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REPORT

Threshold Condition For Nonlinear Tearing Modes In Tokamaks

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THRESHOLD CONDITION FOR NONLINEAR TEARING MODES IN TOKAMAKS

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Abstract

Low-mode-number tearing mode nonlinear evolution is analyzed emphasizing the need for a threshold condition, to account for observations in tokamaks. The discussion is illustrated by two models recently introduced in the literature. The models can be compared with the available data and/or serve as a basis for planning some experiments in order to either test theory (by means of beta-limit scaling laws, as proposed in this paper) or attempt to control undesirable tearing modes. Introducing a threshold condition in the tearing mode stability analysis is found to reveal some bifurcation points and thus domains of intrinsic stability in the island dynamics operational space.

1. INTRODUCTION - MOTIVATIONS

Tearing modes with low mode numbers are known to affect tokamak performance, by degrading confinement [1] and stability [2]. Most importantly, these modes appear as a dominant cause for the so-called β -limit in present-day tokamaks. As a result, understanding their behavior and controlling their evolution are important issues. Despite continuous efforts to clarify the physics of tearing modes since the paper by Furth et al. in 1963 [3], plasma physicists are still left with unanswered questions brought about by new regimes investigated as limits to tokamak performance are pushed forward. On the other hand, improved diagnostics allow additional insights from experimental data.

In recent years, diagnostic improvements and the search for higher performance on the Tokamak Fusion Test Reactor (TFTR) have provided reliable interesting data on low mode-number tearing-modes (typically, $m/n = 2/1, 3/2, 4/3$ or $5/4$, where m and n are the poloidal and toroidal mode number respectively). The latter are observed to degrade confinement during supershots [1].

Recently, an analysis of the available data has led to significant progress in their understanding. Indeed, as reported by Chang et al. [4], evolution and saturation of magnetic islands associated with tearing modes in TFTR supershots have been found to agree with the predictions of the so-called *neoclassical ∇p -driven tearing mode model* [5] (instability driven by bootstrap current). However, this model does not predict a threshold condition (to be overcome before any mode can be excited). This is in contradiction with experiment, since all m/n modes should theoretically be unstable in the absence of a threshold condition while only a few are indeed observed experimentally. Two mechanisms have recently been proposed that could remove this discrepancy:

a) " $\chi_{\perp}/\chi_{\parallel}$ -model": a finite transverse heat conductivity, $\chi_{\perp}$, together with considering large but finite longitudinal conductivity $\chi_{\parallel}$, may prevent the electron pressure from equilibrating on perturbed magnetic surfaces. This is found to modify the source of instability: the required pressure flattening inside the island only occurs for an island width w larger than a threshold w_{th} scaling as $(\chi_{\perp}/\chi_{\parallel})^{1/4}$ [6].

b) " ω^* -model": ion polarization drift effects (ion inertia and finite Larmor radius) provide a stabilizing contribution, potentially dominant for island widths of the order of the ion Larmor radius ρ_i , but rapidly decreasing for larger islands, which introduces a threshold width w_{th} of the order of a few ρ_i [7][8].

Both the $\chi_{\perp}/\chi_{\parallel}$ and ω^* -models predict, under different conditions, the previously missing threshold condition. Although they may well both contribute to the experimentally observed threshold, one can assume that one of them plays a dominant role. To distinguish between these two models, one has to consider the characteristics of the tearing mode excitation process. This task may be complicated by the fact that such valuable experimental data as the magnetic island width, for instance, are less accessible at the time of excitation - when the modes are at small amplitudes - than they are in the later evolution. An alternative, that may provide a more reliable test for the theory, is to analyze the tearing mode excitation in terms of a β -limit scaling law, for instance, as recently investigated by La Haye et al. on DIII-D [9]. Indeed, each of the two models possesses a specific β -threshold scaling, with respect to some dimensionless parameters (related to collisionality, Larmor radius, geometry, profiles...). As a result, a proper data analysis, similar to those done for transport studies [10], supposedly allows discrimination between the $\chi_{\perp}/\chi_{\parallel}$ and ω^* -models. However, complex parametric and/or functional dependencies, to be discussed in the following, impose strong constraints on the such an analysis.

The remainder of this paper is organized as follows. The *neoclassical ∇p -driven tearing mode model* and its comparison to TFTR data are briefly reviewed in Section 2. The $\chi_{\perp}/\chi_{\parallel}$ and ω^* models are presented in Sections 3 and 4 respectively. Finally, these results and their practical consequences are discussed in Section 5.

2. NEOCLASSICAL ∇p -DRIVEN TEARING MODE MODEL AND COMPARISON WITH TFTR DATA: AGREEMENT AND DISCREPANCIES

The *neoclassical ∇p -driven tearing mode model* and its comparison with TFTR data have been extensively discussed in references [5] and [4] respectively. The aim of this section is to highlight the main results of these papers in order to provide a basis for the present work.

