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GRAPHICAL TERRANE CORRECTION FOR GRAVITY GRADIENT

BY

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CONTENTS

ILLUSTRATIONS

GRAPHICAL TERRANE CORRECTION FOR GRAVITY GRADIENT [1]

By Donald C. Barton

INTRODUCTION

The formulas in common use for the terrane correction for the gradient in work with the Eötvös torsion balance—that is, those developed by Von Eötvös [2] and by Schweydar [3] strictly—are valid only for terranes where the ground does not rise more than a few decimeters above the level of the base of the instrument, and these formulas give increasingly erroneous results as the ground rises more than a meter above the level of the base of the instrument. Schweydar [4] recently published an improved formula that gives very much more accurate results where the relief of the terrane is more than a meter above or below the level of the base of the instrument. Although this formula is practical it is a little long and tedious for commercial work. Numerov [5] has suggested a graphic method apparently not applicable to rugged topography, but he does not give his graphs. Jung [6] gives an interesting set of graphs for the determination of the terrane effects; his graphical method apparently can be applied to rugged topography but is slightly complicated for actual use in the field. Heiland has published a new method of calculation of the terrane correction beyond 100 meters which, however, he has not reduced to graphic form.[7] For the calculation of the gradient and differential curvature effects due to any

[1] Work on manuscript completed July, 1928.

[2] Von Eötvös, R., " Bestimmung der Gradienten der Schwerkraft und ihrer Niveauflächen mit Hilfe der Drehwage ": Verhandl. 15 allgem. Konferenz Erdmessung, Budapest, 1906, Teil 1, 1908.

[3] Schweydar, W., " Die topographische Korrektion bei Schweremessungen mit der Torsionswage ": Ztschr. Geophysik, Bd. 1, 1925, pp. 81–89.

[4] Schweydar, W., " Die topographische Korrektion bei Schweremessungen mittels einer Torsionswage ": Ztschr. Geophysik, Bd. 3, 1927, pp. 17–23.

[5] Numerov, B., " Graphische Methode zur Berücksichtigung des topographischen Einflusses und des Einflusses der unterirdischen Massen auf die gravimetrischen Beobachtungen ": Ztschr. Geophysik, Bd. 1, 1925, pp. 367–371.

[6] Jung, Karl, " Diagramme zur Bestimmung der Terrainwirkung für Pendel und Drehwage und zur Bestimmung der Wirkung zwei-dimensionaler Massenanordnungen ": Ztschr. Geophysik, Bd. 3, 1927, pp. 201–212.

[7] Heiland, C. A., A Cartographic Correction for the Eötvös Torsion Balance: Am. Inst. Min. and Met. Eng. Tech. Pub. 52, February, 1928, 16 pp.

subsurface mass Nikiforov [8] devised a graphic method which can be adapted to the calculation of the terrane effects in places of rugged relief; the writer, as yet, has been able to see only the brief English digest appended to a short discussion of the Russian original in a communication from Capt. H. Shaw.

The present paper gives a graphical method that is as accurate as possible under the conditions of ordinary field surveying with the torsion balance, that is rapid, simple, and elastic, and that allows the observer to visualize readily the extent to which rugged topography will affect the gradient at his station. This graphical method is not meant to supersede the Eötvös or older Schweydar methods of calculating the terrane correction for the gradient but to supplement them where the terrane around the station has a relief of more than one-half meter above the level of the base of the instrument or more than several meters below it. The method given does not take into account the distinction made by Eötvös, Heiland, and others between the terrane correction for the zone within 100 meters and the cartographic correction for the topography further than 100 meters from the instrument; it can be used with ease to obtain the gradient terrane correction for all the topography further than 5 meters (5 feet if used with the Lancaster-Jones Shaw terrane formulæ).

BASIS OF EÖTVÖS AND SCHWEYDAR METHODS OF CALCULATING TERRANE CORRECTIONS FOR THE GRADIENT

The Eötvös and the Schweydar methods of calculating the terrane correction for the gradient are based on approximate integrations of the following fundamental formula:

$$\frac{\partial^2 v}{\partial x\, \partial z} = 3K\delta \int_{\alpha_1}^{\alpha_2} \int_{\rho_1}^{\rho_2} \int_{0}^{z_1} \frac{\rho^2\, (\cos\alpha)\,(h-z)\, d\alpha\, d\rho\, dz}{[\rho^2 + (h-z)^2]^{5/2}}, \tag{1}$$

where the base of the instrument is the origin of a system of cylindrical coordinates with the linear axis vertical, where α equals the azimuth of any point; ρ, the horizontal distance of the point from the origin; h, the height of the center of gravity of the instrument above the ground; z, the elevation of the point above the level of the base of the instrument; K, the gravity constant; and δ, the specific gravity of the soil. In areas of flat terrane where z is small the quantity $(h-z)$ differs only slightly from h in value, but in areas of rougher terrane, as z approaches h in value $(h-z)$, and with it the whole expression approaches zero, and as z approaches $2h$ in value, $(h-z)$ approaches $-h$ in value. The value of the integral of

