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FORMULAS, TABLES, AND CURVES FOR COMPUTING THE MUTUAL INDUCTANCE OF TWO COAXIAL CIRCLES

BY

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FORMULAS, TABLES, AND CURVES FOR COMPUTING THE MUTUAL INDUCTANCE OF TWO COAXIAL CIRCLES

By Harvey L. Curtis and C. Matilda Sparks

ABSTRACT

The formulas for the mutual inductance of coaxial circles can be expressed either in terms of the radii of the circles and the distance between them, or in terms of a parameter which is a function of these dimensions. Both types of formulas are developed in this paper, and tables prepared to aid in the computation by each type. Charts are included from which an approximate value of the mutual inductance can be read directly. A number of examples are given to show the methods of computation by the different formulas, tables, and charts.

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I. INTRODUCTION

An exact formula for the mutual inductance of coaxial circles was derived by Maxwell¹ in terms of elliptic integrals. However, it does not readily lend itself to computation either when the coils are close together or when they are far apart.² For this reason a large number of series formulas³ have been developed. Some of these give accurate values of the mutual inductance when the two coils are close together, others when the coils are far apart, and some in intermediate ranges. For any given case it is possible to select one or more formulas which will give an accurate value of the mutual inductance. However, most of these are somewhat complicated, and difficulty is often experienced in applying them to numerical examples.

By using several different formulas to make the necessary computations, the Bureau of Standards has published a table⁴ which provides a simple method for computing the mutual inductance of any two coaxial circles. The formula for use with this table is of the form $M = \sqrt{Aa} C$, the values of C as a function of k' (defined later) being given in the table.

¹ Electricity and Magnetism, 2, art. 701.

² Computation by this formula has been simplified, and its range of usefulness greatly extended by the tables of elliptic integrals recently published by Nagaoka and Sakurai. These tables, entitled "Tables of Theta Functions, Elliptic Integrals K and E , and Associated Coefficients in the Numerical Calculation of Elliptic Functions," were published by the Tokyo Institute of Physical and Chemical Research in December, 1922.

³ These formulas have been collected and their limitations pointed out by Rosa and Grover. B. S. Bull., 8, pp. 6-32, B. S. Sci. Paper No. 169; and 14, pp. 538-544, B. S. Sci. Paper No. 320.

⁴ This is published in B. S. Circular No. 74, entitled "Radio Instruments and Measurements," p. 275. The table was prepared by Dr. F. W. Grover, one of the authors of the circular. However, the method of computing the table is not indicated.

In the present paper formulas are derived and tables of constants given for computing by two methods the mutual inductance of any two coaxial circles. The tables can also be used in solving the inverse problem, viz, determining the dimensions of circles which will give a desired mutual inductance. In addition, charts are given from which an approximate value of the mutual inductance can be read directly, and from which the dimensions of circles to give a desired mutual inductance can be determined by inspection.

II. THE NOMENCLATURE

The constants which are generally measured, and from which the others are derived, are A , the radius of the larger circle; a , the radius of the smaller circle; and D , the distance between the centers of the circles. From these are determined the constants used in computation, viz:

$$r_1^2 = (A + a)^2 + D^2 \quad (1)$$

$$r_2^2 = (A - a)^2 + D^2 \quad (2)$$

$$k^2 = \frac{r_1^2 - r_2^2}{r_1^2} = \frac{4Aa}{(A + a)^2 + D^2} = \frac{\overset{4a \rightarrow a}{\cancel{A}}}{\left(1 + \frac{a}{A}\right)^2 + \frac{D^2}{A^2}} \quad (3)$$

$$k'^2 = 1 - k^2 = \frac{r_2^2}{r_1^2} = \frac{(A - a)^2 + D^2}{(A + a)^2 + D^2} = \frac{\left(1 - \frac{a}{A}\right)^2 + \frac{D^2}{A^2}}{\left(1 + \frac{a}{A}\right)^2 + \frac{D^2}{A^2}} \quad (4)$$

$$k_1 = \frac{4aA}{(r_1 + r_2)^2} = \frac{4aA}{[\sqrt{(A + a)^2 + D^2} + \sqrt{(A - a)^2 + D^2}]^2} \quad (5)$$

FIG. 1.—Diagrammatic cross section of two coaxial circles showing dimensions

When $D = 0$

$$k_1 = \frac{a}{A}$$

When $D \neq 0$

$$k_1 = \frac{\frac{4a}{A}}{\left[\sqrt{\left(1 + \frac{a}{A}\right)^2 + \frac{D^2}{A^2}} + \sqrt{\left(1 - \frac{a}{A}\right)^2 + \frac{D^2}{A^2}} \right]^2} \quad (6)$$

When D is large relative to A

$$k_1 = \frac{\frac{4a}{A} \cdot \frac{A^2}{D^2}}{\left[\sqrt{1 + \frac{A^2}{D^2} \left(1 + \frac{a}{A}\right)^2} + \sqrt{1 + \frac{A^2}{D^2} \left(1 - \frac{a}{A}\right)^2} \right]^2} \quad (7)$$

The above is the nomenclature used by Rosa and Grover in Scientific Paper No. 169, with the exception that D is used for d . The geometric meaning of r_1 and r_2 is shown in Figure 1.

Since $k' = \frac{r_2}{r_1}$, and since r_2 is always less than r_1 , then k' is always less than unity, which value it approaches as a limiting case. Likewise, all values of k lie between zero and unity.

The other terms used are the complete elliptic integrals⁵ K and E . They will be defined where used in terms of series.

The natural logarithm will be indicated by the symbol "logn" while the symbol "log" will be used for common logarithms only.

III. DERIVATION OF SERIES FORMULAS IN TERMS OF k^2 AND k'^2

There are well-known series for expanding the elliptic integrals. However, the application of these series to the elliptic integral formula for mutual inductance is often a tedious process. A simple transformation will simplify the application.

The elliptic integral formula as given by Maxwell is

$$M = 4\pi \sqrt{Aa} \left[\left(\frac{2}{k} - k \right) K - \frac{2}{k} E \right] \quad (8)$$

⁵ The complete elliptic integral of the first kind is represented by K instead of F , which was used in B. S. Sci. Paper No. 169.

From equation (3) $2\sqrt{Aa} = k r_1$. Putting this value in equation (8)

$$M = 2\pi r_1 [(2 - k^2) K - 2 E] \quad (9)$$

If the expansion is to be in terms of k'^2 (equal to $1 - k^2$) then

$$M = 2\pi r_1 [(1 + k'^2) K - 2 E] \quad (10)$$

The series values of K and E in terms of k are ⁶

$$K = \frac{\pi}{2} \sum_{n=0}^{\infty} \frac{(|2n|)^2}{2^{4n} (|n|)^4} k^{2n} \quad (11)$$

$$E = -\frac{\pi}{2} \sum_{n=0}^{\infty} \frac{(|2n|)^2}{2^{4n} (|n|)^4} \cdot \frac{1}{2n-1} k^{2n} \quad (12)$$

Substituting these values in (9)

$$M = \frac{\pi^2 r_1 k^4}{8} \sum_{n=0}^{\infty} \frac{(|2n|)^2}{2^{4n} (|n|)^4} \cdot \frac{2(2n+1)^2}{(n+1)(n+2)} k^{2n} \quad (13)$$

This⁷ may be written as $M = r_1 H$ where H is a function of k^2 . When the coils are far apart, or when one coil is much larger than the other, k^2 is small and the series is rapidly convergent. In this region H is very nearly proportional to k^4 .

The expressions for K and E in terms of k' , given as equation (3) of Scientific Paper No. 169, can be written in the form

$$\begin{aligned} K = & \left(\log n \frac{4}{k'} \right) \left[1 + \frac{1^2}{2^2} k'^2 + \frac{1^2}{2^2} \cdot \frac{3^2}{4^2} k'^4 + \dots \right. \\ & \left. + \frac{1^2}{2^2} \cdot \frac{3^2}{4^2} \dots \frac{(2n-1)^2}{(2n)^2} k'^{2n} + \dots \right] \\ & - \left[\frac{1^2}{2^2} \frac{2}{1 \times 2} k'^2 + \frac{1^2}{2^2} \cdot \frac{3^2}{4^2} \left(\frac{2}{1 \times 2} + \frac{2}{3 \times 4} \right) k'^4 + \dots \right. \\ & \left. + \frac{1^2}{2^2} \dots \frac{(2n-1)^2}{(2n)^2} \left(\frac{2}{1 \times 2} + \frac{2}{3 \times 4} + \dots + \frac{2}{(2n-1)2n} \right) k'^{2n} + \dots \right] \\ = & \sum_{n=0}^{\infty} \frac{(|2n|)^2 k'^{2n}}{2^{4n} (|n|)^4} \left[\frac{1}{2} \log n \frac{16}{k'^2} - \frac{(2n+1)^2 k'^2}{4(n+1)^2} \sum_{s=0}^n \frac{1}{(s+1)(2s+1)} \right] \end{aligned} \quad (14)$$

⁶ B. S. Sci. Paper No. 169, p. 9.

