PRODUCTIVITY AND INJECTIVITY OF HORIZONTAL WELLS

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Management Summary

1. DOE Approval

The DOE approval for the annual renewal of the research grant to the Stanford Project on the *Productivity and Injectivity of Horizontal Wells* was received in early March 1996. The Project formally commenced on March 10, 1993. Mr. Thomas Reid is the DOE Project Manager in Bartlesville and Mr. John Augustine is the DOE Contracts Officer in Pittsburgh.

2. Industrial Affiliates Program

The DOE Project operates in association with an Industrial Affiliates program on horizontal wells, for which oil company membership has also continued during 1996. The membership during this year comprised the following organizations:

Amoco (USA)
AGIP (Italy)
ARTEP/IFP (France)
BP Exploration (USA)
Chevron (USA)
INTEVEP (Venezuela)
Marathon (USA)
Mobil (USA)
Norsk Hydro (Norway)
Petrobras America (Brazil)
Phillips Petroleum (USA)
Texaco (USA)
Union Pacific Resources (USA)
US Department of Interior/
Minerals Management Service (USA)

3. Project Goals

The Project has eight principal goals to be studied and developed over a five year period. These goals are as follows:

TASK 1: Advanced Modeling of Horizontal Wells - Develop special gridding techniques and associated averaging algorithms for accurate simulation of HW-performance.

TASK 2: Investigate and Incorporate the Effects of Reservoir Heterogeneities - Study impacts of various types of heterogeneity and develop methods for incorporating their effects in both fine grid and coarse grid models.

TASK 3: Develop Improved Methods for Calculating Multi-Phase Pressure Drops within the Wellbore - Plan, execute, and interpret two-phase flow experiments at an oil company research facility, and use results to analyze/validate a new two-phase model.

TASK 4: Pseudo-Functions - Define improved methods for computing two-phase pseudo-functions for effective relative permeabilities for coarse grid blocks near an HW - determine sensitivities to heterogeneities, flow conditions, skin factors, etc.

TASK 5: Develop Multi-Well Models - Develop numerical techniques and software in a parallel computing architecture capable of interactively coupling multiple detailed HW-models to a large scale reservoir simulator.

TASK 6: Test HW-Models with Field Examples - Work with affiliate's member companies to establish HW-modeling capabilities from field measurements, particularly for pathological problem cases.

TASK 7: EOR Applications - Provide and implement practical HW aspects into modeling of EOR processes - miscible gas, steam displacement, in-situ combustion.

TASK 8: Application Studies and Their Optimization - Seek field opportunities for HW's and study their best implementation in various reservoir scenarios e.g., multiple laterals, hydraulic fracture variants, etc.

Tasks 1, 2, 3, 4, and 7 were the primary foci of activities in 1996, and this annual report is therefore written around these five tasks, which are also ongoing in 1997. Some of our principal achievements are summarized below.

4. General Single Phase Wellbore Flow Model (Chapter 1)

A general wellbore flow model, which incorporates not only frictional, accelerational, and gravitational pressure drops, but also the pressure drop caused by inflow for single phase flow was developed in 1996. The new wellbore model clearly identifies the relative magnitude of each pressure drop component and is readily applicable to different wellbore perforation patterns and well completions. It is found that the influence of either inflow (production well) or outflow (injection well) depends on the flow regime present in the wellbore. For laminar flow, wall friction increases due to inflow but decreases due to outflow. For turbulent flow, inflow reduces the wall friction, while outflow increases the wall friction. New wall friction correlations for wellbore flows based on our 1995 experiments have been developed.

5. Large-Scale Flow Loop Experiments and Data Analysis (Chapters 2 and 3)

Two-phase flow experiments were continued during the period 1995-96 in the large Wellbore Model at the Marathon Technology Center in Littleton, Colorado. Extensive modifications were made to the rig in order to (1) use 4.5 inch ID pre-packed wire wrapped screen liner (2) measure in situ liquid fractions at three window locations using nuclear densitometers, and (3) enhance the data acquisition system. The inflow section of the model was changed from 75 ft to 66 ft to allow for three transparent windows at which liquid fractions were measured and flow regimes were recorded. A slightly higher gas rate of 1.8 MMscfD was achieved as opposed to 1.7 MMscfD in 1995. The fluids in all the experiments were water and air. After running a few baseline single phase flow experiments, a total of more than 200 two-phase flow experiments were performed. These experiments included single phase axial water flow with the inflow of water or air, two phase core flow with no radial

influx, two-phase core flow with the inflow of water or air, and finally two-phase core flow with various arrangement of the influx of water and air through perforations. In addition to horizontal flow experiments, some of the experiments were also carried out at $+2^\circ$ (upflow) and -2° (downflow). Differential pressures were recorded along the wellbore using Rosemount pressure transmitters. The observed flow regimes as seen through the transparent acrylic windows were recorded on video tape for every experiment. These video images will be used to check the physical flow regimes in the system with screens and compare them with the 1995 experiments where we did not have the liner. The experimental work was financed by the Industrial Consortium for this Project.

Analyses of all the 1995 and 1996 experimental data were performed. Results indicate that inflow alters flow pattern transitions significantly enough so that predictions based on available multiphase flow models could be badly in error. This indicates the need to incorporate the effects of inflow in the new Stanford mechanistic model. Theoretical efforts to accomplish this for each flow regime have already started and will continue during 1997. Advances have been made in the current mechanistic model (with no inflow). The previously developed correlations for the stratified and annular mist flow regimes have been optimized based on the available data in the Stanford Multiphase Flow Database (SMFD). The existing two-phase flow correlations do not contain features designed to represent effects of perpendicular inflow through perforations, and future work will need to include ways to handle these cross-flow perturbations. Various improvements have also been made to the many features of the SMFD.

6. Effect of Pressure Drop and Optimum Well Length (Chapter 4)

To understand the effect of pressure drop on cumulative oil production, a new project on the effect of friction on well productivity was launched in 1995. A semi-analytical model has been developed to evaluate horizontal well productivity with consideration to friction in the wellbore. The model involves coupling Darcy flow in the reservoir to the flow in the wellbore using material balance equations. Results obtained for the single phase oil case indicate that wellbore pressure drop becomes important only when high permeability or high production rates are involved. Both laminar and turbulent flows are considered. By combining the resulting solutions with a simple economic model, a methodology for computing the optimum well length is developed. For laminar flow, an analytical expression is derived for the optimum well length while for the more practical case of turbulent flow, an iterative procedure yields the optimum well length. The code was written in C++, which inherently supports modularity, and is flexible enough to take advantage of any existing or new multiphase flow models, like the one being developed under Task 3. This kind of coupling with a simulator will allow us to study the effects of wellbore pressure drop in complex heterogeneous reservoirs. Initially the well module will be linked to the reservoir simulator in an explicit fashion so that it will not adversely effect the Jacobian matrix of the simulator. In the final phase of this study, the above mentioned well module will be extended to include multi-lateral wellbores as discussed under the previous task. This comprehensive project will continue in 1997 and beyond.

7. Well Indices (Chapters 5 and 6)

Previously we have shown that the correct well index for a typical partially penetrating horizontal well could be very different from the classical Peaceman well index. The correct well index can be calculated in a semi analytical manner in which the well pressure is obtained analytically and the well block pressure is computed numerically. Another approach is to eliminate the need to calculate any well index. In this so called explicit modeling, the well is very finely gridded. We studied a published utilization of this approach and compared its results with fine grid and analytical results. It is shown that this approach gives incorrect results when used with nonuniform grids having high aspect ratios.

8. Effects of Heterogeneities on Prediction Performance (Chapter 7)

In 1996, we also conducted a study in which we explored the reasons why performance prediction of horizontal wells is so difficult. We generated twenty consistent geostatistical descriptions of permeability and porosity that were all constrained to the hard data obtained from a vertical well previously drilled and cored. All stochastic images were created on a $100 \times 50 \times 30$ fine grid. Two sets of upscaled images of the fine grid: a $20 \times 10 \times 10$ medium grid and a $10 \times 5 \times 5$ coarse grid were also generated. The upscaling was done using two commonly used methods namely, flow-based and power-averaged. Different geological descriptions, grids and methods of upscaling gave very different results. Also we showed that many of the analytical techniques published in the literature do not provide reliable predictions.

The effect of other assumptions on performance prediction of horizontal wells such as: well index, pressure drop, analytical cresting models, and productivity models were also investigated. It is concluded that the greatest source of uncertainty is reservoir description and how it is used in simulators. Upscaling introduces errors and the scale at which upscaling is robust is not known in general. Uncertainties in the well index can cause errors in predictions that are of the same magnitude as those caused by reservoir heterogeneities.

9. Specification for EOR Extensions (Chapter 8)

In a recently completed study of the Project in conjunction with the SUPRI-C (Gas Displacement Project at Stanford), sweep efficiencies of many well configurations were studied in a gas injection process by simulation. Comparisons were made at different viscous to gravity ratios as a way to present the effects of gravity on sweep efficiency. It was found that fully penetrating horizontal wells deliver the highest recovery among the seven different configurations studied and their best recovery is at a viscous to gravity ratio of one. For the heterogeneous systems, horizontal wells still delivered the highest recovery but this time recovery improved as the viscous to gravity ratio decreased. For vertical wells, there are severe rate limitations to achieve the gravity drainage benefits, but the use of longer horizontal wells may allow this new technique to become economically attractive. Initial simulation runs have been made to investigate the comparative aspects of sweep efficiency for vertical and horizontal wells. In this sensitivity study, the effects of many parameters such as length and location of both well types on sweep efficiency are being examined. In conjunction with these simulation runs, we are setting up laboratory experiments with horizontal wells and analog three-phase fluids to study vertical sweep, viscous fingering, and local displacement

efficiencies. The newly installed high resolution X-ray CT Scanner with its dual X-ray energy features at Stanford will be used to monitor saturation distributions in these experiments. The experiments will be analyzed using appropriate 3D-simulation methods.

10. Cresting Correlations and Pseudo-Functions (Chapter 9)

Accurate representation of cresting behavior requires fine grid simulations which are costly and not always practical. This issue has been addressed by developing a coarse grid method to accurately predict breakthrough time and post breakthrough performance of horizontal wells. Correlations for breakthrough time, maximum rate, and water/oil or gas/oil ratio have been developed. The correlations are based on numerous simulations runs and some analytical approaches using an appropriate set of dimensionless variables. A procedure to derive pseudo functions either using numerical correlations or coarse grid simulations was also devised. In 1997, we plan to apply the developed correlations to various example problems including case studies using real field data.

11. Generalized Gridding Methods (Chapter 10)

In our flexible grid simulator, FLEX, one can align gridblock boundaries along streamlines. Examples of this technique have shown improvements in water-cut response for 2D areal problems. This improvement was achieved in spite of using vertical isopotential lines. In another recently developed code, this restriction has been removed. The calculated streamlines and actual isopotential lines from this new code are used to align the grids in FLEX. Advantages of this approach are being investigated.

12. Newly Launched Projects

A number of new projects have been recently started. Here, we only introduce them and leave the reporting on them to a later time when reasonable advance has been made in each;

- Optimal Positioning of Multi-Lateral Wells

A simulation study to investigate the productivity of multi-lateral wells in terms of their arrangement and positioning.

- Gas Condensate Reservoirs

A critical evaluation of literature has been made and the underlying equations have been derived. Both black oil and compositional formulations were considered. Derivations have revealed subtle points in the validity of some assumptions in the previous methods.

- A Field Case Study

We are in the process of obtaining data on the Dos Cuadras field from DOI/MMS. This is an offshore field located in the Eastern Santa Barbara Basin in California. We plan to build a geostatistical model for a part of the field and perform flow simulations. There are several horizontal wells in the field. We will test our predictive techniques by using data on these wells.

13. Affiliates Progress Review Meeting

An annual review meeting for member companies in the Stanford Horizontal Well Project was held at Stanford on October 10-11, 1996. This meeting was well attended and member companies made presentations on their horizontal well interests and activities on the second day. The next meeting is scheduled for October 27 and 28, 1997.

14. Papers and Presentations at Conferences, etc.

Three papers were presented:

1. *"General Wellbore Flow Model for Horizontal, Vertical, and Slanted Well Completions"* presented at the 1996 SPE Annual Fall Meeting - Denver, October 6-9
2. *"Why is it so Difficult to Predict the Performance of Non-Conventional Wells"* presented at the 1996 SPE International Conference on Horizontal Well Technology - Calgary, November 18-20
3. *"Effects of Pressure Drop in Horizontal Wells and Optimum Well Length"* presented at the 1997 SPE Production Operation Symposium - Oklahoma City, March 9-11

We held a joint meeting with SUPRI-A (Heavy Oil) on February 12, 1997 in Bakersfield, California to discuss research directions for a project on the use of horizontal wells in thermal recovery of heavy oils. The meeting was well attended by the Affiliate Members of Stanford groups and other interested parties. A number of possible research topics were proposed. We intend to develop several projects dealing with the application of horizontal well technology to heavy oil reservoirs.

We also organized a short course on Reservoir Simulation at Stanford University and offered it free to members of our Industrial Affiliates (July 22-26, 1996)

We also attended the following meetings:

- *"Forecasting Performance of Petroleum Reservoirs: is the Answer Worth the Cost,"* Deans Distinguished Lecture Series, Rice University - Houston, January 31, 1997.
- *"Multiphase Flow in Pipes"* University of Texas, Austin, March 18, 1996.
- Lectures for Norsk Hydro, Norway, on various problems of modeling well performance, May 27 to July 7, 1996.
- Lectures for RIPED, China, on various problems of modeling well performance, July 10 to 12, 1996.
- Lectures for JNOC and JAPEX, Japan, on various problems of modeling well performance, July 15 and 16, 1996.

1. Single Phase Fluid Flow in a Wellbore

by Liang-Biao Ouyang, Sepehr Arbabi, and Khalid Aziz

Abstract

A general wellbore flow model is presented to incorporate not only frictional, accelerational and gravitational pressure drops, but also the pressure drop caused by inflow. Influence of inflow or outflow on the wellbore pressure drop is analyzed. New friction factor correlations accounting for both inflow and outflow are also developed.

1.1 Introduction

Starting with Yuan & Finkelstein in 1956, a number of mechanical engineers have studied single phase fluid flow in porous pipes with mass transfer through the pipe wall. However, This type of flow did not interest petroleum engineers until the horizontal well technology was introduced and widely applied in the oil and gas industry in the 1980s. For fluid flow in horizontal wells, the flow format is quite similar to pipe flow with mass transfer through porous walls. The main differences between these two types of flow are as follows (Ouyang et al., [1]):

- In horizontal wells, the mass transfer is normally through perforations, while in the case of pipe flow it is through pores in the wall. In other words, the effective perforation density is very large (theoretically infinite) for the porous pipe flow case. Nevertheless, in the case of openhole completions, the horizontal well and porous pipe flow problems are conceptually identical.
- The injection rates are usually quite small in the porous pipe flow case, this is not necessarily the case for wellbore flow.
- When there is no mass transfer through the pipe wall, the effective pipe roughness is very different from the actual pipe roughness in a horizontal well due to the effect of perforations on the axial flow (such as flow separation, cavity flow or secondary flow), but it changes only slightly from the actual value for the porous pipe flow case.

Little information is available related to the significance of the above-mentioned differences on the prediction of horizontal well behavior. Although much research work has been completed for the porous pipe flow, these results may not be easily applicable to horizontal wells. Recognizing this fact, petroleum engineers have begun to study the horizontal well flow behavior since the late 1980s. Research efforts are under way for wellbore flow with mass transfer through pipe wall, but more needs to be done due to the following observations:

- No correlations exist for determining the wall friction factors for single phase and multiphase fluid flow in a wellbore with inflow or outflow through perforations.

- The accelerational and inflow-directional pressure drop are neglected in most of the single wellbore flow models or wellbore-reservoir coupling models.
- The wall friction shears are usually evaluated by using the friction factor correlations for pipe flow without mass transfer through pipe wall. Therefore, the impact of mass transfer through pipe wall on the wall friction is not included.
- No mechanistic model has been developed to predict the flow characteristics, such as flow pattern, liquid holdup and pressure drop in multiphase wellbore flow with the presence of inflow or outflow through perforations.

1.2 Influences of Mass Transfer through Pipe Wall

Based on the analysis by Ouyang et al. [1, 2], the influence of mass transfer through pipe wall on velocity profile and wall friction can be summarized as follows:

- *Laminar Flow*: when mass transfer through the pipe wall exists, the parabolic velocity profile is inappropriate for describing the velocity distribution over the pipe cross section. The velocity should increase for inflow (injection) and decrease for outflow (suction) following the law of mass conservation. Although inflow leads to the increasing of axial velocities across the whole pipe section, as can be imagined, the axial velocities near the pipe wall will increase more than those far away from the wall (and thus close to the centerline). Similarly, outflow decreases axial velocities near the wall more significantly than those away from the wall. As a result, the velocity gradient on and near the pipe wall increases due to the inflow while it decreases due to outflow. Correspondingly, the wall friction increases for inflow case but decreases for outflow.
- *Turbulent Flow*: with the presence of mass transfer through perforations, the time average velocity profile for turbulent pipe flow is altered due to the interaction between the axial flow and the perforation flow. Inflow lifts and expands the turbulent boundary layer and thus increases the axial velocity beyond the layer while decreases the velocity within the layer to follow the mass conservation law. As a consequence the axial velocity gradient near the pipe wall decreases and so does the wall friction shear stress. On the contrary, outflow lowers and reduces the boundary layer and thus decreases the average velocity outside the layer but increases the velocity inside the layer, which results in an increase of the axial velocity gradient near the pipe wall and hence the wall friction shear stress. The analysis is consistent with the numerical observations of Kinney & Sparrow [3] for pipe flow with suction through pipe wall.

1.3 Single Phase Wellbore Flow Model

Consider fluid flow in a wellbore as shown in Figure 1.1 and assume single phase flow of an incompressible Newtonian fluid under isothermal conditions with no heat transfer to and from the fluid. Furthermore, assume that no mechanical work is done on or by the fluid during its passage through the pipe. With these assumptions, the momentum balance equation takes the form (see Ouyang et al. [2] for details)

$$\begin{aligned}
[(pA)_2 - (pA)_1] = & \left[\frac{1}{\alpha_1 g_c} \rho A_1 U_1^2 - \frac{1}{\alpha_2 g_c} \rho A_2 U_2^2 \right] \\
& + \frac{n \Delta x}{\alpha_I g_c} \rho A_I U_r U_x - \tau_w S \Delta x \\
& - \rho \bar{A} \Delta x \frac{g}{g_c} \sin \theta
\end{aligned} \tag{1.1}$$

Figure 1.1: Mass and Momentum Transfer along a Wellbore with Mass Transfer through Perforations

which can be rearranged in combination with mass balance equation to give the following pressure gradient equation

$$\begin{aligned}
\frac{dp}{dx} = & -n \frac{A_I}{A} \frac{2\rho}{g_c \alpha} U U_I - \tau_w \frac{S}{A} \\
& + n \frac{A_I}{A} \frac{\rho}{2g_c \alpha_I} U_I^2 \sin 2\gamma - \rho \frac{g}{g_c} \sin \theta
\end{aligned} \tag{1.2}$$

where γ is the injection angle (Figure 1.1), A_I the cross-section area of each perforation.

Eq. 1.2 indicates that the overall pressure gradient consists of four different components:

- The pressure gradient due to kinetic energy change (accelerational effects). This term should be zero for incompressible fluid flow in pipes of constant ID and without inflow or outflow. Obviously, it will not be zero for wellbore flow with influx.
- The frictional pressure gradient. This part of pressure gradient depends on both axial and perforation flows.
- The pressure gradient due to the inflow direction, and it is termed as inflow-directional pressure drop in this paper. It may help or hinder the axial flow depending on the inflow direction.
- The gravitational pressure gradient. It is reasonable to assume that the gravitational pressure gradient is trivial and thus negligible for horizontal wellbore flow.

In order to quantitatively describe the relative importance of different pressure gradient components, the following three dimensionless numbers are introduced:

R_{af} is defined as the ratio of the accelerational pressure gradient and the frictional pressure gradient;

R_{gf} is defined as the ratio of the gravitational pressure gradient and the frictional pressure gradient;

R_{da} is defined as the ratio of the directional pressure gradient and the accelerational pressure gradient.

Therefore, the total pressure gradient can be expressed as:

$$\frac{dp}{dx} = -\frac{4\tau_w}{D}[1 + R_{af}(1 - R_{da}) + R_{gf}] \quad (1.3)$$

Ouyang et al. [1, 2] found that the closer we are to the horizontal well toe ($x = 0$), the smaller the local wellbore production rate q_w , thus the larger the R_{af} and the more important the accelerational pressure gradient. In contrast, near the heel of the horizontal well ($x = L$), the local production rate q_w becomes large and close to the total well production rate, while the specific influx (inflow rate per wellbore length) q_e does not change significantly, so R_{af} is small, and the accelerational pressure gradient is small and may be negligible.

For the uniform inflow case, we have shown in Ref. [2] that:

- In the laminar flow regime ($x \leq L_t$), R_{af} is only dependent on fluid properties, inflow rate and pipe ID, but it is independent of location x and pipe roughness ϵ . The larger the q_e , the larger the R_{af} , but the shorter the laminar flow length L_t .
- In the turbulent flow regime ($x > L_t$), the Fanning friction factor f is dependent on the local Reynolds number, the wall Reynolds number and the relative pipe roughness, as a consequence, the R_{af} depends on location x , pipe geometry (pipe ID and pipe roughness), fluid properties, and inflow rates.

It is anticipated that turbulent flow occurs along almost the whole wellbore section for most practical situations. Furthermore, the momentum correction factor α does not change much for different velocity profiles under turbulent flow conditions (Ouyang & Aziz, [4]), therefore, α can be assumed constant. As a result, Eq. 1.2 can be rewritten as

$$\tau_w = \frac{D}{4} \left[\frac{dp}{dx} - n \frac{A_I}{A} \frac{2\rho}{g_c \alpha} U U_I + n \frac{A_I}{A} \frac{\rho}{2g_c \alpha_I} U_I^2 \sin 2\gamma - \rho \frac{g}{g_c} \sin \theta \right] \quad (1.4)$$

Based on Eq. 1.4, the wall friction shear stress and hence the wall friction factor can be calculated. The wall friction factor is expected to depend on the local axial Reynolds number, the injection Reynolds number, the effective relative pipe roughness and maybe the inflow/axial flow rate ratios.

1.4 New Wall Friction Factor Correlations

The mass transfer through the pipe wall affects wall friction shear. The influence of either inflow or outflow depends on the flow regime present in the wellbore. The inflow (injection) increases the wall friction for laminar flow while decreases it for turbulent flow. On the contrary, outflow (suction) decreases the wall friction for laminar flow while increases the friction for turbulent flow. In other words, the wall friction is altered compared to the pipe flow where no inflow or outflow is present. Therefore, the friction factor correlations for pipe flow without inflow or outflow can not be used for wellbore flow with both axial flow in the pipe and inflow or outflow through perforations.

1.4.1 Laminar Flow in a Porous Pipe

Based on the reduced ordinary differential equation for the dimensionless stream-function and the numerical procedure provided in Kinney [5], the ratios between the local friction factor and the no-wall-flow friction factor corresponding to different wall Reynolds numbers R_{ew} were calculated. These data have been used to develop the following friction factor correlations by means of a fast and effective nonlinear regression procedure (Ouyang et al., [1, 2]).

- For inflow (injection) case

$$f = \frac{16.0}{R_e} \left(1 + 0.04304 R_{ew}^{0.6142} \right) \quad (1.5)$$

- For outflow (suction or extraction) case

$$f = \frac{16.0}{R_e} \left[1 - 0.0625 \frac{(-R_{ew})^{1.3056}}{(R_{ew} + 4.626)^{-0.2724}} \right] \quad (1.6)$$

1.4.2 Turbulent Flow in a Porous Pipe

A new correlation for the local wall friction factor for turbulent flow has also been developed in the present paper by using Olson & Eckert's experimental data [6] for turbulent

air flow in a porous pipe with uniform air injection through the pipe wall. The new correlation is:

$$f = f_0 \left[1 - 29.03 \left(\frac{R_{ew}}{R_e} \right)^{0.8003} \right] \quad (1.7)$$

It is found that the new correlation can provide satisfactory prediction of wall friction factor for porous pipe flow with injection through the pipe wall. The no-wall-flow friction factor f_0 can be determined either from the Colebrook-White [7, 8] equation or from one of its explicit approximations (Ouyang & Aziz, [4]).