[8] Nikiforov, P., ————: Institute of Practical Geophysics of the Supreme Council of Public Economy, U. S. S. R., Bull. 1, Leningrad, 1926, pp. 153–242.

(1) taken $z=0$ to $z=z_n$ is at a maximum when $z_n=h$ and becomes zero when z_n becomes greater than $2h$. In both the Eötvös and the older Schweydar methods the gradient effect, C, produced by any one of the zones in any sector is considered to be

$$C=Mz_n \tag{2}$$

where M is a constant; that is, C is assumed to increase linearly with the increase of the value of z_n, although as a matter of fact C reaches a maximum at $z_n=h$ and decreases indefinitely algebraically through zero at $z_n=2h$ for increasing values of z_n, if $z_n>h$. The relation of (2) is accurate enough for small values of z_n, and when that assumption is valid the assumption also holds valid that

$$C_\alpha - C_{\alpha+\pi} = M(z_\alpha - z_{\alpha+\pi}). \tag{3}$$

In general, however, where the relation of (2) is not valid the relation of (3) likewise does not hold, and if z_α and $z_{\alpha+\pi}$ differ by a moderately small amount it makes considerable difference whether their value is in the neighborhood of 0, h, or $> h$.

DERIVATION OF THE WRITER'S GRAPHICAL METHOD FOR CALCULATION OF THE GRADIENT TERRANE EFFECT

The terrane around the instrument is divided into the N., NE., E., SE., S., SW., W., NW. octants as in the Eötvös and Schweydar methods and into zones bounded by circles of 5, 10, 20, 40, 80, 160, 320, 640 ---------- 640×2^n. Each zone is divided into subzones delimited by circles of radius differing by $\sqrt[4]{2}$.

For each subzone of each octant a curvilinear prism is used; its front and back faces are the vertical surfaces through the 5, $5\times\sqrt[4]{2}$, $5\times\sqrt[4]{2^2}$, $5\sqrt[4]{2^3}$, 10 ---------- or $5\times\sqrt[4]{2^n}$ meter circles; its lateral faces are vertical radial planes.

In the original construction of the author's charts rectangular prisms were used as an approximation for the curvilinear prisms, and the gradient effects were calculated by the ordinary simple formula for the gradient produced by a horizontal rectangular block perpendicular to the axis of reference and bisected by it; that is,

$$\frac{\partial^2 U}{\partial x \partial z} = K\delta \log_e \frac{(\sqrt{x_1^2+y^2+z_1^2})+y}{(\sqrt{x_1^2+y^2+z_1^2})-y} \times \frac{(\sqrt{x_1^2+y^2+z_2^2})-y}{(\sqrt{x_1^2+y^2+z_2^2})+y} \times \frac{(\sqrt{x_2^2+y^2+z_1^2})-y}{(\sqrt{x_2^2+y^2+z_1^2})+y} \times \frac{(\sqrt{x_2^2-y^2-z_2^2})+y}{(\sqrt{x_2^2-y^2-z_2^2})-y}, \tag{4}$$

where K is the gravity constant, the density of the block is taken as 1, and x_1, y, z_1, x_2, and z_2 are the coordinates of the corners of the block. For these calculations

$$x_1=5,\ x_2=5\times\sqrt[4]{2},\ \text{and}\ y=\tfrac{1}{2}(5+5\times\sqrt[4]{2})\ \tan 22\tfrac{1}{2}^\circ.$$

By putting z_1 successively equal to 0, 0.1, 0.2, etc., and $\frac{\partial 2U}{\partial x \partial z}$ equal successively to 0.1, 0.2, 0.3 E, etc., z_2 is left the only unknown in (4), and by solving for z_2 a pile of blocks are obtained which differ successively in their effect on the gradient by 0.1 E.

