the components plus the expected quality loss $E[\text{Loss}] = 250\sigma_y^2$ is given in Table 6 for various tolerance levels settings. Thus, the best tolerances are grade B for R, grade A for L and grade C for C, since this tolerance level setting can minimize the expected quality loss due to variations and the cost for upgrading components. Moreover, if we use Taylor's expansion on Eq. (26) and take the 1st order approximation, the variance of Y can be found by

$$\sigma_y^2 = 7.1 \times 10^{-4} \sigma_R^2 + 88.887 \sigma_L^2 + 44007966 \sigma_C^2 \quad (29)$$

While, if we ignore the quadratic terms and the interactions in the VTF, it gives

$$\sigma_y^2 = 7.5 \times 10^{-4} \sigma_R^2 + 93.49 \sigma_L^2 + 43356487 \sigma_C^2 \quad (30)$$

Eq. (30) is closed to Eq. (29). Theoretically, VTF is closely related to Taylor's expansion.

7. SUMMARY

Tolerance design based on VTF, QLF and DOE methodology presented in this paper is concerned with developing tolerances for components or subsystems of a system. The objective is to minimize the expected quality loss due to variations in the parameters of the components and the cost for upgrading these components. Basically, the expected quality loss is measured by the variance of the quality characteristic of the system. The variations in parameters of the components or subsystems will cause the variations in the quality characteristic. The high-quality components or subsystems can make a high-quality system, but the cost is high. Using the VTF developed in this paper, we can find the variances of quality characteristic for any tolerance levels setting of the components based on the data generated from one setting of tolerances, and further to balance quality and cost.

Table 2. Grades of R, L and C with the associated cost

Component	Grade	Tolerance	Relative Cost
R	A	$\sigma_R=0.667$	0.70
	B	$\sigma_R=1.333$	0.30
	C	$\sigma_R=2.666$	0.00 (Base value)
L	A	$\sigma_L=5.67 \times 10^{-6}$	1.50
	B	$\sigma_L=11.33 \times 10^{-6}$	0.00 (Base value)
C	A	$\sigma_C=1.67 \times 10^{-6}$	1.00
	B	$\sigma_C=3.33 \times 10^{-6}$	0.50
	C	$\sigma_C=6.67 \times 10^{-6}$	0.00 (Base value)

Table 3. The noise levels associated with R, L and C

Component	1st level	2nd level	3rd level
R	$R_1=36.735$	$R_2=40.000$	$R_3=40.265$
L	$L_1=0.1561$	$L_2=0.1700$	$L_3=0.1839$
C	$C_1=91.83 \times 10^{-6}$	$C_2=100 \times 10^{-6}$	$C_3=108.17 \times 10^{-6}$

Table 4. The responses for different noise level combinations

	L_1			L_2			L_3		
	C_1	C_2	C_3	C_1	C_2	C_3	C_1	C_2	C_3
R_1	2.32	2.25	2.19	2.16	2.09	2.04	2.01	1.95	1.90
R_2	2.20	2.14	2.09	2.06	2.00	1.96	1.93	1.88	1.83
R_3	2.09	2.04	1.99	1.97	1.92	1.88	1.86	1.81	1.77

Table 5. ANOVA for the responses (subscript l represents the linear effect; subscript q represents the quadratic effect)

Source	SS	df	MS	F_0	p
R_l	0.13869	1	0.13869	18565***	0.2740
R_q	0.00007	1	0.00007	9.37***	0.0001
L_l	0.31205	1	0.31205	41770***	0.6166
L_q	0.00015	1	0.00015	20.1***	0.0003
C_l	0.05014	1	0.05014	6712***	0.0091
C_q	0.00015	1	0.00015	20.1***	0.0003
$R_l \times L_l$	0.00403	1	0.00403	539***	0.0079
$R_l \times C_l$	0.00053	1	0.00053	70.9***	0.0010
$L_l \times C_l$	0.00013	1	0.00013	17.4***	0.0002
Errors	.000127	17	.00000747		0.0005
Total	0.506067	26	0.019464		1.0000

*** Significant at the level 0.01.

Table 6. The quality loss plus the cost due to upgrading

$\sigma_C (\times 10^6)$	$\sigma_R=5.67 \times 10^{-6}$			$\sigma_L=11.33 \times 10^{-6}$		
	1.67	3.33	6.67	1.67	3.33	6.67
$\sigma_R=0.667$	4.08	3.65	3.53	4.83	4.40	4.28
$\sigma_R=1.333$	3.93	3.50	3.38*	4.68	4.28	4.13
$\sigma_R=2.666$	4.72	4.23	4.08	5.40	5.00	4.88

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