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Wake Fields, Potential Well Distortion and Beam Stability in the LER PEP-II

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Abstract

Longitudinal and transverse wake fields are constructed for LER PEP-II. The effects of potential well distortion and the single bunch longitudinal stability are discussed for LER PEP-II storage ring. The coupled-bunch stability recalculated with the updated impedance.

Model of the Longitudinal Impedance

The low-energy PEP-II ring (LER) is designed for the energy 3.1 GeV and the beam current 2.1 A in 1658 bunches with rms $\sigma_z = 1$ cm. The main LER parameters are given in the CDR report^[1].

The longitudinal impedance is modeled as the sum of narrow-band (NB) and the broad-band (BB) impedances. The NB impedance is given by the NB impedance of six PEP-II RF cavities and two NB modes of the 290 BPMs in the ring. The BB impedance is the sum of the BB high-frequency tail of the RF cavities, resistive wall impedance (RW), and the BB impedance of the components of the vacuum system.

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The NB impedance of a cavity is the sum over ten main modes

$$Z_{cav}^{NB}(\omega) = i \sum_l \kappa_l \left[\frac{1}{\omega - \omega_l + i\gamma_l} + \frac{1}{\omega - \omega_l - i\gamma_l} \right] \quad (1)$$

where a mode loss factor κ_l is related to the shunt impedance R_l and unloaded Q_0 and loaded Q_L quality factors of the mode: $\kappa_l = R_l \omega_l / (2Q_{0,l})$, and $\gamma_l = \omega_l / (2Q_{L,l})$.

The longitudinal wake and the S-function, $S(z) = \int_0^z dz W(z)$ corresponding to the narrow-band impedance is the sum of the wakes of individual modes. For $z > 0$

$$W_l^\delta(z) = \sum_m 2\kappa_m \cos(\omega_m z/c) e^{-\omega_m z/2Q_{L,m}},$$

$$S(z) = \sum_m \frac{2\kappa_m \omega_m c_0}{\omega_m^2 + \gamma_m^2} [\sin(\omega_m z/c) + \frac{\gamma_m}{\omega_m} (1 - \cos(\omega_m z/c)) e^{-\gamma_m z/c_0}]. \quad (2)$$

For $z < 0$, $W^\delta(z) = 0$.

The longitudinal and transverse modes of the RF cavities have been recently measured^[2] and these result are taken for simulations.

Additional to that, simulations and measurements indicate that a 4-button BPM has two NB modes with frequencies $f = 6.8$ and 8.8 GHz, $R = 25$ and $R = 10$ Ohms correspondingly. The loaded $Q_L \simeq 60$ for both modes, $Q_0 \simeq 296$. The total loss factor of a 4-button BPM $\kappa = 2.7 \times 10^{-3}$ V/pC, where $\kappa = 2.0 \times 10^{-3}$ V/pC contributed by the two NB modes. We assume that there are 290 BPM-s per ring.

The total loss-factor of a cavity is $\kappa_{tot} = 0.55 \text{ V/pC}$. The sum of the loss-factors of the NB modes is less than that leaving the rest to contribution of the BB impedance of a cavity. We model the impedance of a cavity as the NB impedance summing up modes with frequencies $\omega < \Omega_0$ and the BB impedance,

$$Z(\omega) = (1 + i) \frac{Z_0 \Lambda}{\sqrt{\omega \sigma / c}} \theta(\omega - \Omega_0) \quad (3)$$

which describes the high-frequency tail at $\omega > \Omega_0$. The parameters Λ can be defined

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from the BB loss-factor

$$\kappa_{tot} = \sum_{\omega_l < \Omega_0} \kappa_l + \frac{2\Lambda}{\sigma} \int_{(\Omega_0 \sigma / c_0)^2}^{\infty} dt t^{-3/4} e^{-t}. \quad (4)$$