Similar calculations were made later with Nikiforov's formula for a curvilinear prism bounded by radii and by cylindrical surfaces concentric with the z (vertical) axis; that is,

$$\frac{\partial^2 U}{\partial x \partial z} = K\delta \log_e (\sin \alpha_1 - \sin \alpha_2)\left[\frac{\rho}{\sqrt{\rho^2+z^2}} - \log_e\left(1+\frac{\sqrt{\rho^2+z^2}}{\rho}\right)_{\rho_2}^{\rho_1}\right]_{z_2}^{z_1} \qquad (5)$$

where K is the gravity constant, the density of the block is taken as 1, α_1 and α_2 are the azimuths of the bounding vertical radial planes, ρ_1 and ρ_2 are the radii of the bounding cylindrical surfaces, and z_1 and z_2 are the upper and lower bounding planes. The results of the calculations by the two methods agreed within the allowable error.

By the law of similar bodies similarly placed in projection of each other, which holds for the gravity gradient and differential curvature, the pile of blocks in the first subzone is expanded successively into the successive subzones; that is, all dimensions in the first subzone multiplied by $\sqrt[4]{2}$ give the corresponding dimensions for the second subzone, multiplied by $\sqrt[4]{4}$ give the dimensions of the third subzone, and multiplied by $\sqrt[4]{8}$ give the dimensions of the fourth subzone. All dimensions of the first zone, multiplied by two, give the second zone; by four, the third zone; ----------; and by 2^n, the dimensions of the (n+1) zone. A representation of the 5 to 10 meter zone of the north octant with its four piles of 0.1 E blocks is given in Figure 1.

The graphic working charts of the method (figs. 2 and 3) represent a vertical section along the axis of an octant. The radial limits of each zone, the 5, 10, 20, ------ 640 meter vertical lines, are spaced logarithmically; that is, with a constant interval. The distances within each zone—that is, from 5 to 10, from 10 to 20, and from $5\times2^{n-1}$ to 5×2^n meters—however, are divided linearly; and the position horizontally in the table of any point between 5 and 10, 10 and 20, 20 and 40, ------ or between $5\times2^{n-1}$ and 5×2^n meters distance from the origin along the radial axis of the octant can be found by simple linear interpolation between the respective 5 and 10, 10 and

20, 20 and 40, ------ or $5\times2^{n-1}$ and 5×2^{n} meter lines. The horizontal lines representing elevations of 0.5, 1, 2, 4, ------ 2^{n} meters above or below the center of gravity of the instrument are given similarly a logarithmic spacing. The distance from 0 to 0.5 meter is taken the same as that from 0.5 to 1 meter. The vertical distances from 0 to 1, 1 to 2, ------ 2^{n-1} to 2^{n}, however, are divided linearly, and the vertical position in the chart of any point between 0 and 1, 1 and 2, ------ between 2^{n-1} and 2^{n} meters elevation in reference to the center of gravity of the instrument can be found by simple linear

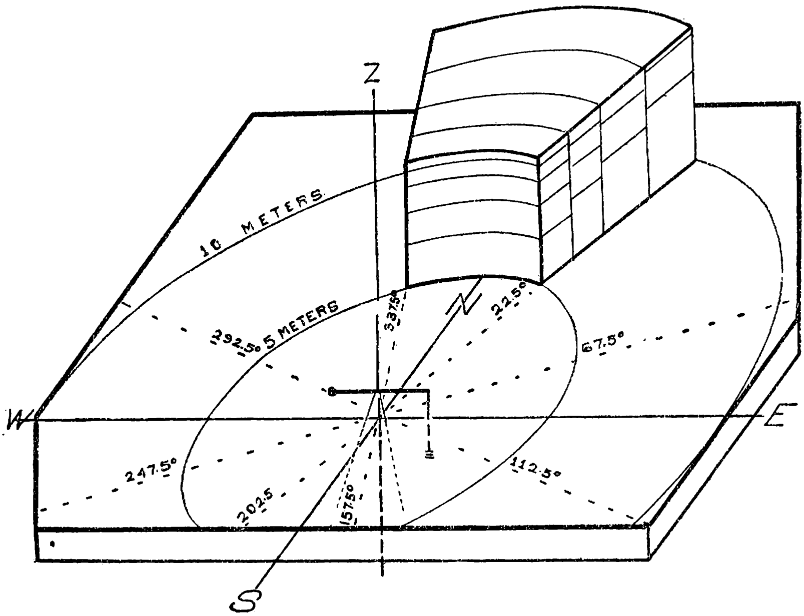

FIG. 1.—Block diagram of terrane around a torsion balance station to illustrate the construction of the chart of Figures 2 and 3

interpolation between the respective 0 and 1, 1 and 2, ------ or 2^{n-1} and 2^{n} meter horizontal lines. The subdivisions, marked from 0 to 1.5 in Figures 2 and 3 of the vertical subzones, represent the cross sections of the constituent blocks of the subzones, each block of which produces an effect of 0.1 E. at the origin, assuming a specific gravity of 1.0.

