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Development of a J-Estimation Scheme for Internal Circumferential and Axial Surface Cracks in Elbows

Prepared by
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Battelle

Prepared for
U.S. Nuclear Regulatory Commission

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Development of a J-Estimation Scheme for Internal Circumferential and Axial Surface Cracks in Elbows

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CONTENTS

	<u>Page</u>
EXECUTIVE SUMMARY	ix
ACKNOWLEDGMENTS	xi
NOMENCLATURE	xiii
PREVIOUS REPORTS IN SERIES	xix
1.0 INTRODUCTION	1-1
1.1 References	1-1
2.0 LINE-SPRING/SHELL THEORY	2-1
2.1 Linear Elastic Line-Spring Model	2-1
2.2 Elastic-Plastic Line-Spring Model	2-3
2.3 References	2-6
3.0 COMPARISONS OF THE PREDICTIONS OF LINE-SPRING/SHELL AND THREE-DIMENSIONAL MODELS	3-1
3.1 Circumferentially Surface-Cracked Straight Pipes	3-1
3.2 Circumferentially Surface-Cracked Elbows	3-4
3.3 Discussion	3-8
3.4 References	3-11
4.0 DETERMINATION OF INITIAL PRESSURE APPLIED TO THE CRACKED ELBOWS	4-1
4.1 References	4-2
5.0 DEVELOPMENT OF J-ESTIMATION SCHEME FOR CRACKED ELBOWS	5-1
5.1 A J-Estimation Scheme for Circumferentially Surface-Cracked Elbows	5-2
5.2 A J-Estimation Scheme for Axially Surface-Cracked Elbows	5-8
5.3 Investigation of Rotations at the Elbow/Straight Pipe Junction of the Cracked Elbows ..	5-10
5.4 Sample Comparisons	5-12
5.5 References	5-14
6.0 SUMMARY AND CONCLUSIONS	6-1

CONTENTS

Page

LIST OF FIGURES

2.1	Illustration of the line-spring concept (a) Cross-section of a part-through circumferential surface crack of length $2l$ and depth a in a pipe of thickness t , (b) Schematic illustration of the line-spring concept which replaces the part-through crack with a through-slit and a series of springs along it	2-2
2.2	Yield surface in the generalized force space assumed for the line-spring model in ABAQUS (From the ABAQUS Theory Manual, page 3.9.4-7, by permission of Hibbitt, Karlsson & Sorensen, Inc.)	2-5
3.1	Stress-strain curve for material DP2-F29	3-2
3.2	Predicted and experimentally measured load versus pipe displacement at the load points	3-3
3.3	Calculated J-integral values versus pipe displacement at the load-point location for the various models studied	3-3
3.4	The predicted variation of the crack-driving force, J, with the applied load assuming a Ramberg-Osgood material ($n = 5$, $\alpha = 1$)	3-5
3.5	Comparisons of the predictions of moment-rotation response of a surface-cracked pipe using a line-spring model with the corresponding response obtained from experimental measurements	3-5
3.6	Schematic sketch of a circumferentially cracked elbow	3-6
3.7	The three-dimensional finite element mesh of the circumferentially cracked 184 mm (7.25 inch) mean radius by 38 mm (1.25 inch) thick elbow	3-6
3.8	The line-spring/shell finite element mesh of the circumferentially cracked elbow (mesh is for mean radius of 184 mm (7.5 inch))	3-7
3.9	Variation of J with normalized bending moment as predicted by the line-spring/shell and three-dimensional models	3-8
3.10	The angular variation of J for the circumferentially cracked elbow as predicted by the line-spring/shell and three-dimensional models at maximum applied load (a) Elastic deformation, (b) Elastic-plastic deformation	3-9
3.11	Effect of internal pressure on the variation of J with applied bending moment for a circumferentially cracked elbow	3-10
3.12	Effect of internal pressure on the variation of J with applied bending moment for a circumferentially cracked straight pipe	3-10

CONTENTS

	<u>Page</u>
5.1 Comparisons of the finite element predictions and the fit using the functional dependence given in Equation 5.4 for a circumferentially cracked elbow (a) $R_m/t = 5$, $a/t = 0.5$, $n = 5$ (b) $R_m/t = 10$, $a/t = 0.5$, $n = 5$	5-6
5.2 The variation of the h_1^M functions with the strain-hardening exponent, n , and the ratio of crack-depth to wall thickness, a/t , obtained using the GE/EPRI (SC.ELB2) and NSC (SC.ELB1) definition of the limit moment, M_o (a) GE/EPRI (SC.ELB2), (b) NSC (SC.ELB1)	5-7
5.3 Schematic sketch of an axially cracked elbow subjected to an in-plane opening moment	5-9
5.4 Comparisons of FEM rotations at the junction of a straight pipe and elbow for uncracked and circumferentially cracked elbows with several a/t ratios and $R_m/t = 10$ for a strain-hardening exponent $n = 5$	5-13
5.5 Comparisons of FEM rotations at the junction of a straight pipe and elbow for uncracked and axially cracked elbows with several a/t ratios and $R_m/t = 10$ for a strain-hardening exponent $n = 5$	5-13
5.6 Variation of J with applied bending moment for axially and circumferentially cracked elbows and circumferentially cracked straight pipe (a) $R_m/t = 10$, and (b) $R_m/t = 20$	5-15

CONTENTS

Page

LIST OF TABLES

4.1	Typical S_m values for several piping steels at 288 C (550 F)	4-1
4.2	Initial pressure applied to elbows	4-2
5.1	F_1 functions for circumferentially cracked elbows (used in both SC.ELB1 and SC.ELB2 methods) [$\theta/\pi = 0.5$, $\psi = 90$ degrees]	5-5
5.2	h_1^M functions for circumferentially cracked elbows using GE/EPRI definition of M_o (SC.ELB2 method) [$\theta/\pi = 0.5$, $\psi = 90$ degrees]	5-5
5.3	h_1^M functions using NSC equation for M_o (SC.ELB1 method) [$\theta/\pi = 0.5$, $\psi = 90$ degrees]	5-5
5.4	F_1 functions for axially cracked elbows (SC.ELB3 method) [total crack angle = 30 degrees, $\psi = 90$ degrees]	5-11
5.5	h_1^{PF} functions for axially cracked elbows (SC.ELB3 method) [total crack angle = 30 degrees, $\psi = 90$ degrees]	5-11
5.6	h_1^M functions for axially cracked elbows (SC.ELB3 method) [total crack angle = 30 degrees, $\psi = 90$ degrees]	5-11

EXECUTIVE SUMMARY

This report summarizes efforts to develop elastic and elastic-plastic fracture mechanics analyses for internal surface cracks in elbows. The analyses involved development of a GE/EPRI type J-estimation scheme which requires an elastic and fully plastic contribution to crack-driving force in terms of the J-integral parameter. The elastic analyses require the development of F-function values to relate the J_e term to applied loads. Similarly, the fully plastic analyses require the development of h-functions to relate the J_p term to the applied loads. The F- and h-functions were determined from a matrix of finite element analyses.

To minimize the cost of the analyses, three-dimensional ABAQUS finite element analyses were compared to a simpler finite element technique called the line-spring method. The line-spring method provides a significant computational savings over the full three-dimensional analysis. The comparison showed excellent agreement between the line-spring and three-dimensional analysis. This experience was consistent with comparisons with circumferential surface-crack analyses in straight pipes during the NRC's Short Cracks in Piping and Piping Welds program.

The matrix of analyses covered the following parameters:

- 90-degree long-radius elbows
- R_m/t ratios of 5, 10, and 20
- combined pressure and bending
- bending in the in-plane closing mode
- determination of separate elastic F-functions for pressure loading and bending loading.

For the fully-plastic h-function development, it was decided to conduct the analyses under combined pressure and bending, where the pressure contribution was fixed to correspond to a hoop stress equal to an average S_m value for typical nuclear piping materials used in the U.S. These results should not be used if the operating hoop stress differs greatly from the average S_m value used.

Further factors included in the analyses were:

- the strain-hardening exponents (n) for the fully-plastic solutions were 3, 5, and 10,
- crack depth-to-thickness (a/t) ratios were 0.3, 0.5, and 0.75,
- the circumferential crack length was limited to one crack length of 50 percent of the circumference with the crack centered on the extrados, and
- the axial crack length was limited to one crack length of 30 degrees of the 90-degree elbow, with the crack on the flank (side or crown) of the elbow.

In the development of the h-function values it was found that the reference moment function used has a significant effect on the smoothness of the h-function values versus a/t , R_m/t , and n . This is important since the discrete tabular values are interpolated for each of these parameters. The reference moment with the smoothest changes in h_1 -function values is given and used in subsequent analyses. For the circumferential extrados surface-crack case, the SC.ELB2 method used the GE/EPRI method for the reference moment which gave the smoothest h_1 -function. The SC.ELB1 analysis was also for a circumferential extrados

Executive Summary

surface-crack, but used the Net-Section-Collapse analysis equations for the reference moment. The SC.ELB1 h_1 -function was not as smooth as the SC.ELB2 h_1 -function.

The results of the work produced a matrix of the required F and h-function values, and these were implemented into a computer code called IP2ELBOW. This code used the NRCPIPE (Windows version) framework, and contains the SC.ELB1 and SC.ELB2 J-estimation schemes for circumferential extrados surface-cracked elbows, as well as the SC.ELB3 J-estimation scheme for axial flank surface-cracked elbow analyses. The computer code was then used to compare the magnitudes of the J versus moment curves for the axial surface crack in an elbow, the circumferential surface crack in an elbow, and a circumferential surface crack in a straight pipe.

These comparisons were on pipe/elbows with the same diameters and thickness, internal pressure, a/t , and same stress-strain curves. R_m/t ratios of 10 and 20 were considered in these analyses.