2.1. Neoclassical ∇p -driven tearing mode model

In the *neoclassical ∇p -driven tearing mode model* magnetic islands are driven by the pressure-gradient-driven bootstrap current, known to exist in sufficiently collisionless tokamak plasmas (relevant to present day experiments). The analysis presented in this paper is essentially based on the island-width evolution-Equation which, in the case of *bootstrap-driven* tearing modes, reduces to

$$\frac{dw}{dt} = K_R \left(\Delta' + \frac{K_b}{w} \right), \quad (1)$$

with

$$K_R = k_R \frac{\eta_{nc}}{\mu_0} \quad k_R \approx 1.22, \quad (2)$$

$$K_b = k_b \beta_p l_p^{-1} \varepsilon^{1/2} s^{-1}, \quad (3)$$

where $k_b=O(1)$ is an order unity constant, w is the island width, Δ' is the usual tearing mode stability parameter [3], $\mu_0=4\pi \cdot 10^{-7}$ (with MKSA units), and, with all quantities evaluated at the resonant surface where $q=m/n$, η_{nc} is the neoclassical plasma resistivity, $\varepsilon=r/R_0$ is the inverse aspect ratio, $\beta_p = p/(B_p^2/2\mu_0)$ is the poloidal beta, and

$$s = \frac{r}{q} \left(\frac{dq}{dr} \right) \quad \text{and} \quad l_p = -\frac{p}{r} \left(\frac{dp}{dr} \right)^{-1} \quad (4)$$

are the magnetic shear and the pressure gradient length (normalized to the local minor radius r), respectively. The first term (associated with Δ') on right hand side of Equation (1) is the one resulting from the well-known Rutherford analysis [11], which is stabilizing (destabilizing) for negative (positive) Δ' . The second term is the destabilizing (we restrict ourselves to situations where s and l_p are positive) perturbed bootstrap current induced by the helical pressure gradient perturbation. In the frame of this simple picture, positive- Δ' modes are indefinitely growing, while negative- Δ' modes saturate at

$$w_{sat} = -k_b/\Delta' = k_b \varepsilon_a^{-1} \beta_p l_p^{-1} \varepsilon^{3/2} s^{-1} \delta^{-1} a, \quad (5)$$

with $\varepsilon_a = a/R_0$ the plasma inverse aspect ratio, $\delta' = r|\Delta'|$ and s' the sign of Δ' .

The absence of a threshold condition for this model is clear from Equation (1), where the destabilizing *bootstrap-term* obviously diverges for smaller island width w , thus making it impossible for this term to be balanced by the finite Δ' -term - whatever its value - as w tends to zero. There are two basic solutions: a saturating island for negative Δ' and a growing island for positive Δ' , that are illustrated in Figure 1a, which shows the *normalized growth rate* $\gamma_b \equiv d\bar{w}_b/d\tau = C_b + 1/\bar{w}_b$ as a function of the dimensionless measure of the island width $\bar{w}_b = w/r$ and the *control parameter* $C_b = \Delta'/K_b = \Sigma k_b^{-1} \beta_p^{-1} l_p \varepsilon^{-1/2} s \delta'$ ($\tau_b = K_R K_b t / r^2$ is a dimensionless *time variable*).

Another way to exhibit the global features of the *neoclassical ∇p -driven tearing mode model* is to make use of the stability diagram shown in Figure 1b. Although this representation is not essential to clarify the dynamics of such a simple model, it is interesting as an introduction to following sections where more refined models are discussed. The idea is to characterize the qualitative features of the island dynamics in terms of a unique dimensionless control parameter. Those readers not familiar with the synthetic representation used in the following can find a basic introduction to the field of "elementary stability and bifurcation theory" in many undergraduate textbooks such as [12]. In the present case, the island width dynamics is described as a function of the *control parameter* $C_b = \Delta'/K_b$ and turns out to be essentially characterized by the *universal* stability boundary of equation $C_b = -1/\bar{w}_b$ (*universal* means that with variables $\bar{w}_b$ and C_b the stability boundary does not depend on any other parameter). This stability boundary corresponds to the stable branch associated with saturation, for negative Δ' . The convention is to represent stable (unstable) branches by continuous (dashed) lines. Note that the natural boundary $\bar{w}_b = 0$ is formally represented here as an unstable branch - although it does not correspond to a stationary solution of Equation (1) - in order to account for island growth from zero size (i.e., the absence of a threshold condition). The vertical arrows indicate the direction of evolution of the island width w : up for growth and down for shrinking. That is, the arrows point from an unstable branch towards a stable branch. These branches can respectively be visualized as the crest and valley of the *stability landscape* illustrating the island width evolution in terms of the familiar motion of a ball on a bumpy terrain; this motion is constrained by the value of the control parameter C_b imposed by the equilibrium features. One advantage of this representation is the clear identification of the two basic stability domains - with positive and negative Δ' , respectively associated with positive and negative C_b , owing to the simple proportionality relationship between these two parameters.

To conclude this section, it should be noted that the above discussion ignores the so-called quasilinear tearing mode evolution [13] which considers the dependence of Δ' on the island width w . This effect will be neglected throughout the rest of this paper since we are mostly interested in the early island evolution - where this effect is small - rather than in its saturation.