USE OF WORKING CHART OF AUTHOR'S METHOD

The fundamental use of the chart is as follows: The elevation of the ground is taken with a level, alidade, or transit usually at about 5, 10, 20, 40, 80, 160, 320, 640 meters distance along the N., NE., E., SE., S., SW., W., NW. radii, but as many points should be taken as necessary to give an accurate enough representation of the topography.

The octants are taken one at a time and the elevations for the octant plotted on the chart; a smooth curve for each zone is then drawn or visualized through the points, and the number of the 0.1 E rectangles included between the curve and the axis of zero elevation is counted and entered without regard to sign in the proper space in line *A* of Figure 4.

I N.	II NE.	III E.	IV SE.	V S.	VI SW.	VII W.	VIII NW.		
	−4.7 −2.0							*A*	Chart values.
	2.3 2.8							*B*	Sp. gr.
	−10.8 − 5.6 −16.4							*C*	A×B.

$-\mathrm{I}_C$ V_C $-\mathrm{II}_C$ $-\mathrm{VIII}_C$ IV_C VI_C	−16.4		$-\mathrm{III}_C$ VII_C $-\mathrm{II}_C$ VIII_C $-\mathrm{IV}_C$ VI_C	−16.4	
0.707×	−16.4	−11.6	0.707×	−16.4	−11.6
$\frac{d^2U}{dxdz}$		−11.6	$\frac{d^2U}{dydz}$		−11.6

FIG. 4.—Suggested form for calculation

The values entered in line *A* of Figure 4 are normally uncorrected for specific gravity and are for a specific gravity of 1.0; they must therefore be multiplied respectively by the specific gravity estimated to be most probable for the particular octants. The products are entered in their respective spaces in line *B*. The total effect is:

$$\frac{\partial^2 U}{\partial x \partial z} = -\mathrm{I}_B - \mathrm{V}_B + 0.707(-\mathrm{II}_B - \mathrm{VIII}_B + \mathrm{IV}_B + \mathrm{VI}_B) \qquad (6)$$

$$\frac{\partial^2 U}{\partial y \partial z} = -\mathrm{III}_B + \mathrm{VII}_B + 0.707(-\mathrm{II}_B + \mathrm{IV}_B + \mathrm{VI}_B + \mathrm{VIII}_B). \qquad (7)$$

The resulting values are then subtracted from the respective observed values.

FLEXIBILITY OF METHOD

1. If the terrane is too irregular to allow a single radial line of elevations down the axis of an octant to give a reasonably accurate representation of the terrane.

(*a*) Any octant may be divided into two equal sectors or into four sectors whose widths are in the ratio of 53 : 47 : 47 : 53. The levels should be run along the radial axis of each subsector. The effect produced by each one-half or one-fourth octant is calculated as for the whole octant, but the effect produced by 0.1 E block of the chart in the first case is 0.05 E and in the second case 0.025 E and the total effect for the one-half or one-fourth octant as read from the chart must be divided by two or by four.

(*b*) If the irregularity of the topography is confined to one or two subzones of one or more octants, one-half octants, or one-fourth octants, it is necessary to apply the procedure only to those subzones within those octants, one-half or one-fourth octants.

2. A high degree of complexity in the variation of the specific gravity can be handled. If necessary, each 0.1 E block of each subzone, or even each 0.05 E or 0.025 E block of the one-half or one-fourth octants, respectively, could be assumed to have a different specific gravity. Such detail would be more exact than the exigencies of the actual situation in the field would warrant, and in most practical work it is necessary only to divide the hills along the division lines of the zones and subzones, along one-fourth, one-half, or whole octants, and along horizontal planes of equal elevation into large-step rectangular blocks, each of which is assumed to have some definite uniform specific gravity. The number of 0.1, 0.05, or 0.025 E blocks included within each of those large blocks can be counted and multiplied by the specific gravity of the block. As a matter of fact, a fairly irregular distribution of specific gravity can be handled if one-fourth octants are used.