For turbulent pipe flow with outflow (suction) through porous pipe wall, the local friction factor can be evaluated by the Kinney-Sparrow-Wallis's correlation [3]:

$$f = f_0 \left[1 - 17.5 \frac{R_{ew}}{R_e^{0.75}} \right] \quad (1.8)$$

1.4.3 Wellbore Flow

Both correlations (Eqs. 1.7 and 1.8) are based on experimental data for porous pipe flow. It is unclear whether they can also be used for normal wellbore flow conditions. We have applied Eq. 1.7 to analyse the single phase wellbore flow data from the 1995 Stanford horizontal wellbore experiments and found that it overpredicts the wall friction factor. Therefore, another new correlation for the local friction factor has been developed for wellbore turbulent flow based on the 1995 Stanford horizontal wellbore experiment data:

$$f = f_0 \left[1 - 0.0153 R_{ew}^{0.3978} \right] \quad (1.9)$$

1.5 Results and Discussions

1.5.1 Flow Regimes Present in a Wellbore

The changes of accelerational to frictional pressure gradient ratio, R_{af} , with distance along the wellbore for uniform-influx horizontal wells with different wellbore lengths, production rates, and oil viscosities indicate that three regions exist along the wellbore, the laminar flow region, the partially-developed turbulent flow region, and the fully-developed turbulent flow region. The dimensionless number R_{af} is a constant in the laminar flow region that depends on fluid properties (viscosity and density), production rate and pipe geometry (pipe ID and pipe roughness). If all other parameters are fixed, then the shorter the horizontal well, or the higher the production rate, or the more viscous the oil, the larger the value of R_{af} , but the shorter the length of the laminar flow region. For the fully-developed turbulent flow region, all the R_{af} curves for different production rates, different fluid properties, and different wellbore lengths, merge into one, which only changes with wellbore location. Between the laminar and the fully-developed turbulent flow region lies the partially-developed turbulent flow region where R_{af} varies with wellbore location, fluid properties, production rate and pipe geometry.

1.5.2 Partially-Perforated Horizontal Well Example

A partially-perforated horizontal well refers to a well where only part of the wellbore is perforated to allow inflow. Also, it is assumed that the wellbore flow rate before the perforated zone, Q_0 , may or may not be zero.

Figures 1.2 and 1.3 show pressure gradients and cumulative pressure drops from the start of the perforations in a partially-perforated horizontal well (well #1, see Table 1.1). Also shown in Figure 1.2 is the frictional pressure gradient determined from the no-wall-flow friction factor f_0 . It is found that there is a nontrivial difference between the actual frictional pressure gradient and the frictional pressure gradient obtained from the no-wall-flow friction factor. The accelerational pressure gradient is comparable to the frictional component, and the dimensionless number R_{af} changes from 1.27 at the starting point of the well perforation to 0.71 at the end of the perforation. The cumulative accelerational pressure drop from the starting point of the perforations is more or less the same as the cumulative frictional pressure drop (Figure 1.3).

Figure 1.2: Pressure Gradients Change with Wellbore Location (well #1)

1.5.3 Fully-Perforated Horizontal Well Example

In contrast to partially-perforated horizontal wells, fully-perforated horizontal wells have perforations over the entire wellbore length.

Figure 1.4 shows the change in pressure gradients with wellbore location for horizontal well #2 (see Table 1.1). In this case, the dimensionless pressure gradient ratio R_{af} ranges from 0.8 to 0.05, and the cumulative accelerational pressure drop is only about 6% of the total pressure drop along the whole wellbore, hence the accelerational pressure gradient is

Figure 1.3: Change in Cumulative Pressure Drops with Wellbore Location (well #1)

quite small compared to the frictional component for this horizontal well and only minor errors will result if the accelerational pressure drop is neglected.

1.5.4 Coupling the New Wellbore Model with a Reservoir Inflow Model

It is assumed in the above discussion that the specific inflow rate (i. e. the inflow rate per unit wellbore length) is the same everywhere along the whole wellbore. This is not necessarily true in a real horizontal well, since the wellbore pressure near the well toe is higher than that at the well heel due to wellbore pressure drop, the pressure drawdown near the well toe is less than that near the well heel, therefore, the specific inflow rate near the well toe should be smaller than that near the well heel. On the one hand, wellbore pressure or pressure drop is needed to determine the specific inflow rate distribution along the wellbore. On the other hand, the specific inflow rate distribution is an indispensable parameter to calculate wellbore pressure or pressure drop. Hence, a model which incorporates both the wellbore fluid flow and the fluid flow from reservoir into wellbore is required for determining specific inflow rate distribution, wellbore pressure drop, and overall well production rate.

Since the reservoir inflow model is not the main issue of this paper, a simple reservoir model as used by Dikken [9] and Novy [10] is applied to describe the reservoir fluid flow:

$$q_e(x) = J_s [p_e - p_w(x)] \quad (1.10)$$

As discussed above, the total pressure gradient for horizontal wells consists of three components, frictional, accelerational and inflow-directional. For the horizontal well #3 (Table 1.1), the frictional component contributes the largest part of the total pressure gradient,

Figure 1.4: Pressure Gradients Change with Wellbore Location (well #2)

the accelerational component contributes in the range of 20% to 80% depending on the wellbore location (R_{af} changes from about 7.0 to 0.25), while the inflow-directional component contributes very little and can be neglected (Figure 1.5). Correspondingly the cumulative pressure drop along the wellbore is primarily caused by wall friction (which is also affected by inflow), while acceleration or kinetic energy changes only contribute about 5% to 10% to the overall pressure drop (Figure 1.6).

Specific inflow rate is not constant along the wellbore due to wellbore pressure drop (Figure 1.7). Significant differences occur between the real specific inflow rate predicted by considering pressure drop along the wellbore and the fictitious specific inflow rate where no pressure drop along the wellbore is assumed. For more accurate prediction of the specific inflow rate distribution, accelerational pressure drop should also be included in the wellbore model, otherwise around 10% overestimation of the specific inflow rate is likely to occur.

The well production rate is proportional to wellbore length if no pressure drop occurs along wellbore. The statement is invalid for many real horizontal wells. Figure 1.8 shows the production rate change with wellbore length. Well production rate decreases significantly due to pressure drop in the wellbore, and is likely to reach a maximum value at certain wellbore length beyond which no additional production is expected, no matter how long the horizontal well is. The well production rate decreases further if both frictional and accelerational pressure drops are included.

1.6 Conclusions

A general single phase wellbore flow model, which incorporates not only frictional, accelerational and gravitational pressure drops, but also the pressure drop caused by inflow, is

Figure 1.5: Pressure Gradients Change with Wellbore Location (well #3)

Figure 1.6: Change in Cumulative Pressure Drops with Wellbore Location (well #3)

Figure 1.7: Specific Inflow Rate Distribution (well #3)

Figure 1.8: Overall Well Production Rate Change with Wellbore Length (well #3)

Table 1.1: Horizontal Well Parameters

<i>Parameter</i>	Well #1	Well #2	Well #3
Wellbore Length (<i>feet</i>)	110	1870	1000
Perforated Interval (<i>feet</i>)	100	1870	1000
Pipe ID (<i>inch</i>)	6.18	4.5	6
Relative Pipe Roughness	2.0×10^{-4}	1.0×10^{-4}	2.0×10^{-4}
Perforation ID (<i>inch</i>)	0.18	0.18	0.18
Perforation Density ($\frac{hole}{feet}$)	8	10	10
Inflow Direction (<i>deg</i>)	90	45	45
Fluid Density ($\frac{lbm}{ft^3}$)	62.4	52.44	62.0
Fluid Viscosity (<i>cp</i>)	0.878	2.5	1.0
Wellbore Rate before Completion Point ($\frac{Bbl}{day}$)	7000	N/A	N/A
Inflow Rate ($\frac{Bbl}{day}$)	7000	7500	N/A
Specific Inflow Rate ($\frac{Bbl}{day \times feet}$)	N/A	N/A	2.0

presented. The new wellbore model is readily applicable to any wellbore perforation patterns and well completions, and can be easily incorporated in reservoir simulators or analytical reservoir inflow models.

Inflow and outflow affect wall friction shear. The influence of either inflow or outflow depends on the flow regime present in the wellbore. Inflow (production wells) increases the wall friction for laminar flow while decreases the friction for turbulent flow. The opposite is true for outflow (injection wells).

New wall friction factor correlations for pipe flow with influx have been developed based on published data and the Stanford horizontal wellbore experiments. They can be applied for wellbore flow to determine wall friction shear and thus frictional pressure drop for either inflow (production well) or outflow (injection well) and for either laminar flow or turbulent flow.

It is shown that accelerational pressure drop may or may not be important compared to the frictional component depending on the specific pipe geometry, fluid properties and flow conditions.

Nomenclature

A	pipe cross-section area
D	pipe internal diameter (ID)
D_I	perforation internal diameter
f	Fanning friction factor
f_0	no-wall-flow Fanning friction factor
f_T	total or apparent Fanning friction factor
g	acceleration due to gravity
g_c	conversion factor
L	wellbore length
L_t	wellbore length for laminar flow region
n	perforation density
p	pressure
q_e	specific inflow rate
q_w	local wellbore flow rate
R_{af}	ratio between accelerational and frictional pressure gradient
R_{gf}	ratio between gravitational and frictional pressure gradient
R_{da}	ratio between inflow-directional and accelerational pressure gradient
Re	Reynolds number
Re_w	wall Reynolds number, positive for injection and negative for suction
S	wellbore perimeter
U	average axial velocity
U_n	average velocity for no inflow or outflow case
U_w	average velocity for inflow or outflow case
U_∞	free stream velocity
V	volume
V_w	wall inflow or outflow velocity
x	axial coordinate

Greek Letters

α	momentum correction factor
γ	inflow direction angle
δ	boundary layer thickness
θ	wellbore inclination angle
μ	fluid viscosity
ν	kinetic fluid viscosity
ρ	fluid density
τ_w	wall friction shear stress

Subscripts

acc	accelerational
ele	elevational or gravitational

fri frictional

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2. Development and Testing of a New Mechanistic Model for Multiphase Flow in Pipes

by Nicholas Petalas and Khalid Aziz

Abstract

A revised stratified flow model is presented including new correlations for the friction factors at the gas/liquid interface and at the wall/liquid interface. The mechanistic model is tested after incorporating the new stratified flow model and the test results are presented and discussed.

2.1. Introduction

The development of the mechanistic model started with a preliminary version that was developed based on the best correlations found in the currently available literature. The model's performance was then evaluated through the use of graphical techniques¹ over the complete range of pipe geometries, fluid flow rates and properties.

To aid in the testing of the model with actual data, the Stanford Multiphase Flow Database⁴ (SMFD), was developed. The model's performance was then tested through comparisons with experimental data from the SMFD and it was also compared to existing mechanistic models and empirical correlations.

2.2. Recent developments

Improvements to the mechanistic model are being implemented by independently analyzing each flow regime using the SMFD and developing, where required, new empirical correlations to replace those previously selected. The first flow pattern to be investigated was the annular-mist regime and new correlations were developed for the gas/liquid interfacial friction factor and the liquid fraction entrained. The resulting model, including the comparisons with existing models, was discussed in a report⁶ issued in April 1996 and presented in a paper⁵ at the summer meeting of the ASME in July 1996.

In the work that has since ensued, the stratified flow regime was examined and the model was revised as described below. In addition, the model was tested using 79 actual field measurements provided by BP and was reportedly accurate to within 6%. The field data, however, have not as of yet been made available to us.

A version of the mechanistic model of Xiao⁷ et. al. was obtained through private communication with J.J. Xiao. This model differs significantly from the published version which, in previous comparisons with the present model, was shown to be quite poor. The new

version has been implemented and although the results are significantly improved, the Stanford model is still superior to all other models tested.

2.3. Revised Stratified Flow Model

The geometrical aspects of stratified flow are shown in Figure 2.1.

FIGURE 2.1 - MOMENTUM BALANCE WITH TWO SEGREGATED FLUIDS

A momentum balance can be written on the liquid and gas phases contained in a control volume. Thus, for the liquid phase:

$$-\frac{p_2 A_L}{\Delta L} + \frac{p_1 A_L}{\Delta L} - \tau_{wL} S_L + \tau_i S_i - \rho_L A_L \frac{g}{g_c} \sin \theta = 0 \quad \text{Eq. (2.1)}$$

Similarly, for the gas phase:

$$-\frac{p_2 A_G}{\Delta L} + \frac{p_1 A_G}{\Delta L} - \tau_{wG} S_G - \tau_i S_i - \rho_G A_G \frac{g}{g_c} \sin \theta = 0 \quad \text{Eq. (2.2)}$$

Note that the interfacial shear stress, τ_i , is positive if the gas phase is flowing faster than the liquid phase, i.e. $V_G > V_L$. After taking the limit as $\Delta L \rightarrow 0$ the two momentum balance equations may be expressed as:

$$-A_L \left(\frac{dp}{dL} \right) - \tau_{wL} S_L + \tau_i S_i - \rho_L A_L \frac{g}{g_c} \sin \theta = 0 \quad \text{Eq. (2.3)}$$

and

$$-A_G \left(\frac{dp}{dL} \right) - \tau_{wG} S_G - \tau_i S_i - \rho_G A_G \frac{g}{g_c} \sin \theta = 0 \quad \text{Eq. (2.4)}$$

The shear stress terms can be expressed in terms of friction factors:

$$\tau_{wL} = \frac{f_L \rho_L V_L^2}{2g_c}, \tau_{wG} = \frac{f_G \rho_G V_G^2}{2g_c}, \text{ and } \tau_i = \frac{f_i \rho_G V_i |V_i|}{2g_c} \quad \text{Eq. (2.5)}$$

The average interface velocity, V_i , is assumed to be given by the difference of the average phase velocities, i.e.

$$V_i = (V_G - V_L) \quad \text{Eq. (2.6)}$$

The friction factor for the gas phase is evaluated from the single phase relationship by using the Colebrook³ correlation with the following definition of Reynolds number and hydraulic diameter:

$$\text{Re}_G = \frac{D_G \rho_G V_G}{\mu_G}, D_G = \frac{4A_G}{S_G + S_i} \quad \text{Eq. (2.7)}$$

In order to determine the friction factors for the gas/liquid interface (f_i) and for the liquid/wall interface (f_L) the following empirically derived correlations are used:

$$f_i = 0.0104(0.004 + 0.5 \times 10^{-6} \text{Re}_{sL}) \text{Fr}_L^{1.335} \left\{ \frac{\rho_L Dg}{\rho_G V_G^2} \right\} \quad \text{Eq. (2.8)}$$

$$f_L = 0.452 f_{sL}^{0.731} \quad \text{Eq. (2.9)}$$

The Froude number is defined as $\text{Fr}_L = \frac{V_L}{\sqrt{gh_L}}$.

The evaluation of these relationships is based on data from the Stanford Multiphase Flow Database. A total of 1,026 stratified smooth and stratified wavy experimental measurements were extracted from the database and the following procedure adopted.

By adding Eq. (2.3) to Eq. (2.4) and dividing by the area, the interfacial shear stress term is eliminated and following equation is obtained:

$$-\frac{dP}{dL} = \frac{\tau_{wL} S_L + \tau_{wG} S_G}{A} + (E_L \rho_L + E_G \rho_G) \frac{g}{g_c} \sin \theta \quad \text{Eq. (2.10)}$$

Using the experimental value of E_L , the “experimental” value of the dimensionless liquid height, $[h_L/D]_{\text{exp}}$, is determined. From this S_L , S_G , and τ_{wG} are obtained. Now using the experimental value for (dP/dL) , the value of τ_{wL} can be obtained, and hence f_L . By iterating, the value of f_i is determined for which the calculated h_L/D matches $[h_L/D]_{\text{exp}}$. This is done for all of the experimental observations and the resulting values of f_i and f_L are correlated to obtain Eq. (2.8) and Eq. (2.9).

2.4. Results

The behavior of the model was examined over a wide range of flow rates and fluid properties using three-dimensional surface plots. This was done over the complete range of upward and downward pipe inclinations and both pressure gradient and volumetric liquid fraction were analyzed. The results indicate much smoother transitions between flow regimes than the previous version of the model.

A comparison with experimental data was also performed. Data were extracted from the Stanford Multiphase Flow Database for which pressure gradient, holdup and flow pattern observations were available. This resulted in a total of 5,951 measurements consisting of variations in fluid properties, pipe diameters, and upward as well as downward inclinations. The model predictions for liquid volume fraction are plotted against the experimental measurements in Figure 2.2. Figure 2.3 shows a similar plot for the pressure gradient calculations.

FIGURE 2.2 - COMPARISON OF CALCULATED LIQUID VOLUME FRACTION WITH EXPERIMENTAL DATA

FIGURE 2.3 - COMPARISON OF CALCULATED PRESSURE GRADIENT WITH EXPERIMENTAL DATA

The total number of pressure gradient predictions to fall within 25% of the experimental observations improved slightly to 3,096 data points versus 2,977 for the previous version of the model. Similarly, the volume fraction liquid prediction resulted in a small improvement: 4,076 of the 5,951 data points were predicted to within 25% of the experimental data, up from 3,986.

Table 2.1 shows how the new stratified model affects the flow pattern predictions. It is seen that further improvement is required here as the correct flow pattern is predicted in only 1,792 of the 5,951 cases. It should be noted, however, that intermittent flow (consisting of elongated bubble, slug, and froth flow) constitute the majority of the experimental observations and this model has yet to be optimized.

A further point to consider is that the stratified smooth and stratified wavy flow regimes are treated as a single flow pattern by the model. A distinction between the two is only made for the purposes of flow pattern prediction. If the transition between smooth and wavy stratified flow is ignored, it is seen that stratified flow was observed in 1,012 of the 5,951 cases. The model predicted stratified flow in 719 cases. Of these, the predictions matched the experimental observations in 464 cases, a significant improvement over the $110+92=202$ reported in Table 2.1. This suggests that the transition mechanism between stratified smooth and wavy flow may need to be modified.

TABLE 2.1 - FLOW PATTERN DISTRIBUTION IN EXPERIMENTAL DATA

Flow Pattern	Predicted	Measured	Matched
Elongated bubble	394	584	106
Bubble	43	426	0
Stratified smooth	131	449	92
Stratified wavy	588	563	110
Slug	1511	2103	627
Annular-Mist	1416	1007	667
Dispersed bubble	498	411	38
Froth I	1323	408	152
Froth II	47	0	0
Total	5951	5951	1792

2.5. Conclusions and Future Work

The improvements to the stratified flow model have resulted in minor improvements to the mechanistic model's ability to predict flow patterns, pressure drop and holdup. There have also been significant reductions in the discontinuities between flow patterns resulting in much smoother transitions.

Significant additional improvements are expected to be achieved by optimizing the intermittent flow model in the same manner as has been done for annular-mist and for stratified flow. Once all of the individual flow patterns have been optimized, the transition mechanisms presently used in the model will be reassessed. The model will also be tested using data from actual wells (approximately 1,800 such measurements are available in the SMFD)..

Testing of the model with data from the Stanford horizontal well experiments has indicated that the accelerational effects due to inflow cannot be ignored. A model to account for these effects is being developed and when it is completed, it will be incorporated in and tested with the mechanistic model.

Nomenclature

A	Cross-sectional area
D	Pipe internal diameter
E	In situ volume fraction
g	Acceleration due to gravity
h_L	Height of liquid (Stratified flow)
L	Length
p	Pressure
Re	Reynolds number
S	Contact perimeter

V_{SG}	Superficial gas velocity
V_{SL}	Superficial liquid velocity
ϵ_k	Pipe roughness
θ	Angle of inclination
μ	Viscosity
ρ	Density
σ	Interfacial (surface) tension
τ	Shear stress
$\tilde{x}$	Dimensionless quantity, x

Subscripts

G	relating to the gas phase
i	relating to the interface
L	relating to the liquid phase
SG	based on superficial gas velocity
SL	based on superficial liquid velocity
wL	relating to the wall-liquid interface
wG	relating to the wall-gas interface

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3. Analysis of the 1995 and 1996 Stanford Horizontal Wellbore Experiments

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Abstract

A brief introduction to the 1996 Stanford Horizontal Wellbore experiments is presented. Different existing mechanistic models, simplified models as well as empirical correlations are applied to analyze the 1995 and 1996 Stanford horizontal wellbore experiments. Finally, comparisons between measurement and prediction are performed, which leads to the conclusion that no existing models can predict sufficiently accurate pressure drops for gas-liquid wellbore flow where wall mass transfer exists.

3.1 Introduction

Over the last decade horizontal wells have become a well established technology for the recovery of oil and gas. They have become increasingly attractive for production from thin reservoirs, naturally fractured reservoirs, reservoirs with gas and water coning problems, offshore environments where various wells are drilled from a central platform, as well as in enhanced oil recovery practices such as steam injection.

The modeling of horizontal wells poses additional challenges, one of which is the determination of the frictional pressure drop along the wellbore. The pressure drop due to friction is usually neglected in classical reservoir simulation models. However, practical examples have shown that in many cases the inclusion of wellbore friction is essential for the correct prediction of well performance. For horizontal wells, the frictional pressure drop becomes even more significant since the drilling technology now allows wells to be several thousands of feet long. Thus, the frictional pressure drop over the production portion of the well will be much larger in horizontal wells compared to that in vertical wells.

Depending on the completion method used in a horizontal well, fluid may enter the wellbore radially through perforations at various locations. This alters the flow behavior along the wellbore and complicates the modeling of the problem. For single phase flow, Ouyang *et al.* [1] have shown that influx through well perforations increases wall friction for the laminar flow regime and decreases wall friction for the turbulent flow regime. This means that the frictional pressure drop can be larger or smaller than that for no influx or no-wall-flow situations, depending on the flow regime present in the wellbore. In addition, as the distance between adjacent perforations may not be large enough to achieve a stabilized velocity profile, the flow characteristics and the pressure gradient in a horizontal wellbore may change with location. The flow behavior in a horizontal wellbore with an increasing flow rate along it due to influx and the relationship between pressure drop and influx from the reservoir are some of the most important but unsolved issues in the engineering analysis of horizontal wells. Furthermore, in certain cases,

gas and liquid flow simultaneously in horizontal wellbores under different flow patterns. Since various flow patterns behave differently when no inflow is present through the pipe walls, with inflow the problem will become more complicated.

A literature survey leads to the following observations:

- No correlations exist for determining wall friction factors for single phase and multiphase fluid flow in wellbores with inflow or outflow through perforations.
- The accelerational and inflow-directional pressure drops are neglected in most of the single wellbore flow models or wellbore-reservoir coupling models.
- The wall friction shear is usually evaluated by using the friction factor correlations for pipe flow without mass transfer through pipe wall. Therefore, the impact of mass transfer through the pipe wall on the wall friction is not included.
- No mechanistic model has been developed to predict the flow characteristics, such as flow pattern, liquid holdup and pressure drop in multiphase wellbore flow with the presence of inflow or outflow through perforations.

In order to investigate the influence of inflow on single phase and multiphase wellbore flow behavior, experiments have been designed and performed by Stanford University in 1994 for oil-nitrogen and in 1995 for air-water flow in a 100 feet-long horizontal wellbore model at the Marathon Oil Company's Littleton facility. For 1996, the wellbore model was modified by inserting stainless steel wire-wrapped screens inside the base acrylic pipe. Air-water flow experiments, similar to the 1995 experiments, were carried out using this setup, as described in more detail in the next section.

3.2 Experimental Setup

The horizontal wellbore flow model is shown in Figure 3.1. The outer acrylic pipe is the same as used during the 1995 experiments. It is 100 feet long and consists of twenty 5-foot sections with an average internal diameter of 6.2 inches. The longer inner pipe is made of three acrylic window sections (denoted by W in Figure 3.1), each about 4 feet long, and three stainless steel wire-wrapped screen sections, each about 21 feet long. Packers are used to eliminate flow in the annular space around the window sections. Inflow only occurs in the screen sections so that the total inflow length is approximately 63 feet, down from 75 feet in the 1995 experiments. Window sections are used to observe and record the flow regimes and they each house a nuclear densitometer to measure the average *in situ* liquid fraction. Figure 3.2 shows the average measured internal diameter of different sections of the inner wellbore. The screen sections have a very small variation in diameter (4.500 to 4.509 in), whereas the window sections show a somewhat larger variation in their diameters (4.450 to 4.476 in). However, both variations are much smaller than the variations in diameters of the sections forming the outer pipe.