2.2. Comparison with TFTR data: agreement and discrepancies

As recently reported by Chang et al. in reference [4], the *neoclassical ∇p -driven tearing mode model* briefly described above is found to be in good agreement with the nonlinear evolution of tearing modes (with $m/n = 3/2, 4/3$ and $5/4$) observed on TFTR during the supershot regime [1] (see Figure 2 in reference [4], where the experimental signal is barely distinguishable from the simulation). However, as mentioned in [4], the comparison between model and experiment reveals two discrepancies.

Firstly, k_b (denoted k_2 in [4]), the only free parameter of the model, is predicted to be an order unity constant by theory, must be adjusted to different values (nonetheless within a narrow range) in order to reproduce the time evolution of different islands, although it does not need to be varied during simulation for a given island. This may reflect the fact that k_b is not actually a constant, but rather a compound parameter whose features are not taken into account in the rather crude *bootstrap-model* it comes from (for instance, the simple dependence with respect to the pressure gradient is actually a combination of both electron and ion density and temperature gradients).

Secondly, the *neoclassical ∇p -driven tearing mode model* predicts instability for all the modes without any condition on mode numbers m and n , owing to the singular nature of the destabilizing *bootstrap-contribution* for $w=0$. This is clearly a major discrepancy with experiment since only a reduced number of modes are indeed observed in TFTR supershots. In particular, the $m/n = 2/1$ mode, predicted to be the "most unstable", is rarely seen in these discharges with higher- m MHD activity. Moreover, although this point has not been fully clarified yet, it seems that the observed islands systematically first appear at a width of the order of 1 cm, which is above the diagnostic sensitivity (≈ 0.3 cm, for Mirnov coils in the present case), suggesting that they coherently arise from above the noise level. Finally, different modes are found to develop in apparently similar discharges, with no clear information about whether it is due to either a different trigger or subtle equilibrium modifications. These are good indications that a necessary threshold condition is missing in the present model. However, any modification should affect the island evolution in its very beginning only, since there is already a reasonable agreement for the later

evolution. This issue and its related consequences for tearing mode behavior in tokamaks are discussed in the remainder of this paper.

3. THE " $\chi_{\perp}/\chi_{\parallel}$ -MODEL"

3.1. Basic features

In a recent paper [6] Fitzpatrick proposed a model, hereafter called the " $\chi_{\perp}/\chi_{\parallel}$ -model", to account for the threshold condition discussed in the previous section. His point is basically that, for small island width (smaller than a critical value to be discussed later), the destabilizing *bootstrap-contribution* is overestimated by the *neoclassical ∇p -driven tearing mode model*, owing to the assumption that temperature and density perturbations associated with the island are flux functions flattened inside the separatrix. The usual assumption is that the ratio $\chi_{\perp}/\chi_{\parallel}$ of the transverse to the longitudinal thermal conductivity is vanishingly small (our comments are restricted to energy transport across the island, i.e. temperature, but the same holds for particle transport, i.e., density). This "ideal" assumption leads to the singular behavior of the *bootstrap-contribution*, found to vary as $1/w$ in the *neoclassical ∇p -driven tearing mode model* (see Equation (1)).

Assuming temperature/density profile-similarity in the vicinity of the island, and calculating the perturbed temperature profile from the heat flow Equation

$$\nabla \cdot \mathbf{q} = 0 \quad \text{with} \quad \mathbf{q} = -\chi_{\parallel} \nabla_{\parallel} T - \chi_{\perp} \nabla_{\perp} T, \quad (6)$$

Fitzpatrick finds the island width evolution Equation

$$\frac{dw}{dt} = K_R \left(\Delta' + \frac{K_b w}{w^2 + w_{\chi}^2} \right) \quad (7)$$

with all quantities defined as in 2.1, and the critical island width

$$w_{\chi} = k_{\chi} \varepsilon_a^{-1} n^{-1/2} \varepsilon^{1/2} s^{-1/2} \varepsilon_{\chi}^{1/4} a, \quad (8)$$

where k_{χ} is a constant, $k_{\chi} \approx 5$, and $\varepsilon_{\chi} = \chi_{\perp}/\chi_{\parallel}$. The critical width scaling is given by the comparison of the transverse and longitudinal terms in the heat flow Equation (6). In the limit where $\varepsilon_{\chi} = \chi_{\perp}/\chi_{\parallel}$ tends to zero, w_{χ} vanishes and this equation leads to the standard *neoclassical ∇p -driven tearing mode model*.

3.2. Qualitative island dynamics

Introducing the control parameter

$$C_x = \frac{\Delta' w_x}{K_b} = \Sigma' k_x k_b^{-1} n^{-1/2} \beta_p^{-1} l_p \epsilon^{-1} s^{1/2} \delta \epsilon_x^{1/4}, \quad (9)$$

the qualitative island dynamics can be described by the stability diagram shown in Figure 2a (with the same conventions as those introduced at the end of section 2, for Figure 1b). It is characterized by a *universal* stability boundary, given by the equation

$$C_x = -\frac{\bar{w}_x}{1 + \bar{w}_x^2}, \quad (10)$$

where $\bar{w}_x = w/w_x$ is a dimensionless measure of the island width.