For typical nuclear materials, the results of the comparisons showed that for an axial surface crack in the elbow, and depending on the R_m/t ratio, the moment values at crack initiation are 1.5 to 2 times lower than for the circumferential surface crack in the elbow. This is a significant difference. Moreover, for the flaw size examined here, the moment values for pressurized straight pipe were somewhat lower than the corresponding values for circumferentially cracked elbow in the case of R_m/t of 10. In the case when R_m/t is 20, the pressurized straight pipe moment value was approximately 1.5 times greater than the circumferential elbow moment value for J values at crack initiation. Additional sensitivity studies could be performed with the IP2ELBOW code to further investigate the differences between the straight pipe and elbow J solutions.

Finally, it should be noted that there is a separate IPIRG-2 program report on elbow experiments and comparisons with analytical predictions.

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NOMENCLATURE

1. SYMBOLS

A	Ligament area
A^{nc}	Area of uncracked region
a	Surface crack depth
C_1, C_2, C_3, C_4	Constants used in definition of yield surface in line-spring model
c	Remaining ligament
D_o	Outer diameter of the pipe
E	Young's modulus of the material
F, F_1, F_i	Functions used in elastic stress intensity factor definition in line-spring model
F_1^{pr}, F_1^M	Functions used in elastic stress intensity factor definition associated with internal pressure and bending
f	Strain-hardening factor used in line-spring model
h, h_1	Coefficient functions used in defining plastic part of crack-driving force
h_1^{pr}, h_1^M	Coefficient functions used in defining plastic part of crack-driving force associated with internal pressure and bending
I	Second moment of inertia of an uncracked cylinder about its neutral axis
J	J-integral fracture parameter
J_e	Elastic component of J-integral
$J_p, \Delta J_p$	Total and incremental plastic component of J-integral
J_e^{pr+M}	Elastic part of J associated with combined internal pressure and bending
J_p^{pr}, J_p^M	Elastic part of J associated with internal pressure and bending
K	Elastic stress intensity factor

Nomenclature

K_I	Mode I elastic stress intensity factor
K_I^{pr}, K_I^M	Mode I stress intensity factor due to internal pressure and bending
ℓ	Total length of axial surface crack
M	Bending moment
M_o	Referenced moment for cracked pipe
$M_{o(GE)}$	Limit moment of circumferentially cracked pipe used in GE/EPRI method
M_o^{nc}	Limit moment for uncracked pipe
$M_{o(NSC)}$	Net-section-collapse moment
M^∞	Far-field applied moment
M_2	Parameter used in the definition of limit pressure
m	Proportionality constant used in the line-spring model
N	Membrane force
N^∞	Far-field applied membrane force
n	Ramberg-Osgood strain-hardening exponent
P_{ij}	Compliance matrix used in the line-spring model
p	Pressure
p_o	Limit pressure
p_a	Allowable pressure
Q	Load parameter
$Q_i (i=1,2)$	Generalized forces used in the line-spring model
Q_i^{Lim}	Generalized limit load
q_i^p	Plastic part of generalized displacements work-conjugate to Q_i used in the line-spring model
$q_i (i=1,2)$	Generalized displacements work-conjugate to Q_i used in the line-spring model

R_{el}	Radius of elbow
R_i	Inside radius
R_m	Mean pipe radius
R_o	Outside radius
S_m	ASME design stress
t	Wall thickness of elbow
t_{pipe}	Wall thickness of pipe
$W^P, \Delta W^P$	Total and incremental plastic work
X, Y	Normalized parameters used in defining yield surface in the line-spring model
x, y, z	Coordinate axes
Υ	Factor used in determining the allowable pressure, p_a
α	Coefficient of Ramberg-Osgood model
β	Parameter used in net-section collapse moment
ΔQ_i	Incremental generalized forces
$\Delta q_i, \Delta q_i^e, \Delta q_i^p$	Incremental generalized total, elastic and plastic displacements
Δq^p	Positive scalar quantity representing material hardening
$\Delta \epsilon_o^p$	Ligament-average plastic strain
$\Delta \delta_{up}^p$	Incremental plastic part of the crack opening displacement
ϵ^T	Total strain
ϵ_o	Yield strain
ϵ^e	Elastic strain
ϵ^p	Plastic strain
$\bar{\epsilon}^p$	Effective plastic strain
δ	Total normal displacements at the crack-tip

Nomenclature

λ	Elbow parameter
Φ, Φ_1, Φ_2	Yield surfaces used in the line spring model
ϕ	Relative rotation at the crack tip
ψ	Elbow angle
σ	Uniaxial stress
σ_f	Flow stress
σ_o	Initial yield stress
$\sigma_o(\bar{\epsilon}^p)$	Current yield stress
θ	Half-crack angle
ν	Poisson's ratio

2. ACRONYMS AND INITIALISMS

AECB	Atomic Energy Control Board, Canada
ANPA	Agenzia Nazionale per la Protezione dell'Ambiente, Italy
ASME	American Society of Mechanical Engineers
CEA	Commissariat A L'Energie Atomique, France
CRIEPI	Central Institute of Electric Power Industry, Japan
CUAEPP	Commercial Use of Atomic Energy for Peaceful Purposes, Bulgaria
EDF	Electricite de France
EPRI	Electric Power Research Institute, U.S.
GE	General Electric, U.S.
GE/EPRI	A through-wall crack J-estimation scheme
HAEC	Hungarian Atomic Energy Commission
HSK	Hauptabteilung für die Sicherheit der Kernanlagen, Switzerland

INER/AEC	Institute of Nuclear Energy Research, Republic of China
IP2ELBOW	Computer code for fracture analysis of circumferentially and axially cracked elbows
IPIRG-2	Second International Piping Integrity Research Group
KINS	Korea Institute of Nuclear Safety
KKL	Kernkraftwerk Leibstadt AG, Switzerland
NRCPIPE	Computer code for fracture analysis of circumferentially through-wall-cracked straight pipe
NRA	Nuclear Regulatory Authority of Slovak Republic
NRI	Nuclear Research Institute, Czech Republic
NSC	Net-Section-Collapse
SC.TNP2	J-estimation method for fracture analysis of circumferentially surface-cracked straight pipe
SC.ELB1	J-estimation method for fracture analysis of circumferential extrados, surface-cracked elbows
SC.ELB2	J-estimation method for fracture analysis of circumferential extrados, surface-cracked elbows with reference moment using the GE/EPRI reference moment method
SC.ELB3	J-estimation method for fracture analysis of axial, flank, surface-cracked elbows
SEN	Single-edge notch specimen
SKI	Statens Kärnkraftinspektion (Swedish Nuclear Power Inspectorate)
SKKU	Sung Kyun Kwan University, Korea
TAG	Technical Advisory Group
USNRC-NRR	United States Nuclear Regulatory Commission - Office of Nuclear Reactor Regulation
USNRC-RES	United States Nuclear Regulatory Commission - Office of Nuclear Reactor Research

Nomenclature

VATESI	State Nuclear Energy Safety Inspection of the Republic of Lithuania
VUJE	Vyskumny Ustav Jadavyeh Elektrarni (Nuclear Power Plant Research Institute), Slovakia

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- "Analysis of Experiments on Stainless Steel Flux Welds," Topical Report, NUREG/CR-4878, February 1987.
- "Experimental and Analytical Assessment of Circumferentially Surface-Cracked Pipes Under Bending," Topical Report, NUREG/CR-4872, April 1987.

Other Related Program Reports

- "Validation of Analysis Methods for Assessing Flawed Piping Subjected to Dynamic Loading," NUREG/CR-6234, August 1994.

Previous Reports in Series

“Approximate Methods for Fracture Analyses of Through-Wall Cracked Pipes,” Topical Report, NUREG/CR-4853, January 1987.

“Assessment of Design Basis for Load-Carrying Capacity of Weld-Overlay Repair,” Topical Report, NUREG/CR-4877, February 1987.

“Analysis of Experiments on Stainless Steel Flux Welds,” Topical Report, NUREG/CR-4878, February 1987.

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“Validation of Analysis Methods for Assessing Flawed Piping Subjected to Dynamic Loading,” NUREG/CR-6234, August 1994.

1.0 INTRODUCTION

In order to assure the structural integrity of nuclear piping systems, several approaches have been developed as part of the ASME Boiler and Pressure Vessel Code. All of these approaches are strictly applicable to flawed straight pipe sections. However, service failures have occurred in pipe bend or elbow sections of piping systems. Simple modifications or extensions of approaches applicable to flawed straight-pipe sections appear quite inadequate for pressurized elbows containing flaws. Accordingly, a simple fracture methodology, based on the spirit of the GE/EPRI estimation scheme (Ref. 1.1), was developed using detailed finite element analyses.

Owing to the large number of finite element calculations needed to develop an estimation scheme, a full three-dimensional finite element analysis computational effort to ensure adequate mesh design would be a quite formidable task. An attractive alternative, which was used in this work, is the line-spring/shell model, which reduces the three-dimensional surface crack problem into a more tractable two-dimensional shell problem. To ensure accuracy of the solution using the line-spring/shell model, detailed comparisons of the predictions were made with the three-dimensional model. Based on these comparisons and past work on straight pipes using the line-spring/shell model, the line-spring/shell model was validated for surface cracks in pipes and elbows.

An estimation scheme was developed for circumferentially cracked and axially cracked pressurized long radius elbows. In the case of a circumferentially cracked elbow, the constant-depth crack was located on the inside surface at the extrados. The total crack angle was 180 degrees and a closing in-plane bending moment was applied to the pressurized cracked elbow. The constant-depth internal axial crack of total length 30 degrees was located at the flanks of the pressurized elbow which was subjected to an opening in-plane bending moment. Solutions were developed for several radius-to-wall thickness ratios as well as crack-depth-to-wall thickness ratios.

