ABSTRACT

The extraction of the modal parameters for closely spaced modes in the frequency domain is a common problem. However, it is made more difficult if the damping for the closely spaced modes is high. Data from a structure with more than three percent viscous damping is presented which exhibits this phenomenon. Traditional experimental techniques failed to identify all the modal parameters of three closely spaced modes. Mode shapes from an analytical model are manipulated to produce a modal filter which is used to calculate enhanced frequency response functions from which the modal parameters can be more readily identified. Discussion of the advantages and disadvantages of this technique as compared with traditional frequency response function enhancement techniques will be presented.

NOMENCLATURE

- $\Psi_s$: Mode shape matrix from analytical model
- $\Psi_r$: Reduced mode shape matrix from model
- $p$: Vector of pseudo-generalized coordinates
- $x$: Vector of displacements at measured degrees of freedom
- FRF: Frequency response function
- $H_p(\omega)$: Vector of FRFs of the measured degrees of freedom
- $H_e(\omega)$: Vector of enhanced FRFs
- $\omega$: Frequency in radians per second
- $\zeta$: Viscous damping ratio
- $m_r$: Modal mass
- a subscript: Associated with analytical model
e subscript: Associated with experimental hardware
ac subscript: Hybrid associated with FEM and experimental hardware
I: Identity matrix
$\Phi$: Eigenvector matrix extracted from $H_{pe}(\omega)$
$q$: Vector of modal coordinates

$\Phi^T$: Matrix of mode shapes extracted using combined results of $\Psi_r$ and $\Phi$

INTRODUCTION AND MOTIVATION

In a modal test of a moderately damped structure, three of the natural frequencies fell within about two percent of each other. Three shakers were being used, but one of the shakers did not excite any of the three modes. One shaker excited one of the modes and the other shaker excited all three modes. Multiple applications of the polyreference and direct parameter estimation algorithms would not yield three distinct roots and shapes. In particular, the second and third roots could not be found distinctly, and the mode shape extractions always appeared as a linear combination of multiple mode shapes. All attempted extractions indicated that the damping was more than three percent for these modes. A finite element model (FEM) indicated that distinct shapes should be realizable, yet no readily available experimental technique was successful in the extraction of the modal parameters. This problem led to the following approach.

APPROACH

The authors have used modal filters previously to reconstruct from acceleration measurements the applied dynamic force to elastic bodies[1,2]. Modal filters are a transformation matrix made up of reciprocal modal vectors which will be explained later. Shelley has done extensive work calculating and using modal filters for control and on-line estimation[3]. As early as 1980, Allemang suggested that enhanced frequency response functions (FRFs) calculated from the product of FEM mode shapes with the reduced mass matrix and experimental FRFs could aid in the estimation of the roots for modal analysis[4]. What follows is the authors’ approach for calculating enhanced FRFs utilizing a modal filter created from FEM mode shapes.

Let $\Psi_s$ be the mode shape matrix from the FEM. The partition of $\Psi_s$ that corresponds exactly to the experimentally measured degrees of freedom (dof) and a truncated set of n modes that span the bandwidth of interest will be called $\Psi_{sr}$, or the reduced $\Psi_r$. Neglecting the effects of modal truncation the modal expansion is written.

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\[ \Psi_{at} \mathbf{p} = \mathbf{x} \]  

with \([\Psi_{at}]_{m \times m}, \{\mathbf{p}\}_{m \times 1}, \text{ and } \{\mathbf{x}\}_{m \times 1}\), where \(\mathbf{x}\) is the vector of \(m\) displacement dofs corresponding to measured dofs, and \(\mathbf{p}\) is a vector of uncoupled generalized coordinates. This is the classic modal substitution. Let \(\mathbf{H}_k(\omega)\) be the vector of FRFs with response at \(x\) due to a single excitation force at one dof, and let \(\mathbf{H}_p(\omega)\) be the corresponding vector of FRFs for the generalized coordinates \(p\). The FRFs of \(x\) and \(p\) are related in the same manner as equation (1).

\[
\mathbf{H}_x(\omega) = \Psi_{at} \mathbf{H}_p(\omega)
\]

Furthermore,

\[
\mathbf{H}_g(\omega) = \Psi_{at}^+ \mathbf{H}_g(\omega)
\]

where \(\Psi_{at}^+\) is the pseudoinverse of \(\Psi_{at}\).