Three basic stability domains can be identified, depending on the value of the control parameter C_x :

- First of all, one can observe that, qualitatively speaking, the dynamics of positive- Δ' modes (i.e., the stability of rational surfaces with positive C_x for monotonic safety factor and pressure profiles) is not affected by $\chi_{\perp}/\chi_{\parallel}$ effects: they are unstable with no threshold since both the Δ' - and *modified-bootstrap*- contributions are then destabilizing.

- On the contrary, negative- Δ' modes (i.e., negative- C_x rational surfaces) are not systematically unstable anymore. Indeed, the destabilizing *bootstrap-contribution*, considerably damped for perturbations such that the related island width w is typically less than the critical width w_x , can be balanced by the stabilizing Δ' effects. As a result, rational surfaces with C_x less than the bifurcation value $C_x^{bif} = -1/2$ are intrinsically tearing-stable: $C_x > C_x^{bif} = -1/2$ is a necessary condition for tearing mode excitation in the frame of the $\chi_{\perp}/\chi_{\parallel}$ -model.

- On the other hand, rational surfaces with negative- C_x greater than the bifurcation value $C_x^{bif} = -1/2$ are potentially tearing-unstable, but require a minimum level of perturbation to be destabilized: $w > w_x$ is a sufficient condition for tearing mode excitation, on rational surfaces satisfying the condition $C_x > C_x^{bif} = -1/2$, within the framework of the $\chi_{\perp}/\chi_{\parallel}$ -model.

The three basic solutions are intrinsic stability for $C_x < -1/2$, saturating island with excitation threshold for $-1/2 < C_x < 0$, and growing island with neither excitation threshold nor saturated state for $C_x > 0$. They are illustrated in Figure 2b, which shows the *normalized growth rate* $\gamma_x \equiv d\bar{w}_x/d\tau_x = C_x + \bar{w}_x/(1 + \bar{w}_x^2)$ as a function of $\bar{w}_x$ ($\tau_x = K_R K_b t / w_x^2$ is a dimensionless *time variable*).

The C_χ -domain $-1/2 < C_\chi < 0$ is supposedly the one of interest for the experimentally observed tearing modes, whose qualitative dynamics is then fully characterized by the stability boundary given by Equation (10) and shown in Figure 2a. It characterizes negative- Δ' tearing modes driven unstable by the bootstrap current effects, which are proportional to β_p . The stability boundary is separated into two branches, respectively associated with threshold and saturation (or crest and valley of the *stability landscape* for a description in terms of the familiar motion of a ball on a bumpy terrain), by the bifurcation point ($C_\chi = -1/2, w = w_\chi$). The stable branch (continuous line), associated with the only accessible saturation state, is not of great interest since it essentially agrees with the *neoclassical ∇p -driven tearing mode model*. We are actually more interested in the unstable branch (dashed line) associated with the threshold condition, which determines, together with the level of perturbation (error field, statistical noise related to sawteeth...), the conditions for tearing mode excitation in the frame of the $\chi_\perp/\chi_\parallel$ -model. In principle, the relevance of this model can be studied by comparing the functional dependence of the threshold it predicts to experimentally measured scalings.

3.3. β -limit scaling for the $\chi_\perp/\chi_\parallel$ -model

The maximum β values achieved in present day tokamaks are often set by low-mode-number tearing modes. As discussed in the preceding paragraph, negative- Δ' tearing modes can be driven unstable by the β -dependent bootstrap-current contribution. As a result, a β -limit scaling analysis may provide a good test for the $\chi_\perp/\chi_\parallel$ -model. Such an analysis has actually already been attempted on DIII-D [9], but was based on too simple a stability criterion: a mode was required to have an island width w greater than the threshold value w_χ to be excited. Equation (10) and Figure 3 clearly indicate that this is a sufficient but not necessary condition for rational surfaces satisfying $C_\chi > C_\chi^{bif} = -1/2$ (i.e., the necessary condition for tearing mode excitation). The effective threshold value w_{th} is rather a function of w_χ and other parameters (for instance and in particular β_p). When a mode can be excited (i.e., for $C_\chi > C_\chi^{bif} = -1/2$), w_χ is precisely the largest finite possible value of w_{th} . Over most of the domain where tearing mode excitation requires a finite threshold island width w_{th} (i.e., $-1/2 < C_\chi < 0$), the latter is actually roughly proportional to $C_\chi w_\chi$.

Considering this more appropriate stability criterion leads to a β -limit scaling-law that significantly differs from the one obtained by La Haye et al. [9]. The analysis is restricted to four dimensionless parameters: the plasma beta $\beta \approx nT/B^2$, the collisionality $\nu_* \approx na/T^2$, the normalized Larmor radius $\rho_* \approx T^{1/2}/aB$ and the perturbation level $\varepsilon_B \approx \delta B_r/B$ (La Haye et al. restricted their analysis to a β -limit scaling with collisionality only, although it is not obvious that ρ_* and ε_B were experimental constants). Geometry and profile parameters ($\varepsilon, \varepsilon_a, l_p, s, \delta' \dots$) are

assumed to be fixed from one experiment to the other (the so-called *similar* discharges). As a result, the stability criterion for the $\chi_{\perp}/\chi_{\parallel}$ -model can be written as