Where the gradient correction is calculated for a valley or other depression and where the distribution of specific gravity is irregular, the effect of each zone or subzone for each one-fourth, one-half, or whole octant must be calculated for the same depth as for the corresponding symmetrically opposite zone or subzone, and instead of using the effect from the line of zero elevation to the top of the ground, according to the directions of a preceding paragraph, the effect must be calculated from the surface to that depth. If a subzone is divided into upper and lower zones of different specific gravities, if E_p is the chart reading in E for the common contact of the two zones, E_s and E_D are the chart readings for the surface and the depth to which the correction is carried, if δ_1 and δ_2 are the respective specific gravities, and if C is the effect for the subzone:

$$C = -[\delta_1(E_p - E_s) + \delta_2(E_D - E_p)] \quad (8)$$

The quantities $(E_p - E_s)$ and $(E_D - E_p)$ are always positive.

3. To carry the calculation of the gradient correction beyond a distance of 640 meters from the instrument, it is necessary by the law of similar bodies similarly placed in projection of each other

merely to divide elevations and distances by any constant number. That is, $\frac{s_1}{m}, \frac{s_2}{m}, \frac{s_2}{m}$ and $\frac{z_1}{m}, \frac{z_2}{m}, \frac{z_3}{m}$ may be used in place of s_1, s_2, s_3 and z_1, z_2, z_3 where s and z are respectively the distance and elevation of any point and m is any number.

4. The charts of Figures 2 and 3 replace the Eötvös or the older and more common Schweydar method for the calculation of the gradient correction at distances greater than 5 meters from the instrument, but if desired either of the older methods may be used to a distance of 10 (or 20) meters and the present chart for greater distances from the instrument. The 5 to 10 (and 10 to 20) meter zones of the chart are neglected in that case and 20 per cent of the effect of the first subzone correction is applied to the first subzone of the 10 to 20 meter zone (or of the 20 to 40 meter zone).

5. This method is directly applicable without regard to the height of the center of gravity of the instrument above the ground. The Eötvös and Schweydar methods use the quantity $(h-z)$ in which h is the height of the center of gravity of the instrument above the level of the ground; that is, a constant, and z, the elevation of the surface of the ground above the level of the base of the instrument, is the variable. In the charts of Figures 2 and 3, the quantity $(h-z)$ is used directly; as a matter of practical field work it is as easy in leveling to refer elevations to the level of the center of gravity of the instrument as it is to refer them to the level of the base of the instrument. If the quantity $(h-z)$ is measured directly, the height of the instrument does not enter the calculations.

If desired, the charts of Figures 2 and 3 can easily be made over into similar charts in which the elevations are referred to the level of the base of the instrument and in which the z of the charts is replaced by $(h-z)$; a new zero line of elevation and of gradient effect is drawn at $-h$ on the old chart, and the elevations and piles of 0.1 E blocks are built up and down from that line; if E'_p and E''_p are the respective values in E on the old and the new charts of any point p and if E'_{-h} is the value of E on the point $-h$ in the same subzone of the old chart:

$$E''_p = E'_{-h} - E'_p \tag{9}$$

where E'_{-h} and E'_p but not E''_p are taken without regard to sign. E''_p is negative from $z = -\infty$ to 0 of the new chart, positive from 0 to $2h''$, negative from $2h''$ to $+\infty$, and at a maximum at $+h''$. The rules of signs in (6) and (7) and in the form of Figure 4 should be reversed.

6. The unit of vertical and horizontal distance in the charts can be taken as any unit of linear measurement, not merely meters, without