A total of 164 experiments have been conducted (Table 3.1) which can be classified as:

- Single phase water flow without inflow – calibration experiment (series 0 and V)
- Single phase water axial flow with water inflow (series IA1–IA3 and IV)
- Air and water two-phase flow

1. Air-water axial flow without inflow (series IIA1–IIA3)
2. Water axial flow with air inflow (series IIB1–IIB3)
3. Air-water axial flow with single phase inflow
  - Water inflow (series IIC1a–IIC1c)
  - Air Inflow (series IIC2a–IIC2b)
4. Air and water axial flow with air and water inflow (series IID1a–IID3b)

For each experimental run, axial flow and inflow flow rates; the liquid fractions at 11.25 feet, 36.25 feet, and 61.25 feet locations; pressure drops every 10 feet and 50 feet; pipe inclination angle; inlet and outlet temperatures and pressures were measured and recorded automatically into a Microsoft Excel file for later analyses. For most experiments, data were recorded for at least 10 minutes after reaching stabilized flow.

Figure 3.1: Wellbore Section Used for the 1996 Horizontal Flow Experiments

Figure 3.2: Average Measured Wellbore Internal Diameter for Each Section

Table 3.1: Experiments Conducted in 1996

Experiment Series	Exp. Number	Angle	Axial Flow		Inflow		Inflow Location	
			water	air	water	air	water	air
0	1	0	17.1		N/A		N/A	
Wellbore Char.	2	0	30		N/A		N/A	
V Wellbore Char.	V0 — V12	0	17.5 — 30		N/A		N/A	
IA1	3	0	6.9		3.4		See Figure 3.1	
Water inflow (L)	4	0	10.3		3.4		See Figure 3.1	
IA2	5	0	6.9		6.9		See Figure 3.1	
Water inflow(M)	6	0	10.3		6.9		See Figure 3.1	
IA3	A5	0	13.7		10.3		See Figure 3.1	
Water inflow(H)	A6	0	17.1		10.3		See Figure 3.1	
IV Water Inflow (Vary Inflow Rate)	IV1 to IV9	0	17.5		1.0 — 9.0		See Figure 3.1	
IIA1	7	0/-2/+2	3.4	0.5	N/A		N/A	
Air core flow	8	0/-2/+2	6.9	0.5	N/A		N/A	
low rate	9	0/-2/+2	10.3	0.5	N/A		N/A	
	10	0/-2/+2	13.7	0.5	N/A		N/A	
IIA2	11	0/-2/+2	3.4	1.0	N/A		N/A	
Air core flow	12	0/-2/+2	6.9	1.0	N/A		N/A	
intermediate	13	0/-2/+2	10.3	1.0	N/A		N/A	
	14	0/-2/+2	13.7	1.0	N/A		N/A	
IIA3	15	0/-2/+2	3.4	1.7	N/A		N/A	
Air core flow	16	0/-2/+2	6.9	1.7	N/A		N/A	
high rate	17	0/-2/+2	0.3	1.7	N/A		N/A	
	18	0/-2/+2	13.7	1.7	N/A		N/A	
	15a	0	3.4	1.8	N/A		N/A	
	16a	0	6.9	1.8	N/A		N/A	
	17a	0	10.3	1.8	N/A		N/A	
	18a	0	13.7	1.8	N/A		N/A	
IIB1	19	0/-2/+2	3.4			0.5	See Figure 3.1	
Air inflow	20	0/-2/+2	6.9			0.5	See Figure 3.1	
low rate	21	0/-2/+2	10.3			0.5	See Figure 3.1	
	22	0/-2/+2	13.7			0.5	See Figure 3.1	
IIB2	23	0/-2/+2	3.4			1.0	See Figure 3.1	
Air inflow	24	0/-2/+2	6.9			1.0	See Figure 3.1	
intermediate	25	0/-2/+2	10.3			1.0	See Figure 3.1	
	26	0/-2/+2	13.7			1.0	See Figure 3.1	
IIB3	27	0	3.4			1.7	See Figure 3.1	
Air inflow	28	0	6.9			1.7	See Figure 3.1	
high rate	29	0	10.3			1.7	See Figure 3.1	
	30	0	13.7			1.7	See Figure 3.1	

Table 3.1: Experiments Conducted in 1996 (continued)

Experiment Series	Exp. Number	Angle	Axial Flow		Inflow		Inflow Location	
			<i>water</i>	<i>air</i>	<i>water</i>	<i>air</i>	<i>water</i>	<i>air</i>
IIC1a	31	0/-2/+2	3.4	0.5	3.4		See Figure 3.1	
Air core flow	32	0/-2/+2	6.9	0.5	3.4		See Figure 3.1	
water inflow	33	0/-2/+2	10.3	0.5	3.4		See Figure 3.1	
	34	0/-2/+2	13.7	0.5	3.4		See Figure 3.1	
IIC1b	35	0/-2/+2	3.4	1.0	3.4		See Figure 3.1	
Air core flow	36	0/-2/+2	6.9	1.0	3.4		See Figure 3.1	
water inflow	37	0/-2/+2	10.3	1.0	3.4		See Figure 3.1	
	38	0/-2/+2	13.7	1.0	3.4		See Figure 3.1	
IIC1c	39	0	3.4	1.7	3.4		See Figure 3.1	
Air core flow	40	0	6.9	1.7	3.4		See Figure 3.1	
water inflow	41	0	10.3	1.7	3.4		See Figure 3.1	
	42	0	13.7	1.7	3.4		See Figure 3.1	
	39a	0	3.4	1.8	3.4		See Figure 3.1	
	40a	0	6.9	1.8	3.4		See Figure 3.1	
	41a	0	0.3	1.8	3.4		See Figure 3.1	
	42a	0	13.7	1.8	3.4		See Figure 3.1	
IIC2a	43	0/-2/+2	3.4	0.5		0.5	See Figure 3.1	
Air core flow	44	0/-2/+2	6.9	0.5		0.5	See Figure 3.1	
air inflow	45	0/-2/+2	10.3	0.5		0.5	See Figure 3.1	
	46	0/-2/+2	13.7	0.5		0.5	See Figure 3.1	
IIC2b	47	0/-2/+2	3.4	0.5		1.0	See Figure 3.1	
Air core flow	48	0/-2/+2	6.9	0.5		1.0	See Figure 3.1	
air inflow	49	0/-2/+2	10.3	0.5		1.0	See Figure 3.1	
	50	0/-2/+2	13.7	0.5		1.0	See Figure 3.1	
IID1a	51	0	3.4	0.5	3.4	0.5	See Figure 3.1	
	52	0	13.7	0.5	3.4	0.5	See Figure 3.1	
IID1b	53	0	3.4	0.5	3.4	0.5	See Figure 3.1	
	54	0	13.7	0.5	3.4	0.5	See Figure 3.1	
IID2a	55	0	3.4	0.5	3.4	0.5	See Figure 3.1	
	56	0	13.7	0.5	3.4	0.5	See Figure 3.1	
IID2b	57	0	3.4	0.5	3.4	0.5	See Figure 3.1	
	58	0	13.7	0.5	3.4	0.5	See Figure 3.1	
IID3a	59	0	3.4	0.5	3.4	0.5	See Figure 3.1	
	60	0	13.7	0.5	3.4	0.5	See Figure 3.1	
IID3b	61	0	3.4	0.5	3.4	1.0	See Figure 3.1	
	62	0	13.7	0.5	3.4	1.0	See Figure 3.1	
Total	164							

3.3 Analysis of Experimental Data

3.3.1 Overview

Preliminary comparison results of the 1995 Stanford horizontal wellbore experiments were given in the 1996 Annual Report to the Department of Energy (DOE) of the United States [2]. Due to time limitation, only part of the 1995 experimental data was analyzed and compared with predictions by simplified models, mechanistic models as well as empirical correlations. In the past year, research associated with the Stanford horizontal wellbore experiments includes: (a) Complete analysis for both 1995 and 1996 experiments; (b) Improvement of the Stanford mechanistic model for gas-liquid two-phase flow in pipes; (c) Development of new models (including a simplified model and mechanistic models) and correlations to account for mass transfer through wellbore perforations. Results from the first area of our research are presented

3.3.2 Model Prediction Procedure

In all, five models are selected for comparison with experimental data:

- the Stanford mechanistic model (Petalas & Aziz, 1996 [3]);
- the Xiao, Shoham & Brill model (1990) [4];
- the Beggs & Brill model (1973, revised 1977) [5];
- the Dukler, Wicks & Cleveland (1964) correlation [6];
- the no-slip homogeneous model (Beattie & Whalley, 1982 [7]).

Among these models, the first three involve a mechanistic approach, the fourth is an empirical correlation, while the last one is a simplified flow model. Note that the models or correlations used do not account for the effects of mass transfer through the pipe wall on wall friction and on the kinetic energy change.

Figure 3.3: Segmented Model Approach

Because of inflow through the perforations, the local gas and liquid flow rates (thus superficial velocities U_{SL} and U_{SG}) vary with wellbore location. Consequently, a segmented model approach as shown in Figure 3.3 must be applied. This approach features three primary steps:

- 1). Divide the entire wellbore into small segments such that the predicted pressure drop along each segment is smaller than a specified tolerance.
- 2). Compute the gas and liquid flow rates entering each segment through the perforations and add these inflow rates to the axial flow rates at the inlet of the segment.
- 3). Determine the flow pattern, if applicable (based on the model being used), and the pressure drop along each segment and sum them up to obtain pressure drops along the entire wellbore or along any section of interest.

3.3.3 Comparison between Measurement and Prediction

1995 Experiments

A total of 134 experimental runs featuring different combinations of axial flow and influx were conducted in the 1995 program [2], including

- single phase water axial flow with water inflow;
- single phase water axial flow with air inflow;
- air and water two-phase axial flow with water inflow;
- air and water two-phase axial flow with air inflow;
- air and water two-phase axial flow and inflow.

Some duplicate experiments were performed, which increases the total number of experimental runs to 200. Among all these experiments, more than 95% fall into air-water two-phase flow category and will be used in the comparison below.

Comparisons between measured and predicted pressure drops for the 1995 experiments are shown in Figures 3.4–3.8. Two plots are included in each figure, where plot (a) compares the pressure drop along the section of pipe from 40 to 50 *feet*, while plot (b) shows the comparison of the pressure drop along the entire wellbore (100 *feet* long). The following observations can be made based on Figures 3.4–3.8:

- 1). Large scatter is seen in Figures 3.4–3.8, indicating that there exist significant differences between experimental data and model predictions. To make things worse, no apparent trends are evident in the data, except that all the models overpredict the pressure drop for downward flow. For horizontal and upward flow, the models overpredict pressure drop for some runs, while underpredicting for others.

- 2). It is found that predictions by the Stanford mechanistic model improve by either ignoring stratified flow or forcing annular-mist flow. Particularly, significant improvements in prediction are found in the case of forcing annular-mist flow, though nontrivial discrepancies are still present in this case.
- 3). The predicted pressure drops along the entire wellbore and the section at 40–50 feet by the Xiao, Shoham & Brill (1990) model [4] become zero at numerous data points (Figure 3.5). Horizontal pipe flow occurs for most of these data points. The underlying reason is that the Xiao, Shoham & Brill (1990) model [4] predicts negative values of pressure drop for these experiments, which is obviously incorrect.

Figures 3.9–3.13 are similar to those listed in Figures 3.4–3.8. The only difference is that the data points are classified based on influx in Figures 3.9–3.13 as opposed to inclination angle. All the data points are divided into four groups: (a) no influx; (b) air influx only; (c) water influx only; (d) both air and water influx. Basically, all the models reflected in Figures 3.9–3.13, were developed for gas-liquid two-phase pipe flow without any influx (we refer to this type of flow as regular pipe flow to distinguish it from pipe flow with wall mass transfer). Unfortunately, the predictions for regular pipe flow based on these models deviate substantially from measurements. One of the possible reasons is that the closure relationships or correlations used in these models were proposed on the basis of experimental data with smaller pipe diameters and therefore may introduce some inaccuracy for large pipe diameter situations such as in the 1995 Stanford experiments. Another possible reason is that pressure drops were not very high, which makes accurate measurement difficult. The models also seem to overpredict pressure drop for air-water two-phase flow without influx (Figures 3.9–3.13). This could indicate that an inappropriate pipe roughness has been used.

1996 Experiments

The 1996 Stanford horizontal wellbore experiments differ from the 1995 experiments primarily in wellbore geometry and inflow format. The average wellbore internal diameter is 4.5 inch in 1996 as compared to 6.18 inch in 1995. Fluid (air and/or water) influx is through a screen in 1996 experiments, whereas it is through perforations, uniformly distributed over the wellbore test section, in 1995 (Ouyang *et al.* [2]). Excluding 52 single phase experiments, there are 162 experimental runs related to air-water two-phase flow if repeated measurements are also counted. As in the analysis for the 1995 experiments, the same models are applied to predict pressure drop along the entire pipe length and comparisons between model predictions and the experimental data are shown in Figure 3.14–3.23. Due to the smaller pipe internal diameter, measured pressure drops are much higher than for experiments in 1995 with the same air and water flow rates. Greater than 10 psia pressure drops along the whole pipe were observed in nearly half of the experimental runs.

As before, the data points are grouped according to pipe inclination angle in Figures 3.14–3.18, while they are grouped by influx in Figures 3.19–3.23. From these figures it is

observed that the model predictions for the 1996 Stanford experiments are much improved over those for the 1995 experiments and less data scatter is seen. The following observations can be made:

- 1) The Stanford mechanistic model provides good predictions for some of the experimental runs (Figure 3.14), although significant differences still exist in other parts of the data. It is interesting to note that for most of the data points, the Stanford mechanistic model substantially overpredicts pressure drop during downward flow. The predictions improve if either stratified flow is ignored or annular-mist flow is forced. Even in these cases, the Stanford mechanistic model slightly underpredicts the pressure drop.
- 2) The pressure drop predictions for the 1996 experiments by the Xiao, Shoham & Brill model, the Dukler, Wicks & Cleveland correlation and the no-slip homogeneous model are similar and they all show a remarkable improvement over the predictions for the 1995 experiments (Figures 3.15, 3.17 and 3.18). For a majority of data points, however, all these models underpredict the pressure drop. It is apparent that improvement in these models is necessary in order to apply them to wellbore flow with influx.
- 3) The worst predictions of pressure drop for the 1996 experiments are observed for the Beggs & Brill model (Figure 3.16), although, here too, predictions show an improvement over those for the 1995 experiments.

The comparisons shown in Figures 3.19–3.23, where data was grouped by influx, are interesting. Fairly clear trends in model predictions can be seen:

- 1) All the models underpredict the pressure drop for cases with gas influx. They either slightly overpredict the pressure drop or give good predictions for cases without influx or with water influx.
- 2) Both the Stanford mechanistic model and the no-slip homogeneous model result in good predictions for cases with no influx.
- 3) The no-slip homogeneous model and the Xiao, Shoham & Brill model are able to provide fairly accurate predictions for most experimental runs with water influx.

3.4 Discussions & Conclusions

A segmented method has been applied to predict flow pattern, pressure drop and liquid holdup by using existing mechanistic models or empirical correlations. Unfortunately, substantial differences between measurements and predictions have been observed. Some of the possible reasons are given below:

- Existing models or correlations do not account for the inflow effect, although the kinetic energy effect can be quite significant due to inflow.
- Correlations for wall friction factors, interfacial friction factor and liquid entrainment fraction were proposed for gas-liquid two-phase pipe flow without inflow, as a consequence they may not be appropriate for air and water wellbore flow where inflow occurs.
- Some transition criteria in existing mechanistic models may be inappropriate for wellbore flow with influx, and the flow patterns predicted may be incorrect.

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(a) Pressure Drop along the Section at 40–50 feet

(b) Entire Pressure Drop

Figure 3.4 Comparison between Measured Pressure Drop and Prediction by Petalas & Aziz (1996) Model (1995 Experiments)

(a) Pressure Drop along the Section at 40–50 feet

(b) Entire Pressure Drop

Figure 3.5 Comparison between Measured Pressure Drop and Prediction by Xiao, Shoham & Brill (1990) Model (1995 Experiments)

(a) Pressure Drop along the Section at 40–50 *feet*

(b) Entire Pressure Drop

Figure 3.6 Comparison between Measured Pressure Drop and Prediction by Beggs & Brill (1973, revised 1977) Model (1995 Experiments)

(a) Pressure Drop along the Section at 40–50 feet

(b) Entire Pressure Drop

Figure 3.7 Comparison between Measured Pressure Drop and Prediction by Dukler, Wicks & Cleveland (1964) Correlation (1995 Experiments)

(a) Pressure Drop along the Section at 40–50 feet

(b) Entire Pressure Drop

Figure 3.8 Comparison between Measured Pressure Drop and Prediction by the Homogeneous Model (1995 Experiments)

(a) Pressure Drop along the Section at 40–50 feet

(b) Entire Pressure Drop

Figure 3.9 Comparison between Measured Pressure Drop and Prediction by Petalas & Aziz (1996) Model (1995 Experiments)

(a) Pressure Drop along the Section at 40–50 feet

(b) Entire Pressure Drop

Figure 3.10 Comparison between Measured Pressure Drop and Prediction by Xiao, Shoham & Brill (1990) Model (1995 Experiments)

(a) Pressure Drop along the Section at 40–50 feet

(b) Entire Pressure Drop

Figure 3.11 Comparison between Measured Pressure Drop and Prediction by Beggs & Brill Model (1973, revised 1977) (1995 Experiments)

(a) Pressure Drop along the Section at 40–50 feet

(b) Entire Pressure Drop

Figure 3.12 Comparison between Measured Pressure Drop and Prediction by Dukler, Wicks & Cleveland (1964) Correlation (1995 Experiments)

(a) Pressure Drop along the Section at 40–50 feet

(b) Entire Pressure Drop

Figure 3.13 Comparison between Measured Pressure Drop and Prediction by the Homogeneous Model (1995 Experiments)

Figure 3.14 Comparison between Measured Pressure Drop and Prediction by Petalas & Aziz (1996) Model (1996 Experiments)

Figure 3.15 Comparison between Measured Pressure Drop and Prediction by Xiao, Shoham & Brill (1990) Model (1996 Experiments)

Figure 3.16 Comparison between Measured Pressure Drop and Prediction by Beggs & Brill (1973, revised 1977) Model (1996 Experiments)

Figure 3.17 Comparison between Measured Pressure Drop and Prediction by Dukler, Wicks & Cleveland (1964) correlation (1996 Experiments)

Figure 3.18 Comparison between Measured Pressure Drop and Prediction by the Homogeneous Model (1996 Experiments)

Figure 3.19 Comparison between Measured Pressure Drop and Prediction by Petalas & Aziz (1996) Model (1996 Experiments)

Figure 3.20 Comparison between Measured Pressure Drop and Prediction by Xiao, Shoham & Brill (1990) Model (1996 Experiments)

Figure 3.21 Comparison between Measured Pressure Drop and Prediction by Beggs & Brill (1973, revised 1977) Model (1996 Experiments)

Figure 3.22 Comparison between Measured Pressure Drop and Prediction by Dukler, Wicks & Cleveland (1964) Correlation (1996 Experiments)

Figure 3.23 Comparison between Measured Pressure Drop and Prediction by the Homogeneous Model (1996 Experiments)

4. Effects of Pressure Drop in Horizontal Wells and Optimum Well Length

by Raju Penmatcha, Sepehr Arbabi, and Khalid Aziz*

Abstract

As the length of a horizontal well is increased, its contact with the reservoir increases. But at the same time, the resistance to flow in the well also increases, which has a direct negative effect on the productivity of the well. The overall performance of a horizontal well depends on the balance of these two opposing factors. No reliable tools are currently available that account for both these factors in the evaluation of horizontal well performance. A semi-analytical well-model is developed which can quantify the effects of both single phase oil and two-phase oil/gas flow pressure loss in the well on the overall well performance. The model is quite flexible and can incorporate any friction factor correlation. A methodology is developed to show the effects of various reservoir, fluid and well parameters on well productivity. We demonstrate that ignoring frictional effects could lead to unrealistically high production estimates and low breakthrough times for water or gas. As a result of pressure drop in the well, breakthroughs occur first at the heel of the well. A methodology is also developed to calculate the optimum horizontal well length.

4.1 Introduction

Most of the analytical work done in the past¹⁻¹⁶ on horizontal well productivity either assumed that the well is infinitely conductive or the flow is uniform along the entire well length. References 1-6 are based on the assumption of steady state flow in the reservoir, 7-9 are for pseudo steady-state flow conditions and 10-16 are for transient flow. The assumption of uniform flow was made purely for mathematical convenience. The infinite conductivity assumption is a good assumption only in certain cases when the pressure drop in the wellbore is very small compared to the drawdown in the reservoir, otherwise, the pressure drop in the wellbore should also be taken into account.

Previous work can be categorized into three types: (1) Models for wells that are infinitely conductive and thus are not influenced by pressure drop in the well, (2) models where the reservoir is represented by an analytical model for single phase flow, and (3) general models that couple multi-phase flow simulators with wells.

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4.1.1 Type1: Infinitely Conductive Well

A horizontal wellbore can be considered to be infinitely conductive if the pressure drop in the wellbore is so small that ignoring it would not result in significant errors. Analytical equations with this assumption are available in the literature^{5,6,11-15}. Based on the nature of the flow, any of the available equations can be used for quick performance evaluation of horizontal wells in single phase, homogeneous reservoirs. A numerical simulator, on the other hand, can be used for a thorough analysis of complex reservoirs. Using an analytical equation or a numerical simulator with the infinite conductivity well assumption for cases with significant pressure drop in the wellbore can lead to errors in the estimates. Most of the commercial simulators either ignore wellbore pressure drop or use inappropriate models to compute this term.

4.1.2 Type2: Wellbore Coupled to an Analytically Approximated Reservoir

Realizing the importance of wellbore pressure drop, a few groups have developed techniques to evaluate the effects of wellbore friction. This section outlines the available methods that couple the wellbore to an analytically approximated reservoir. The behavior of many homogenous, single phase reservoirs can be approximated analytically. Such analyses, though not too general in their applicability to complex reservoirs, give a good understanding of the nature of the problem. None of the approaches presented in this section take into account multiphase flow, or radial inflow effects in the wellbore.

Dikken¹⁷ was the first author to present an analytical model to couple turbulent flow in the horizontal wellbore to the flow in the reservoir. He showed that in most practical situations, a horizontal well will exhibit non-laminar (transition or turbulent) well flow. He considered steady state, single-phase pressure drop in the well due to laminar and turbulent flow in the wellbore and linked the wellbore to the reservoir using a material balance relationship. For turbulent flow, Blasius' formula was used to compute the friction factor. He solved this problem analytically for an infinite length horizontal well and numerically for a finite length horizontal well. Flow in the reservoir was defined using the PI (productivity index) relationship and he assumed that PI was constant along the well length.

Novy¹⁸ generalized Dikken's work by developing equations that could handle both single phase oil and single phase gas flow. In the case of gas flow, he also included the non-Darcy flow term in his analysis. He used a cylindrical reservoir drainage area, and stated that partial penetration or changing the shape of the drainage volume would not significantly alter his results. The problem was formulated as a boundary value problem and was solved using a finite difference scheme. He showed that for laminar flow in the wellbore, frictional effects reduce production rate by at least 10% when the wellbore pressure drop is more than 15% of the drawdown at the heel of the well. He also developed plots that can be used to determine combinations of well length and production rate that are immune to significant production loss due to frictional effects.

Ozkan et al.¹⁹ developed a model similar to that of Dikken, except they used a transient solution for the reservoir based on Green's function. Their model was presented in terms of dimensionless variables.

Landman²⁰ proposed further improvements to Dikken's model by allowing the PI to change along the wellbore. PI can vary along the well due to variation in perforation density, formation permeability or flow characteristics (radial/spherical flow). Using this model, he developed a methodology to calculate optimum perforation density along the well (defined as the density that yields constant specific inflow along the well). Forcing a constant specific inflow into the well delays breakthrough of gas or water.

4.1.3 Type 3: Wellbore Coupled to a Numerically Approximated Reservoir

In the papers discussed here, the wellbore was coupled to a numerical simulator. Since these works are not analytical it is harder to analyze the specific characteristics of the problem. But the advantage is that the numerical simulator can characterize a complex reservoir with multi-phase flow and hence it is more general.

Stone et al.²¹ applied energy, mass, and momentum balance equations both in the reservoir and the wellbore. The reservoir simulator was a time dependent, thermal, three phase, one dimensional model. In the wellbore, stratified, bubbly, slug and mist flow patterns were considered. Reservoir and the wellbore were coupled implicitly.

Islam et al.²² developed a model for radial influx into the wellbore based on experimental and numerical analyses. They developed mass and momentum balance equations for two-phase flow in the wellbore and coupled these to a numerical simulator. They suggested that bubbles created at the perforations due to radial influx and jet-like flow through the perforations induce additional turbulence in the wellbore.

