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**Classical Trajectory versus Quantum Interference.
A Linear Chain Model for
the Origin of Uncertainty Broadening**

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Jau Tang

Chemistry Division, Argonne National Laboratory,
Argonne, IL 60439

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Abstract

A simple linear chain model, as an alternative to the orthodox Schrödinger approach, is proposed to explain the origin of the uncertainty broadening and to improve our physical insight into the difference between classical and quantum worlds. Quantum interference in space is manifested as a result of fast exchange between adjacent particles of different internal degrees of freedom.

I. Introduction

Quantum theory has been successful in explaining many physical and chemical systems, and many stringent tests and predictions have been confirmed. However, the seemingly counter-intuitive quantum behavior in the microscopic world has puzzled many scientists since de Broglie introduced the concept of "matter wave" [1,2]. Even though the quantum theory is well established [3], many debates about the logic, interpretation[4], and origins continue (for a review and references therein see Ref. [5]). The intention of this work is not to settle all these problems. A very simple model is constructed to explain quantum interference using understandable terms, so that our physical insight into such a phenomenon can be improved. It will be shown that the uncertainty broadening and quantum interference in space can be explained as a result of fast exchange interaction between particles of different internal degree of freedom. Such an exchange process has to be short -ranged which causes a loss of distinguishability and leads to quantum interference. In the macroscopic world, because of the large size and mass, the distinguishability is lost and the exchange interaction is small.

II. Linear chain model for quantum interference

First of all, let us consider hopping motion along a one-dimensional N-site chain. The balls are assumed to have an internal degree of freedom which can oscillate between two polarities at a rate U . Here we define the rule of game in the following rate equation for the distribution functions $f_n(t)$ and $g_n(t)$ for two polarities at the n -th sites

$$\begin{aligned}
\frac{d}{dt} f_n &= -\frac{W}{2} (f_{n-1} - f_n) + \frac{W}{2} (f_{n+1} - f_n) + U_n g_n \\
\frac{d}{dt} g_n &= -\frac{W}{2} (g_{n+1} - g_n) + \frac{W}{2} (g_{n-1} - g_n) - U_n^* f_n, \quad (n = 0, 1, 2, \dots, N)
\end{aligned}
\tag{1}$$

In this model it is assumed that the ball with positive polarity can only hop to the right and the other kind of ball of different polarity can only hop to the left with the same hopping rate W . With a given initial distribution of balls at the various site, one would like to determine the distribution of the balls along the chain at a later time.

Using discrete Fourier transform with $F_k(t) = \sum_n f_n(t) \exp(-i2\pi kn/N)$ and $G_k(t) = \sum_n g_n(t) \exp(-i2\pi kn/N)$, eq. (1) can be transformed into a 2×2 rate equation for each k -component ($k = 0, 1, \dots, N-1$)

$$\begin{aligned}
\frac{d}{dt} F_k &= -\frac{W}{2} \left(e^{\frac{-i2\pi kn}{N}} - e^{\frac{i2\pi kn}{N}} \right) F_k + U_k G_k \\
\frac{d}{dt} G_k &= -\frac{W}{2} \left(e^{\frac{i2\pi kn}{N}} - e^{\frac{-i2\pi kn}{N}} \right) G_k - U_k^* F_k
\end{aligned}
\tag{2}$$

where U_k is the Fourier transform of U_n and the cyclic boundary condition is assumed. We have obtained the solution for eq. (2) which can be expressed as

$$\begin{aligned}
F_k(t) &= [\cos(U_k t) + i \sin\theta_k \sin(U_k t)] F_k(0) \\
&\quad + \cos\theta_k \sin(U_k t) G_k(0) \\
G_k(t) &= -\cos\theta_k \sin(U_k t) F_k(0) \\
&\quad + [\cos(U_k t) - i \sin\theta_k \sin(U_k t)] G_k(0)
\end{aligned}
\tag{3}$$

where

$$\begin{aligned}
 V_k &= \sqrt{|U_k|^2 + W_k^2} \\
 W_k &= W \sin\left(\frac{2\pi k}{N}\right) \\
 \theta_k &= \tan^{-1} \left[\frac{W}{|U_k|} \sin\left(\frac{2\pi k}{N}\right) \right] .
 \end{aligned} \tag{4}$$

The distribution in the coordinate space n can be determined from the above equation using inverse Fourier transform $f_n(t) = N^{-1} \sum_k F_k(t) \exp(i2\pi kn/N)$ and $g_n(t) = N^{-1} \sum_k G_k(t) \exp(i2\pi kn/N)$. For a very large N one can replace $2\pi k/N\ell$ by a continuous variable κ .