$$w_{th} \propto C_{\chi} w_{\chi} \text{ (for } -1/2 < C_{\chi} < 0) \Rightarrow \beta^{-1} \varepsilon_{\chi}^{1/2} \varepsilon_B^{-1/2} \approx \text{Constant}, \quad (11)$$

where use has been made of the scaling $w \propto \sqrt{s\varepsilon_B/\varepsilon}$. Then, we take

$$\chi_{\parallel} \equiv \frac{V_{Th}^2}{v_c} \approx \beta^{1/4} v_*^{-5/4} \rho_*^{-1/2} \quad (12)$$

and

$$\chi_{\perp} \equiv \chi_B F(\beta, v_*, \rho_*) \approx \beta^{(x_{\beta}+1/4)} v_*^{(x_v-1/4)} \rho_*^{(x_{\rho}+1/2)}, \quad (13)$$

with χ_B the Bohm diffusion coefficient and x_{β}, x_v and x_{ρ} the exponents, such that

$$F(\beta, v_*, \rho_*) \equiv \beta^{x_{\beta}} v_*^{x_v} \rho_*^{x_{\rho}}, \quad (14)$$

to be adjusted to characterize a given transport model, as in the appendix of the paper by Christiansen et al.[14]. Hence, for the $\chi_{\perp}/\chi_{\parallel}$ -model, the β -limit scaling-law becomes

$$\beta_{\chi}^{limit} \approx v_*^{\frac{1+x_v}{2-x_{\beta}}} \rho_*^{\frac{1+x_{\rho}}{2-x_{\beta}}} \varepsilon_B^{\frac{-1}{2-x_{\beta}}}. \quad (15)$$

Neglecting the dependence with respect to ε_B , one finds, for instance:

- $\beta_{\chi}^{limit} \approx v_*^{0.82} \rho_*^{0.72}$, for the Goldston scaling-law ($x_{\beta} = 0.36$, $x_v = 0.34$, $x_{\rho} = 0.18$ [14]).
- $\beta_{\chi}^{limit} \approx v_*^{0.5} \rho_*$, for the gyro-Bohm scaling-law ($x_{\beta} = 0$, $x_v = 0$, $x_{\rho} = 1$ [14]).

The dependence with respect to ε_B is a tricky point of the analysis. Indeed, this parameter characterizes the perturbation level required to trigger the tearing mode under consideration (i.e., to create an island whose width is above the threshold w_{th}), which is actually expected to depend on some parameters, particularly β , under certain conditions. To illustrate this point, one can consider the DIII-D experiments reported by La Haye et al. [9], where the observed tearing modes are triggered either by sawteeth or ELMs (Edge Localized Modes) whose amplitude is known to statistically depend on various parameters that might affect Equation (15). Similarly, fishbone modes are often observed prior to tearing mode excitation in TFTR supershots [1], and monster sawtooth crashes are observed to trigger low-mode-number tearing modes on Tore Supra [15].

On the other hand, this problem is not relevant to tearing mode excitation by error-fields since the amplitude of the latter can, in principle, be held constant. However, a problem remains when one takes rotation into account: the trigger efficiency, either by sawteeth, ELMs, fishbones

or error fields, crucially depends on plasma rotation via toroidal coupling effects [16]. It is well known that a magnetic perturbation developing at a given rational surface cannot induce a significant reconnection at a toroidally coupled rational surface if the differential rotation between these two surfaces lies above a certain threshold related to several parameters (see Section 2.3.4, particularly Equation 23c, in reference [16], for instance). This makes it difficult to compare discharges with strong variations in plasma rotation, and in particular to extrapolate small tokamak results (i.e., fast rotation) to large tokamaks (i.e., lower spinning rate, owing to larger plasma dimensions).

4. THE “ ω^* -MODEL”

4.1. Basic features

Dealing with the “ ω^* -model” is complicated. First, two regimes exist in the island evolution. Basically, ω^* -effects consist in including the $\mathbf{E} \times \mathbf{B}$ drift contribution in the analysis of the electron dynamics (which primarily determines island stability via the electron current response to the tearing perturbation). This amounts to considering an electromagnetic problem involving an electrostatic potential perturbation $\delta\phi$ in addition to the usual longitudinal vector potential perturbation $\delta A_{\parallel}$ characteristic of tearing modes. The electrostatic potential $\delta\phi$ is calculated from the quasi-neutrality condition, and the ion dynamics then comes into play (via density perturbations). In the nonlinear regime the magnetic island width w is always larger than the electron Larmor radius, but may be comparable to the ion Larmor radius ρ_i in its earlier growth phase. As a result, two limits must be considered:

- in the *small* island limit ($w < \rho_i$), the ion response to the tearing perturbation is adiabatic: $\tilde{n}_i = (e_i n_{i0} / T_{i0}) \tilde{\phi}$ and the island width evolution Equation (including both *Rutherford*-, *bootstrap*- and ω^* - contributions) takes the form [17]

$$\frac{dw}{dt} = K_R \left(\Delta' + \frac{K_b}{w} + \frac{K_*^S}{w} \right) \quad \text{with} \quad K_*^S = k_*^S(\omega, \omega^*) \beta_p I_p^{-2} s^{-2} \quad (16)$$

where $k_*^S(\omega, \omega^*)$ is a function of mode rotation frequency ω and various diamagnetic frequencies, denoted ω^* for convenience but consisting of a combination of $\omega_{Xj}^* = (mT_{j0}/e_j B_0 r)(\partial X_j / X_j \partial r)$, with $X = n$ (density) or T (temperature) and $j = e$ (electron) or i (ion). Note that Fitzpatrick's correction to the *bootstrap-contribution* is neglected in this section, although a realistic model should certainly include both this correction and ω^* -effects.