Generally, Λ depends on the choice of Ω_0 but the dependence is weak. Fig. 1 depicts the sum of the loss factors and Λ vs $f_0 = \Omega_0/2\pi$. We choose Ω_0 corresponding to the nine-th mode $f_0 = 2162$ MHz and take $\Lambda = 0.05$. Expressions for the wake and S-function describing the high-frequency tail of the RF cavities are:

$$W(z) = 4\Lambda \sqrt{\frac{2\pi}{\sigma z}} \left[1 - \sqrt{\frac{2}{\pi}} \int_0^{\sqrt{\Omega_0 z / c}} dt (\cos t^2 + \sin t^2) \right],$$

$$S(z) = 8\Lambda \sqrt{\frac{z}{\sigma}} \int_{\sqrt{\Omega_0 z / c}}^{\infty} \frac{dt}{t^2} [1 - \cos t^2 + \sin t^2]. \quad (5)$$

Components of the vacuum system such as bellows, ante-chamber, RF seals, tapers, screened pumps, etc., see Table 1, give^[3] BB mostly inductive impedance with the total inductance $L = 87$ nH and the total loss $\kappa_{BB} = 3.5$ V/pC calculated for the rms $\sigma = 1$ cm bunch. Subtracting the contribution of the NB modes of the BPM-s, we get $\kappa_{BB} = 2.9$ V/pC.

Table 1. The main contribution to the inductive impedance of PEP-II

	L (nH)	k_l (V/pC)
Dipole screens	0.10	
BPM	11.	0.8
Arc bellow module	13.5	1.41
Collimators	18.9	0.24
Pump slots	0.8	
Flange/gap rings	0.47	0.03
Tapers oct/round	3.6	0.06
IR chamber	5.0	0.12
Feedback kickers	29.8	0.66
Injection port	0.17	0.004
Abort dump port	0.23	0.005
Total	83.3	3.4

These contributions can be described by the sum of pure inductive and pure resistive impedances as it is described in^[3]. However, a constant resistive term may exaggerate the resistive part of the impedance at high frequencies. To avoid this, we tried to model the contribution of the vacuum components by $Q = 1$ BB model taking a single term in Eq. (1) with a resonance frequency ω_r and the loss parameter κ_r . The low frequency limit relates the loss parameter with the inductance $\kappa_r = Lx_r^2/(2\sigma^2)$ where $x_r = \omega_r\sigma/c_0$. To find x_r we calculated

$$\kappa_{BB} = \int \frac{d\omega}{2\pi} Z_{Q=1}(\omega) e^{-(\omega\sigma/c_0)^2} \quad (6)$$

as function of x_r with the loss factor defined from $L = 87$ nH. Dependence of κ_{BB} on x_r is shown in Fig.2. To get $\kappa_{BB} = 2.9$ V/pC, the parameter x_r has to be $x_r = 0.29$

what corresponds to $f_r = 1.4$ GHz. This frequency is well below the 5 GHz frequency spread of 1 cm bunch contradicting the results of simulations that the wake fields of the vacuum components are inductive. Indeed, the wake given by the $Q = 1$ model with parameters defined in this way from $L = 87$ nH and $\kappa_{BB} = 2.9$ V/pC does not look inductive, see Fig. 3. Hence, the $Q = 1$ model is not self-consistent.

For this reason, we tried another model to describe the wakes of the vacuum components. The simplest form of the impedance, which is inductive at low frequencies and rolls off as $1/\sqrt{\omega}$ (result known for a pill-box cavity with attached tubes) is

$$Z(\omega) = \frac{-i\omega L}{(1 - i\omega a/c)^{3/2}}, \quad (7)$$

or, using $x = \omega a/c$,

$$Z(\omega) = Z_0 \frac{Lx}{4a\pi\sqrt{2}(1+x^2)^{3/2}} [\sqrt{(1+x^2)^{3/2} + 3x^2 - 1} - i\sqrt{(1+x^2)^{3/2} - 3x^2 + 1}]. \quad (8)$$

The impedance in this form has two parameters, the same number as $Q = 1$ model has. The impedance for $\omega < 0$ is defined by $Z(-\omega) = Z^*(\omega)$. The wake field is given by an integral

$$W(z) = \int \frac{d\omega}{2\pi} Z(\omega) e^{-i\omega z/c_0} \quad (9)$$

where the contour of integration is above the real axis of ω .