This report is organized as follows. The theory behind the line-spring/shell model is detailed in Section 2.0. Validation of this model against fully three-dimensional models is presented in Section 3.0. The estimation scheme is developed for pressurized cracked elbows. An appropriate choice of internal pressure to be applied to the elbow is required. This pressure is determined from the ASME Boiler and Pressure Vessel Code using the average values of S_m for several ferritic and austenitic steels. This is described in Section 4.0. In Sections 5.0 and 6.0, the estimation scheme and the resulting functions are given for circumferentially and axially cracked elbows, respectively. A summary of the discussions and conclusions is presented in Section 6.0.

1.1 References

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2.0 LINE-SPRING/SHELL THEORY

The line-spring model proposed by Rice and Levy (Reference 2.1) provides an attractive and computationally effective means to analyze part-through cracks in plates and shells. The attractive feature of the line-spring model is that the additional compliance introduced by the presence of a part-through flaw into a shell structure (e.g., pipes and elbows) is accounted for by replacing the part-through flaw with a through-slit of the same length with a foundation or a series of springs connecting the two sides of the through-slit, as schematically illustrated in Figure 2.1. The local compliance of a spring connecting one point of the slit to the corresponding point on the other side (Points A and B in Figure 2.1) is calculated based on known solutions of single-edge-notch specimens (SEN) under plane strain conditions. Thus, knowing the local compliance of a spring, which depends on the depth of the surface crack at the point, the stress intensity factor, K , and the crack-driving force, J , may be calculated. In order to provide reliable estimates of the fracture parameters using a line-spring/shell model, the length of the crack, $2l$, needs to be much larger than the thickness, t , of the plate/shell. In addition, a constant or gradually changing crack depth, a , is required for more accurate predictions.

This method of introducing an additional compliance by means of a series of springs along the through-slit in a shell enables solving the three-dimensional problem by means of a generalized two-dimensional problem. Owing to this feature, the finite element mesh required for the line-spring analysis is simple and the same mesh can be used to solve problems with different crack depths and/or different shell thicknesses. Thus, the line-spring/shell method is computationally inexpensive and provides a valuable tool to analyze a large number of problems with varying crack and pipe geometries.

2.1 Linear Elastic Line-Spring Model

When a loaded, cracked, structure is analyzed using the line-spring/shell model, the generalized shell resultants transmitted by the series of springs are a membrane force, N , and a bending moment, M , per unit length of the foundation. The work-conjugate measures to these generalized resultants are the relative normal displacements, δ , and relative rotation, ϕ . Denoting the membrane force, N , as Q_1 and the bending moment, M , as Q_2 and their work-conjugate measures as q_1 and q_2 , the generalized forces and displacements are related in the elastic region through a compliance matrix, P_{ij} , as follows

$$q_i = P_{ij} Q_j \quad (i,j=1,2) \quad (2-1)$$

where the elastic compliance matrix, P_{ij} , is determined from the stress intensity calibrations of SEN specimens using the energy/compliance relationship proposed by Rice (Ref. 2.2). Using this relationship, the Mode I stress-intensity factors are given by

$$K_I = F_i \left(\frac{a}{t} \right) Q_i \quad (2-2)$$

(a) Cross-section of a part-through circumferential surface crack of length $2l$ and depth a in a pipe of thickness t

(b) Schematic illustration of the line-spring concept which replaces the part-through crack with a through-slit and a series of springs along it

Figure 2.1 Illustration of the line-spring concept

where F_i denotes functions that can be obtained from the fracture handbook by Tada et al. (Ref. 2.3). The J-integral is then related to the stress intensity factor through

$$J = (1 - \nu^2) \frac{K_I^2}{E} \quad (2-3)$$

where E is Young's modulus and ν is Poisson's ratio of the material in consideration. This concept of line-spring can be easily extended to cases involving Mode II and Mode III as well, when considering elastic deformations only.

2.2 Elastic-Plastic Line-Spring Model

For elastic-plastic deformations, an incremental formulation assuming a simple associated flow rule with isotropic hardening is used in the case of Mode I response. Crucial to the development of the incremental elastic-plastic line-spring theory is the concept of a convex yield surface, $\Phi\{Q_i; a, t; \sigma_o(\bar{\epsilon}^p)\}$, where $\sigma_o(\bar{\epsilon}^p)$ is the current uniaxial yield stress at an equivalent plastic strain of $\bar{\epsilon}^p$. This attractive concept of the yield surface and an explicit form for the yield surface under tension-dominant conditions were discussed by Rice (Ref. 2.4). Forms for the yield surface under conditions of bending were later provided by Parks and White (Ref. 2.5) and Shiratori and Miyoshi (Ref. 2.6). More recently, Lee and Parks (Ref. 2.7) have proposed an enhanced yield surface for elastic-plastic line spring models. The yield surface, as implemented in ABAQUS, adopts the surface proposed by Rice (Ref. 2.4) in combination with a surface of polynomial form obtained from a smooth continuation assumption such that the latter smoothly blends with the former surface. The latter surface, though providing a reasonable estimate, can be enhanced with the recent proposition of Lee and Parks (Ref. 2.7). Defining the following quantities

$$X = \frac{\sqrt{3} Q_1}{2 \sigma_o(\bar{\epsilon}^p) (t-a)} \quad (2-4)$$

$$Y = \frac{\sqrt{3} \left(Q_2 + \frac{t}{2} Q_1 \right)}{2 \sigma_o(\bar{\epsilon}^p) (t-a)^2}$$

the yield function used in ABAQUS is given by

$$\begin{aligned}\Phi_1 &= (X - 0.3)^2 + 4.41 \left(Y - \frac{X}{2}\right) - 0.49 = 0 \quad \text{for } X \leq 2Y \\ \Phi_2 &= C_1 X + C_2 Y + C_3 X^3 Y + C_4 XY^3 - 1 \quad \text{for } X > 2Y\end{aligned}\tag{2-5}$$

where C_1 , C_2 , C_3 , and C_4 are constants chosen such that the two surfaces blend continuously at $X = 2Y$. The values of the constants may be found in the ABAQUS Reference Manual (Reference 2.8). These yield surfaces are shown in Figure 2.2. Regions where the line-spring model is not appropriate, owing to crack closure, are indicated in this figure.

Given the yield surface, $\Phi\{Q_i; a, t; \sigma_o(\bar{\epsilon}^p)\} = 0$, in the generalized force space coupled with the simple associated flow rule entailing the isotropic hardening assumption, the formulation of the incremental elastic-plastic line-spring is described below.

An additive decomposition of the total generalized displacement increments, Δq_i , into elastic and plastic parts is assumed, i.e., $\Delta q_i = \Delta q_i^e + \Delta q_i^p$. The elastic part of the generalized displacement increment is related to the generalized force increment, ΔQ_j , through the elastic compliance matrix, P_{ij} , as follows

$$\Delta q_i^e = P_{ij} \Delta Q_j\tag{2-6}$$

When the applied loads are such that the generalized force state lies on the yield surface and plasticity ensues, the plastic part of the generalized displacement increments, assumed to satisfy the normality rule, are taken to be the product of a positive scalar quantity, Δq^p , which represents the incremental hardening parameter calculated from the basic stress-strain data of the material, and the outward normal to the yield surface. Thus

$$\Delta q_i^p = \Delta q^p \frac{\partial \Phi}{\partial Q_i}\tag{2-7}$$

Completion of the formulation now requires a relationship between the plastic part of the generalized incremental displacements, Δq_i^p , and the incremental ligament-average plastic strain, $\Delta \epsilon_o^p$, which can be obtained directly from the known basic true stress-logarithmic strain data of the material. To this end, the plastic work increment per unit length of the flaw is expressed in two different ways. The plastic work increment, ΔW^p per unit thickness of the SEN specimen using the generalized field quantities may be written as

$$\Delta W^p = Q_i \Delta q_i^p\tag{2-8}$$

Figure 2.2 Yield surface in the generalized force space assumed for the line-spring model in ABAQUS (From the ABAQUS Theory Manual, page 3.9.4-7, by permission of Hibbitt, Karlsson & Sorensen, Inc.)

and in terms of the uniaxial field quantities over the area A of the SEN specimen where plastic dissipation occurs as

$$\Delta W^p = \int_A \sigma_o(\bar{\epsilon}^p) \Delta \epsilon_o^p dA \quad (2-9)$$

Parks (Ref. 2.9) reasoned that the plastic dissipation area A for a deep crack would be proportional to the square of the remaining ligament, $(t - a)$, and for shallow cracks this area A is proportional to the square of a "characteristic length", which is still related to $(t - a)$. Then, following Parks (Ref. 2.9) the above integral can be evaluated approximately as

$$\Delta W^p = f \sigma_o(\bar{\epsilon}^p) \Delta \epsilon_o^p (t-a)^2 \quad (2-10)$$

where the dimensionless scalar f is called the strain-hardening factor. Parks and White (Ref. 2.5) suggest that a value of 0.4 for f provides an excellent pointwise matching of the line-spring model to the numerical results of the SEN specimen. This value is used in ABAQUS.