- in the *large* island limit ($w > \rho_i$), the ion response is determined by the continuity Equation $\nabla \cdot (e_i n_i \mathbf{V}_E + \mathbf{J}_{pol}) = 0$, where $\mathbf{V}_E = (\mathbf{E} \times \mathbf{B})/B^2$ is the electric drift velocity and $\mathbf{J}_{pol}$ is the polarization current, including both ion inertia and finite Larmor radius effects. The island width evolution Equation (with *Rutherford*-, *bootstrap*- and ω^* - contributions) takes the form [7][8]

$$\frac{dw}{dt} = K_R \left(\Delta' + \frac{K_b}{w} - \frac{K_*^L \rho_i^2}{w^3} \right) \quad \text{with} \quad K_*^L = k_*^L(\omega, \omega^*) \beta_p l_p^{-2} s^{-2} \quad (17)$$

It is usual, within the framework of what could be called *standard mode frequency conditions*, to consider the above $k_*^{S/L}(\omega, \omega^*)$ as positive functions, so that ω^* -effects are destabilizing (stabilizing) for *small* (*large*) islands. Then, if one first neglects details related to scaling and frequency dependence, the *small*-island dynamics is similar to that of the *neoclassical* ∇p -driven *tearing mode model* (it is just the matter of replacing K_b by $K_b + K_*^S$ in Section 2). We are interested in the *large*-island dynamics, which not only provides the new qualitative feature of the stability analysis, namely the stabilizing ω^* -contribution, but also corresponds to the observable part of the dynamics (sub- ρ_i features cannot be measured by standard diagnostics). If the ratio K_*^L/K_b is high enough (i.e., typically larger than 1), the stabilizing ω^* -contribution can balance the destabilizing *bootstrap-contribution* and account for tearing mode stability for an island width w less than a threshold width of the order of an ion Larmor radius. Moreover, since ω^* -effects rapidly vanish with increasing island width (as $1/w^3$, to be compared with $1/w$ for the destabilizing *bootstrap-contribution*), the asymptotic island dynamics (i.e., w greater than a few ρ_i) is expected to be given by the *neoclassical* ∇p -driven *tearing mode model*, known to do a good job of accounting for TFTR supershot data in this diagnostic-accessible limit [4].

4.2. Qualitative island dynamics

Introducing the critical island width

$$w_* = \sqrt{K_*^L/K_b} \rho_i = k_b^{-1/2} k_*^{L/2} l_p^{-1/2} \rho^* \varepsilon^{-1/4} s^{-1/2} a \quad \text{with} \quad \rho^* = \rho_i/a, \quad (18)$$

and the control parameter

$$C_* = \frac{\Delta' w_*}{K_b} = \Sigma k_b^{-3/2} k_*^{L/2} \varepsilon_a \beta_p^{-1} l_p^{1/2} \rho^* \varepsilon^{-7/4} s^{1/2} \delta, \quad (19)$$

the qualitative *large*-island dynamics associated with Equation (17) can be described by the stability diagram shown in Figure 3a (with the same conventions as those introduced at the end of Section 2, for Figure 1b). It is characterized by a *universal* stability boundary, given by the equation

$$C_* = \frac{1 - \bar{w}_*^2}{\bar{w}_*^3}, \quad (20)$$

where $\bar{w}_* = w/w_*$ is a dimensionless measure of the island width.

This stability diagram has pretty much the same features as that of the $\chi_{\perp}/\chi_{\parallel}$ -model (see Figure 2a). Note that, although we are essentially interested in the observable *large*-island dynamics, the *small*-island dynamics (below the dash-dot line in Figure 3a) is indicated to give a complete picture of the island evolution, although the stable branch associated with it (i.e., the straight line connecting the so-called positive- Δ' bifurcation point and the origin) does not correspond to any particular physical mechanism.

The *large*-island dynamics itself is characterized by a two-branch stability boundary. The unstable branch (dashed line), related to the threshold condition for the ω^* -model, is limited by two bifurcation points: the so-called negative- Δ' and positive- Δ' bifurcation points ($C_* = C_*^{bif-} = -2\sqrt{3}/9 \approx -0.385$, $\bar{w}_* = \sqrt{3} \approx 1.73$) and ($C_* = C_*^{bif+}$, $w \approx \rho_i$), respectively (implicitly assuming monotonic profiles and K_*^L/K_b greater than 1).