One of the parameters, L , defines inductance at small frequencies, another parameter, a defines the loss factor. The loss factor for a bunch with rms $\sigma_B \gg a$ is

$$\kappa_\sigma = \frac{3}{8\sqrt{\pi}} \frac{La}{\sigma_B^3}. \quad (10)$$

Parameters in Table 2 were found in simulations with $\sigma_B = 1$ cm. They define $L = 83$ nH and, in the approximation of Eq. (9), $a = 0.15$ cm. Once parameter of the model are defined, impedance Eq. (7) must define the wake for the arbitrary σ_B . We tested the model for a symmetric circular taper in the round beam pipe with radius 4.5 cm.

The radius of the beam pipe was linearly reduced to 4.0 cm and then went back to 4.5 cm. The total length of the taper was 30 cm. Calculations of the loss parameter and the inductance (the latter was defined from the maximum value of the wake field given by the code, $W_{max} = L/\sigma_B^2\sqrt{2\pi e}$) were performed for different σ_B with the code ABCI. Comparison of the dependence of the loss factor on σ_B predicted by the model (solid line) and given by ABCI (large crosses) is shown in Fig. 4. Results of ABCI for $\sigma = 1$ cm were taken to define parameters L and a . Model works very well for $\sigma > 1$ cm and quite satisfactory for smaller σ , up to 1/3 cm (calculations with ABCI for shorter bunches become very time consuming). The upper curve in Fig. 4 depicts inductance of the taper given by ABCI and shows that the taper is mostly inductive in this range of σ_B -s. It is worth noting that in the $Q = 1$ model the dependence of the loss factor on σ_B is very weak: the loss factor remains almost constant for $\sigma_B < 1$ cm. The wake functions calculated for several σ_B are plotted in Fig. 5 together with the bunch profile. They correspond to the inductive impedance in agreement with the results of ABCI.

The longitudinal resistive wall (RW) impedance is given by

$$\frac{Z_l}{n} = Z_0 \frac{(1-i)\delta}{2} \frac{1}{b} \quad (11)$$

where

$$\delta = \frac{c}{\sqrt{2\pi\sigma_W\omega}}$$

is the skin depth, $\sigma_W = 3.84 \times 10^7 \text{ Ohms}^{-1}\text{m}^{-1}$ is the conductivity of the Al wall. At the bunch frequency, i.e at the revolution harmonic $n_{max} = R/\sigma = 3.5 \times 10^4$ the skin depth $\delta_m = 2.94 \mu\text{m}$ for $b = 2.4$ cm. Eq. (11) has to be corrected for very small (skin depth comparable with the beam pipe thickness) and at very high frequencies^[4], otherwise the integral defining RW wake with such an impedance is divergent at high frequencies. The corrected form of the impedance leads to the following wake and S-function^[5]

$$W_{RW}(z) = \frac{32\pi R}{b^2} \left\{ \frac{1}{3} e^{-\hat{s}} \cos(\hat{s}\sqrt{3}) - \frac{\sqrt{2}}{\pi} \int_0^\infty \frac{dx x^2}{x^6 + 8} e^{-x^2 \hat{s}} \right\},$$

$$S_{RW} = \frac{32\pi R s_0}{b^2} \left\{ \frac{1}{12} [1 - e^{-\hat{s}} \cos(\hat{s}\sqrt{3}) + \sqrt{3}e^{-\hat{s}} \sin(\hat{s}\sqrt{3})] - \frac{\sqrt{2}}{\pi} \int_0^\infty \frac{dx}{x^6 + 8} [1 - e^{-x^2 \hat{s}}] \right\}. \quad (12)$$

