TOLERANCE DESIGN BASED ON VARIATION TRANSFER FUNCTION

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ABSTRACT

Tolerance design presented in this paper minimizes the expected quality loss due to system performance variations from the target value and the cost for controlling the tolerance. A novel framework for controlling the variation of components and subsystems by design is proposed. The method is based on quadratic loss function (QLF) and the variability transfer function (VTF) and design of experiments methodology. The VTF developed in this paper makes it possible to assess the variance of system performance for any allowable or constrained tolerances of the components or subsystems based on the data from only one tolerance level setting with or without experiments or simulations to be conducted. The VTF transfers the variations in the parameters of the components or the subsystems to the variations in the quality characteristic of the system. An optimization model is presented for tolerance design and a method is given to find the coefficients for the VTF. An example is used to demonstrate the method.

Key Words: Design of Experiments, Taguchi, Tolerance Design.

1. INTRODUCTION

Significant progress has been made toward growing quality and productivity improvements over the past few decades. These developments include the exciting and interesting reforms in quality design methodology as well as philosophy. Japan has shown the world that improving quality leads to improved productivity at no impact on cost. In America, the traditional issues of quality control are concerned with the downstream side of a production process or on-line activities, focusing on control charts and inspection schemes. The emphasis now is on moving upstream to the prevention or the activities during the design and development stages of the system. The philosophy of the system can build quality into products and make the products insensitive to environmental variations, deviations, or variations of the system. Many of the previous and current design efforts in the United States are concentrated on this step. Parameter design can achieve significant improvement in quality and cost. Experiments and simulations are performed to search for the optimal level settings for design factors so that the system performance is less sensitive to undesirable variations. Further improvement in quality, components or subsystems can be upgraded to balance quality loss due to variations and cost for controlling tolerances. To do this, Taguchi’s variance transfer function (VTF) is developed to control the variations of the system performance and the relationships of the components or the subsystems with no consideration of interactions between the effects of the components [1]. The equation is intuitively appealing but no practical way of a theoretical basis is given. In practice, the interactions between the effects of the components can be assumed to be fixed and the VTF will present the theoretical basis of the variance transfer function (VTF) that can be used to transfer the variances of the components to the variance of the system. Based on quality loss function (QLF) and the VTF, an economic analysis will be performed to find the components and the subsystems to minimize the total loss to the customers and the producers.

The philosophy of robust design originates from the definition of quality in terms of the lost impact to the society from the time a product is shipped. Viewing quality from a societal perspective is unique since it deals with customers, consumers, and thus with the whole society. Stems from this viewpoint, a quality loss function (QLF) is developed as a measure of quality in robust design. For the case where a symmetrical quadratic QLF is applicable, it is given by

\[ L(y) = \frac{1}{2} (y – \mu)^2 \]

where \( y \) is the quality characteristic, \( \mu \) is the target value, and \( K \) is the variance of the system. Due to the manufacturing or other variations, the parameters of components may vary from the target values. These variations will be reflected to the system performance and that causes the deviation from the target value. A quality loss is incurred because the system failed to precisely implement the desired task. So the characteristic is a random variable denoted by \( Y \). A. A. result, the expected quality loss can be assessed by \( E(L(Y)) = \frac{K}{\theta^2} \), with the assumption that the area has been adjusted to the target, where \( \sigma_2 \) is the variance of the quality characteristic [2]. The model and the approach to implementing tolerance design will be discussed based on the following sections.

2. OPTIMIZATION MODEL FOR TOLERANCE DESIGN

The objective of tolerance design is to balance quality loss due to variations from the target and quality cost for controlling tolerances (precision or allowances). In other words, tolerance design is performed to minimize the total loss that consists of quality loss due to variations from the target and the cost for controlling tolerances. There are conventional approaches for tolerance design [3], one of which is Taylor series method. Assume that a system or a product has a components. The quality characteristic of the system is a function of these components and is given by

\[ Y = f(X_1, X_2, \ldots, X_n) \]

where \( X_1, X_2, \ldots, X_n \) are the parameters of the components that are random variables. Let \( \mu_i, \sigma_i \) be the mean and the variance of \( X_i \) (\( i = 1, 2, \ldots, n \)). By using Taylor series approximation on eq. (1) at the point \( (\mu_1, \mu_2, \ldots, \mu_n) \), the variance of \( Y \) can be found to be

\[ \sigma_y^2 = \sum_{i=1}^{n} \left( \frac{\partial f}{\partial X_i} \right)^2 \sigma_i^2 \]

with the assumption that \( X_1, X_2, \ldots, X_n \) are independent random variables.