Folefac et al.²³ concluded that well length, well diameter and perforated interval have the most significant effect on the magnitude of pressure drop in the wellbore. They used a drift flux model in the wellbore and allowed for inter-phase mass transfer in the wellbore using an equation of state. They stated that production logging showed variation of flow rate along the well, which could be due to pressure drop in the wellbore.

Brekke et al.²⁴ developed a network wellbore simulator, called HOSIM, which can calculate pressure drop with up to three phases in the wellbore. Their approach is modular. However, the network model, representing the wellbore, has to be built on a PC before it can be coupled to the reservoir through the Eclipse simulator. For pressure drop in the well they provide a choice of three multi-phase flow models. The reservoir and the wellbore were connected using the PI approach. The PI was allowed to vary with time and location along the well length. They calculated the PI from the simulator runs by slightly perturbing the flow and then internally calculating dQ/dP during every iteration.

None of the published works show how the magnitude of the effect of various reservoir, fluid and well variables on frictional pressure drop in the wellbore and thereby the well productivity. Here we present a methodology that can be used to assess the effect of various parameters on well productivity. Results are presented for the effects of well length, well flow rate, wellbore roughness, drawdown, fluid viscosity and reservoir permeability on well productivity. Many of the existing simulators neglect frictional pressure drop in the wellbore. Our work gives the reservoir engineer guidelines on determining when friction might be important.

With pressure drop in the well, productivity does not continue to increase as the well length is increased. At some point the additional drilling and maintenance costs do not offset additional revenues. So far the literature does not provide a method of calculating the optimum length of horizontal wells. We present a method to calculate the optimum length of horizontal wells. The results can be expressed explicitly for the case of steady-state, single-phase flow in a homogeneous reservoir.

4.2 Effect of Friction on Productivity

Figure 4.1 shows a horizontal well in a reservoir. Let p_e be the pressure at the outer boundary of the reservoir and $p_w(x)$ be the varying pressure along the wellbore due to frictional pressure drop. The well inflow equation can be written as

$$q_s(x) = J_s(x) [p_e - p_w(x)] \quad (4.1)$$

where $q_s(x)$ is the flow into the well per unit length of the wellbore and $J_s(x)$ is the productivity index per unit length of the wellbore. $J_s(x)$ can vary along the wellbore due to variation in perforation density, formation permeability or flow characteristics (spherical vs. radial flow).

Figure 4.1: Reservoir/Wellbore Coupling

For simplicity, we assume that the reservoir is homogeneous and there is steady state, single phase oil flow in the system. The steady state flow assumption can be used in a reservoir where the oil production rate is equal to the water injection rate in a water flood operation. Since the fluid is single phase liquid, we assume that the density variations of the fluid along the

wellbore are negligibly small. We also assume that the penetration ratio of the well in the drainage area is high enough so that we can practically neglect the end effects. This allows us to assume that $J_s(x)$ is constant along the wellbore. Using these assumptions, Eq.(4.1) can be integrated along the entire well to obtain the total flow rate from the well:

$$Q_{w,fric} = \int_0^L q_s(x)dx = \int_0^L J_s [p_e - p_w(x)]dx = J_s p_e L - J_s \int_0^L p_w(x)dx \quad (4.2)$$

If the pressure drop in the wellbore is neglected, then

$$Q_{w,nof} = \int_{x=0}^L q_s(x)dx = \int_0^L J_s [p_e - p_{w,0}]dx = J_s L [p_e - p_{w,0}] \quad (4.3)$$

where $p_{w,0}$ is the pressure at the heel of the well. We define a variable called Productivity Error, P.E., which gives us the error in the well productivity calculations as a result of neglecting frictional pressure drop in the wellbore. P.E. is defined as:

$$P.E. = \frac{Q_{w,nof} - Q_{w,fric}}{Q_{w,nof}} \quad (4.4)$$

Substituting Eq.(4.2) and Eq.(4.3) in Eq.(4.4), we get

$$P.E. = \frac{J_s L (p_e - p_{w,0}) - J_s p_e L + J_s \int_0^L p_w(x)dx}{J_s L (p_e - p_{w,0})} = \frac{\frac{1}{L} \int_0^L p_w(x)dx - p_{w,0}}{(p_e - p_{w,0})} = \frac{\frac{1}{L} \int_0^L [p_w(x) - p_{w,0}]dx}{(p_e - p_{w,0})} \quad (4.5)$$

Hence,

$$P.E. = \frac{\text{average wellbore pressure drop}}{\text{drawdown at the heel}} \quad (4.6)$$

Eq.(4.6) indicates that pressure drop in the wellbore becomes significant only if it is comparable to the drawdown at the heel of the well. We will use this equation in the sensitivity analysis section later where we will show the effect of various parameters on well productivity.

4.3 Flow Rate vs. Well Length

Mass balance gives the following equation for flow in the wellbore:

$$\frac{d}{dx}[q_w(x)] = -q_s(x) \quad (4.7)$$

where $q_w(x)$ is the flow rate in the wellbore and it varies along the length of the wellbore. After some mathematical manipulation (see Appendix A), we can obtain the following relationship to calculate the required well length after the drawdown at the heel and the required well flow rate are specified.

$$L = \int_0^Q \frac{dq_w(x)}{\sqrt{2J_s \int_Q^{q_w} \left[\frac{dp_w}{dx} \right] dq_w(x) + [J_s \Delta p_0]^2}} \quad (4.8)$$

In the above equation, $\frac{dp_w}{dx}$ is the pressure gradient in the wellbore, L is the length of the well, and Δp_0 is the drawdown at the heel of the well. Equation (4.8) is independent of the method of calculating wellbore pressure drop. This gives us the flexibility of using any single phase or multi-phase correlation to obtain the pressure drop in the wellbore. In this study, the pressure drop in the well due to friction is calculated using the following equations:

$$\frac{dp_w(x)}{dx} = 1.079 \times 10^{-4} f \left[\frac{\rho U^2}{D} \right] = 7.294 \times 10^{-12} f \left[\frac{q_w^2 \rho}{\pi^2 D^5} \right] \quad (4.9)$$

where f is Moody's friction factor. The flow is considered laminar for $N_{Re} < 2100$, where N_{Re} is the Reynolds number. For $N_{Re} \geq 2100$, turbulent flow equations are used.

Reynolds number is calculated as,

$$N_{Re} = 0.1231 \frac{q_w \rho}{\mu D} \quad (4.10)$$

For laminar flow,

$$f = \frac{64}{N_{Re}} \quad (4.11)$$

is used, and for turbulent flow Jain's approximation²⁵ of Colebrook-White equation is used. Typically, the flow rate in the well starts out to be laminar at the toe and then as the flow in the well increases towards the heel it slowly becomes turbulent. An equation that accounts for radial influx into the wellbore, for example, can be easily used in this model, as long as the derivative can be expressed as a function of the variable of integration in Eq.(4.8), q_w .

4.4 Sensitivity Analysis

Using the parameters in Table 4.1 and the equations developed so far, the effects of friction on well productivity are illustrated below. All the cases described below are based on steady state flow in the reservoir with single phase oil in the wellbore. The steady-state equations developed by Economides et al.⁶ are used for describing the flow in the reservoir. The productivity index per unit length (J_s) is obtained by dividing the productivity index of the horizontal well with its length. J_s is then assumed constant along the entire horizontal well length.

Table 4.1: Parameters Used in Example Problem

Horizontal Permeability	3000 mD
Vertical Permeability	300 mD
Reservoir Thickness	50 ft
Outer Boundary Pressure	5000 psia
Oil FVF	1.18
Gas FVF	0.0045
Oil Viscosity	1.0 cP
Gas Viscosity	0.02 cP
Oil Density	40.0 lbm/cu.ft
Gas Density	2.0e-04 lbm/cu.ft
Drainage Area	650 acres
Well Radius	3.0 inch
Skin	0.0
Wellbore Roughness	0.0 ft

4.4.1 Effect of Well Length

Figure 4.2 shows the effect of flow rate and well length on well friction. The solid lines in the plot show the variation of oil production rate with well length for three different drawdowns. The drawdowns are low because of the high permeability of the reservoir. The concave shape of these lines shows the decrease in incremental production with well length. For example, for a 5 psi drawdown the flow rates of 8000 STB/day can never be reached with any well length. If friction were neglected in the wellbore, then the well productivity continues to increase with well length.

This figure also shows lines of constant P.E.. For a given drawdown at the heel of the well (denominator in the RHS of Eq.(4.6) held constant), as the well length increases, pressure drop in the wellbore increases due to the increase in the flow rate. Hence, the longer the well, the larger is the ratio of pressure drops for a given drawdown, and hence the higher the P.E. from Eq.(4.6). As expected, frictional effects become more important as the horizontal well length increases.

Figure 4.2: Effect of Flow Rate and Well Length on Wellbore Friction

4.4.2 Effect of Flow Rate

As discussed, the pressure drop in the wellbore can increase faster than that in the reservoir. The flow rate from a given horizontal well in a reservoir can be altered by changing the drawdown. Keeping all parameters constant except the flow rate from the well and the drawdown at the heel of the well we can show some interesting results.

Assuming for flow in the reservoir to be laminar, from Eq.(4.3),

$$\Delta p_0 \propto Q_{nof} \quad (4.12)$$

Blasius' formula shows that for a smooth pipe with turbulent flow,

$$f = CN_{Re}^{-0.25} \quad (4.13)$$

Combining Eq.(4.13) with Eqs.(9) and (10), we can show that

$$\Delta p_{turbulent} \propto q_w^{1.75} \quad (4.14)$$

Now consider the case when the drawdown at the heel is increased by lowering the pressure at the heel of the well, $p_{w,0}$. Based on Eq.(4.12), Q_{nof} increases linearly due to an increase in drawdown. Let us consider the increase in Q_{fric} due to the increase in drawdown. The increase in drawdown increases the flow rate $q_w(x)$ along the well. And because of turbulent flow in the wellbore pressure drop in the wellbore increases at a higher rate than the drawdown at the heel. Following Eq.(4.2), this causes Q_{fric} to increase at a sub-linear rate with an increase in the drawdown at the heel. Hence, using Eq.(4.4), an increase in drawdown at the heel increases the numerator of Eq.(4.4) super-linearly while the denominator is increasing linearly. This causes the P.E. to increase with an increase in drawdown. Since an increase in drawdown increases the flow rate from the well we can also say that an increase in the well flow rate causes the P.E. to increase and vice-versa.

This result can be illustrated using Figure 4.2. Let us assume that the well length is fixed at 3000 ft. The flow rate from this well can be increased by increasing the drawdown. When the drawdown is 5 psi, the flow rate is about 2600 STB/day and the P.E. is below 5%. By increasing the drawdown to 15 psi, the flow rate can be increased to 7500 STB/day. But the P.E. has also increased and it now lies between 5% and 10%. The P.E. goes above 10% when the drawdown is increased to 25 psi. So for a fixed well length, the higher the flow rate, the more important the frictional pressure drop in the wellbore. This suggests that frictional effects will be higher for a well during its early life when the flow rate is the highest.

4.4.3 Effect of Wellbore Roughness

The effect of wellbore roughness is shown in Figure 4.3. The dashed lines in this figure are calculated using the parameters in Table 4.1. The solid lines are drawn for the case when the wellbore relative roughness is increased from 0.0 to 0.1 (such a high value of effective roughness may be caused by open hole completion or due to fluid influx into the well) and keeping everything else the same as in Table 4.1. Drawdown is 25 psi.

For a well of length 3000 ft, increasing the wellbore roughness does two things. It decreases the production rate and increases the P.E.. The increase in P.E. in this case is from approximately 11% to 26%. It can also be observed that for a fixed production rate, the required well length increases with the increase of wellbore friction.

Figure 4.3: Effect of Well Roughness on Wellbore Friction (Drawdown=25 psi)

4.4.4 Effect of Fluid Viscosity

The flow in the reservoir is almost always laminar, while in the wellbore the flow can be turbulent. Because of this, an increase in fluid viscosity causes the pressure drop in the wellbore to increase less than the increase in the reservoir. This gives the interesting result that the P.E. can be lower for higher viscosity fluids. This effect is illustrated in Figure 4.4. The dashed lines are drawn using the parameters in Table 4.1 with 15 psi drawdown. The bold lines are drawn by increasing the viscosity in Table 4.1 to 10 cP and drawdown to 150 psi. As shown in Figure 4.4, for producing 8000 STB/day, the required horizontal well lengths for the two cases shown are not very different. But the P.E. for the case with 10 cP viscosity is less than 2% while that for the case with 1 cP is much higher at 10%.

Figure 4.4: Effect of Oil Viscosity on Wellbore Friction

4.4.5 Effect of Reservoir Permeability

The effect of reservoir permeability is shown in Figure 4.5. The dashed lines are drawn using the parameters in Table 4.1 for single phase oil flow with 15 psi drawdown. The bold lines are drawn by decreasing the horizontal permeability from 3000 mD to 300 mD and increasing the drawdown to 150 psi. The vertical to horizontal permeability ratio was kept constant at 0.1 in both these cases. This figure clearly indicates that the P.E. increases with increasing reservoir permeability.

Figure 4.5: Effect of Reservoir Permeability on Wellbore Friction

4.5 Two-Phase Flow in the Wellbore

The frictional effects in the wellbore become more pronounced when there are two-phases flowing. The effect of two-phase flow is illustrated by looking at a case where the GOR is 1000 SCF/STB. The gas present is free gas and the GOR is assumed to be constant over the length of the horizontal well. Figure 4.6 shows a comparison of four different cases with a 15 psi drawdown in the reservoir. The four cases considered are: a) single phase oil flowing in the wellbore (the presence of gas is neglected), b) two-phase flow wellbore pressure drop computed by assuming no-slip homogeneous flow, c) two-phase flow wellbore pressure drop calculated by the Beggs & Brill²⁶ model and, d) friction is neglected in the wellbore. Let us look, for example, at the case when the oil rate is 8000 STB/day. If friction is not considered in the wellbore, the calculations show that a horizontal well of length 3000 ft is sufficient to produce this amount. But when frictional effects are considered, a 4500 ft long well is required. Since friction is inevitable, a 3000 ft well would produce only 6700 STB/day. As the graph shows, frictional

effects get worse when there are two phases flowing in the wellbore. The plot also shows that the P.E. is more when the Beggs & Brill correlation is used than when a homogeneous flow model is used. If a new model that gives appropriate consideration to radial influx into the wellbore is used, then the pressure drop in the well would be higher.

Figure 4.6: Effects of Two-Phase Flow in the Well on Wellbore Friction

4.6 Frictional Effects on Water/Gas Breakthrough

Figure 4.7 shows the effects of friction on breakthrough tendencies at the heel of the well. In this figure, the inflow from the reservoir into the well is shown for a 3000 ft horizontal well. The inflow at the heel of the well is taken as a basis and the inflow at other well locations is shown as a fraction of the inflow at the heel. As the figure depicts, when the frictional effects are insignificant, the inflow into the well is uniform along the entire well length. But as frictional effects increase with increasing pipe roughness, the fluid inflow is highest at the heel and decreases towards the toe. The message from this figure is that ignoring frictional effects, when they are significant, can lead to under-estimation of breakthrough times for water and gas.

Figure 4.7: Early Breakthrough at the Heel of the Well due to Wellbore Friction

4.7 Optimum Well Length

Optimum well length is defined here as the length of the horizontal well in a given reservoir that gives maximum revenues, with consideration for frictional pressure drop in the wellbore. Because of the frictional effects, the incremental recovery of oil decreases with increasing well length. But the drilling and completions costs increase almost linearly as the well length is increased. Because of this, there will be a certain length of the well which maximizes revenues.

The total income from a well is given as a product of oil price and flow rate produced from the well.

$$\text{Total Income} = \sum_{i=1}^t \$_{oil}^i Q_{w,fric}^i \frac{1}{(1+I)^i} \quad (4.15)$$

$Q_{w,fric}^i$ is the production rate from the well in STB/day and $\$_{oil}^i$ is the oil price in \$/STB at time i . I is the interest rate and t is the production time in days.

Total expenses for the well for this simplified case are the sum of the cost of drilling and completing the well and the operating cost.

$$\text{Total Expenses} = C_F + C_H L + C_M t \quad (4.16)$$

where C_F is the fixed cost to drill vertical, C_H is the cost to drill one foot of horizontal well, and C_M is the operating cost per day of operation.

Net Revenue, R = Total Income - Total Expenses.

$$R = \left[\sum_{i=1}^t \$_{oil}^i Q_{w,fric}^i \frac{1}{(1+I)^i} \right] - C_F - C_H L - C_M t \quad (4.17)$$

For the sake of simplicity, we assume that the flow is at steady state so that the flow rate remains constant with time and we also assume that oil price is constant in the frame work of our time. Then Eq.(4.17) becomes,

$$R = \$_{oil} Q_{w,fric} F(t, I) - C_F - C_H L - C_M t \quad (4.18)$$

where

$$F(t, I) = \sum_{i=1}^t \frac{1}{(1+I)^i} \quad (4.19)$$

Under the special case when interest rate $I = 0$, $F(t, I)$ will be equal to t .

Dividing both sides by C_F , we get:

$$\frac{R}{C_F} = \frac{\$_{oil} Q_{w,fric} F(t, I)}{C_F} - 1 - \frac{C_H L}{C_F} - \frac{C_M t}{C_F} \quad (4.20)$$

The above equation can be non-dimensionalized using the following definitions:

$$\text{Dimensionless net revenue, } R_D = \frac{R}{C_F}$$

$$\text{Dimensionless production rate, } Q_D = \frac{\$_{oil} Q_{w,fric} F(t, I)}{C_M t}$$

Dimensionless well length, $L_D = \frac{C_H L}{C_F}$

Dimensionless production time, $t_D = \frac{C_M t}{C_F}$

With these definitions Eq.(4.20) becomes:

$$R_D = Q_D t_D - 1 - L_D - t_D \quad (4.21)$$

Taking the derivative with respect to L_D on both sides and setting $\frac{dR_D}{dL_D} = 0$ leads to,

$$t_D \frac{dQ_D}{dL_D} = 1 \quad (4.22)$$

Using the above equation, we can obtain the optimum length of a horizontal well for the cases of laminar and turbulent flow in the wellbore.

With laminar flow in the wellbore, the optimum well length is given by the following equation: (see Appendix B for details)

$$L_{opt} = \frac{1}{\sqrt{J_s R_w}} \cosh^{-1} \left(\sqrt{\frac{\$_{oil} J_s \Delta p_0 F(t, I)}{C_H}} \right) \quad (4.23)$$

where R_w is the resistance in the wellbore. Eq.(4.23) shows that the optimum well length decreases as flow resistance in the wellbore increases. Also the optimum well length increases as the oil price increases, because with a higher oil price an additional barrel of oil produced due to a longer well is worth more.

When there is turbulent flow in the wellbore, the optimum well length is calculated using the following relationship:

$$\frac{dQ}{dL} = \left[\frac{1}{J_s \Delta p_0} + J_s \frac{dp_w}{dx}(Q) M' \right]^{-1} \quad (4.24)$$

where,

$$M' = \int_0^Q \frac{1}{\left[2J_s \int_Q^q \frac{dp_w}{dx} d\epsilon + [J_s \Delta p_0]^2 \right]^{\frac{3}{2}}} dq_w \quad (4.25)$$

Using Eq.(4.24) and Eq.(4.22), we can calculate the flow rate from the well with optimum length. L_{opt} can then be calculated using this flow rate in Eq.(4.8).

Results of an example problem to show the effects of friction on the optimum well length are shown in Figure 4.8. This figure shows that when friction is neglected, the production from the well increases at a constant rate with an increase in the well length. But when turbulent flow in the wellbore is considered, then after a certain well length revenues start to drop. The optimum well length in this example is about 1600 ft.

Figure 4.8: Optimum Well Length Calculation

The radius of the well can also be optimized along with its length. The costs involved in changing the radius of the well are not very direct. We need to account for pipe costs, drilling costs, mud costs and various others. If all those parameters are available, then a similar analysis can also yield the optimum well radius.

4.8 Future Work

In the future, we plan to extend this model to transient flow in the reservoir. We also plan to couple our model to a three phase, three dimensional numerical simulator and develop a procedure for determining optimum well length for more realistic problems.

4.9 Conclusions

1. A flexible reservoir/wellbore coupling model was developed to account for frictional pressure drop in horizontal wells.
2. The ratio of wellbore pressure drop to reservoir drawdown gives a good indication of the frictional effects on well productivity.
3. Ignoring pressure drop in the wellbore can give higher estimates for well productivity and later estimates for breakthrough time.
4. Errors in productivity calculations due to ignoring frictional effects increase with increasing well length, flow rates, wellbore roughness and reservoir permeabilities; and they decrease with higher drawdown or higher fluid viscosities.
5. A methodology to calculate the optimum well length for horizontal wells was also developed.

Acknowledgments

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Nomenclature

C_F	Fixed cost for drilling & completion of the vertical part of the well, \$
C_H	Cost per unit length for drilling & completion of the horizontal part of the well, \$/ft
C_M	Operating cost per day of production, \$/day
D	Diameter of the well, ft
f	Moody friction factor
F	A specified function
I	Annual interest rate
J_s	Productivity index per unit length of the well, rbbl/day/psi/ft
k	Reservoir permeability, mD
L	Horizontal well length, ft
p	pressure, psi
P.E.	Productivity error

$\$_{oil}$	Oil price per barrel, \$/STB
q_s	Inflow into the well per unit length of the well
Q	Oil production rate, rbbl/day
q_w	Flow in the wellbore, rbbl/day
R	Net revenue, \$
R_w	Resistance in the wellbore, rbbl/day
t	Production time, day
U	Superficial oil flow rate in the well, cu.ft/sec
x	Distance along the wellbore coordinate, ft

Greek Letters

ρ	Oil density, lbm/cu.ft
Δp_0	Drawdown at the heel of the well, psi

Subscripts

H	Horizontal
V	Vertical
w	Well
e	External boundary
s	Specific
fric	Frictional effects considered
nof	Frictional effects not considered
opt	Optimum

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Appendix A:

Flow Rate vs. Well Length

Mass balance gives the following equation for flow in the wellbore:

$$\frac{d}{dx} q_w(x) = -q_s(x) \quad (4.A1)$$

where $q_w(x)$ is the flow rate in the wellbore and it varies along the length of the well. Combining Eq.(4.1) and Eq.(4.A1), we have

$$\frac{d}{dx} q_w(x) = -J_s [p_e - p_w(x)] \quad (4.A2)$$

Differentiating Eq.(4.A2), with constant J_s and p_e , results in

$$\frac{d^2 q_w(x)}{dx^2} = J_s \frac{dp_w(x)}{dx} \quad (4.A3)$$

Multiplying both sides by $\frac{dq_w}{dx}$, as Landman²⁰ did, we get

$$\frac{dq_w(x)}{dx} \frac{d}{dx} \left[\frac{dq_w(x)}{dx} \right] = J_s \frac{dp_w(x)}{dx} \frac{dq_w(x)}{dx} \quad (4.A4)$$

$$\frac{1}{2} \frac{d}{dx} \left[\frac{dq_w}{dx} \frac{dq_w}{dx} \right] = J_s \frac{dp_w(x)}{dx} \frac{dq_w(x)}{dx} \quad (4.A5)$$

$$d \left[\left(\frac{dq_w}{dx} \right)^2 \right] = 2 J_s \frac{dp_w(x)}{dx} dq_w(x) \quad (4.A6)$$

Integrating the above equation, we obtain

$$\int_{q_w=Q}^{q_w=q_w} d \left[\left(\frac{dq_w}{dx} \right)^2 \right] = 2 J_s \int_{q_w=Q}^{q_w=q_w} \frac{dp_w(x)}{dx} dq_w(x) \quad (4.A7)$$

$$\left[\frac{dq_w}{dx} \right]_{q_w}^2 - \left[\frac{dq_w}{dx} \right]_{q_w=Q}^2 = 2 J_s \int_Q^{q_w} \frac{dp_w(x)}{dx} dq_w(x) \quad (4.A8)$$

Using boundary condition I, defined as:

$$\begin{aligned} \left[\frac{dq_w}{dx} \right]_{x=0} &= \left[\frac{dq_w}{dx} \right]_{q_w=Q} = -J_s [p_e - p_{w,0}] \\ &= -J_s \Delta p_0 \end{aligned} \quad (4.A9)$$

Eq.(4.A8) becomes

$$\left[\frac{dq_w}{dx} \right]^2 - [-J_s \Delta p_0]^2 = 2J_s \int_Q^{q_w} \frac{dp_w(x)}{dx} dq_w(x) \quad (4.A10)$$

$$\frac{dq_w}{dx} = -\sqrt{2J_s \int_Q^{q_w} \frac{dp_w(x)}{dx} dq_w(x) + [J_s \Delta p_0]^2} \quad (4.A11)$$

The choice of negative for the square root in the above equation comes from the fact that q_w increases as x decreases. Using boundary condition II, defined as

$$q_{w,x=L} = 0 \quad (4.A12)$$

yields,

$$\int_{x=x}^{x=L} dx = - \int_{q_w=q_w}^{q_w=0} \frac{dq_w(x)}{\sqrt{2J_s \int_Q^{q_w} \left[\frac{dp_w}{dx} \right] dq_w(x) + [J_s \Delta p_0]^2}} \quad (4.A13)$$

$$L - x = \int_0^{q_w(x)} \frac{dq_w(x)}{\sqrt{2J_s \int_Q^{q_w} \left[\frac{dp_w}{dx} \right] dq_w(x) + [J_s \Delta p_0]^2}} \quad (4.A14)$$

Since at the heel, $q_w(x=0) = Q$, the above equation can be expressed as,

$$L = \int_0^Q \frac{dq_w(x)}{\sqrt{2J_s \int_Q^{q_w} \left[\frac{dp_w}{dx} \right] dq_w(x) + [J_s \Delta p_0]^2}} \quad (4.A15)$$

Appendix B: Optimum Well Length

Laminar Flow in the Wellbore:

For this case, Dikken derived an expression for the flow rate from the well:

$$Q_{w,fric} = \sqrt{\frac{J_s}{R_w}} \tanh(L\sqrt{J_s R_w}) \Delta p_0 \quad (4.B1)$$

where R_w is the resistance to flow in the wellbore.