Here $2\pi R$ is the length of the beam pipe, b is the beam pipe radius, $\hat{s} = s/s_0$, and s_0 is defined in terms of the skin depth δ_{max} taken at the bunch frequency $(\omega/c_0)_{max} = 1/\sigma_B$,

$$s_0^3 = \frac{b^2 \delta_{max}^2}{\sigma_B}. \quad (13)$$

The total wake field $W(z)$ and $S(z)$ calculated for the PEP-II LER are shown in Fig. 6a for small $0 < z < 0.1$ cm, in Fig. 6b for long-range $0 < z < 125$ cm, and in Fig. 6c for $0 < z < 15$ cm. The lower curves in Fig. 6 are calculated without taking into account resistive wall impedance. Parameters of the LER were used: $n_{cav} = 8$, $n_{BPM} = 290$, $L = 87$ nH, $\kappa_l = 2.9$ V/pC (without the NB modes of the BPMs), $2\pi R = 2.2$ km of Al beam pipe, $\sigma_B = 1$ cm, $b = 2.5$ cm.

The contribution to the total wake from different components are depicted in Fig. 7 (NB wake of a rf cavity and a BPM), Fig. 8 (BB wake of a cavity, inductive components, and resistive wall), and Fig. 9 (total NB and total BB wakes and their sum) in the intermediate range of distances z .

The long-range wake is dominated by the NB contribution of the cavities and BPMs, which contribute about half of the wake. Note slow decay of the wake at large distances. The main contribution to the short-range wake comes from the inductive components and the resistive wall.

The transverse wake defines the transverse kick $c\Delta p_\perp = N_B e^2 x_l W_\perp(s)$ where x_l is offset of the leading particle. The transverse impedance is, usually, defined as Fourier transform of $-iW_\perp(s)$,

$$W_\perp(s) = \int \frac{d\omega}{2\pi} Z_\perp(\omega) e^{-i\omega s/c}. \quad (14)$$

The transverse impedance can be calculated using Panofsky-Wenzel theorem $Z_\perp(s) = Z_l^{m=1}(s)/k$, where $k = \omega/c$ and $Z_l^{m=1}(s)$ is Fourier transform of the dipole longitu-

dinal wake $W_l^{m=1}(s)$, defined by

$$c\Delta p = N_B e^2 [W_l^{m=0} + x_l x_t W_l^{m=1}(s) + \dots]. \quad (15)$$

The dipole wake $W_l^{m=1}(s)$ can be estimated assuming that $W_l^{m=1}(s) = 2W_l^{m=0}(s)/b^2$ where b is some typical transverse dimension. That give $Z_\perp = 2Z_l^0/kb^2$. In the case of the RW this is exact relationship with b equal to the beam pipe radius. For a smooth symmetric taper described above (total length 30 cm, the beam pipe radius 4.5 cm tapered to the 4.0 cm) the ratio $2W_l^{(0)}/W_l^{(1)}$ of the monopole and dipole longitudinal wakes at maximum given by ABCI is $(4.7\text{cm})^2$, in perfect agreement with the estimate.

Transverse wake calculated in this way is shown in Fig.10. The radius b was taken $b = 4.7625$ cm for the contribution of the rf cavities and $b = 2.4$ cm for all other components.

Equilibrium distribution and Threshold of the microwave instability

The potential well distortion changes the bunch length by $\Delta\sigma = \sigma - \sigma_0$. For the pure inductive impedance, σ is given by

$$\left(\frac{\sigma}{\sigma_0}\right)^3 - \frac{\sigma}{\sigma_0} = \frac{\Lambda L}{2}. \quad (16)$$

where $2\pi R$ is ring circumference, and

$$\Lambda = \sqrt{\frac{2}{\pi}} \frac{r_0 N_B}{2\pi R \alpha \gamma \delta^2 \sigma_0}. \quad (17)$$

For LER parameters and $L = 83$ nH we get $\Delta\sigma/\sigma = 0.23$ at $I_b = 3$ A.