This conventional method has a good mathematical explanation. But the disadvantage is that its analytical model and computation of derivatives are needed. Moreover, it may lead to an error and an inaccurate result.

A system, especially an electronic system, may have many components and subsystems. The variabilities in the characteristics of components and subsystems will adversely affect the quality of the system. Reducing variations can reduce the loss to the customer but it may increase the cost to the producer. On the other hand, larger tolerances or larger allowable variations will bring in a larger loss to the customers. To balance this, an optimization model can be formulated. The objective function consists of two portions. The first part is a quality loss due to variations or \( \sigma_i^2 \), which is an increasing function of the allowable tolerance limits for components and subsystems. The second part is the cost for controlling tolerances. For a system having a component, the tolerance control cost for the \( i \) component is a function of its tolerance level or the standard deviation \( \sigma_i \), and the cost is modeled by \( c_i \sigma_i \) where \( c_i \) and \( \sigma_i \) are positive constants. As a result, the optimization model for tolerance design can be formulated as

\[
\text{Minimize } K \left( \sigma_i^{2} + \frac{\sigma_k^{2}}{\sigma_i^{2}} \right) \\
\text{subject to } (\sigma_1, \ldots, \sigma_n > 0)
\]
$c_j$ can be expressed as a function of $c_0, c_1, \ldots, c_{j-1}$ or $c_j^i = \text{VTF}(c_0, c_1, \ldots, c_{j-1})$ where VTF is the variations transfer function that can be used to estimate the variance of the system for any given set of tolerance levels or $(c_0, c_1, \ldots, c_{j-1})$ of components/subsystems. (3) is a nonlinear programming (NLP) problem. The NLPL algorithm may be used to solve the problem. However, due to the structure of the model that consists of polynomial, geometric programming may also be used. In addition, sophisticated NLP algorithms may not always be needed, depending on the nature of the practical problem, such as the example given in Section 6. If the system model is given as (1), then (2) could be used as a VTF. If an analytical model is given, experimental design and simulation may be used to estimate the parameters for VTF, which will be addressed later.

### 3. SIMULATION OF NOISE FACTORS

Manufacturing or other variations may cause the parameters to deviate from the nominal values. These effects are called noise factors. Monte Carlo methods may be utilized to simulate these noise factors, but it is not efficient for the cumbersome computation. In this section, the simulation of noise factors will be discussed. 1. It is well known that the responses on three levels of factors can be evaluated in the linear and the quadratic effects of the factor (7)-(8). Since the discussion is limited to the linear and quadratic effects, three levels should be assigned to each factor to simulate noise associated with this factor. For the n-component system, let's consider the noise factor associated with the ith component. The parameter of this component has a mean $\mu_i$ and a standard deviation $\sigma_i$. For symmetrically-distributed noise factors, three levels can be assigned to this component:

$$
\begin{align*}
\text{the 1st level: } & \mu_i - \sigma_i \\
\text{the 2nd level: } & \mu_i \\
\text{the 3rd level: } & \mu_i + \sigma_i
\end{align*}
$$

where $\sigma$ is a constant. To perform experiments or simulations, we assign the component with the above levels to an orthogonal array (either full factorial or partial factorial). The component has the same number of times to take each value of the three levels in the orthogonal array. Thus, it can be reasonably implied that the parameter of the component has an equal probability 1/3 to take each of the three levels. Perhaps inspired by making the mean and the variance of this discrete distribution equal to those of the true distribution, Taguchi suggests $\lambda=(2\lambda)^{1/2}$ (11). DeRiccio and Zalling [10] propose other selections of the noise levels to give a better approximation to the true distribution, such as $\lambda=\sqrt{3}$ rather than $\sqrt{2}$ (2)

assuming that the component has a probability 1/6 to take the 1st level and 1/3 to take the 2nd level. Under this assumption, this discrete distribution can match the first five moments of the true distribution. However, the implementation of DeRiccio and Zalling's recommendation for design and analysis seems very complicated because one must use unequal-weighted data to compute the sum of squares and perform analysis.