⁷ This equation (13) is the same as equation (5) of B. S. Sci. Paper No. 169.

$$\begin{aligned}
 E = & \left(\log n \frac{4}{k'} \right) \left[\frac{1}{2} k'^2 + \frac{1^2}{2^2} \cdot \frac{3}{4} k'^4 + \dots \right. \\
 & \left. + \frac{1^2}{2^2} \cdot \frac{3^2}{4^2} \dots \frac{(2n-3)^2}{(2n-2)^2} \cdot \frac{2n-1}{2n} k'^{2n} + \dots \right] \\
 & + 1 - \frac{1}{2} \left(\frac{1}{1 \times 2} \right) k'^2 - \frac{1^2}{2^2} \cdot \frac{3}{4} \left(\frac{2}{1 \times 2} + \frac{1}{3 \times 4} \right) k'^4 - \dots \\
 & - \frac{1^2}{2^2} \cdot \frac{3^2}{4^2} \dots \frac{(2n-3)^2}{(2n-2)^2} \frac{(2n-1)}{2n} \left[\frac{2}{1 \times 2} + \frac{2}{3 \times 4} + \dots \right. \\
 & \left. + \frac{2}{(2n-3)(2n-2)} + \frac{1}{(2n-1)2n} \right] k'^{2n} + \dots \\
 = & \sum_{n=0}^{\infty} \frac{(|2n|)^2 k'^{2n}}{2^{4n} (|n|)^4} \left[\frac{n}{2n-1} \log n \frac{16}{k'^2} + \frac{1}{(2n-1)^2} \right. \\
 & \left. - \frac{(2n+1) k'^2}{2(n+1)} \sum_{s=0}^n \frac{1}{(s+1)(2s+1)} \right]
 \end{aligned} \quad (15)$$

Substituting these values in equation (10)

$$\begin{aligned}
 M = & \pi r_1 \left[\left(\log n \frac{16}{k'^2} \right) \left\{ 1 + \frac{1}{2^2} k'^2 + \frac{1}{2^2} \cdot \frac{1^2}{4^2} k'^4 + \frac{1}{2^2} \cdot \frac{1^2}{4^2} \cdot \frac{3^2}{6^2} k'^6 + \dots \right. \right. \\
 & \left. \left. + \frac{1}{2^2} \cdot \frac{1^2}{4^2} \cdot \frac{3^2}{6^2} \dots \frac{(2n-3)^2}{(2n)^2} k'^{2n} + \dots \right\} \right. \\
 & - 2 \left\{ 2 - \frac{k'^2}{4} + \frac{1}{2^2} \cdot \frac{1^2}{4^2} \left(\frac{2}{1 \times 2} - \frac{1}{2} \right) k'^4 + \frac{1}{2^2} \cdot \frac{1^2}{4^2} \cdot \frac{3^2}{6^2} \left(\frac{2}{1 \times 2} + \frac{2}{3 \times 4} - \frac{2}{6} \right) k'^6 \right. \\
 & \left. + \dots + \frac{1}{2^2} \cdot \frac{1^2}{4^2} \cdot \frac{3^2}{6^2} \dots \frac{(2n-3)^2}{(2n)^2} \left(\frac{2}{1 \times 2} + \frac{2}{3 \times 4} + \dots \right. \right. \\
 & \left. \left. + \frac{2}{(2n-3)(2n-2)} - \frac{2}{2n} \right) k'^{2n} + \dots \right\} \\
 = & \pi r_1 \sum_{n=0}^{\infty} \frac{(|2n|)^2 k'^{2n}}{2^{4n} (|n|)^4} \left[\frac{1}{(2n-1)^2} \log n \frac{16}{k'^2} - \frac{4}{(2n-1)^2} \right. \\
 & \left. + \frac{2k'^2}{(n+1)(2n+1)} - \frac{k'^2}{2(n+1)^2} \sum_{s=0}^n \frac{1}{(s+1)(2s+1)} \right] \\
 = & r_1 H
 \end{aligned} \quad (16)$$

where H is a function⁸ of k'^2 and hence of k^2 . This formula can be used for values of k'^2 from 0 to 0.7, provided a large number of terms are used. When k'^2 is small $\log \frac{1}{k'^2}$ becomes large, and the summation depends almost entirely on the value of this logarithm. Hence, in this region H is nearly proportional to $\log k'^2$.

The mutual inductance of any two coaxial circles can be computed either by formula (13) or (16). If k^2 is less than 0.4 (k'^2 greater than 0.6) equation (13) should be used. For larger values of k^2 use equation (16).

By writing the coefficients of the powers of k and k' as decimals, the above equations are in much more convenient form for computation. Equation (13) becomes

$$\frac{M}{r_1} = 1.23370 k^4 \left| \begin{array}{l} 1.00000 \\ +.75000 k^2 \\ +.58594 k^4 \\ +.47852 k^6 \\ +.40375 k^8 \\ +.34895 k^{10} \\ +.30715 k^{12} \\ +.27424 k^{14} \\ +.24767 k^{16} \\ +.22578 k^{18} \\ +.20744 k^{20} \\ + \\ . \\ . \\ . \end{array} \right. \quad (17)$$

⁸ Formula (16) is equivalent to Butterworth's formula D given in *Phil. Mag.* 31, p. 276; 1916. It is given as formula (4A) in Grover's collection, *B. S. Bull.* 14, p. 540. *B. S. Sci. Paper No.* 320.

Writing all numerics as decimals, equation (16) becomes

$$\frac{M}{r_1} = [8.71035 + 7.23378 \log k'^2] \begin{array}{c|c} 1.00000 & -\pi & 4.00000 \\ \hline +.25000 k'^2 & & -.50000 k'^2 \\ +.01562 k'^4 & & +.01562 k'^4 \\ +.00391 k'^6 & & +.00651 k'^6 \\ +.00153 k'^8 & & +.00300 k'^8 \\ +.00075 k'^{10} & & +.00160 k'^{10} \\ +.00042 k'^{12} & & +.00094 k'^{12} \\ +.00026 k'^{14} & & +.00060 k'^{14} \\ +.00017 k'^{16} & & +.00040 k'^{16} \\ +.00012 k'^{18} & & +.00027 k'^{18} \\ +.00008 k'^{20} & & +.00020 k'^{20} \\ +.00006 k'^{22} & & +.00015 k'^{22} \\ +.00005 k'^{24} & & +.00011 k'^{24} \\ +.00004 k'^{26} & & +.00009 k'^{26} \\ +.00003 k'^{28} & & +.00007 k'^{28} \\ +.00002 k'^{30} & & +.00006 k'^{30} \\ + & & + \\ . & & . \\ . & & . \\ . & & . \end{array} \quad (18)$$

It is only in the middle range of values of k and k' that the large number of terms given above will be required, and then only when extreme accuracy is desired.

The principal use of these formulas is to compute the constants of Table 1. This table permits the computation of an inductance with an accuracy of one-tenth per cent, and its use is much simpler than that of either of the above formulas. It is only when higher precision is required that it is necessary to resort to either formula (17) or (18).

IV. FORMULAS INVOLVING ONLY THE RADII OF THE COILS AND THE DISTANCE BETWEEN THEM

Formulas can be derived which give the mutual inductance as an algebraic function of the measured dimensions. Such formulas are suitable for computation in all cases, except when the coils are close together. These formulas, while in themselves quite complicated, permit the computation of tables from which the mutual inductance can easily be determined.

A formula which is very useful in computing tables is obtained by substituting the value of k_1 given in equation (6) above in Grover's series formula.⁹ The value of M is then given by the equation

$$\begin{aligned}
 M &= \frac{2\pi^2 a^2}{A} \sum_{n=0}^{\infty} \frac{(|2n|)^2}{2^{4n} (|n|)^4} \frac{2n+1}{n+1} \left[\frac{2}{\sqrt{\left(1+\frac{a}{A}\right)^2 + \frac{D^2}{A^2}} + \sqrt{\left(1-\frac{a}{A}\right)^2 + \frac{D^2}{A^2}}} \right]^{4n+3} \frac{a^{2n}}{A^{2n}} \\
 &= \frac{2\pi^2 a^2}{A} \left[\frac{2}{\sqrt{\left(1+\frac{a}{A}\right)^2 + \frac{D^2}{A^2}} + \sqrt{\left(1-\frac{a}{A}\right)^2 + \frac{D^2}{A^2}}} \right]^3 \sum_{n=0}^{\infty} \frac{(|2n|)^2}{2^{4n} (|n|)^4} \cdot \frac{2n+1}{n+1} \left[\frac{2\sqrt{\frac{a}{A}}}{\sqrt{\left(1+\frac{a}{A}\right)^2 + \frac{D^2}{A^2}} + \sqrt{\left(1-\frac{a}{A}\right)^2 + \frac{D^2}{A^2}}} \right]^{4n} \\
 &= \frac{a^2}{A} P
 \end{aligned} \tag{19}$$

where P is a function of $\frac{a}{A}$ and $\frac{D}{A}$. While the first form is the shorter of the two, yet the second is more useful in computation.

If the coils are coplanar, so that $D=0$, equation (19) becomes

$$M = \frac{2\pi^2 a^2}{A} \sum_{n=0}^{\infty} \frac{(|2n|)^2}{2^{4n} (|n|)^4} \frac{2n+1}{n+1} \left[\frac{a}{A} \right]^{2n}$$

⁹ Given as equation (6) in B. S. Sci. Paper No. 169. Loc. cit.

By putting all the numerics in the form of decimals, this equation can be written:

$$M = \frac{2\pi^2 a^2}{A} \left| \begin{array}{l} 1.00000 \\ + .37500 a^2/A^2 \\ + .23438 a^4/A^4 \\ + .17090 a^6/A^6 \\ + .13458 a^8/A^8 \\ + .11103 a^{10}/A^{10} \\ + .09451 a^{12}/A^{12} \\ + .08227 a^{14}/A^{14} \\ + .07284 a^{16}/A^{16} \\ + .06535 a^{18}/A^{18} \\ + .05926 a^{20}/A^{20} \\ + \\ . \\ . \\ . \end{array} \right. \quad (20)$$

The numerics of this equation apply also to equation (19) and can advantageously be used when it is necessary to compute by that formula. Equation (19) is convergent for all values of D , except when $a=A$ and $D=0$. It converges very rapidly when $\frac{D}{A}$ is large relative to $\frac{a}{A}$. In general, it may be said that by means of this formula the mutual inductance between any two coaxial circles can be computed. However, when the circles are close together, the convergence is so slow that the computation is very laborious. When the shortest distance between the circumferences is greater than one-third of the radius of the larger circle, an accuracy greater than 1 part in 10,000 will be obtained if the first 11 terms of the series are used. When the distance between the centers is greater than the radius of the larger circle, the same accuracy can be obtained by using the first 3 terms of the series.

Since equation (20) is convergent when $a < A$ and when $a=A$ if $D \neq 0$, it can be used in mathematical derivations where it is necessary to have a formula for the mutual inductance which is applicable to a wide range of circles. For example, it should be possible by means of this formula to compute the skin effect in a circular ring by the integral-equation method. Whether the resulting formula will be sufficiently convergent for practical use can not be predicted.