More accurate result can be obtained considering the steady-state distribution of a bunch given by the Haisinski solution. In the dimensionless variables, $x = z/\sigma_B$,

where σ_B is the zero-th current rms bunch length, the distribution function $\rho(x)$ is

$$\rho_H(x) = \frac{1}{Z} e^{-U(x)/T}, \quad \frac{U(x)}{T} = \frac{x^2}{2} + \lambda_0 \int_0^\infty dx' S(x' \sigma_0) \rho_H(x' + x). \quad (18)$$

The normalization constant Z is defined by the condition $\int dx \rho(x) = 1$, and

$$\lambda_0 = \sqrt{\frac{\pi}{2}} \Lambda \sigma_0. \quad (19)$$

The first term in the self-consistent potential $U(x)$ is the RF potential. Example of the self-consistent solution $\rho(x)$ and of the potential $U(x)$ is shown in Fig. 12 for $N_B = 6.3 \times 10^{10}$ (RW impedance is included in calculations). The potential $U(x)$ can be approximated by a forth order polynomial as it shown in the left plot of Fig. 12. Then synchrotron frequency ω in units of the zero-th current ω_{s0} and the derivative $d\omega/dJ$, where J is the action variable, are determined by the coefficients of the polynomial. Dependence of ω , $d\omega/dJ$, rms position of bunch centroid $\langle x \rangle$ and rms bunch length (both in units of the nominal $\sigma_B = 1$ cm) are shown in Fig. 13. Results of these calculations show that potential well distortion is small: rms bunch length increases by 15% at the maximum LER current of 3 A, $N_B = 8.3 \times 10^{10}$. The variation of the rms is smaller than for the pure inductive impedance.

The bunch centroid is shifted at this current by one σ_0 .

Stability of a bunch can be predicted only in the framework of a dynamic model of the microwave instability or with numerical tracking. Effective impedance is usually used in the Boussard criterion of the single bunch longitudinal microwave instability:

$$2\pi R \frac{\Lambda}{Z_0} \left| \frac{Z}{n} \right|_{eff} = 1, \quad (20)$$

Effective impedance for the m -th azimuthal mode, $m = 1, 2, \dots$, can be defined^[4] as

$$\left| \frac{Z}{n} \right|_{eff} = \left(\frac{\sigma}{R} \right) \sum_{k=-\infty}^{\infty} \frac{Z_l(k + mQ_s)}{k + mQ_s} h_m(k\sigma/R) \quad (21)$$

where

$$h_m(y) = \frac{y^{2m}}{\Gamma(m + 0.5)} \exp(-y^2), \quad (22)$$

$\int dy h_m(y) = 1$. Calculations give for $|Z/n|_{eff} = 0.0762, 0.0750, 0.0722, 0.0692$ Ohms of inductive impedance for $m = 1, 2, 3, 4$ correspondingly. This estimate in agreement with J. Corlett result^[6]. Eight cavities give small capacitive contribution, $|Z/n|_{eff} = 0.0053, 0.00349, 0.00250$ Ohms for $m = 1, 2, 3$ correspondingly. BPMs give even less: all BPMs together give $1.18 \times 10^{-3}, 0.45 \times 10^{-3}$, and 0.35×10^{-3} Ohms of inductive impedance for $m = 1, 2, 3$ correspondingly. The effective impedance for different m is about the same because it is dominated by the inductive part of the impedance Z/n that is about constant in the wide range of frequencies, see Fig. 11.

The criterion Eq. 20 gives the beam threshold current $I_{th} = 4.14$ A for $|Z/n|_{eff} = 0.076$ Ohm and LER parameters $\alpha = 1.31 \times 10^{-3}$, $\delta = 0.81 \times 10^{-3}$, $E = 3.11$ GeV. (Note that $\Lambda = 1.156 \times 10^{-2} \text{ cm}^{-1}$ at $I = 2.14$ A.)

The so-called weak microwave instability might have lower threshold than the (strong) microwave instability considered above. A criteria proposed for the onset of the weak microwave instability requires study only of the steady-state distribution. The instability can be expected at the current where the derivative of the synchrotron motion of a particle in a bunch in respect with the action variable J is zero, $d\omega/dJ = 0$. This parameter, plotted in Fig. 13, gives the threshold number of particles per bunch $N_B = 12.5 \times 10^{10}$, what corresponds to the beam current $I_b = 4.5$ A. It is worth noting that without contribution of the resistive wall ω and $d\omega/dJ$ vary with current more rapidly, but the threshold current is lower than in the case where the resistive wall is taken into account.