### 4. VTF FOR LINEAR EFFECT MODEL

In Taguchi's parameter design experiments, the components with large variations to determine the robust parameter level setting. If the parameter design cannot meet the quality requirement, one must upgrade the components to balance quality and cost. Thus, it is possible to estimate the variance of $Y$ for other tolerance level settings of the components with no more experiments to be conducted. For simplicity, a two-component system will be discussed that has components A and B. The same result can be obtained in a system composed of three or more components. The model can be linear or nonlinear. Assume $\mu_A, \mu_B$ are the mean and the nominal value of A or B respectively, and $\sigma_A, \sigma_B$ are the variances.

For a linear model, the components have the significant linear effects on Y. Since the levels of the noise factors with A and B are fixed in terms of (4), the model is called fixed model [7]-[8]. Assume the model is given as follows for A at the ith level, B at the jth level, for the kth repetition:

$$
Y_{ikj} = \mu_A + \beta_B + \sigma_A \delta_B + \sigma_B \delta_A + \epsilon_{ikj} \quad (i=1,2,3; \quad j=1,2, \ldots, n)
$$

where $\mu_A$ is the effect of A at the ith level, and it may be represented by $\mu_A(A_{ikj})$; $\beta_B$ is the effect of B at the jth level, and it may be represented by $\beta_B(B_{ijk})$; $\sigma_A \delta_B$ is the interaction between the effect of A at the ith level and the effect of B at the jth level, it can be represented by $\sigma_A \delta_B$, $\sigma_B \delta_A$ is the error term, which is normally distributed with a mean zero and a variance $\sigma^2$; $k$, $\lambda$, and $\gamma$ are constants; $\lambda_A$ and $\lambda_B$ are the parameters of A at the ith level, B at the jth level. The total sum of squares for $Y$ can be partitioned as the sums of the squares for effects and interaction [7]-[8]:

$$
SS_T = SS_A + SS_B + SS_{AB} + SS_{\epsilon}
$$

Since the model is a fixed model, the expected mean square (EMS) for each term on the right-hand side of (6) is given by $A_1$ and $B_2$ in terms of (4) into the following equations:

$$
\begin{align*}
\text{EMS}_A &= \frac{SS_A}{n(k-1)} \quad (3) \\
\text{EMS}_B &= \frac{SS_B}{n(l-1)} \quad (4) \\
\text{EMS}_{AB} &= \frac{SS_{AB}}{n kl} \quad (5)
\end{align*}
$$

The following are the variances of the factors.

$$
\begin{align*}
\text{V}(\lambda_A) &= \frac{MS_A}{n(k-1)} \quad (6) \\
\text{V}(\lambda_B) &= \frac{MS_B}{n(l-1)} \quad (7) \\
\text{V}(\lambda_{AB}) &= \frac{MS_{AB}}{n kl} \quad (8)
\end{align*}
$$

For a known or unknown system model, we can estimate these percent contribution ratios and $\sigma^2_Y$ by statistics $MS_A$, $MS_B$, $MS_{AB}$ and $MS_{\epsilon}$ by performing experiment and analysis of variance (ANOVA) on the tolerance level setting (perhaps the low-cost tolerance level setting as suggested by Taguchi). A and B. For another set of tolerance levels of A and B, or $\lambda_A$ and $\lambda_B$, we can find the variance of Y or $\sigma^2_Y$ without any more experiment or simulation to be conducted, using the following VTF, which can be derived from eq. (11):

$$
\begin{align*}
\sigma^2_Y &= \left(\sigma_A^2 + \sigma_B^2 + \sigma_{AB}^2 + 4 \sigma_A \sigma_B \sigma_{AB}\right) + 9(\lambda-1)\lambda^2 \sigma^2 \quad (9)
\end{align*}
$$

The variance of $Y$ of each component can be estimated by $\sigma_A^2$, $\sigma_B^2$, and $\sigma_{AB}^2$ through repeated experiments as $\sigma_A^2$, $\sigma_B^2$, and $\sigma_{AB}^2$ for A and B, or $\lambda_A$ and $\lambda_B$. For the system with three or more components, the VTF can be derived in a similar way.