If D is large relative to A , then by substituting the value of k_1 given as equation (7) above in Grover's series formula

$$M = \frac{2\pi^2 a^2 A^2}{D^3} \sum_{n=0}^{\infty} \frac{(|2n|)^2}{2^{4n} (|n|)^4} \cdot \frac{2n+1}{n+1} \left[\frac{2}{\sqrt{1 + \frac{A^2}{D^2} \left(1 + \frac{a}{A}\right)^2} + \sqrt{1 + \frac{A^2}{D^2} \left(1 - \frac{a}{A}\right)^2}} \right]^{4n+3} \frac{a^{2n}}{A^{2n}} \cdot \frac{A^{4n}}{D^{4n}}$$

$$= \frac{a^2 A^2}{D^3} S \quad (21)$$

Equation (21) is the same as equation (19), but is written in a form which simplifies the construction of tables when the coils are far apart. When D approaches infinity, the value of P approaches zero asymptotically, making interpolation impossible, whereas the value of S approaches unity in a nearly linear manner. Hence, to facilitate computation when the coils are far apart, tables of S are desirable. However, equation (21) should not be used for the direct computation of an inductance as (19) is equivalent and simpler.

V. USE OF TABLES TO COMPUTE AN INDUCTANCE

Two different types of tables are given at the end of this paper, each one of which permits the calculation of the mutual inductance of any two coaxial circles. In the first type (Tables 1, 2, and 3) there is a single parameter which must be determined from the measured dimensions. In the second type (Tables 6, 7, and 8) there are two parameters which come directly from the measured dimensions. In both types of tables the parameters have been chosen so that the interpolation is approximately linear, thus reducing the work involved in making a computation and increasing the accuracy of the result. In the first type of table the parameter is a function of k or k' . The size of the tables for a given accuracy can be decidedly reduced by choosing different parameters for different ranges of k . For this reason three tables have been constructed.

In Table 1, values of $\frac{M}{r_1}$ are given as a function of k^2 (or k'^2) throughout the entire range of values. However, for the ranges $k^2 = 0$ to $k^2 = 0.1$ and $k^2 = 0.85$ to $k^2 = 1$, the values of $\frac{M}{r_1}$ are changing so rapidly that interpolation is not accurate. For the region $k^2 = 0.85$ to $k^2 = 1.0$ ($k'^2 = 0.15$ to $k'^2 = 0$); that is, when the coils are close together, Table 2 will give the necessary accuracy. If k^2 is less than 0.1; that is, if the coils are far apart, Table 3 should be used.

To compute the inductance of any two coaxial circles by these tables, the following procedure should be followed: From the measured dimensions compute r_1 and k^2 . Values of k^2 in terms of the ratios of dimensions are given in Tables 4 and 5. If the value of k^2 lies between 0.1 and 0.85, find the values of H from Table 1. If k^2 is greater than 0.85, then find the cologarithm of k'^2 and use this as the parameter in Table 2 to obtain H . However, if k^2 is less than 0.1, find k^4 and determine the value of H from Table 3.

It is generally simpler to compute the inductance directly from the dimensions by means of Tables 6, 7, and 8. This avoids the necessity of computing the auxiliary constants r_1 and k^2 . Table 6 gives the values when the distance between the circles is less than the radius of the larger circle, Table 7 when the distance between the circles is greater than the radius of the larger circle but less than three times the radius, and Table 8 when the distance between the circles is more than 2.2 times the radius of the larger circle. There is a region where Tables 7 and 8 overlap. In this region computation is simpler by Table 7, but Table 8 gives the greater accuracy.

If the mutual inductance is sought for circles of which the values of the parameters are not those given in the tables, a double interpolation will be necessary. Since the interpolation is approximately linear in both directions, it is only necessary to add to the tabular value the sum of the horizontal and vertical interpolation.

In general, the tables are constructed so that an accuracy of one-tenth per cent can be obtained in the most unfavorable case of interpolation. Where that was not feasible, the tabular differences have been omitted.

VI. DETERMINATION OF COAXIAL CIRCLES TO HAVE A DESIRED MUTUAL INDUCTANCE

In designing apparatus it is often necessary to determine the diameter and distance apart of two circles to have a predetermined inductance. This can readily be done by either set of tables or by the charts.

In using the first set of tables, a value of A must first be chosen. If it is desired to have the coils close together, the value of $\frac{M}{2A}$ must be large, while if they are to be far apart the value must be small. Ordinarily a value of $\frac{M}{2A}$ between 4.0 and 0.1 should be chosen.

Then since one value of r_1 is $2A$, one value of H is found by dividing M by $2A$. To avoid interpolation, select in the proper

table a value of H , which is near the value of $\frac{M}{2A}$, and read the corresponding value of k^2 .

Then $r_1 = \frac{M}{H}$. By equation (3)

$$a = \frac{k^2 r_1^2}{4A}$$

and by equation (1)

$$D = \sqrt{r_1^2 - (A + a)^2}$$

If the above does not give a suitable size and location for the second circle, other values can readily be determined. Suppose that it is desired to have the second circle larger than that given above. Select a value of H which is 10 or 20 per cent smaller than the one which was first used and compute the values of a and D . Should the value of D be imaginary, then too small a value of H has been chosen.

If it is desired to have the second circle of smaller radius than that given by the first computation, then the value of H must be chosen which is larger than the one which was first used. The procedure is then the same as that outlined above. A few trials will give the desired size and location of the second circle.

If the relative size of the coils and distance between them are given, then Tables 6, 7, and 8 can be used advantageously to determine the absolute size to give a desired mutual inductance.

If $\frac{D}{A}$ is less than three, the value of P can be found from Table

6 or Table 7. Then $a = \frac{M}{P} \cdot \frac{A}{a}$, and the values of A and D can

be computed from the ratios initially given. If $\frac{D}{A}$ is greater than three, the value of S can be found from Table 8. Then

$$D = \frac{M}{S} \cdot \frac{A^2}{a^2} \cdot \frac{D^4}{A^4}$$

In many cases this method will be simpler than that by the use of Table 1.

VII. USE OF CHARTS

Three charts (figs. 2, 3, and 4) have been prepared which are useful when approximate solutions of problems connected with the mutual inductance of coaxial circles are sufficient. As three

FIG. 2

MUTUAL INDUCTANCE OF TWO COAXIAL CIRCLES

FIG. 3.—Chart showing relative size of circles and distance between them to give definite values of M/A . Important region in detail

M —Mutual inductance, millimicrohenrys
 D —Distance between circles, centimeters
 A —Radius of large circle, centimeters
 a —Radius of small circle, centimeters
 Numbers on curves indicate the values of M/A

MUTUAL INDUCTANCE OF TWO COAXIAL CIRCLES

Chart showing the relation of M/A to D/A for different values of a/A

M = Mutual inductance, millimicrohenrys
 D = Distance between circles, centimeters
 A = Radius of large circle, centimeters
 a = Radius of small circle, centimeters

coordinates are necessary to completely represent this function, it is only possible on plane coordinate paper to plot contour lines which show the curves of two of the variables when the third has a definite numerical value. In the first two charts (figs. 2 and 3), definite values of $\frac{M}{A}$ are chosen, and curves given for $\frac{a}{A}$ as ordinate, and $\frac{D}{A}$ as abscissa. The only difference between these charts is that the second (fig. 3) covers a smaller region with a larger scale than does the first. The third chart (fig. 4) gives the curves of $\frac{M}{A}$ as ordinate and $\frac{D}{A}$ as abscissa for definite values of $\frac{a}{A}$.

To determine the mutual inductance of given coils, the third chart (fig. 4) should be used. Should the value of $\frac{a}{A}$ be that for which a curve is plotted, then the value of $\frac{M}{A}$ for the given value of $\frac{D}{A}$ can be read quite accurately. In case it is necessary to interpolate between the curves, the accuracy will be considerably reduced.

To determine the size of circles and distance between them to give a desired mutual inductance, the first and second charts (figs. 2 and 3) are useful. In these charts the curves correspond to the lines of magnetic force¹⁰ around a circle, the center of which coincides with the origin, the plane of which is perpendicular to the X axis, and the radius of which is unity. Since the mutual inductance depends on the number of lines of magnetic force from the first circle which cuts the second, the curves show the different sizes and positions that the second circle can have to give a single value of the mutual inductance. Another way of visualizing this is to consider that the diagram is so reduced in scale that the distance $\frac{a}{A} = 1$ is 1 cm. Then if one circle has its center at the origin, its plane perpendicular to the axis, and its radius equal to 1 cm; and if the second circle, having its center on the X axis, has the point where its circumference cuts the paper coincident with one of the curves, then the ordinate and abscissa of this point give the radius of the circle and the

¹⁰ Such a set of curves without any numerical values is given as Fig. 18 in Maxwell's *Electricity and Magnetism*, 2.

distance between the circles, respectively, while the value of mutual inductance in millimicrohenrys is given by the value of $\frac{M}{A}$ on the curve.

To use this chart for determining the size and distance of two circles to give a desired mutual inductance, it is first necessary to choose the radius A of one of the circles which will be used. It is desirable to choose this value of A so that $\frac{M}{A}$ is one of the values for which a curve is given in the charts (figs. 2 or 3.) Then by selecting various points on the curve, a series of values of $\frac{D}{A}$ and the corresponding values $\frac{a}{A}$ can be obtained, any pair of which will give the desired mutual inductance. If an additional condition is imposed, such, for example, as requiring that the circles shall have the same radii ($A=a$), or that the circles shall lie in the same plane ($D=0$), then there are only one pair of values for $\frac{D}{A}$ and $\frac{a}{A}$.

If it is desired to design an inductance in which the radii of the two circles shall have a definite ratio, then the third chart (fig. 4) can be used advantageously. For example, if the circles are to be of equal size, $\frac{a}{A}=1$, then from the curve so labeled a series of values for $\frac{M}{A}$ and the corresponding values of $\frac{D}{A}$ can be determined. This chart is especially useful where the relative dimensions are known. Then if $\frac{a}{A}$ is one of the values for which a curve is plotted, the value of $\frac{M}{A}$ is read directly from the chart.