The position of the boundary between the *large*- and *small*- island domains crucially depends on the value of the ratio K_*^L/K_b . In particular, a threshold condition (i.e., the unstable branch) exists if and if only K_*^L/K_b is greater than 1/3. Similarly, the positive- Δ' bifurcation point exists if and if only K_*^L/K_b is greater than 1 (in the following, we assume that both these conditions hold). The latter characteristic has important implications for the distinction between the $\chi_{\perp}/\chi_{\parallel}$ - and ω^* - models. Indeed, Figure 2a clearly indicates that within the framework of the $\chi_{\perp}/\chi_{\parallel}$ -model, any rational surface with positive Δ' develops a tearing mode, contrary to what happens within the framework of the ω^* -model for rational surfaces with positive Δ' but C_* less than the positive- Δ' bifurcation value C_*^{bif+} . It has been suspected, on the basis of experimental data, that there exist some experimental situations where the $q=2$ surface in the plasma exhibits a positive Δ' without developing the $m/n=2/1$ tearing mode necessarily predicted by the $\chi_{\perp}/\chi_{\parallel}$ -model. If this were to be confirmed, by more accurate Δ' calculations (including precise current density profiles, wall effects...), it would provide significant evidence in favor of the ω^* -model.

It is clear, from the above comments, that the discussion illustrated in Figure 3a strongly depends on the ratio K_*^L/K_b , and thus on the precise value of $k_*^L(\omega, \omega^*)$. This is the source of the second of the earlier mentioned difficulties with the ω^* -model. Despite recent progress in the understanding of the underlying physics of ω^* -effects, the various published results do not agree on the precise expression for $k_*^L(\omega, \omega^*)$. They actually differ by more than an order of magnitude. Historically, Smolyakov [7] was the first to investigate this problem, for collisional plasmas in cylindrical geometry. His estimate of $k_*^L(\omega, \omega^*)$, found to be proportional to $\omega(\omega - \omega_{p_i}^*)$ (with ω being the mode frequency in the reference frame where there is no radial electric field), may

however apparently lead to too weak $a\omega^*$ -contribution to balance the *bootstrap* destabilizing term. More recently, Smolyakov et al. [18] extended this analysis to toroidal geometry. They found $k_*^L(\omega, \omega^*)$ proportional to $\omega(\omega - \omega_{tor}^*)$ (with ω_{tor}^* being the toroidal plasma rotation frequency) and, most importantly, renormalized by a factor $1/\varepsilon^2$ with respect to the previous cylindrical case. Depending on the relative rotation frequency $\omega - \omega_{tor}^*$ between the plasma and the magnetic island (found to be very close to one another in TFTR supershots [4]), this huge renormalization factor may be too large to account for experimental observations. Finally, another result has recently been proposed by Wilson et al. [19], for toroidal plasmas in the collisionless regime, which is a priori more relevant to present-day tokamaks. It leads to a less excessive renormalization which roughly scales as $q^2/\varepsilon^{1/2}$. Owing to this uncertainty in the k_*^L value, no appropriate comparison between data and the ω^* -model has been achieved yet.

To conclude this part, similarly to what was done for the $\chi_\perp/\chi_\parallel$ -model in Figure 2b, the four basic solutions associated with the ω^* -model are illustrated in Figure 3b, showing the *normalized growth rate* $\gamma_* \equiv d\bar{w}_*/d\tau_* = C_* - (1 - \bar{w}_*^2)/\bar{w}_*^3$ as a function of $\bar{w}_*$ ($\tau_* = K_R K_b t / w_*^2$ is a dimensionless *time variable*).

4.3. β -limit scaling for the ω^* -model

Considering toroidal plasmas in the collisionless regime (Wilson et al. [19]), a β -limit scaling analysis similar to that reported in Section 3.3 for the $\chi_\perp/\chi_\parallel$ -model can be performed for the ω^* -model.

From Figure 3a, it is clear that the necessary condition for tearing mode excitation within the framework of the ω^* -model is: $C_* > C_*^{bif-} = -2\sqrt{3}/9 \approx -0.385$. Then, for rational surfaces satisfying this relationship, a sufficient excitation condition is that the perturbation level is such that it creates a magnetic island with $\bar{w}_* > \sqrt{3} \approx 1.73$. For the $\chi_\perp/\chi_\parallel$ -model, a β -limit scaling was easily identifiable since this model basically consists in balancing the *bootstrap*-contribution - proportional to β - by the Δ' -contribution (supposedly independent of β). On the contrary, in the ω^* -model, the threshold condition determining the β -limit is basically given by the comparison between the *bootstrap*- and Δ' - contributions, both proportional to β . As a result, there is no such simple β -limit scaling-law as Equation (15) for the ω^* -model. Rather, the simplest stability criterion that can be found - apparently the only one that does not depend on any of the imprecisely known parameters (such as k_b) - consists in considering that the threshold island width w_{th} is of the order of w_* . This criterion, for lack of a direct β -limit scaling-law, provides an approximate ε_B -limit scaling-law:

$$w_{th} \approx w_* \Rightarrow \varepsilon_B \approx \rho_*^2, \quad (21)$$

which may then be converted into a β -limit scaling-law, provided the scaling of ε_B with respect to β , v_* and ρ_* is known. Such a scaling can only be determined case by case, since it depends on experimental conditions and particularly the type of trigger: sawteeth, ELMs, error fields... Clearly, this situation is more difficult to handle than that reported in Section 3.3.