If there are \(n\) modes in \(\Psi_{at}\), there will be \(n\) FRFs in \(\mathbf{H}_g(\omega)\). The effect of the pseudoinverse of \(\Psi_{at}\) on \(\mathbf{H}_g(\omega)\) is to form new single dof frequency response functions. The effects of all but one mode are filtered out in each FRF of \(\mathbf{H}_g(\omega)\). (The effects of modes outside the bandwidth of \(\Psi_{at}\) are minimal in the bandwidth of interest). For example, in the first FRF, \(\mathbf{H}_g(\omega)\), responses for all modes except \(\Psi_{at}\) will be suppressed. These FRFs can be analyzed one at a time with experimental single-degree-of-freedom extraction techniques to determine natural frequencies and damping for each mode of the system up through mode \(n\). The inverse or pseudoinverse of \(\Psi_{at}\) is called a modal filter or the set of reciprocal modal vectors. Each of the reciprocal modal vectors (the rows of the pseudoinverse of \(\Psi_{at}\)) is orthogonal to the noncorresponding columns of \(\Psi_{at}\) since \(\Psi_{at}^+ \Psi_{at} = \mathbf{I}\). The following two dof example of two masses connected by a rigid beam illustrates the modal filter.

\[
\begin{bmatrix}
\frac{1}{2} & 1/2 \\
1/2 & -1/2
\end{bmatrix}
\]

In this example, \(\Psi\) is

In the x direction on mass 1 can be written as

\[
\begin{bmatrix}
\frac{1}{2} & 1/2 \\
1/2 & -1/2
\end{bmatrix}
\]

where the first column represents the translation and the second column represents the pitch mode shape. The inverse of \(\Psi\) is

\[
\begin{bmatrix}
\frac{1}{2} & 1/2 \\
1/2 & -1/2
\end{bmatrix}
\]

The FRF for a force in the x direction on mass 1 can be written as

\[
\begin{align*}
H_x(\omega) &= \frac{1}{m_1(\omega - \omega_1^2 + j\zeta_1 \omega_1 \omega_1)} + \frac{1}{m_2(\omega - \omega_2^2 + j\zeta_2 \omega_2 \omega_2)} \\
H_y(\omega) &= \frac{1}{m_1(\omega - \omega_1^2 + j\zeta_1 \omega_1 \omega_1)} \frac{1}{m_2(\omega - \omega_2^2 + j\zeta_2 \omega_2 \omega_2)}
\end{align*}
\]

where the superscript on \(\Psi\) is the mode number, the subscript on \(\omega\) and \(\zeta\) indicate the natural frequency and viscous damping ratio associated with that mode number, and \(\omega\) is frequency in radians per second. The FRF for dof \(x2\) can be written similarly.
appropriate to allow individual modes in be measured, and that the locations of the measured dofs must be
Obviously, the approach requires a FEM. The measurements and addition the experimental mode shapes of interest must be linear
This improves the ability of standard modal extraction algorithms
displacement coordinate system, and the measurements must be
measurement degrees of freedom must be chosen to allow the
addition the pseudoinverse of
which requires their mode shape calculation to be
dependent on their own root calculation (i.e. they are not
According to equation (13) these roots may not be compatible with commercial
software which requires their mode shape calculation to be
extracted experimentally, then a completely
requirements of the modal test may be to provide one to one
calculated with their technique before the mode shapes are
calculated. If their root finders cannot uniquely identify the roots
of the closely spaced modes from the new vector of FRFs, \(H_{pe}(\omega)\), then they will not be able to calculate the mode shapes associated
with \(\Phi\). Although the modal test engineer may have easily
calculated the roots using a single-degree-of-freedom root
estimator, these roots may not be compatible with commercial
software which requires their mode shape calculation to be
dependent on their own root calculation (i.e. they are not
compatible with a root calculated from another technique). Single
dof mode shape estimators are not sufficient to find the mode
shapes since they only supply the mode shape term on the diagonal
of \(\Phi\). In order to find the proper linear combination of the mode
shapes \(\Psi_{se}\) to represent the experimental mode shapes accurately,
the full \(\Phi\) matrix must be accurately estimated. One of the authors
has published the results of a multi-dof mode shape estimator [5]
that can be used with roots calculated by any method. However,
this estimator cannot separate perfectly repeated roots. Usually,
there is a slight separation between closely spaced modes. Then
the accuracy with which \(\Phi\) can be calculated is based on the
accuracy of the root finder’s extraction of frequency and damping.
This limitation on the accuracy of the extraction of \(\Phi\) is more
severe than the limitations on finding the roots from \(H_{pe}(\omega)\). These
limitations may not be critical, particularly if the goal of the modal
test is to provide information for correlating a FEM. The primary
requirement of the modal test may be to provide one to one
correspondence of the test and FEM modes and accurate
extractions of the modal frequencies and damping. Such an
evaluation may be obtained simply by observation of the amplitude
of the resonances in \(H_{pe}(\omega)\) as well as the extraction of the roots
from \(H_{pe}(\omega)\) (this was the case for the application that motivated
this work). However, if very accurate measurements of the mode
shapes are required, and \(\Phi\) must be calculated, limitations may be
encountered.