VIII. MUTUAL INDUCTANCE OF COAXIAL COILS

All the formulas for the mutual inductance of coaxial coils are based on the mutual inductance of two coaxial circles, the radii of which are the mean radii of the coils, and the centers of which coincide with the centers of the coils. If the breadth and depth of the coils are small relative to their mean radii and to the distance between them, then

$$M = n_1 n_2 M_o$$

where M is the mutual inductance between the coils, M_0 the mutual inductance between two coaxial circles at the center of the coils, and n_1 and n_2 the number of turns in the two coils, respectively. In this case, all the methods of computation and design which have been given under coaxial circles can be adapted to use with coils by a proper use of the factor $n_1 n_2$.

If the coils are close together, or if their radii are small, correction terms need to be applied¹¹ to the above formula. No simple method of applying these corrections has yet been developed.

IX. EXAMPLES OF USE OF CHARTS, TABLES, AND FORMULAS

To illustrate the use of the tables, charts, and formulas, examples will be selected from those given by Rosa and Grover,¹² who have given the results correct to about 1 part in 1,000,000. For each example, there will, in general, be five solutions by the methods given in this paper, viz, one by the charts, two by tables, and two by formulas. However, the charts can not be used when the circumferences are close together, or when the distance between the coils is great. The second set of tables (Tables 6, 7, and 8) will not give values when the circumferences are close together, and under the same condition the formula—equation (18)—giving the mutual inductance in terms of the measured dimensions is not sufficiently convergent to be practical. Hence, when the circumferences are close together, only two values can be obtained, and when they are far apart only four values.

EXAMPLE 1

Measured dimensions:

$$A = a = 25 \text{ cm} \quad D = 20 \text{ cm}$$

Computed constants:

$$\begin{aligned} r_1^2 &= 2,900 & r_1 &= 53.8516 \\ r_2^2 &= 400 & r_2 &= 20.000 \\ k^2 &= 0.862069 & k'^2 &= 0.137931 \\ \frac{a}{A} &= 1 & \frac{D}{A} &= 0.8 \end{aligned}$$

Rosa and Grover value of mutual inductance is 167.0856 milli-microhenrys.

¹¹ See Rosa and Grover; loc. cit., p. 33.

¹² Loc. cit.

(a) VALUE OF MUTUAL INDUCTANCE FROM THE CHARTS.—

On chart 3 (fig. 4) the value of $\frac{M}{A}$, read from the curve $\frac{a}{A} = 1$, when

$$\frac{D}{A} = 0.8, \text{ is } 6.7$$

Hence

$$M = 168 \text{ millimicrohenrys}$$

The error is less than 1 per cent.

(b) VALUE OF M FROM TABLES USING k^2 AS A PARAMETER.—By interpolating in Table 4, $k^2 = 0.8622$. Using this value, the value of H from Table 1 is 3.103.

$$\therefore M = r_1 H = 167.1 \text{ millimicrohenrys}$$

This result is in error by less than one-tenth of 1 per cent. The same result would have been obtained had a more accurate value of k^2 been used.

(c) COMPUTATION FROM TABLES WHICH USE ONLY THE RATIO OF THE DIMENSIONS.—Interpolating in Table 6, $P = 6.700$

$$\therefore M = \frac{a^2}{A} P = 167.5 \text{ millimicrohenrys}$$

This is in error by a little over two-tenths of 1 per cent.

(d) COMPUTATION BY FORMULA (18) INVOLVING k^2 .—Substituting in equation (18)

$$\frac{M}{53.8516} = [8.71035 + 7.23378 \times 0.860335] \begin{vmatrix} 1.00000 - \pi & 4.00000 \\ +.03448 & -.06897 \\ +.00030 & +.00030 \\ +.00001 & +.00001 \end{vmatrix}$$

$$= 14.93382 \times 1.03479 - 3.14159 \times 3.93135$$

$$\therefore M = 167.082$$

In this case only four terms were necessary to give the values of the series to within 1 part in 100,000. However, as the result is the difference of two quantities which have values nearly alike, the result is not known with that accuracy. This computed result differs from the correct result by less than 3 parts in 100,000.

(e) COMPUTATION BY A FORMULA WHICH INVOLVES ONLY THE MEASURED DIMENSIONS.—Equation (19) is suitable for this computation.

$$\begin{aligned}
 M &= 2\pi^2 \cdot 25 \cdot [0.67703]^3 \cdot \begin{array}{l} 1.00000 \\ +.07879 \\ +.01035 \\ +.00158 \\ +.00026 \\ +.00005 \\ +.00001 \end{array} \\
 &= 493.48 \times 0.31033 \times 1.09104 \\
 &= 167.085
 \end{aligned}$$

This result is correct to within 1 part in 100,000. The convergence of the series is such that no difficulty would be experienced in obtaining a higher accuracy if desired.

EXAMPLE 2

Measured dimensions:

$$A = a = 25 \text{ cm}$$

$$D = 1 \text{ cm}$$

Computed constants:

$$r_1^2 = 2,501$$

$$r_1 = 50.0100$$

$$r_2^2 = 1$$

$$r_2 = 1.$$

$$k'^2 = \frac{r_2^2}{r_1^2} = 0.000399840$$

Rosa and Grover value of mutual inductance is 1036.666 millimicrohenrys.

(a) VALUE FROM CHARTS.—From the third chart (fig. 4), $\frac{M}{A}$ is approximately 40, but the curve is changing so rapidly at this point that the value is little more than an estimate. This gives

$$M = 1,000 \text{ millimicrohenrys}$$

which is in error by nearly 4 per cent.

(b) VALUE FROM TABLE IN k'^2 .—With coils so close together it is impossible to obtain k'^2 with sufficient accuracy from Table 4. Using the value of k'^2 given above

$$\text{colog } k'^2 = 3.3981$$

From Table 2

$$H = 20.728$$

$$\therefore M = 1,036.6$$

This is in error by less than 1 part in 10,000.

(c) COMPUTATION BY TABLES IN $\frac{a}{A}$ AND $\frac{D}{A}$.—Table 6 can not be used when the coils are so close together.

(d) COMPUTATION BY A FORMULA INVOLVING k'^2 .—Using formula (18)

$$\begin{aligned} \frac{M}{r_1} &= \left[8.71035 + 7.23378 \times 3.39811 \right] \left| \begin{array}{c} 1.00000 - \pi \\ .00010 \end{array} \right| \left| \begin{array}{c} 4.00000 \\ - .00020 \end{array} \right| \\ &= 33.29489 \times 1.0001 - \pi \times 3.99980 \\ \therefore M &= 1,036.665 \end{aligned}$$

This formula is especially well suited to this example. Only two terms are required in the series, yet the result is accurate to 1 part in 1,000,000.

(e) COMPUTATION FROM MEASURED DIMENSIONS.—For these constants, equation (19) is not sufficiently convergent to make the computation practicable.

EXAMPLE 3

Measured dimensions:

$$A = a = 10$$

$$D = 100$$

Computed constants:

$$r_1^2 = 10,400$$

$$r_1 = 101.980$$

$$k^2 = 0.0384615$$

$$\frac{a}{A} = 1$$

$$\frac{D}{A} = 10$$

Rosa and Grover value of mutual inductance is 0.1916496 millimicrohenrys.

(a) VALUE OF CHART.—Value of $\frac{D}{A}$ beyond range of chart.

(b) VALUE FROM TABLE IN k^2 .—With coils so far apart, it is impossible to obtain k^2 with sufficient accuracy from Table 5. Using value of k^2 computed above

$$k^4 = 0.001479$$

From Table 3 $H = 0.001881$

$$\therefore M = 0.1918 \text{ millimicrohenry}$$

This is in error less than one-tenth of 1 per cent.

(c) VALUES FROM TABLE USING RATIO OF DIMENSIONS.—From Table 8

$$S = 19.18$$

$$\therefore M = \frac{10^2 \times 10^2}{100^3} \cdot 19.18$$

$$= 0.1918 \text{ millimicrohenry}$$

The error is less than a tenth of 1 per cent.

(d) COMPUTATION BY A FORMULA IN k^2 .—By equation (17)

$$\frac{M}{r_1} = 1.23370 \times 0.00147929 \left| \begin{array}{l} 1.00000 \\ + .02885 \\ + .00087 \\ + .00003 \end{array} \right.$$

$$M = 0.191650$$

The error is less than 1 part in 100,000.

(e) COMPUTATION BY A FORMULA INVOLVING MEASURED DIMENSIONS.—Equation (19) is suitable for this example.

$$M = 20\pi^2 [0.0990195]^3 \left| \begin{array}{l} 1.000000 \\ + .000036 \end{array} \right.$$

$$= 0.191649$$

This result is in error by only 2 parts in 1,000,000. It has been carried to this accuracy to show the ease of computing by this formula when the coils are far apart.

X. SUMMARY

Charts are given from which the mutual inductance of two coaxial circles can be read. Under favorable conditions the result will not be in error by as much as 1 per cent. These charts are also useful for designers.

Tables are given by means of which the computation of the mutual inductance of any two coaxial circles with an accuracy of 0.1 per cent is easily accomplished. These tables can also be used for the inverse problem of finding two circles which will give a desired inductance.

Formulas are given by means of which the mutual inductance of any two coaxial circles can be computed with an accuracy of 0.01 per cent. The coefficients of the terms in these formulas are expressed in decimal form, and they are so arranged that the labor of computation is reduced to a minimum. When the coils are either close together or far apart, these formulas can be used to give an accuracy even greater than 0.01 per cent. In the intermediate range the number of terms required to give an accuracy of even 1 part in 100,000 is very large. For this range, Nagaoka's formula¹³ is very convenient, especially if the tables of Nagaoka and Sakurai¹³ of k^2 are available.

The authors wish to acknowledge assistance from a number of their associates. In particular, Dr. Chester Snow has aided in simplifying the mathematical expressions.

WASHINGTON, April 14, 1924.

¹³ Given as equation (8) in Rosa and Grover's paper.

APPENDIX

TABLES AND CHARTS FOR DETERMINING THE MUTUAL INDUCTANCE OF COAXIAL CIRCLES

Two types of tables are given in this appendix, namely, single entry tables and double entry tables. In the single entry tables the parameter (k^2 , $\log k'^2$, or k^4) is a somewhat complicated function of the measured dimensions (the radii of the circles and the distance between their centers). In the double entry tables each parameter is either the ratio of two measured dimensions or the square of such a ratio. Hence the parameters for the double entry tables are much more easily determined than are those for the single entry tables. However, the interpolation in the double entry tables is more difficult than with the single entry tables.