### 5. VTF FOR NONLINEAR EFFECT MODEL

The problem arising in practice may have significant nonlinear effects. Therefore, nonlinear models must be considered. The discussion is limited to the cases where only linear and quadratic effects are significant, the method is applicable for the model with cubic or higher-order effects, as long as we set appropriate levels to simulate the entire associated with the components, and choose appropriate fractional coefficients. The VTF is a two-component system for discussion. Assume that the levels for the simulation of noise are set as eq. (9) with $k=\lambda=2/2$. Thus, $\lambda=\lambda_1$. For a at the ith level, B at the jth level and the kth repetition, the response of the system for the quadratic model is given by

$$
Y_{ijk} = \mu + \beta_A i + \beta_B j + \gamma_{ij} + \epsilon_{ijk} \quad (10)
$$

where $\mu$, $\beta_A$, $\beta_B$ are parameters depending on the particular model. These parameters may not be necessary known. If the model is unknown, experiments or simulations can be used to collect these data. The total sum of squares for $Y$ can be partitioned as $SS_T$, $SS_A$, $SS_B$, and $SS_{AB}$, which can be further decomposed into the sums of squares for the linear and quadratic effects as well as the interactions given below:

$$
SS_T = SS_A + SS_B + SS_{AB} + SS_{\epsilon}
$$

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As an example, the computation of $SS_{A}$ is discussed. $SS_{A}$ is the sum of squares for the linear effect of $A$ and it can be computed by the contrast $C_{A}=L_{A}Y$, where $L_{A}$ is the row vector of the coefficients of the contrast for the linear effect of $A$ which can be found in [7] or [8]; $Y$ is a vector of the response totals $Y_{ij}$ for each set of the level combination of $A$ and $B$ (Similar notations will be used in the context and bold letters are vector quantities unless otherwise specified). Thus (14),

$$
E_{MS} = \frac{1}{n} \left( \frac{1}{n} \sum_{i}^{n} y_{i}^{2} - \bar{y}^{2} \right)
$$

Taking the expectation of (16) and also using (14), (4) with $h=v(3/2)$, we have

$$
E_{MS} = F_{A} \sigma_{a}^{2} + \epsilon_{a}^{2}.
$$

Similarly, we can derive the EMS for other effects or interactions:

$$
E_{MS} = F_{AB} \sigma_{ab}^{2} + \epsilon_{ab}^{2}.
$$

Generally speaking, the method can be extended to the system which has three or more components. Consider a system that has components 1, 2, ..., $n$ and the interactions between three or more components are insignificant, the VTF can be derived as follow: when we consider the sum of squares for the effect of a certain component or an interaction between two components, the response under the different levels of other components not included in this effect or interaction can be considered as repetitions.

For instance, for the sum of squares for the interaction of the linear effect of component 1 and the linear effect of component 2, the responses under the different levels of the components $3, 4, \ldots, n$ are combined as a response total $Y_{ij}$. In this way, a VTF can be appropriately derived.

6. EXAMPLE

An electronic device consists of three components, R, L, and C. The quality characteristics of the system is the current and given by:

$$
Y = \frac{110}{\sqrt{R} + \sqrt{L + \frac{1}{LC}}}.
$$

The target value of $Y$ is 2.00. Assume the symmetric quadratic QLF is appropriate and given by

$$
L_{os} = 250(Y-2.00)^{2}
$$

In this example, we will select tolerances for R, L, and C based on QLF, VTF, and DOE methodology. Anyone who is interested in Taguchi's parameter design is referred to [11],[11].

The tolerance of $R$ or $C$ can be chosen from three grades with the associated relative cost and the tolerance of $L$ can be chosen from two grades with the associated relative cost (see Table 2). Our objective is to select the best tolerances for $R$, $L$, and $C$ to minimize the quality loss to variations and the cost due to tolerances.