For a smooth pipe, R_w was defined as,

$$R_w = 2.921 \times 10^{-12} \left(\frac{\mu D}{\rho} \right)^{0.25} \frac{\rho}{\pi^{1.75} D^5} \quad (4.B2)$$

Assuming that J_s is a weak function of L , we get

$$\frac{dQ_{w,fric}}{dL} = \sqrt{\frac{J_s}{R_w}} \sqrt{J_s R_w} \operatorname{sech}^2(L\sqrt{J_s R_w}) \Delta p_0 \quad (4.B3)$$

From the definition of dimensionless variables, we have

$$\frac{dQ_D}{dL_D} t_D = \frac{dQ_{w,fric}}{dL} t \frac{\$_{oil} F(t, I)}{C_M t} \frac{C_M}{C_F} \frac{C_F}{C_H} = \frac{dQ_{w,fric}}{dL} \frac{\$_{oil} F(t, I)}{C_H} \quad (4.B4)$$

Substituting the above equation in Eq.(4.22), the optimum well length is obtained,

$$L_{opt} = \frac{1}{\sqrt{J_s R_w}} \cosh^{-1} \left(\sqrt{\frac{\$_{oil} J_s \Delta p_0 F(t, I)}{C_H}} \right) \quad (4.B5)$$

Turbulent Flow in the wellbore:

Eq.(4.A15) defines the flow in the system when there is turbulent flow in the wellbore. We need the derivative of Eq.(4.A15) with respect to L to calculate the optimum well length. We use Leibnitz's rule to accomplish this. Leibnitz's rule states:

$$\text{If } I(\alpha) = \int_{a(\alpha)}^{b(\alpha)} f(x, \alpha) dx, \text{ then}$$

$$\frac{dI}{d\alpha} = I'(\alpha) = \int_{a(\alpha)}^{b(\alpha)} \frac{\partial f(x, \alpha)}{\partial \alpha} dx + f[b(\alpha), \alpha] \frac{db}{d\alpha} - f[a(\alpha), \alpha] \frac{da}{d\alpha} \quad (4.B6)$$

Taking the derivative of Eq.(4.A15) with respect to L, we get

$$1 - \int_0^Q \frac{\partial}{\partial L} \left[\frac{1}{\sqrt{2J_s \int_Q^{q_w} \frac{dp_w}{dx} d\epsilon + [J_s \Delta p_0]^2}} \right] dq_w(x) - \frac{1}{[J_s \Delta p_0]} \frac{dQ}{dL} = 0 \quad (4.B7)$$

In the above equation, ϵ is the variable of integration.

If we set $M = \frac{\partial}{\partial L} \left[\frac{1}{\sqrt{2J_s \int_Q^{q_w} \frac{dp_w}{dx} d\epsilon + [J_s \Delta p_0]^2}} \right]$, and perform the differentiation using the

Leibnitz's rule on the integral, we obtain

$$\begin{aligned} M &= \frac{\partial}{\partial L} \left[\frac{1}{\sqrt{2J_s \int_Q^{q_w} \frac{dp_w}{dx} d\epsilon + [J_s \Delta p_0]^2}} \right] \\ &= -\frac{1}{2} \frac{1}{\left[2J_s \int_Q^{q_w} \frac{dp_w}{dx} d\epsilon + [J_s \Delta p_0]^2 \right]^{\frac{3}{2}}} * \left(2J_s \left[\int_Q^{q_w} \frac{\partial}{\partial L} \left[\frac{dp_w}{dx} \right] d\epsilon + \frac{dp_w}{dx}(q_w) \frac{dq_w}{dL} - \frac{dp_w}{dx}(Q) \frac{dQ}{dL} \right] \right) \quad (4.B8) \\ &\quad + 0 \end{aligned}$$

In the above equation, we can write

$$\frac{\partial}{\partial L} \left[\frac{dp_w}{dx} \right] = \frac{\partial}{\partial \epsilon} \left[\frac{dp_w}{dx} \right] \frac{\partial \epsilon}{\partial L} \quad (4.B9)$$

Since q_w and ϵ are variables of integration,

$$\frac{dq_w}{dL} = \frac{\partial \epsilon}{\partial L} = 0 \quad (4.B10)$$

Using above relations in Eq.(4.B8), we obtain

$$\frac{dQ}{dL} \left[\frac{1}{J_s \Delta p_0} + J_s \frac{dp_w}{dx} (Q) M' \right] = 1 \quad (4.B11)$$

where,

$$M' = \int_0^Q \frac{1}{\left[2J_s \int_Q^{q_w} \frac{dp_w}{dx} d\epsilon + [J_s \Delta p_0]^2 \right]^{\frac{3}{2}}} dq_w \quad (4.B12)$$

Which implies that,

$$\frac{dQ}{dL} = \left[\frac{1}{J_s \Delta p_0} + J_s \frac{dp_w}{dx} (Q) M' \right]^{-1} \quad (4.B13)$$

Using the above equation along with Eq.(4.22), we can calculate the flow rate from the well with optimum length. L_{opt} can then be determined using this flow rate in Eq.(4.A15).

SI Metric Conversion Factors

bbl x 1.589873 E -01	= m ³
cP x 1.0 E + 00	= mPa.s
ft x 3.048 E -01	= m
lbm x 4.535924 E - 01	= kg
psi x 6.894757 E + 00	= kPa

5. Effects of Wellbore Friction Under Transient Conditions

by Raju Penmatcha* and Khalid Aziz

Abstract

We have developed an analytical solution to describe the three dimensional, transient flow in a reservoir with horizontal wells that have infinitely conductive wellbores. A methodology is also developed that couples frictional pressure drop in the wellbore to the analytical solution. Using this model, we can evaluate the effects of wellbore friction on horizontal well productivity under transient flow conditions.

5.1 Introduction

In the preceding Chapter, we demonstrated the effects of friction in the wellbore when there is steady state flow both in the reservoir and the wellbore. In this section, we develop an analytical solution that can be used to study frictional effects under transient conditions. We develop a methodology to analytically describe three dimensional flow in a reservoir with horizontal wells that have infinitely conductive wellbores. The base solution that we use is the one derived by Babu & Odeh [1] for horizontal well productivity calculations in a box shaped reservoir. This model assumes that the reservoir is homogeneous (can be anisotropic), single phase oil flow, all the wells are horizontal wells in the x or y direction, and the reservoir is box shaped with no-flux boundaries. Babu & Odeh's solution assumes that the flux along the well is uniform. Since uniform flux is not always possible, they make this assumption purely for mathematical convenience. We derive a solution for an infinitely conductive well using their base solution. The principle of super-position in space and time is used to arrive at this solution.

5.2 Babu & Odeh's Solution for Horizontal Well Productivity

Babu & Odeh [1] have developed a general solution for the productivity of a horizontal well in a box-shaped drainage volume. As shown in Figure 5.1, the reservoir contains a horizontal well of radius r_w and length L . All the six external boundaries of the drainage volume are of no-flow type. The drainage volume has a height h , length a and width b . As shown in the figure, the well extends from (x_0, y_1, z_0) to (x_0, y_2, z_0) and is parallel to the y-axis. The well can be placed anywhere in the drainage volume but must be parallel to one of the co-ordinate axes and it can be of any length ($L \leq b$). Reservoir should be homogeneous but can be anisotropic. Porosity ϕ is constant. k_x , k_y , and k_z are the reservoir permeabilities in x, y and z directions. There should be single phase, slightly compressible fluid in the reservoir. Initially at time $t=0$, the pressure is uniform throughout the reservoir and is equal to p_i . At time $t = 0^+$, fluid is withdrawn from the well at a constant rate of q .

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*The analytical solution developed in this work is a result of our discussions with D.K. Babu, Mobil

Under these circumstances, they have developed a solution for pressure at any time ($t > 0$), or at any position ($0 \leq x \leq a, 0 \leq y \leq b, 0 \leq z \leq h$) in the system. More details about this model can be found in their paper [1].

Figure 5.1: Horizontal well in a box-shaped reservoir (Babu and Odeh [1])

The flux along the well is assumed to be uniform in the model described above. But the flux can be uniform only if the well is fully penetrating resulting in 2D type of flow. When the well is partially penetrating, the pressure support from the reservoir at the two ends of the well would result in higher flux at the ends of the well as compared to the flux at the center of the well. For the cases where the pressure drop in the wellbore is not very significant compared to the drawdown, an infinitely conductive wellbore is a far better assumption than a uniform flux wellbore. In this section, we develop a solution for this case using Duhamel's principle and the uniform flux solution of Babu & Odeh.

The uniform flux solution by Babu & Odeh is given as:

$$\Delta p \equiv p_i - p(x, y, z, t) = \left[\frac{886.9 B \mu q}{a b h L \alpha} \right] \int_0^t \int_{y_1}^{y_2} (S_1 \cdot S_2 \cdot S_3) dy_0 d\tau \quad (5.1)$$

where S_1 , S_2 , and S_3 are instantaneous point sink functions (Green's functions) located at (x_0, y_0, z_0) and satisfying no-flow boundary conditions at $x=0, a$; $y=0, b$; and $z=0, h$. x_0 , y_0 , and z_0 are the x, y and z coordinates at the center of the well. S_1 , S_2 , and S_3 are defined as,

$$S_1 \equiv S_1(x, x_0, \tau) \equiv 1 + 2 \sum_{n=1}^{\infty} \cos \frac{n\pi x}{a} \cos \frac{n\pi x_0}{a} \exp \left[-\frac{n^2 \pi^2 k_x \tau}{\alpha a^2} \right] \quad (5.2)$$

$$S_2 \equiv S_2(y, y_0, \tau) \equiv 1 + 2 \sum_{m=1}^{\infty} \cos \frac{m\pi y}{b} \cos \frac{m\pi y_0}{b} \exp \left[-\frac{m^2 \pi^2 k_y \tau}{\alpha b^2} \right] \quad (5.3)$$

$$S_3 \equiv S_3(z, z_0, \tau) \equiv 1 + 2 \sum_{l=1}^{\infty} \cos \frac{l\pi z}{h} \cos \frac{l\pi z_0}{h} \exp \left[-\frac{l^2 \pi^2 k_z \tau}{\alpha h^2} \right] \quad (5.4)$$

where $\alpha = 157.952 \phi \mu c_t$, days

They carried out the integration to obtain the pressure drop at any point (x,y,z) in the reservoir at time t.

$$\Delta p = (p_i - p(x, y, z, t)) = \left[\frac{886.9 B q \mu}{a b h \alpha} \right] [t + other terms] \quad (5.5)$$

The missing terms in the above equation can be found in Ref.[1]. Eq. 5.5 can also be written as,

$$\Delta p = (p_i - p) = q \cdot F(t) \quad (5.6)$$

5.3 Analytical Solution: Infinite Conductive Well

In this section we develop an analytical solution, for a horizontal well with single or multi-laterals placed anywhere in a box-shaped reservoir with sealed boundaries. We use the principle of super-position in space and time to accomplish this task. Let us divide the original well into n segments. Well segment 1 is assumed to be closest to the heel of the well, while segment n is closest to the toe of the well. We can now apply the uniform flux solution for each well segment as given by Eq. 5.6. The interaction between each segment can be taken into account by superposition of pressures in space. This type of superposition is possible because of the linearity of the flow in the porous media. Let the mid-point of every segment be the pressure node for that segment. At time Δt , we can write:

$$\Delta p_1 = q_1 F_{1,1}(\Delta t) + q_2 F_{1,2}(\Delta t) + \dots + q_n F_{1,n}(\Delta t) \quad (5.7)$$

where $\Delta p_1 = (p_i - p(x_0, y_{m1}, z_0))$ and it gives the pressure drop at the pressure node of well segment 1 due to the flow from itself and all other well segments. The term $F_{m,n}$ in the above equation stands for the effect of well segment n on well segment m. We can write similar equations for all the other segments. For well segment m, for example,

$$\Delta p_m = q_1 F_{m,1}(\Delta t) + q_2 F_{m,2}(\Delta t) + \dots + q_n F_{m,n}(\Delta t) \quad (5.8)$$

The unknowns in this problem are 'n' q's at the nodes of the n segments. We need 'n' equations to get a closed form for the system of equations. By taking the pressure difference between node 1 and every other node we can form (n-1) equations. For example, the pressure difference between node 1 and node m is given as:

$$\begin{aligned} \Delta p_1 - \Delta p_m = p_m - p_1 = & q_1 [F_{1,1}(\Delta t) - F_{m,1}(\Delta t)] + q_2 [F_{1,2}(\Delta t) - F_{m,2}(\Delta t)] \\ & + \dots + q_n [F_{1,n}(\Delta t) - F_{m,n}(\Delta t)] \end{aligned} \quad (5.9)$$

Because of the infinite conductivity assumption, pressure is constant in the wellbore. Hence the term on the left hand side of Eq. 5.9 is equal to zero. At this point there is only one more equation needed to close the system of equations. Depending on the well constraint it can be either total flow rate specified or BHP specified.

If the total rate q_T is specified, then the last equation will be

$$q_T = q_1 + q_2 + \dots + q_n \quad (5.10)$$

If the BHP pressure is specified, then the last equation will be

$$\Delta p_1 = p_i - p_1 = q_1 F_{1,1}(\Delta t) + q_2 F_{1,2}(\Delta t) + \dots + q_n F_{1,n}(\Delta t) \quad (5.11)$$

where p_1 is the pressure at the node of segment 1 and is equal to the specified BHP.

Solving the system of n equations given above, we can calculate the q 's from each node for the first time step. Superposition in space was applied in the above set of equations. Now we need to apply superposition in time, to march our solution through time. It is done next.

Let us assume that all time steps are of equal length and are equal to Δt . Based on that, when $t = 3\Delta t$ (at the end of third time step), we have

For well segment 1:

$$\begin{aligned} \Delta p_1 = & q_1 F_{1,1}(3\Delta t) + \Delta_2 q_1 F_{1,1}(2\Delta t) + \Delta_3 q_1 F_{1,1}(\Delta t) \\ & + q_2 F_{1,2}(3\Delta t) + \Delta_2 q_2 F_{1,2}(2\Delta t) + \Delta_3 q_2 F_{1,2}(\Delta t) \\ & + \dots + q_n F_{1,n}(3\Delta t) + \Delta_2 q_n F_{1,n}(2\Delta t) + \Delta_3 q_n F_{1,n}(\Delta t) \end{aligned} \quad (5.12)$$

For well segment 2:

$$\begin{aligned} \Delta p_2 = & q_1 F_{2,1}(3\Delta t) + \Delta_2 q_1 F_{2,1}(2\Delta t) + \Delta_3 q_1 F_{2,1}(\Delta t) \\ & + q_2 F_{2,2}(3\Delta t) + \Delta_2 q_2 F_{2,2}(2\Delta t) + \Delta_3 q_2 F_{2,2}(\Delta t) \\ & + \dots + q_n F_{2,n}(3\Delta t) + \Delta_2 q_n F_{2,n}(2\Delta t) + \Delta_3 q_n F_{2,n}(\Delta t) \end{aligned} \quad (5.13)$$

and for well segment n :

$$\begin{aligned} \Delta p_n = & q_1 F_{n,1}(3\Delta t) + \Delta_2 q_1 F_{n,1}(2\Delta t) + \Delta_3 q_1 F_{n,1}(\Delta t) \\ & + q_2 F_{n,2}(3\Delta t) + \Delta_2 q_2 F_{n,2}(2\Delta t) + \Delta_3 q_2 F_{n,2}(\Delta t) \\ & + \dots + q_n F_{n,n}(3\Delta t) + \Delta_2 q_n F_{n,n}(2\Delta t) + \Delta_3 q_n F_{n,n}(\Delta t) \end{aligned} \quad (5.14)$$

In the above array of equations, q_1 is the flow rate from the 1st segment at $t = \Delta t$; $\Delta_2 q_1 = q_1(2\Delta t) - q_1(\Delta t)$ and $\Delta_3 q_1 = q_1(3\Delta t) - q_1(2\Delta t)$.

The last equation for this set is obtained based on type of well constraint that we have at this time.

Solving the above set of equations, as we did for the first time step, we can solve for the flow rate distribution along all the well divisions at the third time step. This procedure can be repeated for all the time steps.

5.4 Frictional Pressure Drop in the Wellbore

Currently we are working on coupling our wellbore friction models to the analytical solution presented above. This would let us analyze the effects of wellbore friction

on horizontal well productivity under transient conditions. One way of accomplishing the wellbore/reservoir coupling is as follows:

1. At the end of each time step, using the analytical solution developed in the previous section, we calculate the productivity index per unit length of the well (J_s) for each well section.
2. For the first time step, we assume that the pressure in the wellbore is same every where. At the end of this step, we need to evaluate the pressure and flow rate at each of the n well sections. For this we need $2n$ equations.
3. By using an appropriate form of Eq. 4.8 from the previous chapter, we can form n equations at the n nodes. By equating Eqs. 4.A2 and 4.A11 from the Appendix of the previous chapter, we can form $(n-1)$ equations that relate the pressure difference between two adjacent nodes to the wellbore pressure drop. For the first time step, the pressure difference between two adjacent nodes will be zero. For the subsequent time steps, the pressure difference between two adjacent nodes will be the pressure difference between these nodes as calculated at the end of previous time step.
4. The final equation required to solve the system of equations is supplied by the well constraint which is either flow rate or BHP specification.
5. The pressure calculated at each node at the end of each time step is kept constant through out the duration of the next time step. They are updated only at the end of the next time step. So this is explicit coupling. At this point we assume that the errors introduced by explicit coupling of the wellbore and the reservoir will be small. We also plan to do implicit coupling so that we can compare the differences between these two methods.

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6. Effects of Grid Systems on Predicting Horizontal Well Productivity

by Jing Wan, Raju Penmatcha, Sepehr Arbabi, and Khalid Aziz

Abstract

This study compares horizontal well performance simulated by a uniform coarse grid, a uniform fine grid and a non-uniform fine grid using the Peaceman well model. An analytical solution is also used to compare with the numerical solutions. We find that when using a coarse grid with conventional well models, the flow rate of the well is underpredicted, which means the well index calculated by Peaceman formula needs to be increased for the numerical simulator to predict the well productivity correctly. Our analysis shows that the explicit modeling of wells, as done in a recent paper, can lead to incorrect results.

6.1 Introduction

When solving the flow equations of a reservoir numerically, the block pressure cannot be assumed to be equal to the wellbore pressure. These two pressures can be related by a numerical, semi-analytical or analytical well model, which gives the appropriate well index. Peaceman [1] has derived expressions for well index under the condition of 2D flow from an isolated well under steady state or pseudo-steady state condition. Babu and Odeh [2, 3] have considered a more general case and have obtained an analytical solution for the steady-state or pseudo-steady state case with an arbitrarily located well in the block, provided the well is along one of the grid lines. Penmatcha *et al.* [4] have extended Babu and Odeh's model to the case of infinite well conductivity, which yields the expected non-uniform flux pattern along the wellbore.

Most current commercial simulators use the Peaceman model in spite of its limitations. Dietrich and Kuo [5] have compared horizontal well performance simulated by a uniform coarse grid with a non-uniform fine grid system using the Peaceman model. They concluded that the well productivity is overpredicted using a coarse grid and the conventional well model proposed by Peaceman. Dietrich and Kuo also applied an explicit modeling technique using a non-uniform grid system, in which the grid was refined around the well to represent the length and the diameter of the horizontal well. They also set the permeability of the wellbore gridblocks at a very high level to establish a negligible pressure gradient along the wellbore. At the same time, a very large well transmissibility connection factor was used for the wellbore at the heel to force the well and the well block pressures to be the same, thus deactivating the well model.

The numerical and analytical solutions presented here show that for the case considered by Dietrich and kuo, well productivity is underpredicted using a coarse grid system with a conventional well index, which is different from the results of Dietrich and Kuo. We also show that the reservoir performance simulated by a non-uniform fine grid can deviate significantly from that simulated by a uniform fine grid and that calculated by the appropriate analytical well model.

6.2 Well Model

The well index (WI), which accounts for the geometric characteristics of the well and the reservoir properties around the well, is used to relate the well bottomhole pressure and the well block pressure.

6.2.1 Peaceman Formula

Assuming single phase flow we can relate well and well block pressures through the radial form of Darcy's law:

$$q_p = WI \frac{k_{rp}}{\mu_p B_p} (P_{p,block} - P_{well})$$
$$WI = \frac{\theta k h}{\ln(\frac{r_o}{r_w}) + S}$$

where θ is the angle open to flow and S is the skin factor.

Peaceman [1] derived the following expression for r_0 (for a well parallel to the x-axis):

$$r_0 = 0.28 \frac{[(k_z/k_y)^{1/2} \Delta y^2 + (k_y/k_z)^{1/2} \Delta z^2]^{1/2}}{(k_z/k_y)^{1/4} + (k_y/k_z)^{1/4}}$$

It is based on the assumptions of single phase flow, homogeneous reservoir, uniform grid, uniform permeability, an isolated well (the well is at least five gridblocks away from all boundaries and ten gridblocks away from other wells [6]), pseudo-steady state or steady state conditions, and block-centered well. Since a horizontal well cannot be far from the top or bottom boundary when the reservoir is thin, the accuracy of Peaceman's well model is restricted. Furthermore the flow around a horizontal well may not be two dimensional.

6.2.2 Babu and Odeh's Analytical Solution

Babu and Odeh [2] considered a more general case, where the well can be arbitrarily located at any position in a block, provided that it is parallel to one of the axes or gridlines. This analytical solution is based on steady-state (or pseudo-steady state)

However, Babu's analytical solution is based on the assumption of uniform flux along the wellbore. In reality, the flux pattern along the well is non-uniform, which is also the case for the problem considered by Dietrich and Kuo [5]. Recently, Penmatcha *at al.* [4] have developed a solution for a well with non-uniform influx and infinite conductivity. The wellbore is assumed to have a uniform pressure, resulting in non-uniform flux along the well length. Later we will use this analytical solution to evaluate the adequacy of numerically computed well productivity.

6.3 Grid Systems

6.3.1 Uniform Grids

In the previous section, we discussed the fact that the Peaceman well model is based on a uniform grid. For homogeneous, uniform permeability, single phase flow reservoir simulation, it is appropriate to use this model.

In practice, the limitation of computational resources requires that the grid be as coarse as possible with refinement in some regions; therefore we need to ensure that the well model is correct. All well models are based on some assumptions, so the model used needs to be appropriate for the problem being considered. We will show in the next section how this is accomplished.

6.3.2 Non-uniform Grids

The use of non-uniform grid spacing is often required in reservoir simulation. For example, it is necessary for 3-D simulations of stratified reservoirs or for well coning calculations [5]. Dietrich and Kuo [5] used the approach of grid refinement and deactivating the wellbore to realize explicit well modeling. In this approach, the wellbore is represented by a series of very small gridblocks scaled to represent the length and diameter of the horizontal well, the permeability of the wellbore is set at a high level and the transmissibility connection factor for the wellbore at the heel is also made very large, so that the well model itself can be removed from the problem and the effect of the well index is trivial. Then they assumed that this scheme will yield accurate solutions for horizontal well productivity.