Now we would like to explore the physical meanings of this model. From eqs. (3) and (4), one can see that if $U_k = 0$ and $F_k(0) = 0$, then $G_k(t) = 0$. It means that if there is no exchange interaction U_k , the internal degree of freedom is distinguishable. The hopping process of balls are completely determined with a given initial condition, and the evolution of the distribution behaves classically. Let us first consider the case of no exchange interaction U_k with $G_k(0) = 0$. Assuming a Gaussian distribution for $f_c(x, t=0) = \exp(-x^2/2L^2)/(2\pi)^{1/2}L$ with $\int dx f_c(x, t=0) = 1$, one has $F_k(0) = \exp(-\kappa^2 L^2/2)$. It can be shown from eq. (3) that

$$f_c(x, t) = \frac{1}{\sqrt{2\pi L^2}} e^{-(x-WLt)^2/2L^2} \tag{5}$$

which indicates that $f_c(x, t)$ exhibits a shape-preserved classical packet traveling at a speed of WL . It means that if U_k is short-ranged (negligibly small for a small κ) so that $W^2 L^2 \kappa^2 \gg |U_k|^2$, the particles behave classically.

Now let us examine the other extreme in the microscopic world (a large κ) so that $|U_\kappa|^2 \gg W^2 L^2 \kappa^2$, eq. (3) can be reduced to

$$\begin{aligned} F_\kappa(t) &= [\cos(V_\kappa t) + i \sin\theta_\kappa \sin(V_\kappa t)] F_\kappa(0) \\ &\quad + \cos\theta_\kappa \sin(V_\kappa t) G_\kappa(0) \\ G_\kappa(t) &= -\cos\theta_\kappa \sin(V_\kappa t) F_\kappa(0) \\ &\quad + [\cos(V_\kappa t) - i \sin\theta_\kappa \sin(V_\kappa t)] G_\kappa(0) \end{aligned} \quad (6)$$

where $V_\kappa = [|U_\kappa|^2 + W^2 \kappa^2 L^2]^{1/2}$, $\theta_\kappa = \tan^{-1}(W\kappa L / |U_\kappa|)$. Now let us consider the effects of fast exchange of such an internal degree of freedom and also assume U_κ approaching U , a constant at a high κ . With a Gaussian distribution for $f(x, t=0) = \exp(-x^2/4L^2)/(2\pi)^{1/2} L^{1/2}$ with $\int dx |f(x, t=0)|^2 = 1$, one has $F_\kappa(0) = \exp(-2\kappa^2 L^2)$ and $G_\kappa(0) = 0$. Eq. (6) indicates a mixing between $F_\kappa(t)$ and $G_\kappa(t)$ as a result of the exchange interaction U , and $f(x)$ is no longer a real function. It is found that $F_\kappa(t) + iG_\kappa(t) = \exp[-it(U + 2W^2 \kappa^2 R^2) - 2\kappa^2 L^2]$ and $F_\kappa(t) - G_\kappa(t) = \exp[it(U + 2W^2 \kappa^2 R^2/U) \exp(-2\kappa^2 L^2)]$. Because of the fast exchange between internal degree of freedom, only $|f(x, t)|^2 + |g(x, t)|^2$ is measured. One can show

$$|f(x, t)|^2 + |g(x, t)|^2 = \frac{1}{\sqrt{2\pi L^2 (1 + W^4 t^2 / U^2)}} e^{-(x - WLt)^2 / 2L^2 [1 + W^4 t^2 / U^2]} \quad (7)$$

This result, showing a traveling but spreading wave packet, is very different from the shape-preserved $f_c(x, t)$ of eq. (5) in the non-exchange case for classical particles. The uncertainty of the position is indicated by the second moment of the packet $\sigma^2(t) = L^2(1 + t^2 W^4 / U^2)$ which grows in time as illustrated in fig. 1. This

is exactly the same results as the well-known Heisenberg's uncertainty principle. The change of a real distribution function $f(x, t=0)$ into a complex function and the spreading of the second moment $\sigma^2(t)$ obtained here explains much clearer the origins of the well-known quantum behavior [6]. In the macroscopic world, if U_k/W_k^2L is very small as compared to the observation time t then the classical picture is still a very good approximation. This indicates the exchange interaction is short-ranged and is very small in the macroscopic world. Such an interaction decreases due to the large size and mass of a macroscopic object. However, in the microscopic world U_k is strong and U_k/W_k^2L can be very large, the quantum interference is important. The strong exchange interaction and the indistinguishability of the particles make quantum behavior prominent. Thus, this simple model described in this work provides an interesting physical insight into the difference between the macroscopic classical world and the peculiar quantum interference in the microscopic world.