Generally speaking, the method can be extended to the system which has three or more components. Consider a system that has components 1, 2, ..., $n$ and the interactions between three or more components are insignificant, the VTF can be derived as follow: when we consider the sum of squares for the effect of a certain component or an interaction between two components, the response under the different levels of other components not included in this effect or interaction can be considered as repetitions.

For instance, for the sum of squares for the interaction of the linear effect of component 1 and the linear effect of component 2, the responses under the different levels of the components $3, 4, \ldots, n$ are combined as a response total $Y_{ij}$. In this way, a VTF can be appropriately derived.

### Table 1. ANOVA for 2-Component System

<table>
<thead>
<tr>
<th>Source</th>
<th>df</th>
<th>SS</th>
<th>MS</th>
<th>EMS</th>
<th>Estimated value of $\sigma^{2}$</th>
<th>Estimated value of $\omega$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_{1}$</td>
<td>1</td>
<td>$SS_{A}$</td>
<td>$MS_{A}$</td>
<td>$\sigma_{a}^{2}$</td>
<td>$(MS_{A}$)$MS_{A}^{2}$</td>
<td>$SS_{A}$</td>
</tr>
<tr>
<td>$A_{2}$</td>
<td>1</td>
<td>$SS_{A}$</td>
<td>$MS_{A}$</td>
<td>$\sigma_{a}^{2}$</td>
<td>$(MS_{A}$)$MS_{A}^{2}$</td>
<td>$SS_{A}$</td>
</tr>
<tr>
<td>$B_{1}$</td>
<td>1</td>
<td>$SS_{B}$</td>
<td>$MS_{B}$</td>
<td>$\sigma_{b}^{2}$</td>
<td>$MS_{B}$</td>
<td>$MS_{B}^{2}$</td>
</tr>
<tr>
<td>$B_{2}$</td>
<td>1</td>
<td>$SS_{B}$</td>
<td>$MS_{B}$</td>
<td>$\sigma_{b}^{2}$</td>
<td>$MS_{B}$</td>
<td>$MS_{B}^{2}$</td>
</tr>
<tr>
<td>$A_{1}B_{1}$</td>
<td>1</td>
<td>$SS_{AB}$</td>
<td>$MS_{AB}$</td>
<td>$\sigma_{ab}^{2}$</td>
<td>$MS_{AB}$</td>
<td>$MS_{AB}^{2}$</td>
</tr>
<tr>
<td>$A_{1}B_{2}$</td>
<td>1</td>
<td>$SS_{AB}$</td>
<td>$MS_{AB}$</td>
<td>$\sigma_{ab}^{2}$</td>
<td>$MS_{AB}$</td>
<td>$MS_{AB}^{2}$</td>
</tr>
<tr>
<td>$A_{2}B_{1}$</td>
<td>1</td>
<td>$SS_{AB}$</td>
<td>$MS_{AB}$</td>
<td>$\sigma_{ab}^{2}$</td>
<td>$MS_{AB}$</td>
<td>$MS_{AB}^{2}$</td>
</tr>
<tr>
<td>$A_{2}B_{2}$</td>
<td>1</td>
<td>$SS_{AB}$</td>
<td>$MS_{AB}$</td>
<td>$\sigma_{ab}^{2}$</td>
<td>$MS_{AB}$</td>
<td>$MS_{AB}^{2}$</td>
</tr>
<tr>
<td>Error</td>
<td>9</td>
<td>1</td>
<td>$SS_{E}$</td>
<td>$MS_{E}$</td>
<td>$\sigma_{e}^{2}$</td>
<td>1-sum of the above</td>
</tr>
</tbody>
</table>