So far as possible the tables have been constructed so as to give an accuracy of one-tenth of 1 per cent in the value of the computed mutual inductance. However, when it is necessary to determine the inductance with this accuracy, it is suggested that computations be made by both types of tables. Should they fail to check with the required accuracy a third computation should be made, using one of the formulas given in the text.

TABLE 1.—Values of H in the Equation $M=r_1 H$

METHOD OF USING TABLE TO COMPUTE AN INDUCTANCE.—Given A and a , the radii of two coaxial circles, and D , the distance between their centers. Compute

$$k^2 = \frac{4Aa}{(A+a)^2 + D^2}$$

and

$$r_1 = \sqrt{(A+a)^2 + D^2}$$

If the value of k^2 lies between 0.1 and 0.85, obtain the value of H from this table, using linear interpolation if necessary. Then $M=r_1 H$.

METHOD OF USING TABLE TO FIND SIZE AND POSITION OF CIRCLES WHICH WILL GIVE A DESIRED INDUCTANCE.—Choose a value for A . Select a value of H near the value given by $\frac{M}{2A}$, for which a value of k^2 is given in the table. Then

$$r_1 = \frac{M}{H}$$

$$a = \frac{k^2 r_1^2}{4A}$$

$$D = \sqrt{r_1^2 - (A+a)^2}$$

If D is imaginary, the value of H has been taken too far from the value given by $\frac{M}{2A}$.

UNITS.—Values of A , a , and D are in centimeters. Value of M in millimicrohenrys. If A , a , and D are in inches, multiply the value of H by 2.54 to give M in millimicrohenrys (thousandths of a microhenry).

ACCURACY.—For values of k^2 between 0.10 and 0.85, values of H accurate to one-tenth of 1 per cent can be obtained by linear interpolation.

k^1	H	Difference	k^1	k^2	H	Difference	k^2
0	0	0.0001243	1.00	0.50	0.5016	0.0271	0.50
.01	.0001243	.0003767	.99	.51	.5287	.0284	.49
.02	.0005010	.000635	.98	.52	.5571	.0295	.48
.03	.001136	.000899	.97	.53	.5866	.0309	.47
.04	.002035	.001170	.96	.54	.6175	.0321	.46
.05	.003205	.001446	.95	.55	.6496	.0336	.45
.06	.004651	.001730	.94	.56	.6832	.0350	.44
.07	.006381	.002020	.93	.57	.7182	.0366	.43
.08	.008401	.00232	.92	.58	.7548	.0381	.42
.09	.01072	.00262	.91	.59	.7929	.0399	.41
.10	.01334	.00294	.90	.60	.8328	.0416	.40
.11	.01628	.00325	.89	.61	.8744	.0435	.39
.12	.01953	.00358	.88	.62	.9179	.0455	.38
.13	.02311	.00392	.87	.63	.9634	.0476	.37
.14	.02703	.00427	.86	.64	1.0110	.050	.36
.15	.03130	.00462	.85	.65	1.061	.052	.35
.16	.03592	.00498	.84	.66	1.113	.055	.34
.17	.04090	.00536	.83	.67	1.168	.057	.33
.18	.04626	.00574	.82	.68	1.225	.060	.32
.19	.05200	.00613	.81	.69	1.285	.064	.31
.20	.05813	.00654	.80	.70	1.349	.066	.30
.21	.06467	.00696	.79	.71	1.415	.070	.29
.22	.07163	.00739	.78	.72	1.485	.074	.28
.23	.07902	.00782	.77	.73	1.559	.078	.27
.24	.08684	.00828	.76	.74	1.637	.082	.26
.25	.09512	.00878	.75	.75	1.719	.087	.25
.26	.1039	.0092	.74	.76	1.806	.092	.24
.27	.1131	.0097	.73	.77	1.898	.097	.23
.28	.1228	.0102	.72	.78	1.995	.104	.22
.29	.1330	.0108	.71	.79	2.099	.111	.21
.30	.1438	.0113	.70	.80	2.210	.118	.20
.31	.1551	.0119	.69	.81	2.328	.126	.19
.32	.1670	.0124	.68	.82	2.454	.136	.18
.33	.1794	.0130	.67	.83	2.590	.146	.17
.34	.1924	.0137	.66	.84	2.736	.158	.16
.35	.2061	.0142	.65	.85	2.894	.171	.15
.36	.2203	.0150	.64	.86	3.065	.187	.14
.37	.2353	.0156	.63	.87	3.252	.205	.13
.38	.2509	.0163	.62	.88	3.457	.227	.12
.39	.2672	.0171	.61	.89	3.684	.251	.11
.40	.2843	.0178	.60	.90	3.935	.283	.10
.41	.3021	.0186	.59	.91	4.218	.321	.09
.42	.3207	.0194	.58	.92	4.539	.369	.08
.43	.3401	.0203	.57				
.44	.3604	.0211	.56	.93	4.908	.433	.07
.45	.3815	.0220	.55	.94	5.341	.520	.06
.46	.4035	.0230	.54	.95	5.861	.647	.05
.47	.4265	.0240	.53	.96	6.508	.848	.04
.48	.4505	.0250	.52	.97	7.356	1.215	.03
.49	.4755	.0261	.51	.98	8.571	2.114	.02
.50	.5016	.0271	.50	.99	10.685	∞	.01
				1.00	∞	-----	0.

TABLE 2.—Values of H in the Equation $M=r_1 H$ When the Circles are Close Together ($k^2 > 0.85$)

METHOD OF USING TABLE TO COMPUTE AN INDUCTANCE.—Compute k^2 and r_1 as in Table 1. Obtain k'^2 from the relation $k'^2 = 1 - k^2$, or compute from the equation

$$k'^2 = \frac{(A-a)^2 + D^2}{(A+a)^2 + D^2}$$

Find $\text{colog } k'^2$. Use linear interpolation to get value of H from table. Then $M = r_1 H$.

METHOD OF USING TABLE TO FIND SIZE AND POSITION OF CIRCLES WHICH WILL GIVE A DESIRED INDUCTANCE.—Method identical with that given under Table 1. Use this table only when necessary to interpolate between values given in Table 1.

UNITS.— A , a , and D in centimeters; M in millimicrohenrys. If A , a , and D are in inches, multiply the value of H by 2.54 to give M in millimicrohenrys (thousandths of a microhenry).

ACCURACY.—Throughout the table, an accuracy of one-tenth of 1 per cent can be obtained by linear interpolation. For values of $\text{colog } k'^2$ greater than 2.0, an accuracy of one one-hundredth of 1 per cent can be obtained by linear interpolation.

$\text{Colog } k'^2$	H	Difference	k'^2	k^2	$\text{Colog } k'^2$	H	Difference	k'^2	k^2
0.8	2.759	675	0.1585	0.8415	2.5	14.255	718	0.003162	0.9968
.9	3.334	601	.1259	.8741	2.6	14.973	719	.002512	.9975
1.0	3.935	623	.1000	.9000	2.7	15.692	720	.001995	.9980
1.1	4.558	641	.07943	.9206					
1.2	5.199	655	.06310	.9369	2.8	16.413	721	.001585	.9984
					2.9	17.133	720	.001259	.9987
1.3	5.854	668	.05012	.9799	3.0	17.855	722	.001000	.9990
1.4	6.522	676	.03981	.9602	3.1	18.576	721	.0007943	.9992
1.5	7.198	687	.03162	.9684	3.2	19.298	722	.0006310	.9994
1.6	7.885	693	.02512	.9749					
1.7	8.578	698	.01995	.9801	3.3	20.020	722	.0005012	.9995
					3.4	20.742	723	.0003981	.9996
1.8	9.276	703	.01585	.9842	3.5	21.465	723	.0003162	.9997
1.9	9.979	706	.01259	.9874	3.6	22.188	723	.0002512	.9997
2.0	10.685	710	.01000	.9900					
2.1	11.395	712	.007943	.9921	3.7	22.911	723	.0001995	.9998
2.2	12.107	714	.006310	.9937	3.8	23.634	723	.0001585	.9998
					3.9	24.357	723	.0001259	.9999
2.3	12.821	716	.005012	.9950	4.0	25.080	-----	.0001000	.9999
2.4	13.537	718	.003981	.9960					

TABLE 3.—Values of H in the Equation $M=r_1H$ When the Circles Are Far Apart ($k^2<0.1$)

METHOD OF USING TABLE TO COMPUTE AN INDUCTANCE.—Compute k^2 and r_1 as in Table 1. Obtain k^4 . Use linear interpolation to get value of H . Then $M=r_1H$.

METHOD OF USING TABLE TO OBTAIN SIZE AND POSITION OF CIRCLES WHICH GIVE A DESIRED INDUCTANCE.—Method identical with that given under Table 1. Use this table only when necessary to interpolate between values given in Table 1.

UNITS.— A , a , and D in centimeters; in millimicrohenrys. If A , a , and D are in inches, multiply the value of H by 2.54 to give M in millimicrohenrys (thousandths of a microhenry).