The transverse effective impedance is usually used to define the threshold of the transverse mode coupling instability

$$I_{bunch} = \frac{4Q_s(E/e)}{\beta_{\perp}|Z_{\perp}|_{eff}} \frac{4\sqrt{\pi}}{3} \frac{\sigma}{R}. \quad (23)$$

The transverse effective impedance has been weighted with the transverse β -

function and defined as

$$|\beta_{\perp} Z_{\perp}|_{eff} = \frac{\sigma}{R} \sum_{k=-\infty}^{\infty} \beta_{\perp} Z_{\perp}(k + \nu_{\perp} + mQ_s) h_m\left(\frac{\sigma}{R}[k + \nu_{\perp} - \frac{\xi}{\alpha}]\right). \quad (24)$$

The average $\langle \beta_y \rangle = 17.87$ m and $\beta_y = 14.5$ m at RF cavities were taken for calculations. Chromaticity at this moment was put to zero, the vertical tune $\nu_y = 34.64$ was used and only weak dependence of results on the tune was found.

Calculations give for this quantity: $|\beta_{\perp} Z_{\perp}|_{eff} = -1.92 \times 10^4 - i4.10 \times 10^5$ for $m = 0$, $-77.45 - i1.68 \times 10^5$ for $m = 1$, $-108.85 - i1.40 \times 10^5$ for $m = 2$, $-90.4 \times 10^4 - i1.15 \times 10^5$ for $m = 3$ Ohms.

Taking Z_{eff} the sum of the effective impedances for $m = 0$ and $m = 1$ modes, $\beta_y Z_{eff} = 5.8 \times 10^5$ Ohms, and the synchrotron tune $Q_s = 3.71 \times 10^{-2}$, we get the threshold bunch current $I_{bunch} = 53.7$ mA, or 89.1 A beam average current. According to S. Berg calculations^[7], bunch coupling substantially reduces this threshold.

The HOM power was calculated accordingly to

$$P = I_{tot}^2 \sum_{m=-\infty}^{\infty} Z_l(mn_b \omega_0) e^{-(mn_b \sigma/R)^2}. \quad (25)$$

The coefficient for LER with 8 cavities is $P/I_{tot}^2 = 53.3$ kOhms, only 4.45 kOhms is the contribution of 8 rf cavities excluding the fundamental mode. The total HOM power at 3 A current is 480 kW in agreement with the previous estimate^[3].

Coupled-bunch Instabilities.

The impedance defines the coupled-bunch modes and their stability. The mode frequencies $\Omega_{l,m}$ are calculated in Wang's formalism

$$\begin{aligned} \frac{\Omega_{k,m}^l}{\omega_s} &= i\kappa_l \frac{\Gamma(m+0.5)}{2^{m-1}\Gamma(m)} \left(\frac{R}{\sigma}\right)^2 \sum_{p=-\infty}^{\infty} h(y_k) \frac{Z_l(pn_b + k + mQ_s)}{pn_b + k + mQ_s} \\ &+ \kappa_l \frac{\Gamma(m+0.5)}{2^{m-1}\Gamma(m)} \left(\frac{R}{\sigma}\right)^2 \sum_{p=-\infty}^{\infty} h\left(\frac{\sigma}{R}pn_b\right) Im \frac{Z_l(pn_b)}{pn_b}, \end{aligned} \quad (26)$$

where $y_k = (pn_b + k)\sigma/R$ and

$$\kappa_l = \frac{\alpha e I_{beam}}{4\pi E Q_s^2}. \quad (27)$$

A mode is unstable provided $\tau_d^l/\tau = (\omega_s \tau_d^l) I m \Omega_{lm} > 0$. For LER, $\kappa_l = 7.306 \times 10^{-11} \Omega^{-1}$ at 3 A current, and $\omega_s \tau_d^l = 640.4$. Mode frequencies and the growth rate are shown for the modes $m = 1$ and $m = 2$ in Fig. 14 for two train of bunches: with each second and with each third buckets filled. results are different from the previous calculations^[6] due to corrected parameters of the rf modes. For HER the rate τ_d/τ scales as $\tau_d n_{cav} \alpha / E Q_s$ and is larger than in Fig. 13 by a factor of 1.03 at 3 A current.