This approach can now be coupled with an additive decomposition of the crack-driving force, J , into elastic and plastic parts, J_e and J_p to provide estimates of J . The elastic part of the driving force can be

easily calculated using Equation (2-3), while the plastic part of the driving force for the perfectly plastic case is given by

$$\Delta J_p = - \frac{\partial Q_i^{Lim}}{\partial a} \Delta q_i^p = \Delta q^p \frac{\partial \Phi}{\partial Q_i} \left(\frac{\partial Q_i^{Lim}}{\partial (t-a)} \right)_{q_i^p} \quad (2-11)$$

Though the above equation formally applies in the case of perfect plasticity, it can be used in the case of strain hardening by accounting for these effects through the non-negative scalar quantity, Δq^p . A different approach for calculating the plastic part of the J-integral, as used in ABAQUS, is based on the linear relationship between ΔJ_p and the incremental plastic part of the crack-tip opening, $\Delta \delta_{tip}^p$, which can be determined once the plastic strain increments are known. This relationship is given by

$$\Delta J_p = m \sigma_o(\epsilon^p) \Delta \delta_{tip}^p \quad (2-12)$$

where the proportionality constant, m , is given by (Ref. 2.5)

$$m = \frac{\frac{\partial \Phi}{\partial a}}{\sigma_o(\epsilon^p) \left[\frac{\partial \Phi}{\partial Q_1} + (t/2 - a) \frac{\partial \Phi}{\partial Q_2} \right]} \quad (2-13)$$

Note that in the above expression Φ is either Φ_1 or Φ_2 depending on the state of deformation. Though quite accurate in the fully plastic region, the resulting form of the driving force, J , is quite approximate in the transition region of elastic to fully plastic conditions. It will be shown in the next section that this approximation is reasonable by comparing the predictions of the line-spring model with the fully three-dimensional model.

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3.0 COMPARISONS OF THE PREDICTIONS OF LINE-SPRING/ SHELL AND THREE-DIMENSIONAL MODELS

The line-spring/shell model has been previously used by Kumar, and others (Ref. 3.1) to develop an estimation scheme for circumferentially surface-cracked straight pipes. In their report, very good comparisons between the line-spring and three-dimensional models were shown for selected cases. It should be noted that the line-spring/shell model as implemented in ABAQUS takes advantage of the enhancements proposed by Parks and White (Ref. 3.2). This can be seen in the improved predictions using the line-spring/shell model in ABAQUS obtained by Brickstadt (Ref. 3.3), in comparison to the work of Kumar, and others (Ref. 3.1). More recently, Sauter, and others (Ref. 3.4) used the line spring/shell model in ABAQUS to analyze axial cracks in elbows. However, they did not make detailed comparisons of this model's predictions with the three-dimensional model predictions in the elastic-plastic regime.

Battelle has performed detailed analyses using both the line-spring/shell and three-dimensional models for circumferentially surface-cracked straight pipes, as part of the U.S. NRC "Short Cracks in Piping and Piping Welds Program" (Ref. 3.5). These results are reviewed briefly in this section. In addition, the line-spring/shell model is validated against the three-dimensional model for circumferentially surface-cracked elbows. The results shown in this section demonstrate that reasonably accurate predictions of the line-spring model for straight pipes as well as elbows are provided.

3.1 Circumferentially Surface-Cracked Straight Pipes

As part of the NRC/Battelle "Short Cracks in Piping and Piping Welds Program", a mesh refinement study was conducted to ensure accurate prediction of the crack-tip fields and crack-driving force, J . Details of this study can be found in Reference 3.7. Elasto-plastic simulations of a circumferentially surface-cracked pipe experiment from the Degraded Piping Program (Experiment 4112-8), References 3.6 and 3.7, were carried out using three 3D finite-element models using 8-noded brick elements (Meshes 1, 2, and 3) and a line-spring/shell finite-element model with the crack being modeled with line-spring elements. The stress-strain behavior of the material is shown in Figure 3.1. For the 3D finite element model a fan-like mesh focused at the crack-tip was employed in all cases, since this yields more accurate solutions for quasi-static crack problems. There were three levels of mesh refinement that could be examined, namely, refinement along the thickness direction of the cracked and uncracked ligaments, refinement along the crack front direction, and refinement along the circumferential direction of the fan-like mesh focussed at the crack tip. Since the most critical location in circumferentially surface-cracked pipes is at the center of the flaw, the mesh refinement examined was confined to the thickness direction of the crack and the remaining ligament of the pipe.¹ There were 13 elements used along the circumferential crack front in all cases. Mesh 1 consists of eight elements radially along the crack and seven elements radially along the ligament. Mesh 1 may be considered as a moderately refined model. Mesh 2 was coarser than Mesh 1, consisting of only

¹ In all cases, the crack was modeled with a fan-like mesh with uniform grading in the circumferential direction (giving an angular resolution of 22.5 degrees) and with geometrically increasing radial grading with varying number of rings for each mesh.

Figure 3.1 Stress-strain curve for material DP2-F29

five elements along the crack and four along the ligament. Mesh 3 was more refined than Mesh 1, consisting of 14 elements along the crack and 13 elements along the ligament. Further details of these meshes may be found in Reference 3.7.

The predicted load versus pipe displacement at the load point using all the models was compared with the experimentally measured data in Figure 3.2. There were negligible differences between the predictions for the various models. This was not surprising since the load point, being sufficiently far away from the crack, was relatively unaffected by details of crack-tip refinement. The difference between the predicted and measured values increased beyond a pipe displacement of 95.25 mm (3.75 inch). The predicted load at crack initiation was more than 9 percent higher than the experimentally measured load.

The calculated J-integral versus pipe displacement values at the load point are shown in Figure 3.3. Although the various models showed little difference in the load versus pipe displacement at the load-point location (see Figure 3.2), they exhibited a notable difference in the J integral values with load-point displacement relationship. The calculated values of the J-integral for the three 3D finite element meshes were almost path independent. It can be noted that the line-spring model predictions compare well with the predictions using Mesh 3 up to J values of 300 kN/m (1,713 lb/in). The departure of the variation predicted by the line-spring model from that predicted using Mesh 3 at higher J values may in large part be due to the non-uniform hardening of the material as demonstrated in the stress-strain curve (Figure 3.1). To examine this further, the above simulations were carried out using the line-spring/shell model and a 3D model (Mesh 3) using a Ramberg-Osgood material (with $n = 5$ and $\alpha = 1$). As mentioned in the previous section, the line-spring/shell model assumes associated flow theory. In the case of the 3D model, both the associated flow theory as well as deformation plasticity theory were used. The variation of the

COMPARISONS OF THE PREDICTIONS OF LINE-SPRING/SHELL
AND THREE-DIMENSIONAL MODELS

Figure 3.2 Predicted and experimentally measured load versus pipe displacement at the load points

Figure 3.3 Calculated J-integral values versus pipe displacement at the load-point location for the various models studied

crack-driving force, J , with the applied load as predicted by the various models is shown in Figure 3.4. It can be observed that the line-spring and 3D models predict nearly identical variations even at very large applied load values. It is also interesting to note that the line-spring predictions are much closer to the three-dimensional model predictions using the associated flow theory. Thus, for a uniformly hardening material, the line-spring model is quite accurate.

Further validation of the line-spring model is established by the simulation of a surface-cracked pipe experiment conducted at Battelle (Experiment Number 1.2.1.20). The outer diameter and thickness of the pipe were 407.0 mm (16.01 inch) and 9.6 mm (0.376 inch), respectively. The crack geometry is defined by a depth-to-thickness ratio of 0.476 and total crack length-to-circumference of 0.25. The stress-strain response used in the analysis was obtained from a Ramberg-Osgood fit of the actual stress-strain response of the material. The Ramberg-Osgood parameters were: $\sigma_o = 224$ MPa (32.5 ksi), $n = 4.94$ and $\alpha = 4.21$. The cracked pipe was loaded under an internal pressure of 1.5 MPa (225 psi) prior to applying a bending moment. The pipe rotation was measured at a distance of 559 mm (22 inch) away from the crack. The good agreement between the experimental moment-rotation response and the line-spring/shell model predictions, shown in Figure 3.5, enhances the applicability of line-spring/shell model in the analyses of surface-cracked pipes.

3.2 Circumferentially Surface-Cracked Elbows

Though the line-spring/shell model appears to be reasonably accurate for surface-cracked straight pipes, the same inference cannot be made in the case of surface-cracked elbows. This is due to the fact that the geometry of the elbow is more complex because of its double curvature. Accordingly, detailed analyses using both the 3D and line-spring/shell model were performed for a 90-degree, long radius ($R_e/R_m = 3$) elbow with an internal constant depth surface-crack (crack depth-to-wall thickness ratio of 0.667 and a total crack length-to-circumference of 0.5) located at the extrados, subjected to a closing bending moment. A schematic sketch of the cracked elbow is shown in Figure 3.6.

The mean radius of the 90-degree elbow was 184 mm (7.25 inch) and the wall thickness was 38 mm (1.25 inch). The length of straight section of the pipe was taken to be 3.048 m (10 feet). Owing to symmetry one-quarter of the elbow was modeled. The planes of symmetry lie parallel to the x-y plane and parallel to the plane whose normal is perpendicular to the z-axis and inclined at $\pi/4$ with respect to the x-axis. Two 3D finite element meshes were employed, one with 8-noded and the other with 20-noded isoparametric brick elements with reduced integration. In both cases, a focussed mesh at the crack-tip was used with 13 elements along the crack front and 8 elements along the thickness direction of the pipe in the cracked region. The total number of elements for these two models was 2,486. Initially, the plane of the crack was at an angle of $\pi/4$ to the x-y plane in the finite element model. In the process of performing the calculations, it was discovered that the calculation of crack driving force, J , was ill-suited when the crack does not lie on any of the global planes in ABAQUS. Thus, the finite element mesh was rotated such that the crack lies on the x-z plane. The 8-noded finite element mesh of the cracked elbow is shown in Figure 3.7. The 20-noded finite element mesh is similar to this, except that the elements have 20 nodes each. The line-spring/shell finite element, shown in Figure 3.8, contained 400 8-noded shell elements with 13 3-noded line-spring elements. The material properties assumed in these calculations were: $E = 206$ GPa (29,877 ksi), $\nu = 0.3$, $\sigma_o = 241$ MPa (35 ksi), $n = 3$, and $\alpha = 1$.