5. DISCUSSION - CONCLUSION

Low-mode-number tearing modes have been a problem in tokamaks and other fusion devices for a long time. Apart from Rutherford's model [11], which provides the basis for their analysis (together with quasilinear corrections proposed by White et al. [13]), few models have been successfully compared with experiment. The recent comparison between TFTR supershot data and the so-called *neoclassical ∇p -driven tearing mode model* [5], by Chang et al. [4], has revealed a remarkable agreement between theory and experiment for the evolution of the observed magnetic islands with $m/n = 3/2, 4/3$ and $5/4$. However, this model is unable to predict why some modes are observed while others are not, owing to its lack of a threshold condition for tearing mode excitation. Consequences of such a threshold condition on magnetic island dynamics have been discussed in this paper and illustrated by two particular models: the $\chi_\perp/\chi_\parallel$ and ω^* models presented in section 3 and 4, respectively. The main results of the above analysis are:

- Despite some different characteristics, both the $\chi_\perp/\chi_\parallel$ and ω^* models produce a threshold condition that is missing in the standard *neoclassical ∇p -driven tearing mode model*

- Within the framework of these two models, the qualitative magnetic island dynamics is fully described in terms of a unique dimensionless control parameter C (C_χ and C_ω respectively) which is a function of local equilibrium parameters (related to the rational surface of interest). Depending on the value of the control parameter C on a given rational surface, the latter is either intrinsically stable with respect to tearing modes (for C less than a bifurcation value) or may develop a magnetic island. In the latter case, the magnetic island saturation or indefinite growth (depending on whether Δ' is negative or positive) is well described by the *neoclassical ∇p -driven tearing mode model*, but the conditions for the island excitation (whether or not there is a threshold and what its nature is if there is one) are different between the two models. In particular, only the ω^* -model can account for positive- Δ' mode stability.

- A dimensionless β -limit scaling-law can be deduced from these models and compared to "dimensionally similar" experimental data, since low-mode-number tearing modes often set the experimental β -limit in present day tokamak discharges.

- Given the simplicity of the description in terms of a unique control parameter, each model suggests specific feedback scenarios to be applied to occurring magnetic islands, in order to drive the control parameter down to the bifurcation value below which intrinsic stability is achieved.

Note, however, that the hysteresis feature exhibited by the two models - see Figures 2a and 3a - makes it potentially difficult to get rid of an existing island.

The theoretical basis of this paper still needs to be improved in order to clarify uncertainties about some free parameters such as k_b and k_*^L . However, theoretical issues have been reduced to the most elementary level in order to allow comparison to the largest possible variety of situations.

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Figure caption

Figure 1: The *neoclassical ∇p -driven tearing mode model*.

(a) Qualitative stability diagram (normalized island width, $\bar{w}_b \equiv w/r$, evolution as a function of the unique control parameter C_b). The stable branch (continuous line) is associated with the saturated state for negative- Δ' modes ($C_b < 0$). The horizontal axis ($\bar{w}_b = 0$) is formally represented as an unstable branch (dashed line) to account for island growth from zero width. Arrows indicate the island evolution direction: $\uparrow$ for growth, $\downarrow$ for decrease.

(b) The two basic solutions (growth-rate versus island-width):

- $C_b = -1$, negative- Δ' ($C_b < 0$) island with saturated state but no excitation threshold,
- $C_b = 1$, positive- Δ' ($C_b > 0$) island with neither excitation threshold nor saturated state.

Figure 2: The $\chi_\perp/\chi_\parallel$ -model.

(a) Qualitative stability diagram (normalized island width $\bar{w}_\chi \equiv w/w_\chi$ evolution as a function of the unique control parameter C_χ).

(b) The three basic solutions (growth-rate versus island-width):

- $C_\chi = -3/4$, intrinsic stability ($C_\chi < C_\chi^{bif}$),
- $C_\chi = -1/4$, saturating island ($C_\chi < 0$) with excitation threshold ($C_\chi > C_\chi^{bif}$),
- $C_\chi = 1/4$, positive- Δ' ($C_\chi > 0$) island with neither excitation threshold nor saturated state.

Figure 3: The ω^* -model.

(a) Qualitative stability diagram (normalized island width $\bar{w}_* \equiv w/w_*$ evolution as a function of the unique control parameter C_*).

(b) The four basic solutions (growth-rate versus island-width):

- $C_* = -1/2$, intrinsic stability ($C_* < C_*^{bif-}$),
- $C_* = -1/4$, saturating island ($C_* < 0$) with excitation threshold ($C_* > C_*^{bif-}$),
- $C_* = 1/10$, non-saturating island ($C_* > 0$) with $\Delta' > 0$ but excitation threshold ($C_* < C_*^{bif+}$),
- $C_* = 2/5$, island with neither excitation threshold nor saturated state ($C_* > C_*^{bif+}$).

Figure 1

Figure 2

Figure 3

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