The frequencies of the transverse coupled modes $\Omega_{k,m}^t$ are given in Wang's formalism

$$\frac{\Omega_{k,m}^t}{\omega_s} = -i\kappa^t \frac{\Gamma(m+0.5)}{2^m \Gamma(m+1)} \sum_{p=-\infty}^{\infty} h(y_k) \beta_{\perp} Z_{\perp}(pn_b + l + \nu_{\perp} + mQ_s), \quad (28)$$

where $y_k = (pn_b + k + \nu_{\perp} - \xi/\alpha)\sigma/R$, ξ is chromaticity, and

$$\kappa^t = \frac{e I_{beam}}{4\pi E Q_s}. \quad (29)$$

For LER, $\kappa^t = 2.07 \times 10^{-9} \Omega^{-1}$ at 3 A current.

Mode frequencies and the growth rate are shown for the modes $m = 0$ and $m = 1$ in Fig. 15 for two filling patterns of the beam. Results agree with S. Berg's calculations^[7] except a small difference in the contribution of the RW impedance.

The growth rate for the most unstable modes in all cases is larger than the corresponding damping time: by the order of magnitude for the modes describing the bunch centroid motion (and may be corrected by the feedbacks), and by a factor of two for modes coupled with the internal motion ($m = 2$ in the longitudinal case, $m = 1$ in the transverse case). The growth rate in the train with each third bucket filled is larger than for a train with each second bucket filled. This example shows that the gap in the train of bunches can change the beam stability changing the beam spectrum compared to the spectrum of the beam with equi-distant bunch separation considered in this simulations.

Additional to the synchrotron radiation, Landau damping can give additional beam stability provided that the tune shift of a coherent mode is within the synchrotron tune spread of single particle motion in a bunch. The tune spread in the bunch $\Delta\omega_s/\omega_s$ is caused by the nonlinearity of the RF bucket (giving $\Delta\omega_s/\omega_s \simeq 6\%$ at the rms amplitude) and by the nonlinearity of the PWD due to the longitudinal wake field. The last effect is larger, giving about 15% tune spread, see the upper-left Fig. 13, but the two contributions have, unfortunately, the opposite signs. Nevertheless, the total spread is of the order of $\Delta\omega_s/\omega_s = 10\%$ at the 3A beam current. It may give additional damping

$$\left(\frac{\tau_d}{\tau}\right)_{LD} = (\tau_d * \omega_s) \left(\frac{\Delta\omega_s}{\omega_s}\right). \quad (30)$$

This damping is large, $(\tau_d/\tau)_{LD} \simeq 6.4$ due to the large factor $\tau_d * \omega_s = 640$ in the case of LER PEP-II. If the Landau damping takes place it strong enough to suppress instability in the cases which can not be stabilized by the feedbacks. The coherent frequency shift, calculated according to Eq. 26 and depicted in Fig. 13, is small compared to the tune spread, what is the necessary condition for Landau damping. It has to be noticed, however, that the relatively small coherent tune shift is the result of cancellation of the contributions of the two sums in Eq. 26, while each one of them would give the tune shift by a factor of ten larger then the total tune shift. For example, the last sum in Eq. 26 gives the tune shift $\Omega/\omega_s = 0.118, 0.088$ for the $m = 1, 2$ longitudinal coupled-bunch modes correspondingly.

The higher order modes ($m > 2$) can have the growth rate comparable with the lower order modes due to very slow roll-off of the effective impedance at large frequencies. Landau damping for these modes is proportional to m and large enough to make them stable.

Figure Captions

Fig. 1 Calculation of the parameter Λ , Eq. 3, for the BB impedance of the cavities.

Fig. 2 dependence of the loss factor on $\omega\sigma/c$ for $Q = 1$ model.

Fig. 3 Wake potential $W(z)$ and $\rho(z)$ for $Q = 1$ model.