Figure 3.4 The predicted variation of the crack-driving force, J , with the applied load assuming a Ramberg-Osgood material ($n = 5$, $\alpha = 1$)

Figure 3.5 Comparisons of the predictions of moment-rotation response of a surface-cracked pipe using a line-spring model with the corresponding response obtained from experimental measurements

Figure 3.6 Schematic sketch of a circumferentially cracked elbow

Figure 3.7 The three-dimensional finite element mesh of the circumferentially cracked 184 mm (7.25 inch) mean radius by 38 mm (1.25 inch) thick elbow

Figure 3.8 The line-spring/shell finite element mesh of the circumferentially cracked elbow (mesh is for mean radius of 184 mm (7.5 inch))

Initial attempts made with this line-spring/shell finite element model yielded physically meaningless predictions of the crack-tip behavior, indicating that the crack would close under a closing in-plane bending moment. Detailed investigations of mesh refinement, rotation of the mesh, and transformation of boundary conditions did not change the predictions. It was then discovered that the shell normals, calculated internally by ABAQUS, were in error for some nodes on the x-y symmetry and x-z symmetry planes. Surprisingly, this apparently small error in calculating the normals at the shell section, at and in the vicinity of the crack, caused significant error in the predictions. Later, the exact values of the normals, at and in the vicinity of the crack, were prescribed by the user, which resulted in meaningful predictions.

The variation of the crack-driving force, J , with applied bending moment normalized with the limit moment of a straight pipe with an equivalent flaw is shown in Figure 3.9, as predicted by the line-spring/shell and 3D models. The limit moment, $M_{o(NSC)}$, based on the Net-Section-Collapse approach corresponds to the limit moment of a surface-cracked straight pipe with an equivalent flaw. It can be seen that the predictions of the line-spring model agree very well with the predictions of the 3D models except for extremely high normalized moment values. It is noted that the predictions at very high normalized moment values (greater than 2.0) are of less significance for practical applications. Quite interestingly, unlike circumferentially cracked straight pipes subjected to pure bending, the maximum value of J for the cracked elbow is attained not at the center of the crack-front but approximately 40 degrees away from the center of the crack-front. This is illustrated in Figures 3.10 (a) and (b) in the case of elastic and elastic-plastic deformations, respectively. Again, it is seen that the line-spring/shell model predictions are in good agreement with those of the 3D model.

Since the estimation scheme for cracked elbows was to be developed for pressurized cracked elbows, the effect of pressure on the fracture behavior of cracked elbow under bending was studied using the

Figure 3.9 Variation of J with normalized bending moment as predicted by the line-spring/shell and three-dimensional models

line-spring/shell model. The elbow and crack geometry were identical to the one discussed above. An internal pressure of 15.5 MPa (2,250 psi) along with the corresponding end-cap loads were applied to the cracked elbow prior to the application of the bending moment. The variation of the crack-driving force, J , with subsequent applied bending moment is shown in Figure 3.11, which also shows a similar variation for the unpressurized elbow. That the effect of the internal pressure is rather minimal is illustrated in this figure. In contrast, the effect of pressure is more significant in the case of straight pipes. This is illustrated in Figure 3.12 for a circumferentially cracked straight pipe with $\theta/\pi = 0.5$ and $a/t = 0.5$.

3.3 Discussion

Though the line-spring/shell model idealizes the 3D surface-crack problem as a two-dimensional shell problem, the above results show that this model provides a very reasonable estimate of the fracture parameters of interest. In addition to its capability of providing reasonably accurate estimates of the fracture parameters, the line-spring/shell model is shown to compare well with experimental data. Thus, owing to the ease and inexpensive nature of computation, the line-spring model is ideally suited for the analysis of surface-cracked pipes and elbows. In any approximation of the actual surface-crack problem, such as the line-spring/shell model or the GE/EPRI estimation scheme, there is concern about the predictions of these models in the transition between the elastic and fully plastic regions. To overcome this discrepancy the GE/EPRI estimation scheme employs a plastic-zone correction. This is quite reasonable if the surface crack experiences dominantly either a generalized bending moment or a generalized membrane force. However, most of the surface-cracked structures of interest impart both a generalized membrane force and a generalized bending moment on the crack. The ratio of these two can be expected to have a

COMPARISONS OF THE PREDICTIONS OF LINE-SPRING/SHELL
AND THREE-DIMENSIONAL MODELS

(a) Elastic deformation

(b) Elastic-plastic deformation

Figure 3.10 The angular variation of J for the circumferentially cracked elbow as predicted by the line-spring/shell and three-dimensional models at maximum applied load

Figure 3.11 Effect of internal pressure on the variation of J with applied bending moment for a circumferentially cracked elbow

Figure 3.12 Effect of internal pressure on the variation of J with applied bending moment for a circumferentially cracked straight pipe

constant value under elastic or fully plastic conditions. However, the ratio of these two changes as the crack-tip region undergoes a transition from elastic to fully plastic conditions. Owing to the nature of its formulation, the line-spring/shell model can capture this transition.

3.4 References

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4.0 DETERMINATION OF INITIAL PRESSURE APPLIED TO THE CRACKED ELBOWS

As mentioned earlier, the estimation scheme was developed for cracked elbows subjected to in-plane bending under internal pressure conditions. In addition to its applicability for various pipe and crack geometries, the estimation scheme needs to be applicable for a wide variety of materials, namely ferritic steels and austenitic stainless steels. However, the allowable pressure, p_a , for nuclear piping as proposed in ASME Section III, Article NB-3640 (Ref. 4.1), depends on the design stress intensity value, S_m , for a given material under operating conditions. Since it is a formidable task to develop the estimation scheme for cracked elbows with several initial pressure levels depending on the piping material and operating temperature, the applied pressure level in this work is calculated from an average value of S_m obtained from S_m values for various piping materials at an operating temperature of 288 C (550 F). Table 4.1 summarizes the typical S_m values of various ferritic and austenitic nuclear piping steels at 288 C (550 F) and the average S_m value from Reference 2.2.

Table 4.1 Typical S_m values for several piping steels at 288 C (550 F)

Material	S_m , MPa	S_m , ksi
SA333 Gr 6	124.8	18.10
SA516 Gr 70	135.1	19.60
SA106 Gr B	124.8	18.10
SA106 Gr C	142.0	20.60
SA304	116.9	16.95
SA304L	99.30	14.40
SA304LN	116.5	16.90
SA316LN	120.7	17.50
Average	122.5	17.77

The average of the S_m values at 288 C (550 F) was used in determining the allowable pressure to be applied to the elbow. The allowable pressure, p_a , for straight pipes according to the ASME Section III, Article NB-3640, is given by

$$p_a = \frac{2S_m t_{\text{pipe}}}{D_o - 2\Upsilon t_{\text{pipe}}} \quad (4-1)$$

where D_o is the outer diameter of the pipe, t_{pipe} is the actual thickness of the pipe, and the constant Υ is taken to be 0.4. The above equation is modified for elbows by replacing t_{pipe} with the thickness, t , of a long-radius elbow corresponding to the same schedule as the straight pipe. For example, a 16-inch Schedule 100 straight pipe has a typical thickness of 26.2 mm (1.03 inch), while the thickness of a Schedule 100 elbow has a typical thickness of 33.0 mm (1.30 inch). Thus, t_{pipe} in Equation 4-1, is replaced

by 0.8t, along with the average S_m value to calculate the initial pressure applied to the elbows. The result of the calculations are summarized in Table 4.2.

Table 4.2 Initial pressure applied to elbows

R_m/t	Pressure, MPa (psi)
5	20 (2,900)
10	10 (1,440)
20	5 (720)

The above pressure levels were used in the analyses to develop the estimation scheme for cracked elbows.

4.1 References

- 4.1 1995 ASME Boiler and Pressure Vessel Code, Section III, Division 1, Subsection NB, Class 1 Components.
- 4.2 1995 ASME Boiler and Pressure Vessel Code, Section II, Part D - Properties.

5.0 DEVELOPMENT OF J-ESTIMATION SCHEME FOR CRACKED ELBOWS

In this section, the results of the J-estimation scheme for circumferentially and axially cracked elbows are presented. The geometry of an elbow can be completely characterized by three geometric parameters. These are the angle of the elbow, ψ , the ratio of the radius of the elbow and the radius of the pipe, R_{el}/R_m , and the ratio of the mean radius of the pipe and the wall thickness of the pipe, R_m/t (see Figure 3.6). Conversely, as shown by Griffiths (Ref. 5.1) an elbow parameter, λ , given below, appears to characterize the deformation behavior of an elbow better than the ratio, R_{el}/R_m .

$$\lambda = \frac{R_{el}t}{R_m^2} \quad (5-1)$$

Accordingly, this elbow parameter was used in the development of the J-estimation scheme. The material behavior was idealized as a Ramberg-Osgood material, whose uniaxial response is represented by

$$\epsilon^T = \epsilon^e + \epsilon^p = \frac{\sigma}{E} + \alpha \epsilon_0 \left[\frac{\sigma}{\sigma_0} \right]^n \quad (5-2)$$

where ϵ^T , ϵ^e , and ϵ^p are the total, elastic, and plastic strains, σ_0 and ϵ_0 are the initial yield stress and yield strain, and α and n are the strain-hardening coefficient and exponent, respectively. The analyses were conducted using a rate-independent plasticity model assuming an associated flow rule. With this assumption, when the crack-tip region is in the elastic domain, plasticity is precluded unlike in the case of deformation plasticity theory. Thus, the prediction of the plastic part of the J-integral is less approximate.