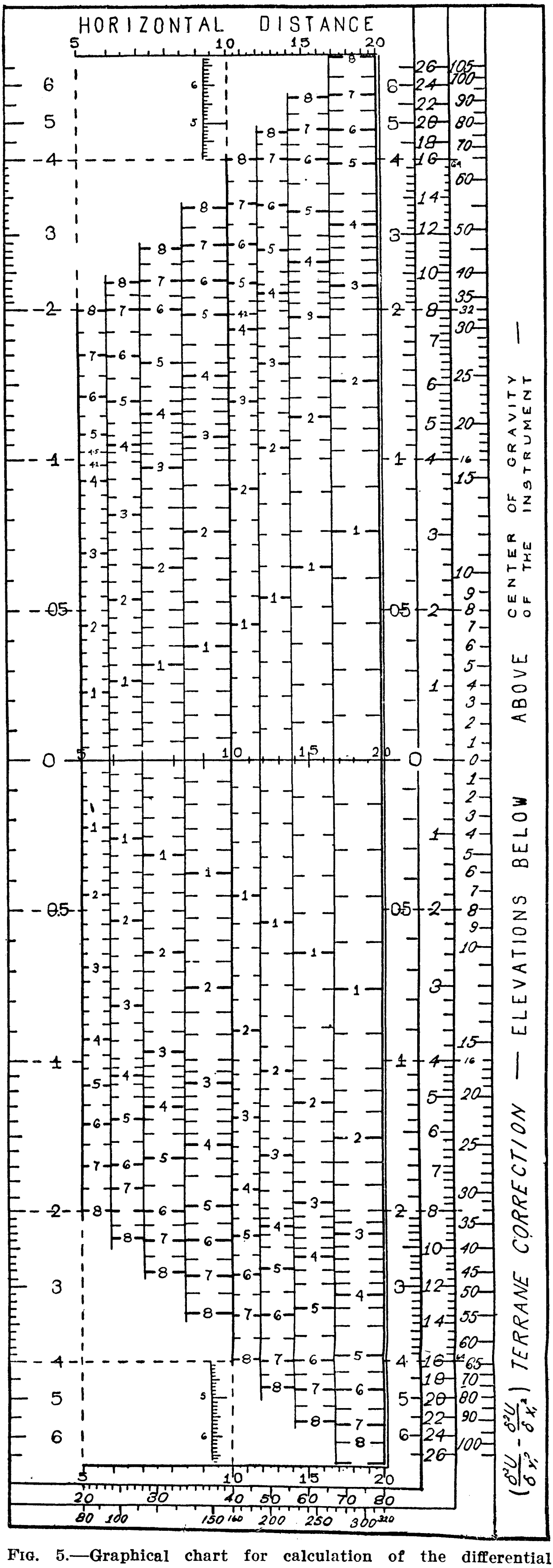

FIG. 5.—Graphical chart for calculation of the differential curvature terraine correction

otherwise affecting the charts. According to the law of similar bodies similarly placed in projection of one another, the gradient effect at the point x_p, y_p units from the origin produced by a body whose dimensions are x_m to x_n, y_o to y_q, and z_r to z_s units of linear distance, is the same whether the unit of measurement is inches, feet, yards, centimeters, meters, or kilometers. The charts, therefore, can be used to supplement the Lancaster Jones-Shaw gradient terrane correction formulæ by taking the measurements in feet instead of meters and by erasing the word "meter" in the charts and substituting for it "feet."

7. The distance to which the gradient correction should be taken and the degree of accuracy necessary in leveling and in measuring the distances can be determined by inspection of the charts, Figures 2 and 3. The effect of the topography should be corrected as far out as $100 \times n$ meters wherever the elevation at $100 \times n$ meters rises or falls more than $2.5 \times n$ meters above or below the center of gravity of the instrument. At a distance of $5 \times n$ meters the levels at elevations of $0.3 \times n$ to $2 \times n$ meters should be accurate to $0.05 \times n$ meters.

ADVANTAGES OF METHOD

Various graphical terrane correction charts similar in general type to the present charts but differing in details of design can be constructed and for special situations will have special features of advantage. For example, where a series of observations are to be taken down a long linear valley or in front of a long straight scarp there would be considerable advantage in using a chart or charts based on infinitely long horizontal rectangular prisms, each of which produced a constant gradient effect at the origin, $\frac{\partial^2 U}{\partial s \, \partial z}$, along the perpendicular from the station to the edge of the valley or of the scarp; $\frac{\partial^2 U}{\partial s \, \partial z}$ and $\frac{\partial^2 U}{\partial y \, \partial z}$ would then be determined trigonometrically from $\frac{\partial^2 U}{\partial s \, \partial z}$.

DIFFERENTIAL CURVATURE CHART

For the calculation of the differential curvature correction it is possible to construct an analogous graphical chart, such as the chart of Figure 5. There is, however, little practical advantage to such a chart; the value of the differential curvature effect of a small block or prism moving vertically is very nearly constant within 15° of the horizontal plane through the center of gravity of the instrument, and for all practical purposes the effect of a block n units high is n times the effect if the block is only one unit high, as long as the block is wholly within about 15° of the horizontal plane through the center of gravity of the instrument. The Eötvös and Schweydar methods of calculating the terrane correction for the differential curvature give fully as accurate and easily obtained results as the chart of Figure 5. The advantage of this chart, however, is that it allows a ready visualization of the very serious effect of rugged topography on the differential curvature and a visualization of the fact that, on account of the difficulty of making an accurate calculation of the effect of the terrane, the differential curvature, much more than the gradient, becomes increasingly unreliable with increasing ruggedness of the topography.