* $F_{A}$, $F_{B}$, $F_{AB}$ are coefficients not related to tolerance levels of $A$ and $B$. ** Since the levels of the components are set to simulate the noise factors, then $MS_{T}$ can be used as the estimator of $\sigma_{T}^{2}$.
where $\sigma_R$, $\sigma_L$, and $\sigma_C$ are tolerances of the lowest-grade components or $\sigma_R=2.666$, $\sigma_L=11.333x10^{-6}$, $\sigma_C=6.57x10^{-6}$. Since ANOVA or Table 5 is obtained by simulating the noises associated with the components, $\sigma_L$ can be estimated by MS_L. Using this VTF, we can compute the estimated values of variance of $Y$ for different $\sigma_L$, $\sigma_C$, $\sigma_C$ without conducting more experiments. The total loss or the relative cost of the components plus the expected quality loss $E[Loss]=250\sigma_L^2$ is given in Table 6 for various tolerance levels settings. Thus, the best tolerances are grade B for $R$, grade A for $L$, and grade C for $C$, since this tolerance level setting can minimize the expected quality loss due to variations and the cost for upgrading components. Moreover, if we use Taylor’s expansion on Eq. (26) and take the 1st order approximation, the variance of $Y$ can be found as

$$\sigma_y^2 = 7.1x10^{-4}\sigma_R^2 + 8.8876\sigma_C^2 + 4.600786\sigma_L^2$$

(29)

While, if we ignore the quadratic terms and the interactions in the VTF, it gives

$$\sigma_y^2 = 7.5x10^{-4}\sigma_R^2 + 9.49\sigma_C^2 + 4.568487\sigma_L^2$$

(30)

Eq. (30) is closed to Eq. (29). Theoretically, VTF is closely related to Taylor’s expansion.

$$\sigma_y^2 = 9.57x10^{-4}\sigma_R^2 + 9.49\sigma_C^2 + 4.568487\sigma_L^2$$

(31)

Eq. (31) is also closed to Eq. (29). Theoretically, VTF is closely related to Taylor’s expansion.

7. SUMMARY

Tolerance design based on VTF, QLF and DOE methodology presented in this paper is concerned with developing tolerances for components or subsystems of a system. The objective is to minimize the expected quality loss due to variations in the parameters of the components and the cost for upgrading these components. Basically, the expected quality loss is measured by the variance of the quality characteristic of the system. The variation in parameters of the components or subsystems will cause the variations in the quality characteristic. The high-quality components or subsystems can make a high-quality system, but the cost is high. Using the VTF developed in this paper, we can find the variations of quality characteristic for any tolerance levels setting of the components based on the data generated from one setting of tolerances, and further to balance the quality and cost.

Table 2. Grades of $R$, $L$ and $C$ with the associated cost

<table>
<thead>
<tr>
<th>Component</th>
<th>Grade</th>
<th>Tolerance</th>
<th>Relative Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R$</td>
<td>A</td>
<td>$\sigma_R=0.567$</td>
<td>0.70</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>$\sigma_R=1.333$</td>
<td>0.30</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>$\sigma_R=2.666$</td>
<td>0.00 (Base value)</td>
</tr>
<tr>
<td>$L$</td>
<td>A</td>
<td>$\sigma_L=5.67x10^{-6}$</td>
<td>1.50</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>$\sigma_L=11.33x10^{-6}$</td>
<td>0.00 (Base value)</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>$\sigma_L=6.87x10^{-6}$</td>
<td>0.00 (Base value)</td>
</tr>
<tr>
<td>$C$</td>
<td>A</td>
<td>$\sigma_C=6.7x10^{-6}$</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>$\sigma_C=3.3x10^{-6}$</td>
<td>0.50</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>$\sigma_C=6.87x10^{-6}$</td>
<td>0.00 (Base value)</td>
</tr>
</tbody>
</table>

Table 3. The noise levels associated with $R$, $L$ and $C$

<table>
<thead>
<tr>
<th>Component</th>
<th>1st level</th>
<th>2nd level</th>
<th>3rd level</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R$</td>
<td>$R_1=36.735$</td>
<td>$R_2=40.000$</td>
<td>$R_3=40.265$</td>
</tr>
<tr>
<td>$L$</td>
<td>$L_1=0.1591$</td>
<td>$L_2=1.0000$</td>
<td>$L_3=0.1839$</td>
</tr>
<tr>
<td>$C$</td>
<td>$C_1=91.83x10^{-6}$</td>
<td>$C_2=100x10^{-6}$</td>
<td>$C_3=125x10^{-6}$</td>
</tr>
</tbody>
</table>