The spirit of the determination of the crack-driving force is similar to that using the line-spring or GE/EPRI method, where an additive decomposition of the crack-driving force into an elastic and plastic part is assumed. The calculation of the elastic part of the crack-driving force, J_e , is quite similar using these two methods. The plastic part of the crack-driving force, J_p , was calculated incrementally using Equations 2-11 through 2-13. Using the line-spring/shell model, a normalization approach is used in the GE/EPRI method. This normalization method allows the determination of J_p with a knowledge of a coefficient function, namely h_1 , which, for a given loading type, depends on the pipe and crack geometry parameters and on only one of the material property parameters, namely the strain-hardening exponent, n . Strictly valid in the case of deformation theory of plasticity and under incompressibility and fully plastic conditions, the above normalization is an extension of the well-known Ilyushin postulate (Ref. 5.2). Ilyushin showed that the solution to a given boundary value problem involving a single monotonically increasing load parameter, say Q , reveals that all local stress related quantities are proportional to Q , while all local strain related quantities are proportional to Q^n , where n is the strain-hardening exponent. In addition, these quantities are proportional to a dimensional coefficient function that depends on the local spatial position and only the strain-hardening exponent, n . Using this postulate, Goldman and Hutchinson (Ref. 5.3) showed that the crack-driving force under fully plastic conditions is proportional to Q^{n+1} and a coefficient function that depends on the geometry of the cracked structure and the strain-hardening

exponent, n . A similar procedure was adopted in determining the other relevant fracture parameters. This procedure is extended to account for elastic-plastic deformations in the GE/EPRI estimation scheme (Ref. 5.4). This scheme is aimed at providing an interpolation over the entire range of elastic-plastic deformations from small-scale to fully plastic yield conditions. For reasons mentioned in Section 3.3, this scheme is quite approximate in the transition region of elastic to fully plastic conditions. Despite this shortcoming, the simple nature of the scheme provides an attractive and inexpensive alternative to numerical solutions of full field boundary value problems involving surface-cracked structures. Additionally, the scheme eminently lends itself to developing Codes/Standards used to ensure structural integrity of nuclear piping systems. Accordingly, the results of the line-spring/shell model for cracked elbows were utilized in developing a GE/EPRI type estimation scheme. In so doing, a slight departure from the standard way of developing J-estimation schemes or more exactly determining the coefficient function, h_1 , adopted in this study is explained below.

Typically, in developing a GE/EPRI type J-estimation scheme from the results of finite element analyses, the cracked structure is loaded to very high loads such that the value of the h_1 function determined using the total J predicted by the finite element model does not vary much (or is essentially constant) with increasing loads. This saturated value defines the coefficient function h_1 for the given cracked geometry and loading under fully plastic conditions. Additional information of such calculations may be found in Reference 5.5. It is noted, however, that the small-displacement gradient assumption adopted in the finite element procedure may well be violated when the cracked structure is loaded to such high loads which render an essentially constant h_1 value. It is not clear whether this violation may significantly affect the h_1 values. In contrast, the line-spring/shell model calculates both the elastic part and plastic part of the crack-driving force using the procedure explained in Section 2.0. The results of J_p predicted by the line-spring/shell model were used to determine the h_1 function value over the entire range of applied load using a curve-fitting program. Some examples of this fit, to be shown later, demonstrate its validity. It is believed that this procedure improves the predictions of the estimation scheme in the transition region between elastic and fully plastic conditions. For these reasons, in the scheme described below a plastic-zone correction to the elastic part of the crack-driving force is not incorporated. However, it is relatively easy to include the plastic-zone correction if a user so desires.

5.1 A J-Estimation Scheme for Circumferentially Surface-Cracked Elbows

The line-spring shell model was used in developing a J-estimation scheme for circumferentially surface-cracked pressurized elbows subjected to in-plane bending. An end-cap loading consistent with the applied internal pressure was assumed. The applied internal pressure was calculated from an average S_m value as discussed in Section 4.0. This scheme is restricted to long-radius ($R_e/R_m = 3$), 90-degree ($\psi = 90$ degrees) elbows subjected to in-plane closing bending moments. A constant depth crack with a ratio of crack angle to circumference, θ/π , of 0.5 located at the inside surface of the extrados of the elbow was assumed. The estimation scheme is developed for the maximum J along the crack front assuming self-similar growth. With these assumptions, solutions of F_1 and h_1 functions were developed for several R_m/t and a/t ratios as well as several strain-hardening exponent values. During the course of the line-spring/shell finite element analyses, it was observed that the initial pressure and end-cap loading for all the cracked elbow geometries considered here experienced pure elastic loading, thus obviating the need to calculate the h_1 function associated with internal pressure loading. Owing to the combined loading conditions that persist during

subsequent bending, the h_1 function associated with bending has an implicit dependence on the internal applied pressure, though this dependence is expected to be small as discussed in Section 3.3. Nonetheless, the estimation scheme developed for circumferentially cracked elbows must be used with caution when the applied internal pressure is significantly different from those given in Section 4.0.

The equations pertaining to the J-estimation scheme for circumferentially surface-cracked elbows are given below. The crack-driving force, J , is decomposed into elastic and plastic parts. The elastic part of J is obtained through the knowledge of the stress intensity factor associated with internal pressure, K_I^{pr} , and bending moment, K_I^M . These stress intensity factors are in turn obtained from F_1 functions as described below

$$\begin{aligned}
 J &= J_e + J_p \\
 J_e &= \frac{(1-\nu^2)}{E} \left[\left(K_I^{pr} \right)^2 + \left(K_I^M \right)^2 \right] \\
 K_I^{pr} &= \frac{p R_m}{2t} \sqrt{\pi a} F_1^{pr} \left(\psi, \lambda, \frac{R_m}{t}, \frac{\theta}{\pi}, \frac{a}{t} \right) \\
 K_I^M &= \frac{M R_m}{I} \sqrt{\pi a} F_1^M \left(\psi, \lambda, \frac{R_m}{t}, \frac{\theta}{\pi}, \frac{a}{t} \right)
 \end{aligned} \tag{5-3}$$

The plastic part of the crack-driving force is given by

$$J_p = \alpha \epsilon_o \sigma_o c \left(\frac{R_m}{t \lambda} \right) h_1^M \left[\frac{M}{M_o} \right]^{(n+1)}; c = t - a \tag{5-4}$$

where M_o is some reference moment of a cracked elbow and the coefficient function, h_1 , is given by

$$h_1^M = h_1^M \left(\psi, \lambda, \frac{R_m}{t}, \frac{a}{t}, \frac{\theta}{\pi}; n; p \right) \tag{5-5}$$

Though all relevant dependencies are shown in the h_1 function, it is emphasized that the dependency on ψ and circumferential crack length, θ/π , are frozen since we only consider $\psi = 90$ degrees and $\theta/\pi = 0.5$ here, and the dependency on the internal pressure, p , is limited as explained earlier. The choice of the reference moment, M_o , used in Equation 5.4 is quite arbitrary. However, appropriate, h_1^M function values amenable to interpolation are obtained only with a proper choice of reference moment, M_o . Typically, the limit-load moment of the cracked component is used for normalization purposes. Owing to the fact that no simple expression for the limit moment for circumferentially cracked elbows is available in the published literature, two different expressions for M_o , both of which pertain to a straight pipe with an equivalent crack were used. The two different expressions for M_o were used to examine the effect of the normalization on the resulting variation of h_1^M functions. The following expressions were used for M_o :

(I) GE/EPRI approach (used for surface-cracked straight pipes and termed the SC.ELB2 method)

$$M_o = M_o^{nc} \frac{A^{nc}}{A} \quad (5-6)$$

where

A = total area of cross-section,
 A^{nc} = area of uncracked cross-section, and
 $M_o^{nc} = 2\sigma_o t (R_o^2 + R_i^2)$.

(II) Net-Section Collapse (NSC) Equation (used for surface-cracked straight pipes and termed the SC.ELB1 method)

For $\beta \leq \pi - \theta$

$$M_{o(NSC)} = 2R_m^2 t \sigma_f \left[2\sin\beta - \frac{a}{t} \sin\theta \right] \quad (5-7)$$

$$\beta = \frac{\pi - \theta(a/t)}{2} - \frac{\pi R_i^2 p}{4R_m t \sigma_f}$$

For $\beta > \pi - \theta$

$$M_{o(NSC)} = 2R_m^2 t \sigma_f \left[2\sin\beta - \frac{a}{t} \sin\theta \right] \quad (5-8)$$

$$\beta = \frac{\pi}{2 - a/t} \left[1 - a/t - \frac{R_i^2 p}{2R_m t \sigma_f} \right]$$

Using the above expressions, the F_1 and h_1^M functions, given in Tables 5.1 through 5.3, were obtained from FEM results. Comparisons of the finite element results of the plastic part of the crack-driving force with the functional dependence as given in Equation 5.4, shown in Figures 5.1 (a) and (b) for selected cases, illustrate the reasonable accuracy of the fit.