EXAMPLE OF APPLICATION OF CHART

A station is taken at the foot of the northwest-southeast fault-line scarp facing southwest. The surface trace of the fault lies about 80 meters northeast of the station. The fault plane dips about 60° to the northeast. More or less horizontal limestone on the northeast of the fault has been faulted down against shales on the southwest. The specific gravity of the limestone is 2.8 and of the shales, 2.3.

To illustrate the use of the charts, the terrane effect is calculated for the northeast octant. A topographic map covering the northeast octant is given in Figure 6. The elevations at 80, 160, 320, 640, and beyond, for use in the calculations, can be taken from the map. The average elevation of the northeast octant at 80 meters can be seen to be about 70 feet plus or minus a foot and a half or about 21 meters. As a variation of 1 meter in elevation produces a difference in effect of (sp. gr. × 0.08) E at 80 meters distance and 20 meters elevation, it is about a toss-up whether or not to run the instrumental leveling out to 80 meters. At 160, 320, and 640 meters distance the average elevation for the octant, respectively, is 85, 103, 106 plus or minus about a foot in each case, or about 26, 31, and 32 meters, respectively. At those elevations and distances a difference in elevation of 1 meter produces a negligible difference in the terrane effect, so that it is unnecessary to carry the instrumental leveling to 160 meters or beyond. As the elevation beyond 640 meters is about constant or shows an indication of decreasing, it is not necessary to obtain the elevation at 1,280 meters, for the chart shows that a mass extending out infinitely from 640 meters and not rising above 35 meters has a practically negligible effect.

Elevations are taken with an alidade, transit, or level at distances of 5, 10, 20, 40, and, if desired, 80 meters along the central axis of the octant. The rod positions are shifted slightly, however, to get the best average value of the elevation of the octant at the various distances. The elevations determined in this case are 0.32, 2.0, 3.5, 11.0, and 21.1 meters.