Table 4. The responses for different noise level combinations

<table>
<thead>
<tr>
<th>$L_1$</th>
<th>$L_2$</th>
<th>$L_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_1$</td>
<td>$C_2$</td>
<td>$C_3$</td>
</tr>
<tr>
<td>$R_1$</td>
<td>2.53</td>
<td>2.25</td>
</tr>
<tr>
<td>$R_2$</td>
<td>2.19</td>
<td>2.99</td>
</tr>
<tr>
<td>$R_3$</td>
<td>2.09</td>
<td>2.04</td>
</tr>
</tbody>
</table>

Table 5. ANOVA for the responses (subscript represents the linear effect; subscript $q$ represents the quadratic effect)

<table>
<thead>
<tr>
<th>Source</th>
<th>SS</th>
<th>df</th>
<th>MS</th>
<th>$F_0$</th>
<th>$p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_1$</td>
<td>0.13669</td>
<td>1</td>
<td>0.13669</td>
<td>18565***</td>
<td>0.034</td>
</tr>
<tr>
<td>$R_2$</td>
<td>0.00007</td>
<td>1</td>
<td>0.00007</td>
<td>9.77***</td>
<td>0.001</td>
</tr>
<tr>
<td>$L_1$</td>
<td>0.31205</td>
<td>1</td>
<td>0.31205</td>
<td>41700**</td>
<td>0.046</td>
</tr>
<tr>
<td>$L_2$</td>
<td>0.00015</td>
<td>1</td>
<td>0.00015</td>
<td>20.1***</td>
<td>0.003</td>
</tr>
<tr>
<td>$C_1$</td>
<td>0.05014</td>
<td>1</td>
<td>0.05014</td>
<td>6772**</td>
<td>0.009</td>
</tr>
<tr>
<td>$C_2$</td>
<td>0.00015</td>
<td>1</td>
<td>0.00015</td>
<td>20.1***</td>
<td>0.003</td>
</tr>
<tr>
<td>$R_CL$</td>
<td>0.00403</td>
<td>1</td>
<td>0.00403</td>
<td>550***</td>
<td>0.007</td>
</tr>
<tr>
<td>$R_CL$</td>
<td>0.00033</td>
<td>1</td>
<td>0.00033</td>
<td>70.3***</td>
<td>0.001</td>
</tr>
<tr>
<td>$L_2C_1$</td>
<td>0.00013</td>
<td>1</td>
<td>0.00013</td>
<td>17.4***</td>
<td>0.002</td>
</tr>
<tr>
<td>$L_2C_1$</td>
<td>0.00027</td>
<td>1</td>
<td>0.00027</td>
<td>50.666</td>
<td>0.009</td>
</tr>
</tbody>
</table>

Total 0.50667 26 0.01966 1.000

*** Significant at the level 0.01.

Table 6. The quality loss plus the cost due to upgrading

<table>
<thead>
<tr>
<th>$\sigma$</th>
<th>$\frac{5.67x10^{-6}}{1.10}$</th>
<th>$\frac{11.33x10^{-6}}{1.20}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_R$</td>
<td>1.67</td>
<td>3.33</td>
</tr>
<tr>
<td>$\sigma_L$</td>
<td>6.67</td>
<td>3.65</td>
</tr>
<tr>
<td>$\sigma_C$</td>
<td>3.93</td>
<td>3.50</td>
</tr>
</tbody>
</table>

$\sigma_R$ = 6.67 (Base value)

| $\sigma_R$ | 4.08                        | 3.65                        |
| $\sigma_L$ | 3.93                        | 3.50                        |
| $\sigma_C$ | 4.68                        | 4.28                        |

$\sigma_R$ = 13.33 (Base value)

| $\sigma_R$ | 4.23                        | 4.08                        |
| $\sigma_L$ | 4.23                        | 4.08                        |
| $\sigma_C$ | 5.40                        | 5.00                        |

$\sigma_R$ = 2.566 (Base value)

| $\sigma_R$ | 4.72                        | 4.23                        |
| $\sigma_L$ | 4.72                        | 4.23                        |
| $\sigma_C$ | 5.40                        | 5.00                        |

$\sigma_R$ = 11.33 (Base value)

ACKNOWLEDGMENT

This research was partially supported by NRC - ARISE HECU Faculty Research Participation Program.

REFERENCES

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