Fig. 4 Dependence of the loss factor on the rms bunch length σ in the model Eq. 7 (solid line) for a symmetric taper, see text. ABCI results are shown by crosses. The upper curve shows that the inductance calculated from ABCI wakes is almost constant up to $\sigma = 0.3$ cm.

Fig. 3 Wake potential $W(z)$ and $\rho(z)$ for the model Eq. 7.

Fig. 6 Wake potential $W(z)$ and $S(z)$ for LER PEP-II. Lower curves don't take into account the resistive wall impedance.

Fig. 7 Narrow-band wake of a rf cavity (a) and of a 4-button BPM (b).

Fig. 8 Broad-band wake of a rf cavity (a), inductive components (b), and resistive wall (c and d)

Fig. 9 Total narrow-band (a), broad-band (b), and their sum (c).

Fig. 10 Transverse wake potential.

Fig. 11 Impedance Z/n in Ohms vs mode number $n = \omega/\omega_0$.

Fig. 12 Example of the Haiisinskii solution: self-consistent potential (left) and the distribution function (right) at $N_B = 6.3 \times 10^{10}$.

Fig. 13 Dependence of parameters of the self-consistent potential on N_B (bunch centroid $x = z/\sigma_0$ and rms bunch length $\sqrt{\langle x^2 \rangle}$ are in units of the nominal $\sigma_0 = 1$ cm).

Fig. 14 Longitudinal bunch-coupling instability: growth rate (upper plot) and coherent tune (lower plot) for a bunch in each second bucket (Fig. (a), $m = 1$ and (b), $m = 2$) and for a bunch in each third bucket (Fig. (c), $m = 1$ and (d), $m = 2$).

Fig.5 Transverse bunch-coupled instability. Notations are the same as in Fig. 14.

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Fig 1

Fig. 2

Fig. 3

Loss factor for model impedance (squares).
ABCI: loss factor (crosses), inductance (diamonds)

Fig. 4

Fig. 5

Total short-range wake V/pC (a) and total S-function (b)

Dotted line in $S(z)$: resistive wall is not included

Fig. 6a

Fig. 6b

Total wake V/pC (a) and total S function (b)
Dotted lines in $S(z)$: resistive wall is not included

Fig. 6c

Fig. 7

Fig. 8

a. wNBtot, b. wBBtot c. wtot

Fig. 9

Fig. 10a

Fig. 10b

Transverse wakes for LER
(a)total NB, (b) total BB, (c) total, $V/pC/cm$

Fig 10C

Fig. 11

Fig. 12

Potential well distortion. N_b in units $E+10$.

Dotted lines: wake excluding resistive wall

Fig. 13

Longitud. coupled bunch instab. $m=1$ $n_b=1746$
 tune shift (a) and rise time (b) vs mode number. LER, 8 cav., $I=3$ A

Fig. 14a

Longitud. coupled bunch instab. $m=1$ $n_b=1164$

tune shift (a) and rise time (b) vs mode number. LER, 8 cav., $I=3$ A

Fig. 14b

Longitud. coupled bunch instab. $m=2$ $n_b=1746$
 tune shift (a) and rise time (b) vs mode number. LER, 8 cav., $I=3$ A

Fig 14c

Longitud. coupled bunch instab. $m=2$ $n_b=1164$

tune shift (a) and rise time (b) vs mode number. LER, 8 cav., $I=3$ A

Fig. 14d

Transv. coupled bunch instab. $m=0$, $n_b=1746$
 tune shift (a) and rise time (b) vs mode number. LER, 8 cav., $I=3$ A

Fig. 15a

Transv. coupled bunch instab. $m=0$, $n_b=1164$

tune shift (a) and rise time (b) vs mode number. LER, 8 cav., $I=3$ A

Fig 15b

Transv. coupled bunch instab. $m=1$, $n_b=1746$
 tune shift (a) and rise time (b) vs mode number. LER, 8 cav., $I=3$ A

Fig. 15c

Transv. coupled bunch instab. $m=1$, $n_b=1164$
 tune shift (a) and rise time (b) vs mode number. LER, 8 cav., $I=3$ A

Fig. 15d