However, we have found that this approach has two significant disadvantages:

1. Numerical Approximation:

Aziz and Settari [6] showed some disadvantages of irregular grids considering the accuracy of finite-difference approximation. In most of current commercial simulators, the block-centered grids are used. Aziz and Settari have shown that this type of grid causes higher numerical errors than point-distributed grids. In the example below, we will show the divergence of the results by a non-uniform fine grid from both a uniform fine grid and an analytical solution.

2. Physical Restrictions:

Palagi [7] discussed well models for Cartesian and Voronoi grids. The numerical results of block pressure obtained with large grid aspect ratios do not agree well with analytical solutions for locations close to the wells. In spite of this, the Peaceman's model provides the correct well pressure. The discrepancy of the pressure of gridblocks adjacent to the well is due to the assumption of linear flow used to derive geometric factors for grid connections in the Cartesian system, even in areas of predominantly radial flow like regions around the well. It is clear that this effect is more significant for large aspect ratios.

Dietrich and Kuo's approach of using explicit well modeling actually uses the block pressure as the well pressure. This is possible because by explicit well modeling and the approach to remove the wellbore, the block pressure and the well pressure are the same. However, without the correct well index, and because the block pressure for large aspect ratio grid system will deviate a lot from the correct one, large errors are introduced.

6.4 Example Problem

We tried one example to test the effects of grid systems on predicting horizontal well productivity.

6.4.1 Model Description

Table 6.1: Reservoir and Fluid Properties

K_x, K_y	= 100 mD (isotropic)
K_z	= 100 mD (isotropic)
K_x, K_y	= 150 mD (anisotropic)
K_z	= 5 mD (anisotropic)
Φ	= 0.2
C_t	= $3.0 \times 10^{-5} \text{ psi}^{-1}$ at P_{ini}
q_{max}	= 10000 STB/d
bhp_{min}	= 2001.521 psia
μ	= 1 cP
B	= 1 RB/STB
P_{ini}	= 3596.94 psi
Drainage area	= $2640 * 2640 \text{ ft}^2$
Thickness	= 100 ft
L_w	= 1000 ft
S	= 0.0
r_w	= 0.1875 ft

We used a three-dimensional, homogeneous closed reservoir. A single horizontal producer is centered in the reservoir and partially penetrates the reservoir in the y-direction. We assume only oil phase exists in the reservoir, so it is a single phase problem. Both constant bottomhole pressure and constant flow rate constraints are used at the horizontal well. Horizontal well production rates are all evaluated under pseudo-steady state when the production rate is constrained. But when the bottomhole pressure is constrained, it is not possible to reach pseudo-steady state, we will elaborate further on this point later.

The properties and controlling parameters in simulation are shown in Table 6.1. The configuration of the non-uniform fine grid is exactly the same as that used in reference [5].

6.4.2 Analysis of Results

All the numerical results were obtained using a homogeneous single phase flow model in the Eclipse reservoir simulator. The analytical solution is used as the yardstick to measure the adequacy of simulation results.

Constant Bottomhole Pressure Constraint on the Well

(i) Results by the uniform fine grid system and the analytical solution

Figure 6.1 shows that simulations with the coarse grid underpredict well productivity over most of the simulated period. Figure 6.2 depicts an expanded region of Figure 6.1 from 20 to 25 days to better show differences among various results. Figure 6.3 is a typical plot used in well testing. We find that there is no unit slope line, which means pseudo-steady state was never reached in this case. Nevertheless, we will use this case to compare our results with those of Dietrich and Kuo's. We can see from Figures 6.1 and 6.2 that the curve of the uniform fine grid is closest to that of the analytical solution. In this case, the Grid Aspect Ratio (GAR) is equal to 6. It should be mentioned that simulations were done with no gravity in order to be consistent with the currently available analytical solution.

Figure 6.4 shows the flow rate ratio vs. well index ratio. WI_0 is the default well index value calculated by Eclipse for the coarse grid simulation, and increased WI is obtained by multiplying it by a factor. The flow rate ratio is the ratio of the coarse to uniform fine grid or coarse to analytical solution. We note that the Peaceman's well index, which is the default in Eclipse, has to be multiplied by a factor corresponding to the flow rate ratio of 1 to yield the correct coarse grid solution. In this case, the correct well index ratio is flow rate dependent and varies from 1.24 to 1.44. In Figure 6.4, we observe two different trends, one is when the well index ratio increases with increasing oil rate ratios, which corresponds to the period that the pressure derivative curve has a negative slope; the other is that when the well index ratio increases while the oil rate ratio decreases, it corresponds to the period when the derivative curve has a positive slope.

(ii) Different Schemes to Obtain Irregular Grids and Explicit Modeling Results

Figures 6.1 and 6.3 show the production rate of the horizontal well calculated by different grids and well models. There are three curves referring to the flow rates simulated by non-uniform fine grids. By setting high permeability to the wellbore and high WI to the heel, we can get slightly better results than those from the non-uniform fine grid with no adjustment of K or WI. While these results are close to those simulated by coarse grids at late times, they are still far from those of uniform fine grids and analytical solutions.

Dietrich and Kuo also evaluated coarse grid simulation by this method, the difference is that they used explicit modeling plus a non-uniform fine grid for their reference solution, which we have demonstrated to be wrong. One reason is that the Grid Aspect Ratio for their problem is about 66. It is large enough to cause a significant error in the pressure of grid blocks near the well. So their result that the coarse grid overpredicts well productivity is completely different from ours, since we see that coarse grid simulation underpredicts well productivity.

Constant Flow Rate Constraint on the Well

In this case, the slope of the pressure derivative curve is exactly unity (Figure 6.5), so we can identify the period after about 10 days as pseudo-steady state. Figures 6.6 and 6.7 show that the coarse grid simulation underpredicts productivity at the end of the pseudo-steady state period. Again to better show details in results, an expanded region of Figure 6.7 is displayed in Figure 6.8. Figure 6.9 shows that in coarse grid simulation, to obtain the correct horizontal well productivity, the well index should be multiplied by a factor of 1.25 for the isotropic case and by a factor of 1.07 for the anisotropic case. Figure 6.10 demonstrates similar results by the use of bottomhole pressure ratio.

6.5 Conclusions

An explicit modeling procedure employing high resolution grid systems near the well has been shown to be even worse than the coarse grid simulation results. Only when using the approach to deactivate the well model, are reasonable results are obtained, but they are still far from the correct solutions. Numerical approximations and physical restrictions are responsible for this deviation.

Horizontal well rates computed using a uniform coarse grid system and a conventional well model will under-predict well productivity for a well of the type considered.

The correct well index to be used in coarse grid systems can be computed and it is problem dependent.

6.6 Nomenclature

Q	Flow rate, STB/day
B	Formation volume factor, bbl/STB
k_r	Relative permeability
k_x	Permeability in the x-direction, mD
k_y	Permeability in the y-direction, mD
k_z	Permeability in the z-direction, mD
r_w	Wellbore radius, ft
r_0	Effective well radius, ft
S	Skin factor
h	Thickness, ft
C_t	Total compressibility, psi^{-1}
Δx	Gridblock size in the x-direction, ft
Δy	Gridblock size in the y-direction, ft
Δz	Gridblock size in the z-direction, ft
Φ	Porosity
bhp	Bottomhole pressure, psi
P	Pressure, psi
P_{ini}	Initial pressure, psi
L_{well}	Well length, ft
μ	Viscosity, cP

Subscripts

p	Phase
w	Well

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Figure 6.1: Influence of Grid System on Production Ratio for the Case of Constant Well Pressure Constraint

Figure 6.2: An Expanded Portion of Figure 6.1 from 20 to 25 Days

Figure 6.3: Approach to Pseudo Steady State for the Case of Constant Well Pressure Constraint

Figure 6.4: Adjustment Factor for WI to obtain Correct Flow Rate - Isotropic Case

Figure 6.5: Approach to Pseudo Steady State for the Constant Flow Rate Case

Figure 6.6: Results of Constant Flow Rate Set Initially Followed by a Period of Constant Wellbore Pressure Constraint

Figure 6.7: Bottomhole Pressure in the Case of Constant Flow Rate Constraint

Figure 6.8: An Expanded Portion of Figure 6.7 from 15 to 20 Days

Figure 6.9: Variation of Oil Rate with Well Index for both Isotropic and Anisotropic Cases

Figure 6.10: Variation of Bottomhole Pressure with Well Index for both Isotropic and Anisotropic Case

7. Effect of Reservoir Heterogeneities and other Parameters on Performance Prediction of Non-Conventional Wells

by Khald Aziz, Sepehr Arbabi, and Clayton V. Deutsch

Abstract

Non-conventional wells that include, horizontal, highly deviated and multilateral wells – often called designer wells – are becoming more and more common. Both analytical and numerical tools have been developed and continue to be developed for predicting their performance. Unfortunately, predictions made using these tools rarely match actual performance, except in cases where sufficient production data are available for history matching and the model used for making predictions is selected carefully. Even then the predictions are generally good only for a limited time. In this paper we explore reasons for our inability to make accurate predictions. We consider a case where a vertical well has been drilled and cored. Then, we generate twenty consistent geostatistical descriptions of permeability and porosity that are all constrained to the hard data obtained from the vertical well. Simulations with these realizations show large differences in production rate, WOR and GOR predictions as a result of variations in reservoir properties. It is also shown that the effect of well index (WI) on simulation results is large. Furthermore, for the example considered, analytical models for critical rate and productivity calculations were found to be of limited practical use.

7.1 Introduction

Wells drilled in petroleum reservoirs interact in a complex way with the reservoir. In order to predict their behavior we must be able to model multiphase flow in the well and the reservoir. In this problem there are many sources of uncertainties, some of these are explored in this paper. The most serious of these is the limited data about the reservoir itself.

As with everything else in nature, in the end the best we can hope to do is to predict the performance of horizontal wells only in a probabilistic sense and reduce the uncertainty to a manageable level. In a fascinating paper Beliveau (1995)¹ has analyzed the performance of 1,306 horizontal wells from 230 fields around the world. He shows that the productivity improvement factors (PIF) – defined as the initial stable oil or gas production rate of a horizontal well divided by the current production of an offsetting vertical well – have an approximately log-normal distribution because “most reservoir parameters are log-normally distributed about their mean.” He shows that horizontal wells in conventional reservoirs have a mode (most likely) PIF of 2, a median of (50/50) 3 and an average (or mean) of 4. In heavy oil reservoirs the mode was about 5 and in fractured reservoir about 6. Beliveau also compared forecasts of well performance with actual results. Here he found that in a 13-well program in a North Sea field actual results were 14% higher than initial forecasts; however, the average of the absolute individual well errors was much higher at 43%. Only 8 of the 13 producers had actual results within $\pm 50\%$ of the forecasts. Furthermore the predicted ranking of wells was not preserved

during their production phase. Beliveau¹ also summarizes Shell Canada's experience with 75 horizontal wells in several fields: Actual average results exceeded forecasts by about 20%; while the average absolute error (error bar) was almost 60%. Fewer than half the wells performed within $\pm 50\%$ of their forecasts. While Beliveau makes repeated reference to the importance of geological heterogeneity, he does not quantitatively investigate its effect on well performance.

In another study by Mullane et al. (1996)² performance of 29 horizontal wells was compared with predictions. While in this case the estimated oil rate was hit exactly, the average absolute error was 47%. Predictions of only about half the wells were within $\pm 50\%$ of actual performance. They used a "calibrated" version of Joshi's deliverability equation (Joshi, 1986)³ to predict oil rate.

In published field scale evaluations, oil rate at a given drawdown appears to be the most common predicted quantity that is compared with field performance. In this paper we will show how more detailed well performance predictions differ with different geological descriptions constrained to the same data, different grid block sizes, and different methods of upscaling absolute permeability. Here we will consider time for which plateau production can be sustained, WOR and GOR. These factors are much more difficult to predict than fluid rate. In addition we will show the effect of techniques used for modeling wells in simulators. In particular we see the effect of the well index and the pressure drop in the well. Finally we will compare analytical and numerical results for water cresting into a horizontal well and well productivity.

7.2 What is Needed for Predicting Performance

Predicting the performance of non-conventional (including horizontal wells) or conventional wells requires that we be able to model transient multiphase flows in systems consisting of one or more connected petroleum reservoirs and several wells, that are produced under specified constraints on pressures or rates imposed on individual wells or groups of wells. In order to predict the performance of a well in such complex systems we must be able to

- describe each reservoir (geometry, permeability and porosity distribution);
- describe each well (profile, completion details and the internal condition of the well ;
- model the artificial lift system used in the field;
- model multiphase, multicomponent flows in heterogeneous porous rocks;
- adequately characterize reservoir fluids, and mixtures of reservoir fluids and any fluids that are injected into the reservoir;
- model changes in fluid properties with changes in pressure and temperature (PVT models);
- model the influx of fluids into wells as result of drawdown along wells;
- model multiphase, multicomponent flow in wells; and
- specify the initial state (saturation and pressures) and the boundary conditions (water influx from aquifers) of each reservoir.

None of the above requirements is ever fully met and yet we must make predictions. Furthermore, three-phase, multicomponent flow modeling requires many of assumptions and simplifications that may not be always justified. The job of the engineer is to understand the sources of uncertainty and make the best possible predictions to provide guidance to management in making decisions to meet overall

corporate objectives. Assessment of uncertainty associated with predictions can be used to associate risk with each option.

Models of various degrees of sophistication may be available or possible. But the level of sophistication must be balanced with (Aziz 1989, 1995)^{4,5}:

- available resources,
- available information, and
- the objective of the predictions.

It would be ridiculous to try to model the whole reservoir by modeling the flow in individual pores of a network that represents the reservoir. Yet a pore network model of a small portion of the reservoir may provide very useful information about the average behavior of a network of pores, that is represented in reservoir engineering through absolute and relative permeabilities and porosities. This is why our interest spans models of various kinds and complexity – from models for flow in a single pore to full-field models.

In the remainder of this paper we will show how performance predictions are affected by major sources of uncertainty. We will restrict our discussion to predicting the performance of a single horizontal well. First we will discuss the effect on well performance predictions of reservoir description and how it is used in simulators. Next we will show the effect of uncertainty associated with some of the assumptions made in models used for predicting horizontal well performance.

7.3 Reservoir Description and Simulation Grid

It is generally acknowledged that lack of knowledge about reservoir heterogeneity is a major cause of the “error bars” associated with performance predictions. Here we will consider a hypothetical example of a horizontal well that is based on data from a real reservoir. The drainage volume associated with the well is 10,000×5,000×100 ft.

We will assume that a single vertical well has been drilled. With information from this well and other sources a stochastic model is constructed to produce multiple permeability and porosity images of the drainage volume.

The synthetic reservoir data was fashioned after a fluvial sandstone reservoir with a 70% net-to-gross ratio. Sequential indicator simulation was used to construct a sandstone/shale lithofacies model. An indicator variogram with a vertical range of 10 ft. and an isotropic horizontal range of 100 ft. was considered. This resulted in shales that generally extend over less than 100×100×10 ft. Porosity and permeability in the shales was set to 0.1 and 1.0 mD. The sandstone porosity model was created by sequential Gaussian simulation with a normal scores variogram with a vertical range of 10 ft. and an isotropic horizontal range of 2,500 ft. Sequential Gaussian simulation with the collocated cokriging option in GSLIB (Deutsch and Journel, 1992)⁶ was used for the permeability model. The normal scores of porosity were correlated with permeability with a correlation of 0.7. The normal scores variogram of permeability had a vertical range of 3.33 ft. and an isotropic horizontal range of 1,500 ft. The data representing a vertical well that goes through the heel of the horizontal well was extracted from an initial unconditional geostatistical realization. The lithofacies, porosity and permeability data from this vertical well were honored in all twenty geostatistical models. The geostatistical parameters for all realizations are identical and the images appear to be similar. For the medium grid the mean shale fraction is 0.311

(std. dev. of 0.007), mean porosity 0.261 (std. dev. of 0.003), and mean horizontal permeability 430.04 (std. dev. 5.71). The horizontal well is aligned along the X-axis and placed in the middle of the drainage volume.

All stochastic images were constrained to the same permeability and porosity at the location of the vertical well and are created on a 100×50×30 grid (150,000 uniform grid blocks, each block is 100×100×3.33 ft) which is considered to be the *fine* grid in this study. For the purposes of simulation we have created two sets of upscaled images of the fine grid: a 20×10×10 *medium* grid (500×500×10 ft blocks), and a 10×5×5 *coarse* grid (1,000×1000×20 ft blocks).

The grid is nearly uniform in all cases. The upscaling from the *fine* grid to the other two grids was done using two commonly used methods:

- numerical single phase flow matching in each of the three directions for each upscaled block and the *fine* grid blocks contained in that block (referred to as *medium-f* and *coarse-f*), and
- power law averaging (referred to as *medium-p* and *coarse-p*).

Histograms of horizontal and vertical permeability for all three scales are shown in Figure 7.1. The first of the flow based upscaled realizations was used to estimate the power law averaging exponent for the horizontal and vertical directions. Exponents of 0.71 and 0.02 were obtained for the horizontal and vertical directions for the *medium* grid, and 0.73 and -0.18 for the *coarse* grid. This gives a total of 100 images to process (20 *fine* grid, 40 *medium* grid and 40 *coarse* grid).

Figures 7.2 through 7.5 show XZ (vertical permeability) and XY (horizontal permeability) slices going through the horizontal well for four selected realizations. These figures compare fine grid descriptions with corresponding upscaled images for the flow based (*medium-f* and *coarse-f*) and power averaging (*medium-p*) methods. These images were selected because they showed extreme behavior during performance predictions.

In addition to the grid blocks generated by this process, a layer of homogeneous grid blocks with huge pore volumes (25 x 10¹² cubic feet) were added to the top and bottom of the reservoir to simulate a large gas cap and a large aquifer. All other data (also extracted from a real field study) are shown in Table 7.1.

Table 7.1: Summary of Rock and Fluid Properties Used

Initial pressure, P_i , at GOC	2294 psia
Rock compressibility, at P_i	1.2E-5 psia ⁻¹
Oil viscosity	1.3-1.5 cP
Gas viscosity	0.016-0.020 cP
Connate water saturation	0.07
Critical gas saturation	0.10
Residual oil sat. to water	0.25
Residual oil sat. to oil	0.07
Capillary pressure	None

The single 2,000 ft horizontal well nearly located in the middle of the drainage volume is produced at

a rate of 5,000 barrels per day with a minimum bottom-hole pressure of 1,500 psia. The results for oil production rate, GOR and WOR for the *medium-f* grid are shown in Figures 7.6 to 7.8. Based on these and other results, a single realization (identified on figures showing results) was selected as the base case. Figure 7.9 compares production rates predicted for this base cases by *fine*, *medium-f*, *medium-p*, *coarse-f* and *coarse-p* grids. Figures 7.10 and 7.11 provide corresponding results for GOR and WOR. Because of the large computational time required, it was not possible to process all of the fine grid images. Results of all of the simulations are summarized in Table 7.2. Here values of maximum, minimum, mean, and spread (maximum-minimum) are presented for:

1. Cumulative oil production during the plateau period (when the oil rate is constant at 5,000 barrels per day),
2. Time for the oil rate to drop to 3,000 barrels per day (40% of the specified rate),
3. GOR at 6,000 days,
4. WOR at 6,000 days. and
5. Bottomhole pressure at 2,000 days.

We have also performed simulations for the base case at the *medium* grid by using the upscaled geological description from the corresponding *coarse* grid. This was done to see the effect of grid block size (discretization errors). The results for this case are referred to as *medium-f-c* and they are compared with other cases on Figures 7.9 to 7.11.

Fig. 7.1: Histograms of horizontal and vertical permeability for fine, medium, and coarse resolutions. A fixed multiplier of 0.1 is applied to the vertical permeability data in the simulations.

Fig. 7.2: Horizontal and vertical permeability maps of realization 3 for *fine*, *medium-f*, *coarse-f*, and *medium-p* grids. Slices are for the plane of the well.

Fig. 7.3: Horizontal and vertical permeability maps of realization 9 for *fine*, *medium-f*, *coarse-f*, and *medium-p* grids. Slices are for the plane of the well.

Fig. 7.4: Horizontal and vertical permeability maps of realization 12 for *fine*, *medium-f*, *coarse-f*, and *medium-p* grids. Slices are for the plane of the well.

Fig. 7.5: Horizontal and vertical permeability maps of realization 19 (base case) for *fine*, *medium-f*, *coarse-f*, and *medium-p* grids. Slices are for the plane of the well

Fig. 7.6: Oil production rate for 20 realizations on *Medium-f* grid. Symbols identify the four images from Figures 7.2-7.5. - • is realization 3, Δ is realization 9, $\square$ is realization 12, and $\blacksquare$ is realization 19 which is the base case

Fig. 7.7: GOR for 20 realizations on *Medium-f* grid. Symbols identify the four images from Figures 7.2-7.5. - • is realization 3, Δ is realization 9, $\square$ is realization 12, and $\blacksquare$ is realization 19 which is the base case

Fig. 7.8: WOR for 20 realizations on *Medium-f* grid. Symbols identify the four images from Figures 7.2-7.5. - • is realization 3, Δ is realization 9, $\square$ is realization 12, and $\blacksquare$ is realization 19 which is the base case

Fig. 7.9: Comparison of oil production rates from different grids for the base case (realization 19)

Fig. 7.10: Comparison of gas/oil ratio (GOR) from different grids for the base case (realization 19)

Fig. 7.11: Comparison of water/oil ratio (WOR) from different grids for the base case (realization 19)

Table 7.2: Summary of Simulation Results

	Cumulative Oil Production (plateau period) MMSTB	Time to Reach Oil Rate of 3,000 STB/Day Days	GOR at 6000 Days MSCF/STB	WOR at 6000 Days	WBHP at 2000 Days Psia	
Maximum	30.94	9726	384	7.44	2109	Coarse Grid flow based
Minimum	7.40	3753	74	0.89	1500	
Mean	18.09	6363	168	1.96	1823	
Spread	23.54	5973	310	6.55	609	
Maximum	34.35	9706	426	8.23	2159	Medium Grid flow based
Minimum	6.95	2618	67	1.23	1500	
Mean	18.20	5948	213	3.10	1856	
Spread	27.40	7088	359	7.10	659	
Maximum	22.97	7565	488	7.62	2036	Coarse Grid power avg
Minimum	7.16	3382	186	2.21	1500	
Mean	13.84	4922	293	4.00	1764	
Spread	15.81	4183	302	5.41	536	
Maximum	23.61	8275	806	11.13	2118	Medium Grid power avg
Minimum	7.41	2601	172	3.59	1500	
Mean	12.76	4467	426	6.90	1730	
Spread	16.20	5674	634	7.54	818	

These results show that the oil rate is more sensitive to upscaling than to gridblock size. However, GOR and WOR are highly sensitive to both the block size and upscaling. This is confirmed by other work (not reported here) where we have found that cresting calculations require very fine grids. It is also clear that the geological description has a huge influence on results. In particular, we see that the spread in results decreases as reservoir parameters are upscaled to coarser and coarser grids, and power law averaging reduces the spread as compared to flow based upscaling. The spread in predictions based on just 20 stochastic images is huge and it is likely to increase as more images are processed. The other interesting observation is that a smaller spread does not necessarily mean lower uncertainty. In other words, the distribution of uncertainty generated by repeated flow simulations may not span the *true* or *full* uncertainty because of the assumptions made in the stochastic model and/or the method used for upscaling. In the example discussed here, the flow based upscaling (considered to be the more reliable method) gives a bigger spread in results than simpler power law averaging. Neither of the two upscaling methods is exact.

7.4 Influence of Model Assumptions

7.4.1 Well Models

All simulators use simplified models to relate the wellblock pressure to the pressure of the well in that block. Here we will show the effect of using an inappropriate well model. The sources of uncertainty in the well model are: saturation gradients that cause the effective phase permeabilities for the well region to be different from the corresponding values for the well grid block, effective absolute permeabilities for the well region may be different from the average values for the block, the effective block radius (r_o) may be in error because default procedures in simulators are based on assumptions that are more suitable for typical vertical wells than for horizontal wells, and the effective skin may not be known. All of these factors are not likely to cause the well index to increase or decrease by a factor of more than 5 times the default value. We have done simulations by changing the well index by a factor of 5, 0.2, 1.2 and 0.8. As expected the oil production rate, GOR, WOR are all highly sensitive to the WI. The effect of changes in WI on production rate and GOR for the base case *medium-f* grid are shown in Figure 7.12.