Here we would like to extend the linear chain model to a more general case. In the continuum limit, eq. (2) can be expressed as

$$\frac{d}{dt} \begin{pmatrix} F_k \\ G_k \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} F_k \\ G_k \end{pmatrix} \quad (8)$$

For the probability to be conserved, we have found out that A, B, C and D has to satisfy the following relations

$$\begin{aligned} AB + BD &= CA + DC = 0 \\ AA + BC &= CB + DD = V_x^2 \end{aligned} \quad (9)$$

There are a few choices for these A , B , C and D . If they are commutable, one can have $A = -D = W_k$ as the W_k in eq. (4), $B = |U_k| = -C$. If they are not commutable, two possibilities were found: 1) $A = -D$, $[B, C] = 0$ and $\{A, B\} = \{A, C\} = 0$. In this case, one may have $A = \sum_{\{i=x,y,z\}} W_i(k) \sigma_i = -D$ and $B = -C = -|U_k|$. 2) Another possibility is that $A = D$, $[B, C] = 0$ and $\{A, B\} = \{A, C\} = 0$. One may choose $A_i = D_i = \sum_{\{i=x,y,z\}} \epsilon_{ijk} W_j(k) \sigma_k$ and $B_i = -C_i = U \sigma_i$. Here we have derived a very interesting result showing that eq. (8) can lead to a requirement for the particle to own a spin (1/2), as indicated by presence of the Pauli's matrices σ_i 's. The dimension of A , B , C and D depends on the scale of a physical process. In the relativistic regime, if one assumes $A = ck \cdot \sigma = -D$ and $B = -C = -m_0 c^2 / \hbar$, eq. (8) is equivalent to the Dirac's equation [3]; and if one assumes $A = ck = -D$ and $B = -C = -m_0 c^2 / \hbar$, then eq. (8) is equivalent to the Klein-Gordon equation [6].

III. Conclusions

In this work, we have presented a simple linear chain model to explain the origins for quantum interference in space and the uncertainty broadening. As an alternative to the orthodox Schrödinger equation approach, better physical understanding for such an peculiar process can be obtained. The quantum interference and uncertainty broadening is explained as a result of fast exchange interaction within the adjacent particles of different internal degree of freedom. Fast exchange process causes a loss of distinguishability and leads to quantum interference. The exchange interaction is short-ranged. If it exceeds kinetic energy of particles ($U_k^2 \gg W^2 L^2 k^2$), it can introduce quantum behavior. In the macroscopic world, on the other hand, the exchange interaction is weak. Because it's short-ranged, the kinetic energy of particles is much larger than the exchange energy,

and the distinguishability and the quantum interference are lost. The internal degree of freedom is related to a particle and its antiparticle. The source for such an exchange interaction in the atomic scale is electromagnetic interaction. At a short distance in the microscopic world, it is strong and there may be a small percentage of component for antiparticle character, and the charge separation is noticeable. At a large scale in the macroscopic world, the tiny separation is almost invisible and the electrostatic potential has been neutralized. Thus, people would not be able to observe quantum motion of billiard balls on a table.

In addition to the simple linear chain model, we have extended it to a general three-dimensional case. The conservation of the probability demands interesting commutator and anti-commutator rules for the operators A, B, C and D. One particular solution of these operators leads to the Dirac's equation for a spin-1/2 particle which was derived much differently, based on more complicate quantum rules and special relativity. Thus, eq. (1) of the proposed simple linear chain model, or eq. (7) of a three-dimensional model seems to be quite general. By varying U_k/W , one can smoothly connect the classical trajectory regime with the quantum world. Further work on applying this model to harmonic potential, periodic wells, or Coulomb interaction will be published elsewhere.

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Fig. 1. The spreading of the population distribution due to uncertainty broadening at t/T . T is defined as $m_e/m_0 L^2$, where m_e/m_0 is the ratio of the electron mass and the particle's mass.

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