It can be seen from Tables 5.2 and 5.3, that the h_1^M functions using the GE/EPRI definition of M_o (SC.ELB2 method) renders a smoother variation of the functions, thus enabling better interpolation. The nature of the variation of the h_1^M functions with the strain-hardening exponent, n , and the ratio of crack-depth to wall thickness, a/t , obtained using the GE/EPRI and NSC definition of the limit moment, M_o , is shown in Figures 5.2 (a) and (b), respectively, for $R_m/t = 10$. These figures clearly demonstrate the non-smooth variation of the h_1^M functions obtained using the NSC definition of M_o (SC.ELB1 method), which causes some concern in performing interpolation. This is in contrast to the corresponding smooth variation

Table 5.1 F_1 functions for circumferentially cracked elbows (used in both SC.ELB1 and SC.ELB2 methods) [$\theta/\pi = 0.5$, $\psi = 90$ degrees]

R_m/t	a/t	F_1^{Pr}	F_1^M
5	0.3	1.42	1.01
5	0.5	1.68	1.22
5	0.75	1.98	1.56
10	0.3	1.21	1.65
10	0.5	1.49	2.14
10	0.75	1.97	2.81
20	0.3	1.26	3.09
20	0.5	1.74	4.13
20	0.75	2.29	5.77

Table 5.2 h_1^M functions for circumferentially cracked elbows using GE/EPRI definition of M_o (SC.ELB2 method) [$\theta/\pi = 0.5$, $\psi = 90$ degrees]

λ	R_m/t	a/t	$R_m/\lambda t$	3	5	10
0.6	5	0.3	8.33	0.24	0.49	1.16
		0.5	8.33	0.91	1.45	1.86
		0.75	8.33	4.17	4.65	39.9
0.3	10	0.3	33.3	0.26	1.04	9.50
		0.5	33.3	1.04	2.74	12.3
		0.75	33.3	3.24	9.99	65.4
0.15	20	0.3	133	0.37	2.97	151
		0.5	133	2.42	11.9	245
		0.75	133	17.4	43.5	521

Table 5.3 h_1^M functions using NSC equation for M_o (SC.ELB1 method) [$\theta/\pi = 0.5$, $\psi = 90$ degrees]

λ	R_m/t	a/t	$R_m/\lambda t$	$n=3$	$n=5$	$n=10$
0.6	5	0.3	8.33	0.12	0.17	0.16
		0.5	8.33	0.22	0.17	0.03
		0.75	8.33	0.16	0.03	0.01
0.3	10	0.3	33.3	0.13	0.34	1.27
		0.5	33.3	0.24	0.30	0.21
		0.75	33.3	0.01	0.05	0.01
0.15	20	0.3	133	0.17	0.95	18.9
		0.5	133	0.53	1.21	3.72
		0.75	133	0.43	0.17	0.02

(a) $R_m/t = 5$, $a/t = 0.5$, $n = 5$

(b) $R_m/t = 10$, $a/t = 0.5$, $n = 5$

Figure 5.1 Comparisons of the finite element predictions and the fit using the functional dependence given in Equation 5.4 for a circumferentially cracked elbow

(a) GE/EPRI (SC.ELB2)

(b) NSC (SC.ELB1)

Figure 5.2 The variation of the h_1^M functions with the strain-hardening exponent, n , and the ratio of crack-depth to wall thickness, a/t , obtained using the GE/EPRI (SC.ELB2) and NSC (SC.ELB1) definition of the limit moment, M_o

obtained using the GE/EPRI definition (SC.ELB2 method). An estimation scheme for crack-mouth-opening displacements is not possible using the line-spring/shell model, since these displacements cannot be obtained due to the two-dimensional idealization of the surface-cracked elbow geometry.

5.2 A J-estimation Scheme for Axially Surface-Cracked Elbows

A schematic sketch of an axially surface-cracked elbow is shown in Figure 5.3. Two constant-depth cracks are assumed to be present on the internal surface of the pipe at the flanks (or crowns) of the elbow. A 30-degree long flaw is assumed. Owing to this geometry, the opening of the crack will occur when an in-plane opening moment is applied, the sign of which is opposite to that for circumferentially cracked elbows. The initial internal pressure and the corresponding end-cap loading are identical to those assumed for the circumferentially cracked elbows. This scheme is restricted to a long-radius, 90-degree, axially cracked elbow. Unlike in the case of circumferentially cracked elbows, the presence of internal pressure in the axially cracked elbows causes significant yielding at the crack-tip. Hence, there is a need to develop h_1 functions for pressure loading in addition to developing h_1 functions for bending. This scheme should be used with caution when the internal pressure that persists in an axially cracked elbow is significantly different from those given in Section 4.0. It is assumed that the total crack-driving force, J , for a pressurized axially cracked elbow subjected to bending can be written as the sum of the elastic part due to internal pressure and bending, J_e^{pr+M} , the plastic part due to internal pressure, J_p^{pr} , and the plastic part due to subsequent bending, J_p^M , as given by

$$J = J_e^{pr+M} + J_p^{pr} + J_p^M \quad (5-9)$$

with the elastic part given by

$$J_e^{pr+M} = \frac{[1-\nu^2]}{E} [K_I^{pr} + K_I^M]^2 \quad (5-10)$$

It is noted that the non-linear nature of combined pressure and bending for J_p^M is accounted for since the pressure is constant and is present when J_p^M is calculated.

$$K_I^{pr} = p \frac{R_m}{t} \sqrt{\pi a} F_I^{pr} \left(\psi, \lambda, \frac{R_m}{t}; \ell, \frac{a}{t} \right) \\ \text{and} \quad (5-11) \\ K_I^M = \frac{MR_m}{I} \sqrt{\pi a} F_I^M \left(\psi, \lambda, \frac{R_m}{t}; \ell, \frac{a}{t} \right)$$

Figure 5.3 Schematic sketch of an axially cracked elbow subjected to an in-plane opening moment

The plastic part of the crack-driving force due to pressure loading is assumed to be represented by

$$J_p^{pr} = \alpha \sigma_o \epsilon_o c \lambda h_1^{pr} \left[\frac{p}{p_o} \right]^{n+1}; \quad c = t - a \quad (5-12)$$

$$h_1^{pr} = h_1^{pr} \left(\psi, \lambda, \frac{R_m}{t}; \ell, \frac{a}{t}; n \right)$$

where p_o is the limit pressure of a corresponding straight pipe with an equivalent axial flaw. The limit pressure, p_o , is calculated using the following expression given in the ASME Section XI Boiler and Pressure Vessel Code (Ref. 5.6) and based on past Battelle work (Ref. 5.7)

$$p_o = \frac{\sigma_o t}{R_m} \frac{[t/a - 1]}{\left[t/a - \frac{1}{M_2} \right]} \quad (5-13)$$

with

$$M_2 = \left[1 + \left(\frac{1.61}{4R_m t} \right) \ell^2 \right]^{0.5} \quad (5-14)$$

where ℓ is the total length of the axial crack. The plastic part of the crack-driving force due to subsequent bending is represented by

$$J_p^M = \alpha \sigma_o \epsilon_o c \left[\frac{R_m}{t \lambda} \right] h_1^M \left[\frac{M}{M_o} \right]^{n+1}; \quad c = t - a \quad (5-15)$$

and

$$h_1^M = h_1^M \left(\psi, \lambda, \frac{R_m}{t}, \frac{\theta}{\pi}, \frac{a}{t}; n; p \right)$$

$$M_o = 0.935 \sigma_o (8 R_m^2 t) \lambda^{2/3} \quad (5-16)$$

In the above expression, M_o denotes twice the limit moment for an uncracked elbow proposed by Griffiths (Ref. 5.1)*. Using the above expressions and the FEM results the F_1 and h_1 functions were generated and are presented in Tables 5.4 through 5.6. This J-estimation scheme is called SC.ELB3.

Owing to the unavailability of other representations of p_o and M_o for the axially cracked elbow, variations of h_1 functions smoother than those given in Tables 5.5 and 5.6 could not be obtained.

5.3 Investigation of Rotations at the Elbow/Straight Pipe Junction of the Cracked Elbows

In addition to the estimation scheme for the crack-driving force in cracked elbows, an estimation scheme for rotations of the cracked elbow is useful for predicting the global behavior of cracked elbows which can serve as a valuable tool in calibrating experiments. In order to develop such an estimation scheme the rotations due to the presence of the crack are determined by eliminating the rotation contribution of an identical uncracked elbow from that of a cracked elbow at some given location. In the case of elbows an ideal location is at the junction of the elbow and straight pipe section. In addition to the calculations performed on the cracked elbows, analyses of pressurized uncracked elbows subjected to both in-plane

* This choice of the reference moment, M_o , rendered the h_1^M functions more amenable to interpolation.

Table 5.4 F_1 functions for axially cracked elbows (SC.ELB3 method)
[total crack angle = 30 degrees, $\psi = 90$ degrees]

R_m/t	a/t	F_1^{pr}	F_1^M
5	0.3	2.491	3.586
	0.5	2.907	3.634
	0.75	3.133	2.976
10	0.3	2.278	3.118
	0.5	3.054	3.549
	0.75	4.133	3.709
20	0.3	2.017	2.553
	0.5	2.944	3.145
	0.75	5.153	4.210

Table 5.5 h_1^{pr} functions for axially cracked elbows (SC.ELB3 method)
[total crack angle = 30 degrees, $\psi = 90$ degrees]

R_m/t	λ	a/t	n		
			3	5	10
5	0.6	0.3	5.513	9.296	18.52
		0.5	12.51	9.705	5.274
		0.75	5.235	1.322	0.094
10	0.3	0.3	6.122	8.910	9.221
		0.5	19.81	12.90	2.759
		0.75	10.04	1.784	0.057
20	0.15	0.3	7.807	10.41	0.849
		0.5	32.46	21.79	2.956
		0.75	26.48	3.919	0.141

Table 5.6 h_1^M functions for axially cracked elbows (SC.ELB3 method)
[total crack angle = 30 degrees, $\psi = 90$ degrees]

R_m/t	λ	$R_m/\lambda t$	a/t	n		
				3	5	10
5	0.6	8.33	0.3	11.84	67.96	3350
			0.5	49.85	194.5	9366
			0.75	117.1	983.3	44748
10	0.3	33.3	0.3	2.193	9.744	226.9
			0.5	11.54	28.79	537.6
			0.75	28.59	64.63	1862
20	0.15	133	0.3	0.539	1.465	12.58
			0.5	3.118	5.813	33.61
			0.75	5.823	11.28	76.58

closing and opening bending were conducted. Comparisons of the rotations of the uncracked elbow with the corresponding predictions for circumferentially or axially cracked elbows revealed little difference between them. This is not surprising since the remaining ligament of the surface-cracked elbow can sustain loads, thereby minimizing the difference in rotations at locations further away from the crack. Figures 5.4 and 5.5 show the negligible effect of the presence of the crack on the rotations at the junction of the elbow and straight pipe for selected cases of circumferentially and axially cracked elbows. Consequently, an h_4 function for rotation due to the crack could not be determined. Additionally, as demonstrated by these results, an η -factor J-estimation scheme integrating the moment-rotation curve for surface-cracked elbows is not a viable method since rotations due to the crack are very small or negligible when compared to the uncracked elbow rotations. An η -factor solution for surface-cracked elbows would have to involve use of moment versus the crack-mouth-opening displacement relationships to develop a sensitive η -factor function. However, the line-spring approach employed here does not give a crack-mouth-opening result for development of an h_2 function which could be used in establishing an η -factor function. 3D analyses are needed to develop the h_2 function.