APPLICATION AND RESULTS

As explained in the motivation section, there were three closely
spaced modes with moderate damping. One of those modes could
be extracted well from the FRFs associated with a single shaker,
but the other two could not, because they were very close in
frequency and excited by only one of the three shakers. The modal
parameter extraction of the two difficult modes always yielded a
linear combination of two of the FEM mode shapes, and the two roots could not be uniquely identified. The mode shape matrix $\Psi_m$ consisted of 13 modes and 73 dof. Its pseudoinverse was multiplied into the 73 FRFs associated with the single shaker that excited all three closely spaced modes. Thirteen resulting FRFs were generated as $H_m(o)$.

Figure 1 shows all of the FRFs of $H_m(o)$ overlaid. It can be seen that most of the FRFs are predominantly exhibiting single dof response, indicating that most mode shapes in $\Psi_m$ are fairly accurate.

Discussions will now center on the three closely spaced modes. Figures 2, 3 and 4 show the three FRFs that corresponded to the three closely spaced modes. In the mode shape matrix $\Psi_m$, these modes corresponded to the fifth, sixth and seventh mode shapes. In Figure 2, it can be seen that two resonances are present. This indicates that two of the experimental mode shapes involved mode five of $\Psi_m$. However, one of the resonances is dominant, indicating that its shape most closely represents the shape of mode five of $\Psi_m$. The direct parameter extraction of the frequency and damping of this resonance is indicated in the plot. Circle fitting gave almost identical results. In Figures 3 and 4, the frequencies for the primary resonance associated with modes six and seven of $\Psi_m$ are very close together. This validated the authors' opinion that the roots were so close as to make it difficult for the commercial algorithms to separate them. This enhancement technique worked well in this particular application since the two closely spaced modes were significantly different in shape, and the FEM shapes were fairly accurate in representing these shapes. Figure 5 shows all three plots overlaid. The amplitude of the resonances is an indication of the relative excitation level of the three modes from the shaker.

COMPARISON WITH OTHER METHODS

An exhaustive search has not been made, but the most reliable and accepted method of developing enhanced FRFs is through the application of vectors called force patterns to provide linear combinations of columns of the FRF matrix. Force patterns are usually calculated as the eigenvectors of the multivariate mode indicator function eigenvalue problem. The multivariate mode indicator function was developed by Williams, Crowley and Vold[6]. It is recognized as being extremely valuable in identifying repeated roots. Since it is entirely dependent on experimental data, no FEM has to be generated. The necessary requirement for identifying a repeated root is that one exciter must excite each additional mode with a repeated root. For example, if their are three repeated roots, then there must be at least three exciters, and each mode must be exercised by a different exciter. One exciter may excite more than one of the modes, but at least one exciter must excite each mode. If this is the case, the polyreference technique or other multiple reference techniques may be able to extract the roots and the shapes quite well. The application of the force patterns of the multivariate mode indicator function was shown by the Williams, Crowley and Vold to be able to create enhanced FRFs that separated the mode shapes of the repeated roots quite well. This is contingent on exciter placement. The force pattern vectors are the length of the number of exciters. If the placement of the shakers is not "orthogonal enough" to the mode shapes of interest, the separation of the modal responses in the enhanced FRFs may not be accomplished. Also, the force pattern is applicable only at the frequency line of the minimum value in the associated multivariate mode indicator function. This is fine for perfectly repeated roots, but the force pattern may not take into account nearby roots. The modal filter, on the other hand, suppresses the undesired modal response throughout the frequency band of $\Psi_m$, not just the modal response of the repeated root. If there are three modes excited by only two shakers, as in the application cited for this paper, the multivariate mode indicator function, polyreference or force patterns applied to create enhanced FRFs cannot isolate all three modes distinctly. In that case the technique described in this paper is of value.

It has been suggested that simply multiplying the FRFs by the value of the mode shape of the particular mode that is being extracted can enhance the FRF, but this does not use the orthogonality relationship to actually suppress the undesired modal response of all other modes. The resulting FRFs will still be a coupled mixture of the original FRFs and the resulting shapes will be a mixture of the multiple shapes weighted by the FEM shape.

CONCLUSIONS

A modal test of a moderately damped structure had three closely spaced resonant frequencies. The natural frequencies, damping and correspondence of the experimental mode shapes to mode shapes of a FEM were desired. For this application, the experimental setup was not sufficient to extract unique roots and shapes for two of the closely spaced modes, even with the use of multiple reference extraction methods and force patterns from the multivariate mode indicator function. The modal filter that was calculated from the FEM mode shapes was robust enough to calculate enhanced FRFs from which the roots of the closely spaced modes could be easily extracted. Also, the amplitude of the strongest resonance in the enhanced FRFs indicated which mode of the FEM most strongly correlated with a resonance from the experiment.

Enhanced FRFs can be created experimentally using the force patterns from the multivariate mode indicator function. The effectiveness of the force patterns is dependent on the modal density, number and placement of exciters. Enhanced FRFs can be created analytically using the modal filter in conjunction with experimentally measured FRFs. The effectiveness of the modal filter is dependent on the FEM mode shapes spanning the experimental mode shape space of interest.
REFERENCES


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