ACCURACY.—Throughout the range of this table, H is nearly a linear function of k^4 . However, as both k^4 and H start at zero, the percentage error in H may be large when k^4 is very small. Hence when k^4 is less than 0.001, it is better to compute the mutual inductance by the formula

$$M=1.2337 r_1 k^4 (1+\frac{3}{4}k^2)$$

This formula will give an accuracy greater than one-tenth of 1 per cent when k^4 is less than 0.001. When k^4 is greater than 0.001 and less than 0.010, an accuracy greater than one-tenth of 1 per cent can be obtained by the use of the table.

k^4	H	Difference	k^2	k'^2	k^4	H	Difference	k^2	k'^2
0	0	1, 264	0	1. 000	0. 006	0. 007860	1, 356	0. 07746	0. 9225
.001	.001264	1, 289	.03162	.9684	.007	.009216	1, 366	.08367	.9163
.002	.002553	1, 307	.04472	.9553	.008	.010582	1, 375	.08944	.9106
.003	.003860	1, 321	.05477	.9452	.009	.011957	1, 384	.09467	.9053
.004	.005181	1, 334	.06325	.9368	.010	.013341	-----	.10000	.9000
.005	.006515	1, 345	.07071	.9297					

TABLE 4.—Values of k^2 When D is Less Than A —Circles Close Together

METHOD OF USING TABLE.—Given two coaxial circles, let

A =radius of larger circle

a =radius of smaller circle

D =distance between the centers of the circles

Compute $\frac{a}{A}$ and $\frac{D^2}{A^2}$. If these correspond to a value given in the table, the value of k^2 is read directly. If they do not correspond to any value in the table, interpolate as follows: Find the value of k^2 (designated by k^2_1) which corresponds to $\frac{a_1}{A_1}$ and $\frac{D^2_1}{A^2_1}$, where $\frac{a_1}{A_1}$ and $\frac{D^2_1}{A^2_1}$ are smaller than $\frac{a}{A}$ and $\frac{D^2}{A^2}$, respectively, by an amount which is less than the tabular difference of the parameters. Let $\frac{a}{A} - \frac{a_1}{A_1} = \alpha$ and $\frac{D^2}{A^2} - \frac{D^2_1}{A^2_1} = \beta$. Then since linear interpolation is assumed $k^2 = k^2_1 + \alpha\delta + \beta\Delta$, where δ is the horizontal tabular difference and Δ is the vertical tabular difference. Care must be exercised in regard to the sign of the tabular difference.

Example: Let $A=10$ $a=9.5$ $D=7$ $\frac{a}{A}=0.95$ $\frac{D^2}{A^2}=0.49$ $k^2=0.8978+0.5 \times 0.0113 - 0.9 \times 0.0219 = 0.8837$.

ACCURACY.—In many parts of this table it is not possible to obtain an accuracy of 0.1 per cent, except when the values of the parameters are near those given in the table. To obtain this accuracy would require the use of second and third differences, which is impractical.

$\frac{D^2}{A^2}$	$\frac{a}{A}$																				
		0	δ	0.1	δ	0.2	δ	0.3	δ	0.4	δ	0.5	δ	0.6	δ	0.7	δ	0.8	δ	0.9	δ
0	0	3306	0.3306	2250	0.5556	1545	0.7101	1062	0.8163	726	0.8889	486	0.9375	314	0.9689	188	0.9877	95	0.9972	28	1.0000
Δ			—253		—361		—397		—396		—378		—352		—324		—296		—269		—244
0.1	0	3053	.3053	2142	.5195	1509	.6704	1063	.7767	744	.8511	512	.9023	342	.9365	216	.9581	122	.9703	53	.9756
Δ			—216		—317		—355		—360		—348		—327		—304		—279		—254		—232
0.2	0	2837	.2837	2041	.4878	1471	.6349	1058	.7407	756	.8163	533	.8696	365	.9061	241	.9302	147	.9449	75	.9524
Δ			—183		—280		—319		—327		—320		—304		—284		—262		—242		—222
0.3	0	2649	.2649	1949	.4598	1432	.6030	1050	.7080	763	.7843	549	.8392	385	.8777	263	.9040	167	.9207	95	.9302
Δ			—164		—250		—288		—300		—296		—284		—266		—249		—229		—211
0.4	0	2485	.2485	1863	.4348	1394	.5742	1038	.6780	767	.7547	561	.8108	403	.8511	280	.8791	187	.8978	113	.9091
Δ			—146		—224		—263		—276		—274		—266		—251		—235		—219		—202
0.5	0	2339	.2339	1785	.4124	1355	.5479	1025	.6504	769	.7273	570	.7843	417	.8260	296	.8556	203	.8759	130	.8889
Δ			—129		—202		—239		—254		—256		—248		—237		—223		—208		—193
0.6	0	2210	.2210	1712	.3922	1318	.5240	1010	.6250	768	.7018	577	.7595	428	.8023	310	.8333	218	.8551	145	.8696
Δ			—116		—184		—219		—235		—238		—233		—224		—211		—198		—185
0.7	0	2094	.2094	1644	.3738	1283	.5021	994	.6015	765	.6780	582	.7362	437	.7799	323	.8122	231	.8353	158	.8511
Δ			—104		—167		—202		—218		—223		—219		—211		—201		—190		—178
0.8	0	1990	.1990	1581	.3571	1248	.4819	978	.5797	760	.6557	586	.7143	445	.7588	333	.7921	242	.8163	170	.8333
Δ			—94		—152		—186		—203		—208		—206		—200		—191		—181		—170
0.9	0	1896	.1896	1523	.3419	1214	.4633	961	.5594	755	.6349	588	.6937	451	.7388	342	.7730	252	.7982	181	.8163
Δ			—86		—140		—172		—189		—196		—196		—190		—183		—173		—163
1.0	0	1810	.1810	1469	.3279	1182	.4461	944	.5405	749	.6154	588	.6742	456	.7198	349	.7547	262	.7809	191	.8000

TABLE 5.—Values of k^2 When D is Greater Than A —Circles Far Apart

METHOD OF USING TABLE.—The method of using this table is the same as for the preceding table except that one computes $\frac{A^2}{D^2}$ instead of $\frac{D^2}{A^2}$.

ACCURACY.—In many parts of this table it is not possible to obtain an accuracy of 0.1 per cent, except when the values of the parameters are near those given in the table. To obtain this accuracy would require the use of second and third differences, which is impractical.

$\frac{A^2}{D^2}$	$\frac{A}{D}$	0	δ	0.1	δ	0.2	δ	0.3	δ	0.4	δ	0.5	δ	0.6	δ	0.7	δ	0.8	δ	0.9	δ	1.0
0	0	0		0		0		0		0		0		0		0		0		0		0
Δ				.357		.699		.1027		.1338		.1633		.1911		.2172		.2417		.2645		.2857
0.1	0	357		.0357	342	.0699	328	.1027	311	.1338	295	.1633	278	.1911	261	.2172	245	.2417	228	.2645	212	.2857
Δ				.287		.543		.767		.961		1.126		1.264		1.377		1.466		1.536		1.587
0.2	0	644		.0644	598	.1242	552	.1794	505	.2299	460	.2759	416	.3175	374	.3549	334	.3883	298	.4181	263	.4444
Δ				.236		.434		.596		.724		.823		.897		.960		.985		1.004		1.011
0.3	0	880		.0880	796	.1676	713	.2389	634	.3023	559	.3582	490	.4072	427	.4499	369	.4868	317	.5185	270	.5455
Δ				.198		.354		.475		.564		.629		.671		.696		.707		.707		.699
0.4	0	1078		.1078	952	.2030	834	.2864	723	.3587	624	.4211	532	.4743	452	.5195	380	.5575	317	.5892	262	.6154
Δ				.168		.298		.388		.453		.496		.520		.531		.532		.525		.513
0.5	0	1246		.1246	1080	.2326	926	.3252	788	.4040	666	.4706	557	.5263	463	.5726	381	.6107	310	.6417	250	.6667
Δ				.146		.249		.323		.372		.400		.416		.419		.416		.406		.392
0.6	0	1391		.1391	1184	.2575	1000	.3575	837	.4412	694	.5106	572	.5678	467	.6145	377	.6522	300	.6822	237	.7059
Δ				.126		.214		.273		.310		.331		.339		.339		.332		.323		.309
0.7	0	1516		.1516	1273	.2789	1059	.3848	874	.4722	715	.5437	580	.6017	467	.6484	370	.6854	291	.7145	223	.7368
Δ				.110		.186		.234		.262		.277		.282		.279		.273		.262		.251
0.8	0	1626		.1626	1348	.2974	1108	.4082	902	.4984	730	.5714	585	.6299	464	.6763	364	.7127	280	.7407	212	.7619
Δ				.97		.162		.202		.226		.236		.239		.235		.227		.218		.207
0.9	0	1723		.1723	1413	.3136	1148	.4284	926	.5210	740	.5950	588	.6538	460	.6998	356	.7354	271	.7625	201	.7826
Δ				.87		.143		.177		.195		.204		.204		.200		.193		.184		.174
1.0	0	1810		.1810	1469	.3279	1182	.4461	944	.5405	749	.6154	588	.6742	456	.7198	349	.7547	262	.7809	191	.8000

TABLE 6.—Values of P in the Equation $M=\frac{a^2}{A} P$ When D is Less Than A —Circles Near Together

The mutual inductance of two coaxial circles whose distance apart is less than three times the radius of the larger circle can be obtained by the formula

$$M=\frac{a^2}{A} P$$

If $D \geq A$, use this table (6) to obtain P ; if $A < D < 3A$ use Table 7.

METHOD OF USING TABLE TO COMPUTE AN INDUCTANCE.—Given two coaxial circles, let

A = the radius of the larger circle

a = the radius of the smaller circle

D = the distance between the centers of the circles

Compute a/A and D^2/A^2 . If these correspond to a value given in the table, the value of P is read directly. If they do not correspond to any value given in the table, interpolate as follows: Find the value of P (designated by P_1), which corresponds to a_1/A_1 and D_1^2/A_1^2 , where a_1/A_1 and D_1^2/A_1^2 are smaller than a/A and D^2/A^2 , respectively, by an amount which is less than the respective tabular difference of these parameters. Designate by a_2/A_2 and D_2^2/A_2^2 , the next higher values of the respective parameters which are given in the table.

$$\text{Let } a/A - a_1/A_1 = \alpha \quad \text{and } D^2/A^2 - D_1^2/A_1^2 = \beta$$

$$\text{Also let } a_2/A_2 - a_1/A_1 = \xi \quad \text{and } D_2^2/A_2^2 - D_1^2/A_1^2 = \eta$$

$$\text{Then } P = P_1 + \alpha\delta/\xi + \beta\Delta/\eta$$

where δ is the horizontal tabular difference of P and Δ is the vertical tabular difference. Use the proper sign of the tabular difference.

Example: Let $A=10$ $a=9.5$ and $D=7$ Then $a/A=0.95$ and $D^2/A^2=0.49$ $P=8.971-0.533 \times 0.04/0.05 - 0.732 \times 0.05/0.10=8.179$.