Fig. 7.12: Effect of well index (WI) on oil production rate and GOR for the base case (realization 19)

7.4.2 Effect of Wellbore Pressure Drop on Well Performance

Another factor in well modeling that is often ignored is the pressure drop in the well. This is only important in cases where the reservoir permeability is high and the drawdown is small. We have used a high pipe roughness of 1 mm in the *medium-f* grid base case to see the effect of well pressure drop on results. The pressure drop calculation method is the homogeneous (no-slip) model in the Eclipse (1995)⁸ simulator. These two values of roughness cause a pressure drop in the well at 6,000 days that is approximately 18% of drawdown. The maximum reduction in oil rate is about 12% when friction is included over when friction is ignored. Clearly not as major a factor in this case as reservoir heterogeneities. The greatest effect of wellbore pressure drop is on GOR. While in the homogeneous case the wellbore pressure drop causes the gas to breakthrough earlier than when this pressure drop is ignored, the behavior for the base case is opposite.

7.4.3 Analytical Cresting Models

Often analytical models are used to assess the tendency of water and gas to crest into a horizontal well. This is done because the simulation of cresting requires very fine grids. The analytical cresting models are based on assumptions that are generally different from those in simulation studies or real fields. Also, analytical models can only produce critical rates or time for

breakthrough, not the behavior for super-critical rates. Arbabi and Fayers (1995)⁷ have shown that different semi-analytical models presented in the literature can produce results for critical rates that are different by a factor of 24. They have proposed a new model that produces essentially the same results as careful simulations that mimic the assumptions in the analytical model. Here we present an example where we have used the technique developed by Arbabi and Fayers (1995)⁷ to calculate the critical rate for our problem, under the assumption of steady-state, using average permeabilities of 433 mD in the horizontal direction and 14 mD in the vertical direction. The optimum well location, defined as the location that gives the same critical rate for both the gas and water interfaces, is predicted to be about 30 ft from the gas/oil contact with a critical oil rate of about 68 barrels per day. Theory shows that for the no-flow boundaries used in simulation, there is no critical rate. Simulations with a rate of 50 barrels per day (critical rate of the well in the middle of the reservoir) show that indeed this is true, but the gas and water breakthroughs occur after 3,000 years for the “optimum” well location, and when the well is located in the middle of the reservoir water breakthrough occurs at about 1,800 years and gas at about 4,300 years. As expected, when the simulations are done with *medium-f* grid breakthroughs occur earlier. We have also done simulations with the production rate of 5,000 barrels per day and the two well locations. Again, as expected, moving the well to the 30 ft location delays water breakthrough and reduces gas breakthrough times. The overall conclusion from this part of our work is that critical cresting rate solutions available in the literature have limited practical utility, because real situations normally have different boundary conditions and furthermore economical production rates are usually much higher than critical rates.

7.4.4 Productivity Models

The most common approach used to compare the performance of horizontal and vertical wells is the use of single phase analytical solutions for steady-state or pseudo steady-state flow in homogeneous media. The most popular of these is Joshi’s equation.^{2,3} Since our problem does not reach steady-state it is not appropriate to directly use Joshi’s equation. A more general transient analytical model based on the work of Babu and Odeh⁹ is used here with infinite conductivity in the well. In Figure 7.13 we compare analytical and simulation results. As expected the agreement between the numerical and analytical models for the single phase case is excellent. However, for the three-phase case (with the gas cap and the aquifer included) the simulation results (homogeneous properties, *medium* grid) are very different from the single phase results. The PI’s calculated for various situations are given in Table 7.3. The PI depends on the model (analytical or numerical, boundary conditions, single or multiphase flow) used and the conditions in the reservoir (transient, pseudo steady-state, steady-state). These results clearly show the inappropriateness of using a steady-state analytical model for predicting horizontal well performance. Furthermore, we observe that the results are very different when pressure support is provided by the gas and water zones.

Fig. 7.13: Comparison between single phase (analytical and simulation) and three-phase *medium grid* simulation with homogeneous properties

Table 7.3—PI's Calculated from Various Methods

Method & Problem	PI (STB/Day/psi)		
	100 Days	1,400 Days	4,000 Days
Babu and Odeh, 1-phase (Transient)	68.7*	67.5	+
Simulation, 1-phase (Transient)	65.3*	63.9	+
Joshi, 1-phase (Steady State)	18.9*	18.9	+
Simulation, 3-phase (Homogeneous Case)	139.7*	15.2*	3.7

+ no oil production at the fixed well bottom hole pressure of 1500 psi

* oil production at constant rate of 5000 STB/Day

7.4.5 Effect of Well Type

In this section, we present and discuss results obtained when the horizontal well is replaced by a vertical well. The vertical producer is fully completed in the oil zone and two situations are considered: (1) the vertical well passes through the heel of the horizontal well discussed previously, and (2) the vertical well is moved to another location near the upper right hand corner of the reservoir. In each case, simulations with the previous twenty realizations were repeated on *medium-f* grids. Figures 7.14-7.16 display the oil production rate, GOR, and WOR for the first case. In this case, all fine stochastic images have the same permeability and porosity at the well. The upscaled images do not have this property in common. Nonetheless, a much narrower spread is seen among various realizations in comparison with the results from the horizontal well (Figs. 7.6-7.8). This narrower spread is somewhat expected but perhaps what is surprising are lower GOR and WOR of the vertical well compared to the horizontal well. One reason for this behavior is provided by oil production rates in Figs. 7.8 and 7.14. From Fig. 7.14, it can be seen that all vertical wells have switched to bottomhole pressure control after 2178 days. On the other hand, horizontal wells can sustain the specified constant production rate of 5000 barrels per day up to 6870 days (Fig. 7.8). This means that vertical wells have lower production rates than horizontal wells. Obviously, this implies lower WOR and GOR for vertical wells. In fact, realizations with similar production rate history indicate lower or close WOR and GOR with horizontal well. Our base case happens to be such a realization. The base case with horizontal well goes under the bottomhole pressure control at 1495 days while it does the same at 773 days with the vertical well. The production rates at the end of 20 years are respectively 1650 STB/day and 1633 STB/day with horizontal and vertical wells. The corresponding GOR and WOR are 84 STB/MSCF and 3.77 for the horizontal well and 318 STB/MSCF and 3.63 for the vertical well.

Fig. 7.14: Oil production rate for 20 realizations on *Medium-f* grid for vertical well case (1). Symbols identify the four images from Figures 7.2-7.5. - • is realization 3, Δ is realization 9, $\square$ is realization 12, and $\blacksquare$ is realization 19 which is the base case

Fig. 7.15: GOR for 20 realizations on *Medium-f* grid for vertical well case (1). Symbols identify the four images from Figures 7.2-7.5. - • is realization3, Δ is realization 9, $\square$ is realization 12, and $\blacksquare$ is realization 19 which is the base case.

Fig. 7.16: WOR for 20 realizations on *Medium-f* grid for vertical well case (1). Symbols identify the four images from Figures 7.2-7.5. - • is realization3, Δ is realization 9, $\square$ is realization 12, and $\blacksquare$ is realization 19 which is the base case.

In the second case, the vertical well is moved to location $x=7500$ ft and $y=1500$ ft near the upper right hand corner of the reservoir. Figures 7.17-7.19 show the results of the twenty realizations. Now every fine stochastic image is different at the well location. As a result the upscaled images show greater variability. This larger variation manifests itself in wider spread among different realizations as depicted in Figs. 7.17-7.19. One interesting observation is that the spread in results for this case is still less than that of the horizontal well. This points to the important role of geology along the length of a horizontal well, and emphasizes the need for proper modeling of this geology to reduce uncertainty.

Fig. 7.17: Oil production rate for 20 realizations on *Medium-f* grid for vertical well case (2). Symbols identify the four images from Figures 7.2-7.5. - • is realization 3, Δ is realization 9, $\square$ is realization 12, and $\blacksquare$ is realization 19 which is the base case

Fig. 7.18: GOR for 20 realizations on *Medium-f* grid for vertical well case (2). Symbols identify the four images from Figures 7.2-7.5. - • is realization3, Δ is realization 9, $\square$ is realization 12, and $\blacksquare$ is realization 19 which is the base case.

Fig. 7.19–GOR for 20 realizations on *Medium-f* grid for vertical well case (2). Symbols identify the four images from Figures 7.2-7.5. - • is realization3, Δ is realization 9, $\square$ is realization 12, and $\blacksquare$ is realization 19 which is the base

7.4.6 Other Factors

Not all sources of errors or uncertainty have been investigated in this paper. Some of the other factors that may influence predictions but were not considered in our study are mentioned below:

1. Exact location of the well and the condition of the wellbore are often not known precisely.
2. Net-to-gross ratio, pore volumes and hydrocarbon volumes were all fixed.
3. Aquifer and gas cap sizes were fixed.
4. Influence of other wells in the field was not considered.
5. A very simple no-slip model for multiphase flow in the horizontal well was used.
6. Wellbore hydraulic calculations during multiphase flow in the well can be in error. In this study well pressure constrained at the well heel.
7. Only two upscaling techniques for absolute permeability were considered.
8. Relative permeabilities were not upscaled.
9. Grid alignment with the well can be a serious problem in multiple well studies.
10. The reservoir was assumed to not have any fractures.
11. A black-oil fluid description was used.

7.4.7 Concluding Remarks

The greatest source of uncertainty is reservoir description and how it is used in simulators. Integration of data through geostatistical techniques leads to multiple descriptions that all honor available data. The reality is never known. The only way to reduce this uncertainty is to use more data from geological studies, formation evaluation, high resolution seismic, well tests and production history to constrain stochastic images.

Even with a perfect knowledge about reservoir geology, current models cannot do routine simulations at a fine enough scale. Furthermore, we normally don't know what scale is fine enough. Upscaling introduces errors and masks some of the physical phenomenon that we are trying to model. The scale at which upscaling is robust is not known and it is probably smaller in most cases than the scale actually used for predicting performance. Uncertainties in the well index (WI) can cause errors in predictions that are of the same magnitude as those caused by reservoir heterogeneities.

Simplified semi-analytical models for cresting behavior and productivity predictions can be very misleading.

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8. Analysis of Gravity Drainage in Horizontal Wells

*by Mahadevan Venkataraman, Dengen Zhou, Sepehr Arbabi, and
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Abstract

One of the major forces affecting the recovery of oil from reservoirs through horizontal wells is gravity. In this report, the effect of gas injection on the gravity drainage process in horizontal wells is discussed. The sweep efficiency of the gas injection process for different configurations of horizontal wells will be studied using a CT scanner. The viscous and gravitational forces coming into play in the reservoir are discussed. The experimental setup for this project is given and some results of simulation runs based on this physical model are provided.

8.1 Introduction

Horizontal wells are used extensively in recovering oil from reservoirs as they expose a larger amount of the formation to the well than vertical wells. One of the important forces in the recovery of oil using horizontal wells is gravity. This is especially important in case of heavy oils where the reservoir pressure is not sufficiently high to move the viscous oil towards the wellbore. Gravitational forces are most effective when the horizontal well is placed close to the bottom of the reservoir and the distance for the flow of oil through the porous medium is minimized.

Gas injection has been used widely to improve the recovery of oil. It can be used for maintaining pressure in the reservoir which in turn contributes to an increase in productivity or for miscible displacement. Nitrogen and carbon dioxide are the two commonly used non-hydrocarbon gases.

The significance of the gravity effects as well as the viscous effects in the gravity drainage processes are studied and a sensitivity analysis on the different factors affecting the production using a horizontal well is presented.

8.2 Literature Survey

Gravity drainage in horizontal wells for the recovery of oil has been studied with regard to the optimal positioning of the well and its advantages over the use of vertical wells. Meszaros et al.[1] experimentally studied the drainage of oil into horizontal wells using inert gas injection with oil viscosities varying from 1000 cp to 4000 cp and found that oil recovery increased from 50% to about 70%. The stability of the gas front was also studied for two horizontal wells placed one on top of the other. Gas was injected through the upper well and the stability of the front was studied using a physical model. It was also found that it is more difficult to maintain a stable gas front in a 3-D model than in a 2-D model. The production was mainly due to gravity drainage at low flow rates and the oil recovery was high. In all the experiments, the initial production rate was found to be high due to gravity drainage and as

the front moved down, the effect of gravity reduced and hence the production of oil reduced considerably. At very low injection pressure, the production rate of oil was very slow but a steady gas front was maintained. Also as the injection pressure was increased, the recovery of oil decreased. Both carbon dioxide and nitrogen were used in the study and in both cases the dissolution of the gas in oil was neglected.

Dykstra and Dickinson[2] studied the effect of dip, reservoir thickness, oil mobility and well spacing on production and recovery from horizontal and vertical wells. In this study, mobility ratio was considered to be uniform for both the horizontal as well as the vertical well case even though this assumption is not valid. The head of the fluid flowing into the well was calculated as the height from the wellbore to the gas oil contact. This head and flow rate were found to decrease with time. In the case of horizontal formations (no dip), there was no effect on the ratio of horizontal and vertical well flow rates with time. It was found that for very thick formations, vertical wells performed better than horizontal wells. For inclined formations, horizontal wells were found to produce more than vertical wells. The thickness affected the flow rate ratios of horizontal to the vertical wells. Also, in case of vertical wells the well spacing was found to be important.

Williams[3] studied the effects of horizontal well placement and gravity on sweep efficiency for a gas injection process. In the cases considered here, the injection rate and the production rate were made equal. Two cases considered were staggered pattern horizontal wells and two parallel horizontal wells. A viscous to gravity ratio was defined and the pore volumes produced were evaluated as a function of this ratio. It was found that sweep efficiency of a staggered pattern was less than that of adjacent horizontal wells. Enhancement of the gravity drainage process through hydrocarbon additives were also studied by Nasr et al.[4]. The recovery of oil by the use of horizontal wells were found to be always greater than 50% and the addition of Naptha to the steam injection process increased the recovery to 82.5%. Extensive literature also exist on steam assisted gravity drainage and inert gas injection[5-7].

In most of the above cases, saturation profiles at different instances of time were not identified. The experiments were done using oil with high viscosities. The saturation profiles inside the models were not studied in these cases. The saturation profiles would indicate the regions where the oil recovery is high and where the residual oil saturations are high. In this report, the experimental setup to be used for studying the saturation profiles during the gravity drainage process using a CT scanner is discussed along with some simulation results based on the physical model.

8.3 Viscous and Gravitational Forces

In a gravity drainage process, the gravitational force is used to overcome the resistance to flow. The gravitational forces acting on the fluid are characterized by the gravity number which is the ratio of gravitational forces to viscous forces. On the other hand, capillary number is the ratio of capillary to viscous forces. The gravity and capillary numbers (Zhou et. al.[8]) are given as;

$$N_g = \frac{kA\delta\rho g}{\mu_0 q_t}$$

$$N_c = \frac{kA\delta p_c}{q_t\mu_0 L}$$

where k is the permeability of the medium, A is the cross-sectional area open to flow, $\delta\rho$ is the difference in the densities of oil and gas, μ_0 is the oil viscosity, q_t is the total flow rate, and δp_c is the capillary pressure between oil and gas at a given oil saturation.

In a gravity drainage process, the gravity number should be much greater than the capillary number. The flow rate in the model should be adjusted such that $N_g \gg N_c$. The gravity and capillary numbers are also used to scale up from the model to the field.

8.4 Experimental Setup and Procedure

The main aim of this study is to obtain sweep efficiencies for different horizontal well configurations. A Berea sandstone core (4 inches diameter and 2 feet long) will be used for this study and the core will be coated with epoxy. Inlet and outlet ports will be provided on diametrically opposite ends of the core as shown in Figure 8.1. The ends of the core will be sealed with a flow distributor. Decane will be used for the initial case. The core will be saturated with decane and the permeability of the core will be measured.

The gravity drainage process by itself would take a very long time. This is because as the height of the fluid decreases, the pressure head drops and it takes a very long time to drain the oil out. Instead, gas will be injected at a very low flow rate to speed up the recovery. Care would be taken to make sure that flow is dominated by gravitational forces. The gas used in this case is Nitrogen.

A CT scanner will be used to scan the core for saturations and the saturation profiles will be obtained at different time intervals and at different depth of the cores. These patterns will be matched with the initial dry pattern of the core and the saturation of oil will be obtained.

8.5 Simulation of the Experiment

Experiments which are to be conducted were first simulated with the help of the Eclipse commercial simulator. A lab scale study was done in order to estimate the injection flow rate of Nitrogen. The size of the core was 4 inches in diameter and 60 cms in length. This was modeled using cubic grids of 1 cm x 1 cm x 1 cm each and no flow boundary conditions were applied to the boundary blocks. Transmissibilities of some boundary blocks were set to zero in order to approximate the cylindrical shape of the core. A horizontal injector was completed at the top and a horizontal producer was used at the bottom. The producing well was kept at atmospheric pressure and the pressure of the injection well was kept just above the atmospheric pressure (0.01 psi difference). The experiments were simulated using different permeabilities and injection pressures and the results that are provided are for the 300 mD case. The typical values of porosity for Berea sandstone varies between 0.1 to 0.3. For this simulation, the value of porosity is taken to be 0.1. Sensitivity runs were also done for different horizontal well configuration such as adjacent and staggered patterns.

A typical saturation profile after 0.4 days of gas injection is given in Figure 8.2. The production rate of oil for horizontal injector placed at the top and horizontal producer placed

Figure 8.1: Experimental Setup

at the bottom is given in Figure 8.3. The shape of the production curve obtained was found to match with that of the general trend of production curves in gravity drainage processes. There is a sharp increase in the production rate at early times and the production falls off with time. In this case, horizontal wells were also modeled as a series of vertical wells. The results were found to be similar for both the cases.

Simulations were run using one vertical producer and one vertical injector. It can be seen that the sweep efficiency of the vertical wells is very low compared to the sweep efficiency of the horizontal well by comparing Figure 8.4 and 8.5. The sweep efficiency was found to be good at the top layers as the gas swept through all the top layers. But after gas break-through, the production of oil is reduced and residual oil saturation is high in some regions. The saturation profile indicating the sweep efficiency of the vertical wells can be seen in Figure 8.5.

The production rates for oil and gas are given in Figures 8.6 and 8.7. It can be seen that there is an increase in the production rate till gas breakthrough and after gas breakthrough there is a sharp decrease in the production rate of oil.

The saturation profiles obtained from the CT scanner will be matched to the saturation profiles from simulation. From the differences in the two saturation profile, it can be observed as to how the heterogeneities affect the sweep efficiency. Also, the effects of heterogeneities can be studied using a simulator. Sensitivity studies were also done using simulations and a similar trend in the production rates and saturation profiles were observed for different viscosities and permeabilities.

8.6 Future Work

Experiments are to be performed for the vertical wells case, staggered horizontal wells pattern and adjacent horizontal wells pattern. The saturation profiles from the experiment are to be compared with the profiles obtained from the simulation runs. The physical model results will be up-scaled to the field scale. Also the effect of heterogeneity on the gravity drainage process will be studied using the simulator.

Figure 8.2: A Typical Saturation Profile after 0.4 Days

Figure 8.3: Production Rate of Oil versus Time for Parallel Horizontal Injector and Producer

Figure 8.4: Saturation Profile in X-Z Direction for Horizontal Injector and Producer

Figure 8.5: Saturation Profile in X-Z Direction for Vertical Injector and Producer

Figure 8.6: Oil Production Rate for Vertical Injector and Producer

Figure 8.7: Gas Production Rate for Vertical Injector and Producer

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9. Correlations for Cresting Behavior in Horizontal Wells

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Abstract

Water and gas cresting in horizontal wells are important phenomena in reservoirs that have an aquifer and/or a gas-cap. In practical situations, many reservoirs are produced under supercritical rates and a breakthrough of the displacing phase becomes inevitable. At the beginning of a reservoir simulation study, it is a common procedure to make an estimate of the breakthrough time and the post-breakthrough behavior. Grid sensitivity runs are also made to obtain the best grid block sizes to use, since an accurate representation of cresting behavior requires fine grid simulations, which are costly and not always practical. In this work a procedure was developed to obtain accurate breakthrough times using just coarse grid simulations. The flow equations were written in dimensionless form and important parameters affecting multiphase flow were identified. Simple correlations for a quick estimate of breakthrough time, maximum oil rate and post-breakthrough behavior were derived based on an appropriate set of dimensionless variables and an extensive number of simulation runs. The effects of gridblock size and grid pattern were investigated in detail. Effects of rate, mobility ratio, well drainage area, well height, and end-points and shapes of relative permeability curves were also included. A procedure to derive pseudofunctions either using numerical correlations or coarse grid simulations is also presented. These pseudofunctions can be used to improve the performance of coarse grid simulations. An optimum grid pattern to start a reservoir simulation study is proposed. Results for fields with several horizontal wells are also discussed. Application to a real field example shows that the correlations and procedures derived are reliable and accurate, and can be used for field situations before starting a reservoir simulation study.

9.1 Introduction

Petroleum reservoirs often have a gas cap and/or an aquifer. In these situations they are subjected to rapid gas and/or water movement towards the well, created by a sharp pressure gradient in the well direction. As the production begins, the interface between the fluids, that is, gas-oil contact or water-oil contact, deforms from its initial plane shape to a cone or crest. When a field is developed by vertical wells, the shape of the deformation is called a cone, but when it is developed by horizontal wells, this deformation is better described as a crest.

Figure 9.1 shows the general three phase model of the cresting behavior, with the two crests approaching the well as the production continues.

Several analytical solutions for coning behavior for vertical wells and more recently for horizontal wells have been reported in the literature [1, 2, 3, 4, 5, 6, 7]. Most of those solutions were developed for the steady-state condition reached by the cone with a constant potential boundary condition at the lateral boundaries of the reservoir. This is normally

Figure 9.1: General model for cresting behavior

done to calculate the critical rate, for which several different approaches have been used. These solutions are useful when the injection pattern in the reservoir creates a constant potential boundary between production and injection wells.

There are also analytical solutions available for infinite-acting reservoirs and for closed-boundary reservoirs, the latter being specially useful when there are several production wells in a line, creating a no-flow boundary [8, 9, 10, 11, 12]. In those situations, it becomes important to know the breakthrough time of the displacing phase (gas or water) and the post-breakthrough behavior for supercritical rates.

Analytical solutions are useful for several cases shown in the literature, since they can give a good estimate, are easy to apply and, more importantly, they do work for some field situations. For these reasons, despite their limitations, they are still being developed and used, at least during the early stages of a reservoir study.

The assumptions used to derive analytical solutions limit their applicability. These limitations have led to the use of numerical simulators to study coning and cresting behavior. Simulators solve the complete set of equations, along with the correct boundary conditions, they can handle important reservoir complexities, and their use has become a standard practice in reservoir engineering.

However, simulators have their own limitations. The numerical diffusion created by the approximations made during the discretization of time and spatial terms leads to the requirement of using a large number of blocks to achieve accurate results. The number of blocks are limited by computer storage and CPU time consideration which are ultimately limited by hardware and economic constraints.

New correlations are developed here for breakthrough time and post-breakthrough behavior of horizontal wells. A procedure to obtain an accurate breakthrough time using just coarse grid simulations is also proposed.

9.2 Assumptions and Dimensionless Equations

Correlations are more general and reliable when derived based on the fundamental flow equations, so that the important physical mechanisms are automatically incorporated in the correlations. The flow equations, however, have a set of dimensional variables that, with some assumptions, can be easily incorporated in dimensionless variables, reducing the number of variables to be used to develop correlations. We first state the assumptions used in this study. The main assumptions are:

1. Homogeneous medium (constant permeability in each direction).
2. Non-deformable formation ($c_{rc} = 0$) and constant porosity.
3. Isotropic medium (same permeability all directions).
4. Constant viscosity for each phase.
5. Absence of capillary pressures.
6. Two-dimensional flow : For vertical wells, this implies radial flow and the problem is solved in r-z coordinates. For horizontal wells, this implies a cross-sectional model, with the well penetrating the entire formation. In this situation, the pressure drop in the well is also neglected.
7. Undersaturated reservoir ($p > p_{bp}$).
8. Constant production rate.
9. Two-phase flow.
10. Symmetry for the well position : Assume that the lateral boundary size is the same for both sides of the well.