5.4 Sample Comparisons

The estimation schemes developed for circumferentially and axially cracked elbows were implemented into a computer code called IP2ELBOW. This code used the basic aspects of a recent windows version of the NRCPIPES Code (Reference 5.8). This code performs initiation and instability analyses in either load or displacement control using the J-estimation schemes developed in this report.

The IP2ELBOW program uses the acronyms SC.ELB1 and SC.ELB2 for the J-estimation schemes for circumferential, extrados, surface-cracked, pressurized elbows. The SC.ELB1 J-estimation scheme used the reference moment, M_o , definition based on the original Net-Section-Collapse equation. The SC.ELB2 J-estimation scheme used the reference moment, M_o , definition based on a GE/EPRI type of formulation. The IP2ELBOW program uses the acronym SC.ELB3 (M_o definition based on the Griffith equation) for the J-estimation scheme for axial, flank, surface-cracked, pressurized elbows.

Sample calculations were made using the IP2ELBOW code to compare J versus moment curves for the axial and circumferential surface-cracked elbow solutions to the solution for a circumferential surface-crack in a straight pipe. The elbow, extrados, circumferential, surface-crack, J-estimation scheme used was the SC.ELB2 method. The elbow, flank, axial, surface-crack, J-estimation scheme used was the SC.ELB3 method. The straight-pipe, circumferential, surface-crack, J-estimation scheme used was the SC.TNP2 analysis. The SC.TNP2 scheme was compared to several FEM results and found to be highly accurate (Reference 5.8) in the case of pure bending. The sample calculations were performed for circumferentially cracked pressurized elbows, pressurized straight pipes and unpressurized straight pipes with the same diameter-to-thickness ratio as well as the same flaw geometry. For the circumferential crack case the flaw length was 50 percent of the circumference in the elbow and straight pipe analyses, but the elbow axial crack case the flaw is 30-degrees along the 90-degree elbow. This actual length corresponds to a circumferential crack that could be 32 percent of the circumference. The applied pressure was 10 MPa (1,440 psi). The calculations were performed for two R_m/t ratios. The material properties used in these calculations pertain to a typical TP304 stainless steel hot-bent elbow, where $E = 21.0$ GPa (30,000 ksi), $\nu = 0.3$, $\sigma_o = 174$ MPa (25.24 ksi), $n = 4.565$ and $\alpha = 4.647$.

Figure 5.4 Comparisons of FEM rotations at the junction of a straight pipe and elbow for uncracked and circumferentially cracked elbows with several a/t ratios and $R_m/t = 10$ for a strain-hardening exponent $n = 5$

Figure 5.5 Comparisons of FEM rotations at the junction of a straight pipe and elbow for uncracked and axially cracked elbows with several a/t ratios and $R_m/t = 10$ for a strain-hardening exponent $n = 5$

Figures 5.6a and 5.6b show the variation of J with applied bending moment for $R_m/t = 10$ and $R_m/t = 20$, respectively. Two main conclusions may be drawn from these figures. First, the moment values at crack initiation for an axial surface crack in the elbow are 1.5 to 2 times higher than for the circumferential surface crack in the elbow for typical nuclear materials (depending on the R_m/t ratio). This is a significant difference. Second, for the flaw size examined here, the moment values for pressurized straight pipe were somewhat lower than the corresponding values for circumferentially cracked elbows in the case of R_m/t of 10. In the case when R_m/t is 20, the pressurized straight pipe moment value was approximately 1.5 times greater than the circumferential elbow moment value for J values at crack initiation. Note also the significant effect of pressure in the case of straight pipes demonstrated by the SC.TNP2 method. In contrast, the effect of pressure is minimal for circumferentially cracked elbows as discussed in Section 3.0.

Additional sensitivity studies and comparisons with pipe fracture experiments will be presented in a separate IPIRG-2 program report.

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(a) $R_m/t = 10$ (b) $R_m/t = 20$

Figure 5.6 Variation of J with applied bending moment for axially and circumferentially cracked elbows and circumferentially cracked straight pipe

6.0 SUMMARY AND CONCLUSIONS

Using the line-spring/shell model, estimation schemes for determining the crack-driving force for circumferentially as well as axially cracked pressurized elbows have been presented. Though the line-spring/shell model idealizes the three-dimensional surface-crack problem as a two-dimensional shell problem, the results presented in Section 3.0 show that this model provides a very reasonable estimate of the fracture parameters of interest. In addition to its capability of providing a reasonably accurate estimate of the fracture parameters, the line-spring/shell model is shown to compare well with experimental data. Thus, owing to the ease and inexpensive nature of computation, the line-spring model is ideally suited for the analysis of surface-cracked pipes and elbows.

The estimation scheme follows the spirit of the GE/EPRI scheme and applies to in-plane bending of cracked elbows. A closing in-plane moment is assumed for an internal circumferential surface crack located at the extrado of the elbow, while an opening in-plane moment is assumed for an internal axial surface crack located at the crown of the elbow. The applied internal pressure was determined using an average S_m value at 288 C (550 F) of various ferritic piping steels and austenitic stainless piping steels used in the U.S.

The elastic F functions for pressure and bending are tabulated in Section 5.0 for a circumferentially cracked elbow. Two different representations of a reference moment, M_o , were used to obtain the coefficient functions, h_1 , for estimating the plastic part of the crack-driving force due to bending under constant applied pressure. The SC.ELB2 J-estimation scheme, which used the GE/EPRI reference moment formula, yielded a fairly smooth distribution of the h_1 function, in contrast to the distribution obtained using the SC.ELB1 J-estimation scheme with the reference moment based on the Net-Section Collapse formula. In order to ensure reliable estimates of J , which in many instances involve interpolation of h_1 values, it is important to have a smooth variation of these functions with changes in pipe and crack geometries as well as material hardening exponent, n .

For axially surface-cracked elbows the elastic F functions for pressure and bending, as well as the plastic h_1 functions for pressure loading are tabulated in Section 5.0. In addition, the coefficient functions, h_1 , for estimating the plastic part of the crack-driving force during subsequent bending (under constant pressure) are calculated using a reference limit moment, M_o , for an uncracked elbow. Owing to the non availability of other representations for a reference pressure, p_o , and a reference moment, M_o , for axially cracked elbows, smoother variations than those given in Section 5.0 could not be obtained. This J-estimation scheme was named SC.ELB3.

The three estimation schemes (SC.ELB1, SC.ELB2, and SC.ELB3) developed were implemented into a computer code called IP2ELBOW. Sample calculations for circumferentially and axially cracked elbows were made using this code for two different R_m/t ratios for $a/t = 0.5$ and $n = 5$. The results of these calculations were compared with those obtained using SC.TNP2 for circumferentially cracked straight pipes using the same pipe and flaw geometry as in the case of circumferentially cracked elbows. For typical nuclear materials, the results of the comparisons showed that for an axial surface crack in the elbow, and depending the R_m/t ratio, the moment values at crack initiation are 1.5 to 2 times lower than for the circumferential surface crack in the elbow. This is a significant difference. Moreover, for the flaw size examined here, the moment values for pressurized straight pipe were somewhat lower than the

corresponding values for circumferentially cracked elbow in the case of R_m/t of 10. In the case when R_m/t is 20, the pressurized straight pipe moment value was approximately 1.5 times greater than the circumferential elbow moment value for J values at crack initiation.

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10. SUPPLEMENTARY NOTES

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11. ABSTRACT (200 words or less)

This report summarizes efforts to develop elastic and elastic-plastic fracture mechanics analyses for internal surface cracks in elbows. The analyses involved development of a GE/EPRI type J-estimation scheme from a matrix of finite element analyses. The following parameters were covered; 90-degree long-radius elbows, various R_m/t ratios, and combined pressure and in-plane bending. For the combined pressure and bending analyses, the hoop stress was fixed to correspond to an average S_m value for typical U.S. nuclear piping materials. Further factors included in the analyses were various strain-hardening exponents and a/t values; however, the circumferential cracked elbow was limited to one crack length, and the axial cracked elbow was limited to one crack length on the elbow flank. These analyses were implemented into a computer code called IP2ELBOW. The results from the computer code calculations showed that the moment values at crack initiation were 1.5 to 2 times lower for the axially cracked elbow than for the circumferentially cracked elbow. The moment values for pressurized straight pipe were somewhat lower than for circumferentially cracked elbows for R_m/t of 10, but were 1.5 greater for R_m/t of 20.

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