ACCURACY.—While the values given in this table are computed to be accurate in the third decimal place, yet in the region where the circumferences are close together, interpolated values will not give an accuracy of 0.1 per cent. To indicate this, the differences have been omitted in this part of the table.

The greatest error in an interpolated value will generally occur when the desired value of the parameter is about midway between values which are given in the table. Then the error is approximately one-eighth of the second difference. Hence, in the table the differences have been omitted when one-eighth of the second difference is greater than 0.2 per cent of the corresponding tabular difference. This does not mean that interpolation must not be undertaken in this region, but that caution must be used.

METHOD OF USING TABLE TO FIND SIZE AND POSITION OF CIRCLES WHICH WILL GIVE A DESIRED INDUCTANCE.—Select relative dimensions; that is, determine suitable values for a/A and D^2/A^2 . From the table obtain the value of P , interpolating if necessary. Then

$$a = \frac{A}{a} \cdot \frac{M}{P}$$

From the chosen ratios the other dimensions can be computed.

UNITS.—Values of A , a , and D in centimeters. Value of M in millimicrohenrys. If A , a , and D are in inches, multiply the value of P by 2.54 to give M in millimicrohenrys (thousandths of a microhenry).

$\frac{D^2}{A^2}$ \ $\frac{a}{A}$	0		0.1		0.2		0.3		0.4		0.5		0.6		0.7		0.8		0.9		1.0
		δ		δ		δ		δ		δ		δ		δ		δ		δ		δ	
0.00	19.739	74	19.813	230	20.043	402	20.445	—	21.058	—	21.945	—	23.225	—	25.135	—	28.246	—	34.406	—	—
Δ	—578	—	—587	—	—622	—	—685	—	—792	—	—971	—	—1285	—	—	—	—	—	—	—	—
0.02	19.161	65	19.226	195	19.421	339	19.760	506	20.266	—	20.974	—	21.940	—	23.233	—	24.887	—	26.515	—	25.753
Δ	—649	—	—660	—	—689	—	—746	—	—736	—	—883	—	—1127	—	—	—	—	—	—	—	—
0.04	18.612	54	18.666	166	18.832	283	19.115	415	19.530	561	20.091	—	20.813	—	21.680	—	22.563	—	22.927	—	21.538
Δ	—625	—	—633	—	—659	—	—697	—	—686	—	—809	—	—1002	—	—	—	—	—	—	—	—
0.06	18.087	46	18.133	140	18.273	235	18.508	336	18.844	438	19.282	529	19.811	—	20.374	—	20.793	—	20.609	—	19.120
Δ	—499	—	—507	—	—532	—	—574	—	—641	—	—742	—	—896	—	—1118	—	—	—	—	—	—
0.08	17.588	38	17.626	115	17.741	193	17.934	269	18.203	337	18.540	375	18.915	341	19.256	—	19.374	—	18.909	—	17.432
Δ	—478	—	—485	—	—506	—	—542	—	—600	—	—687	—	—810	—	—980	—	—1180	—	—	—	—
0.10	17.110	31	17.141	95	17.236	156	17.392	211	17.603	250	17.853	252	18.105	171	18.276	—	18.194	—	17.575	—	16.145
Δ	—466	—	—462	—	—481	—	—514	—	—564	—	—637	—	—736	—	—867	—	—1005	—	—	—	—
0.12	16.654	25	16.679	76	16.755	123	16.878	161	17.039	177	17.216	153	17.369	40	17.409	—	17.189	—	16.481	—	15.109
Δ	—437	—	—442	—	—458	—	—487	—	—533	—	—593	—	—674	—	—771	—	—872	—	—923	—	—864
0.14	16.217	20	16.237	60	16.297	94	16.391	115	16.506	117	16.623	72	16.695	—57	16.638	—	16.317	—	15.558	—	14.245
Δ	—417	—	—422	—	—437	—	—462	—	—499	—	—553	—	—621	—	—699	—	—768	—	—794	—	—737
0.16	15.800	15	15.815	45	15.860	69	15.929	78	16.007	63	16.070	4	16.074	—135	15.939	—	15.549	—	14.764	—	13.508
Δ	—400	—	—404	—	—417	—	—440	—	—472	—	—517	—	—573	—	—633	—	—684	—	—696	—	—642
0.18	15.400	11	15.411	32	15.443	46	15.489	46	15.553	18	15.553	—52	15.501	—195	15.306	—	14.865	—	14.068	—	12.866
Δ	—385	—	—387	—	—399	—	—419	—	—448	—	—485	—	—531	—	—579	—	—615	—	—617	—	—667
0.20	15.015	9	15.024	20	15.044	26	15.070	17	15.087	—19	15.068	—98	14.970	—243	14.727	—	14.250	—	13.451	—	12.299

0.20	15.015	9	15.024	20	15.044	26	15.070	17	15.087	-19	15.068	-98	14.970	-243	14.727	-----	14.250	-----	13.451	-----	12.296	-----
Δ	-891	-----	-900	-----	-923	-----	-963	-----	-1021	-----	-1093	-----	-1176	-----	-1256	-----	-1305	-----	-1286	-----	-----	-----
0.25	14.124	0	14.124	-3	14.121	-14	14.107	-41	14.066	-91	13.975	-181	13.794	-323	13.471	-----	12.945	-----	12.165	-----	11.127	-----
Δ	-807	-----	-813	-----	-831	-----	-880	-----	-900	-----	-950	-----	-1001	-----	-1043	-----	-1059	-----	-1025	-----	-980	-----
0.30	13.317	-6	13.311	-21	13.290	-43	13.247	-81	13.166	-141	13.025	-232	12.793	-365	12.428	-----	11.886	-----	11.140	-----	10.197	-----
Δ	-733	-----	-737	-----	-750	-----	-772	-----	-801	-----	-833	-----	-865	-----	-886	-----	-883	-----	-844	-----	-763	-----
0.35	12.584	-10	12.574	-34	12.540	-65	12.475	-110	12.365	-173	12.192	-264	11.928	-386	11.542	-539	11.003	-----	10.296	-----	9.434	-----
Δ	-668	-----	-672	-----	-681	-----	-697	-----	-716	-----	-738	-----	-756	-----	-765	-----	-752	-----	-712	-----	-643	-----
0.40	11.916	-14	11.902	-43	11.859	-81	11.778	-129	11.649	-195	11.454	-282	11.172	-395	10.777	-526	10.251	-667	9.584	-793	8.791	-----
Δ	-611	-----	-613	-----	-621	-----	-632	-----	-645	-----	-659	-----	-668	-----	-667	-----	-651	-----	-613	-----	-552	-----
0.45	11.305	-16	11.289	-51	11.238	-92	11.146	-142	11.004	-209	10.795	-291	10.504	-394	10.110	-510	9.600	-629	8.971	-732	8.239	-----
Δ	-560	-----	-562	-----	-567	-----	-575	-----	-584	-----	-591	-----	-595	-----	-589	-----	-570	-----	-533	-----	-480	-----
0.50	10.745	-18	10.727	-56	10.671	-100	10.571	-151	10.420	-216	10.204	-295	9.909	-388	9.521	-491	9.030	-592	8.438	-679	7.759	-----
Δ	-516	-----	-517	-----	-521	-----	-525	-----	-531	-----	-535	-----	-534	-----	-525	-----	-504	-----	-471	-----	-424	-----
0.55	10.229	-19	10.210	-60	10.150	-104	10.046	-157	9.889	-220	9.669	-294	9.375	-379	8.996	-470	8.526	-559	7.967	-632	7.335	-----
Δ	-476	-----	-477	-----	-479	-----	-482	-----	-485	-----	-485	-----	-482	-----	-471	-----	-451	-----	-419	-----	-377	-----
0.60	9.753	-20	9.733	-62	9.671	-107	9.564	-160	9.404	-220	9.184	-291	8.893	-368	8.525	-450	8.075	-527	7.548	-590	6.958	-----
Δ	-847	-----	-848	-----	-850	-----	-853	-----	-853	-----	-849	-----	-836	-----	-812	-----	-772	-----	-717	-----	-646	-----
0.7	8.906	-21	8.885	-64	8.821	-110	8.711	-160	8.551	-216	8.335	-278	8.057	-344	7.713	-410	7.303	-472	6.831	-519	6.312	-----
Δ	-732	-----	-732	-----	-731	-----	-729	-----	-725	-----	-717	-----	-701	-----	-677	-----	-641	-----	-592	-----	-534	-----
0.8	8.174	-21	8.153	-63	8.090	-108	7.982	-156	7.826	-208	7.618	-262	7.356	-320	7.036	-374	6.662	-423	6.239	-461	5.778	-----
Δ	-637	-----	-636	-----	-635	-----	-632	-----	-625	-----	-614	-----	-598	-----	-572	-----	-540	-----	-499	-----	-451	-----
0.9	7.537	-20	7.517	-62	7.455	-105	7.350	-149	7.201	-197	7.004	-246	6.758	-294	6.464	-342	6.122	-382	5.740	-413	5.327	-----
Δ	-553	-----	-553	-----	-555	-----	-550	-----	-543	-----	-531	-----	-514	-----	-492	-----	-458	-----	-427	-----	-386	-----
1.0	6.979	-20	6.959	-59	6.900	-100	6.800	-142	6.658	-185	6.473	-229	6.244	-272	5.972	-313	5.659	-346	5.313	-372	4.941	-----

TABLE 7.—Values of P in the Equation $M = \frac{a^2}{A} P$ When D is Greater Than A and Less Than Three Times A

This table covers a region twice as large as the preceding table. Both parameters have been changed to obtain a closer approximation to linear interpolation. Use this table if $A \leq D \leq 3A$.

METHOD OF USING TABLE TO COMPUTE AN INDUCTANCE.—From the measured dimensions compute a^2/A^2 and A^2/D^2 . Find the value of P , interpolating, if necessary, by the method given under the preceding table. Then

$$M = \frac{a^2}{A} P$$

ACCURACY.—In no part of the table will an error as great as 0.2 per cent be introduced by interpolation, except when A^2/D^2 is less than 0.2, and then the error will be but slightly larger than this. In this region a more accurate value of M can be obtained from the following table but the computation is a little more difficult.