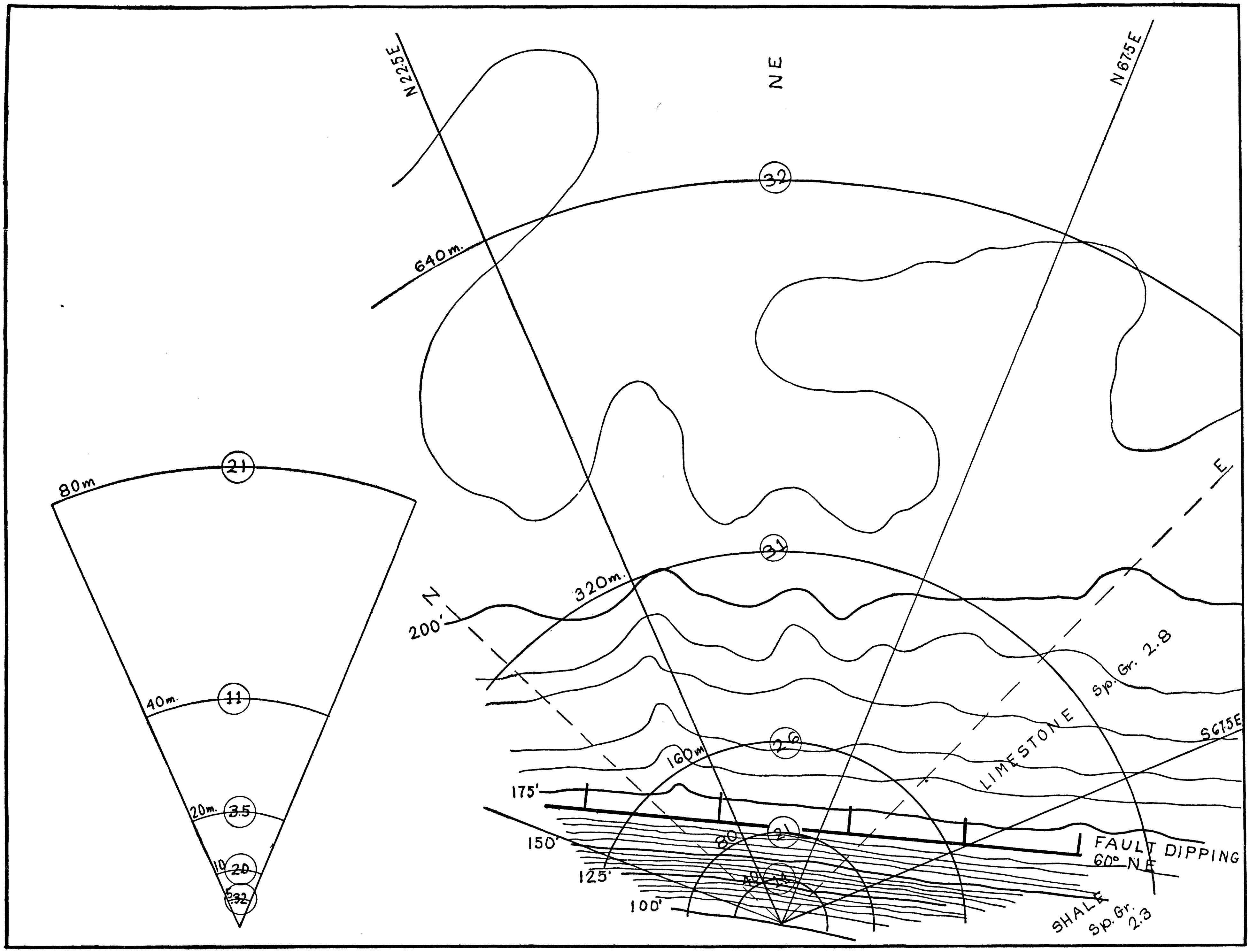

FIG. 6.—Topographic map of terrane effect for northeast octant

The elevations are referred to the base of the instrument as zero. They might have been referred to the center of gravity of the instrument as zero, but as most of the terrane corrections at this station are calculated by the ordinary Eötvös or Schweydar formulas, it seems preferable to conform to the use of the base of the instrument as zero.

Figure 7 represents a piece of tracing paper or celluloid superimposed over a chart of Figure 2.

The elevations at 5, 10, 20, 40, 80, 160, 320, and 640 meters distance are plotted on the sheet of Figure 7 at their respective positions. As the center of gravity of the suspension of the balance used is 1 meter above the base of the instrument, 1 meter must be subtracted from each of the numerical values obtained for the elevations to plot the points on the chart.

The points are then connected by straight lines, for there is essentially a straight-line slope between the various contiguous points.

The block *BAC*, rising above the plane of the base of the instrument at −1, really belongs on the chart of Figure 3, but as the two charts are symmetrical above and below the zero line the point *B* may be replotted at the symetrically corresponding point *B′* and the effect of the block *B′AC* to +1 calculated in place of the effect of the block *BAC* to −1.

The number of unit squares of 0.1 E effect between *B′AC* and (1+1), between the profile and the trace of the fault plane, and between the latter and the profile beyond 80 meters are counted and sum up as follows:

Unit squares of 0.1 E effect

B′AC to +1		Profile to fault		Fault to profile beyond 80 meters	
5 to 10 meters	−0.99	5 to 10 meters	0.10	80 to 160 meters	0.93
10 to 20 meters	−.28	10 to 20 meters	.60	160 to 320 meters	.77
20 meters	−.24	20 to 40 meters	1.53	320 to 640 meters	.25
		40 to 80 meters	2.82	640 meters	.06
	−1.3	80 to 160 meters	.97		
		160 meters	.05		2.0
			6.0		

The total chart effect, therefore, is 4.7 southwest of the fault and 2.0 northeast of the fault.

This pair of values are entered in the chart of Figure 4 under NE. octant and on line *A*. The pair of respective specific gravities are entered on line *B* under NE. By multiplying the chart effect by the respective specific gravity and adding the products the total effect, 16.4, of the octant is obtained, referred to N. 45 E. as the axis of reference. The rest of the calculation on the chart, as shown in Figure 4, is carried on as if it were necessary to calculate the effect of only this one octant. The final result both for $\frac{\partial^2 U}{\partial x \partial z}$ and for $\frac{\partial^2 U}{\partial x \partial z}$ is the gradient due to that irregularity of topography. It is an effect and not a correction. To obtain the terrane correction it is necessary to change the sign of the effect and add or subtract it.

If this had been an actual station, the effect of the north, east, and southeast octants would have been obtained in the same manner. The effect within the 5-meter zone and in the south, southwest, west, and northwest octants would be obtained by the ordinary Eötvös or Schweydar methods. The sum of all these effects is the effect that is subtracted from the observed value of the gradient.

O