METHOD OF USING TABLE TO FIND SIZE AND POSITION OF CIRCLES WHICH WILL GIVE A DESIRED INDUCTANCE.—Method same as in preceding table except that values of a^2/A^2 and A^2/D^2 are now selected.

UNITS.—Values of A , a , and D in centimeters. Value of M in millimicrohenrys. If A , a , and D are in inches, multiply the value of P by 2.54 to give M in millimicrohenrys (thousandths of a microhenry).

$\frac{A^2}{D^2}$	0	δ	0.1	δ	0.2	δ	0.3	δ	0.4	δ	0.5	δ	0.6	δ	0.7	δ	0.8	δ	0.9	δ	1.0
0.10	0.541	-7	0.534	-6	0.528	-6	0.522	-6	0.516	-6	0.510	-6	0.504	-6	0.498	-6	0.492	-5	0.487	-6	0.481
Δ	389		381		372		364		356		348		340		333		326		319		313
0.15	.930	-15	.915	-15	.900	-14	.886	-14	.872	-14	.858	-14	.844	-13	.831	-13	.818	-12	.806	-12	.794
Δ	413		403		391		380		370		360		351		342		333		324		316
0.20	1.343	-26	1.317	-26	1.291	-25	1.266	-24	1.242	-24	1.218	-23	1.195	-22	1.173	-22	1.151	-21	1.130	-21	1.109
Δ	423		409		397		385		373		362		351		340		329		319		310
0.25	1.766	-40	1.726	-38	1.688	-37	1.651	-36	1.615	-35	1.580	-34	1.546	-33	1.513	-33	1.480	-31	1.449	-30	1.419
Δ	422		409		395		382		369		357		346		333		322		311		300
0.30	2.188	-53	2.135	-52	2.083	-50	2.033	-49	1.984	-47	1.937	-46	1.891	-45	1.846	-44	1.802	-42	1.760	-41	1.719
Δ	418		403		389		376		362		348		336		324		313		301		289
0.35	2.606	-68	2.538	-66	2.472	-64	2.408	-62	2.346	-61	2.285	-58	2.227	-57	2.170	-55	2.115	-54	2.061	-53	2.008
Δ	408		396		381		367		353		340		326		314		302		291		281
0.40	3.014	-81	2.933	-80	2.853	-78	2.775	-76	2.699	-74	2.625	-72	2.553	-69	2.484	-67	2.417	-65	2.352	-63	2.289
Δ	399		383		369		356		342		329		317		304		291		279		268
0.45	3.413	-96	3.316	-94	3.222	-91	3.131	-90	3.041	-87	2.954	-84	2.870	-82	2.788	-80	2.708	-77	2.631	-74	2.557
Δ	386		373		359		346		333		319		306		294		282		270		258
0.50	3.799	-110	3.689	-108	3.581	-105	3.476	-102	3.374	-101	3.273	-97	3.176	-94	3.082	-92	2.990	-89	2.901	-86	2.815
0.5	3.799	-110	3.689	-108	3.581	-105	3.476	-102	3.374	-101	3.273	-97	3.176	-94	3.082	-92	2.990	-89	2.901	-86	2.815
Δ	734		709		684		658		632		608		582		557		533		510		487
0.6	4.533	-135	4.398	-133	4.265	-131	4.134	-128	4.006	-125	3.881	-123	3.758	-119	3.639	-116	3.523	-112	3.411	-109	3.302
Δ	681		639		599		561		522		483		445		408		371		334		297
0.7	5.215	-156	5.059	-155	4.904	-154	4.750	-152	4.598	-149	4.449	-146	4.303	-143	4.160	-139	4.021	-135	3.886	-131	3.755
Δ	634		596		558		520		482		444		406		368		330		292		254
0.8	5.849	-174	5.675	-175	5.500	-174	5.326	-173	5.153	-172	4.981	-168	4.813	-165	4.648	-162	4.486	-157	4.329	-153	4.176
Δ	586		547		508		469		430		391		352		313		274		235		196
0.9	6.435	-188	6.247	-191	6.056	-192	5.864	-192	5.672	-191	5.481	-189	5.292	-187	5.105	-182	4.923	-179	4.744	-173	4.571
Δ	544		505		466		427		388		349		310		271		232		193		154
1.0	6.979	-199	6.780	-204	6.576	-207	6.369	-209	6.160	-209	5.951	-209	5.742	-206	5.536	-203	5.333	-199	5.134	-193	4.941

TABLE 8.—Values of S in the Equation $M = \frac{a^2 A^2}{D^3} S$ When D is Greater Than 3 A —Circles Far Apart

When the distance between the circles is greater than three times the radius of the larger circle, the mutual inductance can be most easily computed by the formula

$$M = \frac{a^2 A^2}{D^3} S$$

This table covers the range from $\frac{9A}{4} < D < \infty$

METHOD OF USING TABLE TO COMPUTE AN INDUCTANCE.—Given the measured dimensions as under Table 6; compute a^2/A^2 and A^2/D^2 . Obtain S from the table, interpolating if necessary by the same method as described under Table 6. Then

$$M = \frac{a^2 A^2}{D^3} S$$

ACCURACY.—There is only one place in this table where an error as great as 0.1 per cent can be introduced by interpolation. This is between the values of 0 and 0.02 of A^2/D^2 when $a^2/A^2 = 1$. Even here the maximum error is less than 0.2 per cent.

METHOD OF USING TABLE TO FIND SIZE AND POSITION OF CIRCLES TO GIVE A DESIRED INDUCTANCE.—Choose suitable relative dimensions, but with $D > 3A$ compute a^2/A^2 and A^2/D^2 . Find the value of S in the table. Then

$$D = \frac{MA^2 D^4}{S a^2 A^4}$$

From the relative dimensions the values of a and A can be computed.

UNITS.—Values of A , a , and D in centimeters. Value of M in millimicrohenrys. If A , a , and D are in inches, multiply the value of S by 2.54 to give M in millimicrohenrys (thousandths of a microhenry).

$\frac{A^2}{D^2}$	$\frac{a^2}{A^2}$	0	δ	0.1	δ	0.2	δ	0.3	δ	0.4	δ	0.5	δ	0.6	δ	0.7	δ	0.8	δ	0.9	δ	1.0
0		19.739	0	19.739	0	19.739	0	19.739	0	19.739	0	19.739	0	19.739	0	19.739	0	19.739	0	19.739	0	19.739
Δ		678		633		687		741		795		849		903		957		1010		1062		1114
0.02		19.161	55	19.106	54	19.052	54	18.998	54	18.944	54	18.890	54	18.836	54	18.782	53	18.729	52	18.677	52	18.625
Δ		649		596		643		689		734		779		823		866		908		951		993
0.04		18.612	102	18.510	101	18.409	100	18.309	99	18.210	99	18.111	98	18.013	97	17.916	95	17.821	95	17.726	94	17.632
Δ		624		564		604		643		681		716		751		786		822		856		888
0.06		18.088	142	17.946	141	17.805	139	17.666	137	17.529	134	17.395	133	17.262	132	17.130	131	16.999	129	16.870	128	16.744
Δ		601		535		567		599		630		661		692		720		748		776		802
0.08		17.587	176	17.411	173	17.238	171	17.067	168	16.899	165	16.734	164	16.570	160	16.410	159	16.251	157	16.094	152	15.942
Δ		477		508		535		562		588		613		637		661		683		704		727
0.10		17.110	205	16.905	202	16.703	198	16.505	194	16.311	190	16.121	188	15.933	184	15.749	181	15.568	178	15.390	175	15.215
Δ		457		481		503		526		548		570		590		609		628		647		663
0.12		16.653	229	16.424	224	16.200	221	15.979	216	15.763	212	15.551	208	15.343	203	15.140	200	14.940	197	14.743	191	14.552
Δ		436		457		478		496		516		531		547		563		579		592		606
0.14		16.217	250	15.967	245	15.722	239	15.483	235	15.248	228	15.020	224	14.796	219	14.577	216	14.361	210	14.151	205	13.946
Δ		417		435		451		467		481		496		509		522		534		547		558
0.16		15.800	268	15.532	261	15.271	255	15.016	249	14.767	243	14.524	237	14.287	232	14.055	228	13.827	223	13.604	216	13.388
Δ		401		414		428		441		453		465		477		488		496		504		514
0.18		15.399	281	15.118	275	14.843	268	14.575	261	14.314	255	14.059	249	13.810	243	13.567	236	13.331	231	13.100	226	12.874
Δ		388		395		406		417		427		437		445		454		462		469		476
0.20		15.016	293	14.723	286	14.437	279	14.158	271	13.887	265	13.622	257	13.365	252	13.113	244	12.869	238	12.631	233	12.398

TABLE 9.—Values of $\frac{(2n)^2}{2^{4n} (n)^4}$

The above expression appears in all the formulas for mutual inductance given in this paper. By the use of this table, the series can be easily extended to the twenty-sixth term. Let

$$\frac{(2n)^2}{2^{4n} (n)^4} = \psi$$

<i>n</i>	log $\psi + 10$	ψ	<i>n</i>	log $\psi + 10$	ψ
0-----	10. 000000	1. 0000000	13-----	8. 380558	0. 0240192
1-----	9. 397940	0. 2500000	14-----	8. 348970	. 0223342
2-----	9. 148062	. 1406250	15-----	8. 319520	. 0208699
3-----	8. 989700	. 0976563	16-----	8. 291944	. 0195859
4-----	8. 873718	. 0747684	17-----	8. 266014	. 0184507
5-----	8. 782202	. 0605623	18-----	8. 241546	. 0174400
6-----	8. 706626	. 0508892	19-----	8. 218380	. 0165341
7-----	8. 642252	. 0438785	20-----	8. 196390	. 0157177
8-----	8. 586196	. 0385652	21-----	8. 175460	. 0149782
9-----	8. 536550	. 0343993	22-----	8. 155490	. 0143051
10-----	8. 491998	. 0310455	23-----	8. 136404	. 0136900
11-----	8. 451590	. 0282872	24-----	8. 118114	. 0131254
12-----	8. 414624	. 0259791	25-----	8. 100566	. 0126057