For this study, the following set of dimensionless variables can be defined:

1. Lateral dimensions:

$$x_D = \frac{x}{h_o} \sqrt{\left(\frac{k_v}{k_h}\right)} \quad (9.1)$$

2. Vertical dimension:

$$z_D = \frac{z}{h_o} \quad (9.2)$$

$$\lambda_o = \frac{k_{rocw}}{\mu_o} \quad (9.3)$$

3. Potential:

$$\Phi_D = \frac{\Phi}{\Delta\rho g h_o} \quad (9.4)$$

4. Well drainage distance: using Equation 9.1:

$$x_{eD} = \frac{x_e}{h_o} \sqrt{\left(\frac{k_v}{k_h}\right)} \quad (9.5)$$

5. Well height:

$$h_{wD} = \frac{h_w}{h_o} \quad (9.6)$$

6. Mobility ratio:

$$M = \frac{k_{rwro} \mu_o}{k_{rocw} \mu_w} \quad (9.7)$$

7. Dimensionless time

$$t_D = \frac{\Delta\rho g \lambda_o k_v t}{\phi_{ef} h_o} \quad (9.8)$$

where $\Delta\rho$ is the initial density difference at water-oil contact at reservoir conditions, and is given by:

$$\Delta\rho = \rho_w - \rho_o \quad (9.9)$$

with

$$\rho_w = \frac{\rho_{wstd}}{B_w} \quad (9.10)$$

and

$$\rho_o = \frac{\rho_{ostd} + \rho_{gstd} R_s}{B_o} \quad (9.11)$$

The oil mobility λ is given by Equation 9.3

8. Dimensionless rate

$$q_D = \frac{q_T}{2\pi \Delta\rho g \lambda_o \sqrt{k_v k_h} L h_o} \quad (9.12)$$

where q_T is the total rate at reservoir conditions.

9. Dimensionless cumulative oil recovery

$$N_{pD} = \frac{N_p \sqrt{\frac{k_v}{k_h}}}{2\pi \phi_{ef} L h_o^2} \quad (9.13)$$

10. Dimensionless pressure

$$p_D = \frac{p - p_{STD}}{\Delta \rho g h_o} \quad (9.14)$$

11. Dimensionless compressibility

$$c_{lD} = c_l \Delta \rho g h_o, \quad l = o, w \quad (9.15)$$

12. Normalized saturations

$$S_w^* = \frac{S_w - S_{wc}}{1 - S_{wc} - S_{or}} \quad (9.16)$$

$$S_o^* = \frac{S_o - S_{or}}{1 - S_{wc} - S_{or}} \quad (9.17)$$

so that

$$S_o^* + S_w^* = 1 \quad (9.18)$$

13. Normalized relative permeabilities

Relative permeabilities can be written as

$$k_{rw} = k_{rwo} k_{rw}^* \quad (9.19)$$

$$k_{ro} = k_{rocw} k_{ro}^* \quad (9.20)$$

The normalized relative permeabilities k_{rw}^* and k_{ro}^* are given by power functions of saturations as

$$k_{rw}^* = (S_w^*)^{n_w} \quad (9.21)$$

$$k_{ro}^* = (S_o^*)^{n_o} \quad (9.22)$$

Using this set of dimensionless variables, the dimensionless equations for oil and water can be written as, [13]

Oil:

$$\frac{\partial}{\partial x_D} \left[\frac{k_{ro}^*}{B_o} \frac{\partial \Phi_{oD}}{\partial x_D} \right] + \frac{\partial}{\partial z_D} \left[\frac{k_{ro}^*}{B_o} \frac{\partial \Phi_{oD}}{\partial z_D} \right] - \frac{2\pi q_{oD}}{B_o} = \frac{\partial}{\partial t_D} \left(\frac{S_o^*}{B_o} \right) \quad (9.23)$$

Water:

$$\frac{\partial}{\partial x_D} \left[\frac{k_{rw}^*}{B_w} \frac{\partial \Phi_{wD}}{\partial x_D} \right] + \frac{\partial}{\partial z_D} \left[\frac{k_{rw}^*}{B_w} \frac{\partial \Phi_{wD}}{\partial z_D} \right] - \frac{2\pi q_{wD}}{B_w} = \frac{1}{M} \frac{\partial}{\partial t_D} \left(\frac{S_w^*}{B_w} \right) \quad (9.24)$$

Where:

$$q_{lD} = \frac{q_l}{2\pi \Delta \rho g \lambda_l \sqrt{k_v k_h} L h_o}, \quad l = o, w \quad (9.25)$$

or

$$q_{lD} = \frac{q_D k_{rl}^*}{k_{ro}^* + M k_{rw}^*}, \quad l = o, w \quad (9.26)$$

and

$$q_{oD} + M q_{wD} = q_D \quad (9.27)$$

The mobilities are given by:

$$\lambda_l = \frac{k_{rwro}}{\mu_w} \quad (9.28)$$

These dimensionless equations show the important dimensionless variables that should be used to develop correlations for water cresting behavior. Those dimensionless variables are t_D , q_D , M and two dimensionless variables relating the shape of the relative permeabilities curves, which are chosen here as the exponents n_o and n_w . The dimensionless variable h_{wD} relating the location of the well given by Equation 9.6 and the well drainage distance given by Equation 9.5 are also used. The compressibility effects of B_o and B_w are given by Equations 9.10 and 9.11, which also consider the effect of a constant R_s for undersaturated reservoir.

The boundary conditions are no-flow on the lateral edge and on the top of the reservoir. A large aquifer to give pressure support is placed at the bottom of the reservoir.

An aquifer size of 200 times the reservoir size must be used to maintain the reservoir pressure without any injection. Smaller aquifers result in rapid decline of reservoir pressure, which normally results in changing the well control from constant production rate to specified bottom hole pressure. This completely changes the breakthrough time, since the assumption of constant production rate is no longer valid.

A large aquifer, however, can also be modeled with the same results using a line of injectors at the bottom of the aquifer with total injection matching total production, for an aquifer size of 20 times the reservoir size. This line of injectors was simulated here using a horizontal well injector in the transverse direction.

Also, in order to avoid high temporal numerical diffusion effects, the dimensionless time-step size should not be larger than one hundredth of the dimensionless breakthrough time ($\Delta t_D = 0.01 t_{bD}$).

9.3 Simulations

Equations 9.23 and 9.24 derived in Section 9.2 can be solved using a commercial simulator with the appropriate input data. The ECLIPSE 100 simulator [14], version 95a, was used for the simulations.

Figure 9.2 shows an example of the grid used to make the runs. The horizontal producer is represented by the point close to the top of the reservoir, drilled in the orthogonal direction y of the plane xz . The horizontal injector crosses the entire reservoir in the x direction and is located at the bottom of the aquifer.

Figure 9.2: Grid example for the simulations.

Since the main idea of the work is to derive correlations for breakthrough time and post-breakthrough behavior as functions of the gridblock sizes and the dimensionless variables described in the last section, several initial runs were performed in order to verify their sensitivities. This was done by varying horizontal and vertical gridblock sizes, lateral boundary size, aquifer size, time-step size and the dimensionless variables. The procedures used and initial conclusions reached are described next.

9.3.1 Grid Block Size

An initial set of runs demonstrated that breakthrough time and WOR are more affected by vertical gridblock size dimensions than by horizontal ones. This was expected since the crest moves from the water-oil contact to the well mainly in the vertical direction, so that the numerical diffusion effect in this direction is higher than in the horizontal one.

Assuming a constant horizontal pattern, a plot of the dimensionless breakthrough time (t_{btD}) against the number of blocks used in the vertical direction (n_{bz}) gives the behavior shown in Figure 9.3. This curve suggests an asymptotic shape tending to a value of t_{btD}^0 ,

which would represent the dimensionless breakthrough time when the number of blocks in the vertical direction tends to infinity.

Figure 9.3: Effect of number of blocks used between the top of the formation and water-oil contact on breakthrough time.

One of the simplest functions that can approximate this behavior is the following:

$$t_{btD} = t_{btD}^0 e^{-\frac{\alpha}{n_{bz}}} \quad (9.29)$$

If the vertical gridblock size remains constant throughout the oil column, Δz_D will be the inverse of n_{bz} and Equation 9.29 can be written as

$$t_{btD} = t_{btD}^0 e^{-\alpha \Delta z_D} \quad (9.30)$$

which gives a straight line in a plot of $\ln t_{btD}$ vs Δz_D . The parameters t_{btD}^0 and α are functions of the dimensionless variables described earlier. Two assumptions, however, must be made to write the correlations in such a way: a constant vertical gridblock size for each simulation and a horizontal grid pattern that minimizes the effects of the gridblock sizes on flow in that direction. These assumptions are analyzed next.

9.3.2 Vertical Gridblock Size Analysis

In the vertical direction, uniform grid with refinement close to the well works best. We found that $\Delta Z_D = 2 \times 0.01, 0.02, 2 \times 0.03, 9 \times 0.1$ was almost as good as a uniform grid of $\Delta Z_D = 0.01$.

9.3.3 Horizontal Gridblock Size Analysis

In the horizontal direction, the suggested pattern is to use the same size (smallest gridblock size in the vertical direction) for the first two blocks and then start doubling the

Oil column, h	100 m
Horizontal permeability, k_h	1 d
Vertical permeability, k_v	1 d
Porosity, ϕ	0.1
Oil density, ρ_o (standard)	1000 kg/m^3
Water density, ρ_w (standard)	900 kg/m^3
Gas density, ρ_g (standard)	1 kg/m^3
Gas-oil solubility ratio Rs	0
Oil viscosity, μ_o	1 cp
Water viscosity, μ_w	1 cp
Oil compressibility, c_o	1e-5 bar^{-1}
Water compressibility, c_w	1e-5 bar^{-1}
Rock compressibility, c_r	0.0
Connate water saturation, S_{wc}	0
Residual oil saturation, S_{or}	0
Oil relative permeability exponent, n_o	1
Water relative permeability exponent, n_w	1
Oil relative permeability at S_{wc} , k_{rocw}	1
Water relative permeability at S_{or} , k_{rwro}	1
Production rate, $q(q_D = 0.1)$	5428.74 m^3/day

Table 9.1: Base case data set for simulation runs

gridblock size up to the lateral boundary. For example, if the smallest vertical gridblock size is 0.01, then the horizontal gridblock sizes are $\Delta x_D = 0.01, 0.01, 0.02, 0.04, 0.08, 0.16, \dots$. The size of the last block is adjusted for the location of the lateral boundary. The results based on this pattern were compared with four other patterns and only minor differences were seen. However, the proposed pattern has the advantage that it gives the best approach to a straight line and therefore gives the best match with Equation 9.30.

9.4 Water Cresting Correlations

The base case for the runs was defined for the situation when the reservoir behaves as infinite, with mobility equal to one. The well is at the top of the reservoir and relative permeability curves are straight lines. The input data for the ECLIPSE simulator [14] for the base case are shown in Table 9.1.

From this base case, each dimensionless variable was varied over a large range of practical situations, as shown in Table 9.2. For example, the dimensionless rate varies from 0.005 to 1.0, which using the ECLIPSE data shown in Table 9.1, corresponds to a variation in the dimensional rate from 250 m^3/day to 50000 m^3/day .

For each value of dimensionless variable, six vertical gridblock size Δz_D were used, with n_{bz} varying from 20 to 200, covering the vertical range for most of the practical simulations. Table 9.3 shows this range, with the horizontal gridblock sizes following the pattern

DIMENSIONLESS VARIABLE	BASE CASE	VARIATION
q_D	0.1	0.005 to 1
M	1	0.1 to 100
x_{eD}	20	2.5 to 20
h_{wD}	1	0.2 to 1
n_o	1	1 to 7
n_w	1	1 to 3

Table 9.2: Base case for the simulations and range of variation of each dimensionless variable.

Δz_D	n_{bz}	Δx_D	n_{bx}
0.005	200	0.005, 0.005, 0.01, 0.02, 0.04, 0.08, 0.16, ..., 5.12, 9.76	13
0.01	100	0.01, 0.01, 0.02, 0.04, 0.08, 0.16, ..., 5.12, 9.76	12
0.02	50	0.02, 0.02, 0.04, 0.08, 0.16, ..., 5.12, 9.76	11
0.025	40	0.02, 0.02, 0.04, 0.08, 0.16, ..., 5.12, 9.76	11
0.04	25	0.04, 0.04, 0.08, 0.16, ..., 5.12, 9.76	10
0.05	20	0.04, 0.04, 0.08, 0.16, ..., 5.12, 9.76	10

Table 9.3: Grid patterns for the simulations.

suggested in the last chapter.

The next sections describe the specific procedures used to obtain the correlations. It took about 5000 runs with the ECLIPSE simulator [14] to obtain the results of this work. Each run varied from 1 minute to 4 hours of CPU time in an alpha machine, with a total of around 3000 CPU hours.

9.4.1 Breakthrough Time

The correlations for t_{btD} were derived using Equation 9.30, which can be rewritten as:

$$\ln \frac{t_{btD}^0}{t_{btD}} = \alpha \Delta z_D \quad (9.31)$$

which means that $\ln t_{btD}$ is a linear function of Δz_D , with t_{btD}^0 and α being the parameters to be found.

The procedure used was to record the values of breakthrough time for each run, defined to be the time to reach a water-cut of 0.001. Then, a linear regression with the six points of Δz_D and t_{btD} was performed, using Equation 9.31. Figure 9.4 shows one example of such a match, for $q_D = 0.5$.

With this procedure, for each dimensionless variable, the values of t_{btD}^0 and α were obtained. These values are shown in Table 9.4, together with parameter m for post-breakthrough behavior defined in Section 9.4.2.

q_D	M	x_{eD}	h_{wD}	n_o	n_w	t_{btD}^0	α	m
0.1	1	20	1	1	1	4.3889	1.6048	0.5170
0.15						2.4227	2.3122	0.5058
0.2						1.6310	2.9727	0.5000
0.3						0.9653	4.0448	0.4866
0.5						0.5243	6.9362	0.4678
0.7						0.3445	9.0971	0.4447
1.0						0.2300	11.1761	0.4366
0.07						7.7275	1.2328	0.5353
0.05						13.5931	0.8270	0.5576
0.03						33.9145	0.5843	0.6013
0.02						72.0729	0.4114	0.6531
0.01						254.285	0.1537	0.9917
0.005						729.435	0.1444	1.6030
	2					3.6525	1.5111	0.5538
	5					3.1642	1.5884	0.5897
	10					2.9799	1.4732	0.6094
	50					2.8323	1.5454	0.6335
	100					2.8003	1.4203	0.6371
	0.5					5.6395	1.3949	0.4745
	0.1					12.6440	1.3741	0.4062
		10				4.3514	1.6373	0.5329
		5				4.1576	1.5188	0.6129
		2.5				3.5721	1.6658	0.9256
			0.8			3.5447	2.4520	0.4451
			0.6			2.2080	3.5495	0.3634
			0.4			0.9943	6.7327	0.2715
			0.2			0.2495	29.6500	0.2074
				2		4.1237	2.1519	0.4938
				3		3.6852	2.6456	0.4668
				4		3.2572	2.8951	0.4492
				5		2.9260	3.2076	0.4410
				6		2.6515	3.0973	0.4355
					2	4.4433	0.7300	0.5196
					3	4.4626	0.3964	0.5201

Table 9.4: Values of t_{btD}^0, α for breakthrough time and m for q_{oD} for each dimensionless variable simulated. A blank entry indicates that dimensionless variable has its base value (first row).

Figure 9.4: Linear regression of t_{btD} vs Δz_D for $q_D = 0.5$

Next, the correlations for the parameters t_{btD}^0 and α as functions of the dimensionless variables were developed. The variations in dimensionless variables were made from the base case, that is, only one dimensionless variable was varied for each set of Δz_D . It was assumed that each dimensionless variable could be matched independently, so that the final correlation is a combination of several functions of just one variable. This assumption implies that the effect of simultaneous variations in two or more dimensionless variables can be calculated as a combination of separate variations in each dimensionless variable from the base case. The runs showed that this is a good assumption for most combinations of dimensionless variables except for simultaneous variations of q_D with M , x_{eD} or n_o . For this reason, an extra set of runs varying these dimensionless variables was made and the values of t_{btD}^0 and α for these runs are shown in Tables 9.5 and 9.6. Extra functions of these dimensionless variables were attached to each correlation to account for this.

The values of q_D were matched with each value of t_{btD}^0 and α . The other parameters were matched as correction factors, that is, the ratio between each value and its correspondent base case value. For example, each value of t_{btD}^0 for the well location $h_{wD} = 0.8, 0.6, 0.4$ and 0.2 was divided by 4.3889 (see Table 9.4), which is the value of t_{btD}^0 for the base case. These ratios were then used in the regression with the h_{wD} values. In this way, any dimensionless variable in the following correlations, except for q_D , can be skipped for a quick estimate of breakthrough time, if this dimensionless variable has its base case value. For example, the base case value for mobility ratio is $M=1$. If this value can be assumed for a real field situation, the factors for M in Equations 9.34 and 9.35 are equal to one and can be skipped from the correlations.

It could also be seen that in some situations a dimensionless variable has a small effect on results. In these situations, the dimensionless variable was not considered in the correlations.

A combination of linear regression and the polytope procedure [15] was used for

q_D	M	x_{eD}	t_{btD}^0	α	m
0.2	2		1.2476	3.2477	0.5481
0.2	5		0.9673	3.8930	0.6008
0.2	10		0.8432	4.4230	0.6119
0.2	50		0.7190	4.8453	0.6385
0.2	0.5		2.2655	2.2006	0.4498
0.5	2		0.3388	11.8824	0.4356
0.5	5		1.1401	14.0348	0.0744
0.5	0.5		0.7745	4.0770	0.4309
0.05	2		12.2626	0.8370	0.6038
0.05	5		11.3972	0.8720	0.6405
0.05	10		11.0968	0.8493	0.6559
0.05	50		10.8592	0.8819	0.6733
0.05	100		10.8220	0.8669	0.6789
0.05	0.5		16.0048	0.7757	0.5154
0.02	2		69.6013	0.5326	0.6697
0.02	5		68.0938	0.6324	0.6849
0.02	10		67.5586	0.6661	0.6922
0.02	50		67.0587	0.5439	0.6938
0.02	0.5		76.7792	0.3609	0.6321
0.2		10	1.6250	2.9871	0.5081
0.2		5	1.5842	3.0518	0.5499
0.2		2.5	1.4496	3.3277	0.7285
0.5		10	0.5196	6.7655	0.4730
0.5		5	0.5137	7.0519	0.5011
0.5		2.5	0.4815	7.4504	0.6114
0.05		10	13.3166	0.7036	0.5953
0.05		5	12.1040	0.7787	0.8287
0.05		2.5	9.0261	0.8581	1.3244
0.02		10	64.7916	0.1272	0.9707
0.02		5	46.3915	0.2128	1.4986
0.02		2.5	28.2599	0.2392	2.1657
0.01		10	183.151	0.1444	1.5506
0.01		5	111.778	0.1945	2.2586
0.01		2.5	62.973	0.1296	3.2044

Table 9.5: Values of t_{btD}^0, α for breakthrough time and m for q_{oD} for simultaneous variations of q_D with M or x_{eD} . h_{wD} , n_o and n_w are all equal to 1, their base values. Also a blank entry indicates that dimensionless variable has its base value.

q_D	n_o	t_{btD}^0	α	m
0.2	2	1.4872	4.0277	0.4711
0.2	4	1.1382	4.3968	0.4215
0.2	6	0.9393	4.4910	0.4153
0.5	2	0.4549	7.2575	0.4778
0.5	4	0.3507	6.4689	0.3952
0.5	6	0.2921	5.7655	0.4011
0.05	2	13.191	1.2828	0.5409
0.05	4	10.968	1.9781	0.4935
0.05	6	9.007	2.2315	0.4704
0.02	2	71.1536	0.4904	0.6466
0.02	4	63.4909	0.9991	0.6064
0.02	6	53.9141	1.3018	0.5805
0.01	2	253.433	0.3534	0.9822
0.01	4	237.871	0.8563	0.9048
0.01	6	207.192	1.0771	0.8847

Table 9.6: Values of t_{btD}^0, α for breakthrough time and m for q_{oD} for simultaneous variations of q_D with n_o . $x_{eD} = 20$ and M, h_{wD} , and n_o are equal to 1, their base values.

developing correlations. Since more than one function could normally match the points, the criteria used to select the best function for each match was a maximum error of 5% in the simplest possible function. All the functions can be categorized in two main groups:

1. Polynomial functions:

$$f(y) = A + Bf(x) + Cf(x)^2 + Df(x)^3 \quad (9.32)$$

with $f(x) = x$ or $\ln(x)$ and $f(y) = y$ or $\ln(y)$.

2. Extensions of the linear functions:

$$f(y) = A(Bf(x) + C)^D + E \quad (9.33)$$

with $f(x) = x^a$ or $\ln(x + a)$ or e^{ax^b} and $f(y) = y$.

The parameters t_{btD}^0 and α can be written as:

$$t_{btD}^0 = ft(q_D)ft(M)ft(x_{eD})ft(h_{wD})ft(n_o)ft(n_w)ft(q_D, M)ft(q_D, x_{eD})ft(q_D, n_o) \quad (9.34)$$

and

$$\alpha = fa(q_D)fa(h_{wD})fa(n_o)fa(n_w)fa(q_D, M) \quad (9.35)$$

where the functions ft and fa are given by either Equation 9.32 or Equation 9.33. The equations and functions to be used for each factor are given in Table 9.7. The values of each constant are given in Table 9.8.

For example, to calculate $ft(q_D)$ in Equation 9.34 for $q_D \geq 0.1$, Table 9.7 gives the equation to be used as:

$$ft(q_D) = A(Bq_D^a + C)^D + E \quad (9.36)$$

with the constants given by the values shown in the first line of Table 9.8.

The extra correction factors are given by:

$$ft(q_D, M) = a ft(M) + b ft(M) \ln(q_D) \quad (9.37)$$

$$ft(q_D, x_{eD}) = a ft(q_D) + b ft(q_D) \ln(x_{eD}) \quad (9.38)$$

$$ft(q_D, n_o) = a ft(n_o) + b ft(n_o) \ln(q_D) \quad (9.39)$$

$$fa(q_D, M) = a fa(M) + b fa(M) q_D \quad (9.40)$$

where the factors are also shown in Tables 9.7 and 9.8.

9.4.2 Post-Breakthrough Behavior

The correlations for post-breakthrough time behavior were derived using a solution based on vertical equilibrium, that can be written as [12]:

$$q_{oD} = q_D \left(\frac{t_{btD}}{2t_D - t_{btD}} \right)^m \quad (9.41)$$

where m is the parameter to be calculated by a nonlinear regression with the dimensionless variables and is equal to 0.5 for the vertical equilibrium assumption used by Tiefertal [12].

The water-cut f_w is given by:

$$f_w = 1 - \frac{q_{oD}}{q_D} \quad (9.42)$$

and WOR at standard conditions by:

$$WOR = \frac{B_o}{B_w} \left(\frac{q_D}{q_{oD}} - 1 \right) \quad (9.43)$$

The simulations were performed normally up to $t_D = 3 t_{btD}$ and for this range m showed only small variations with the gridblock sizes, so that the results of the most refined grid of the pattern, that is, $\Delta z_D = 0.005$, were used for each value of dimensionless variable to perform the regressions. The Gauss-Newton method [15] was used for these computations. These values are shown in Tables 9.4 and 9.5. When the dimensionless variables were not far from the base case, the runs demonstrated that the values of m calculated by these matches can normally be used for t_D of up to $50 t_{btD}$.

Function	Equation	f(x)	f(y)
$ft(q_D), q_D \geq 0.1$	9.33	x^a	y
$ft(q_D), q_D < 0.1$	9.32	$\ln x$	$\ln y$
$ft(M)$	9.32	$\ln x$	$\ln y$
$ft(x_{eD})$	9.33	$\ln(x + a)$	y
$ft(h_{wD})$	9.32	x	y
$ft(n_o)$	9.33	e^{ax^b}	y
$ft(n_w)$	9.33	e^{ax^b}	y
$aft(M)$	9.32	$\ln x$	$\ln y$
$bft(M)$	9.32	$\ln x$	y
$aft(q_D)$	9.32	$\ln x$	y
$bft(q_D)$	9.32	$\ln x$	y
$aft(n_o)$	9.32	$\ln x$	y
$bft(n_o)$	9.32	$\ln x$	y
$fa(q_D)$	9.32	$\ln x$	$\ln y$
$fa(h_{wD})$	9.33	x^a	y
$fa(n_o)$	9.33	x^a	y
$fa(n_w)$	9.33	x^a	y
$afa(M)$	9.33	x^a	y
$bfa(M)$	9.32	$\ln x$	y
$fm(q_D)$	9.32	$\ln x$	$\ln y$
$fm(M)$	9.32	$\ln x$	y
$fm(x_{eD})$	9.32	$\ln x$	$\ln y$
$fm(h_{wD})$	9.32	x	y
$fm(n_o)$	9.33	e^{ax^b}	y
$afm(q_D)$	9.32	$\ln x$	$\ln y$
$bfm(q_D)$	9.32	$\ln x$	y

Table 9.7: Equations and functions for the correlations of t_{btD}^0 , α and $m(q_{oD})$.

Function	a	b	A	B	C	D	E
$ft(q_D), q_D \geq 0.1$	-0.36187	-	0.63589	0.32543	0.55970	7.20687	-0.02716
$ft(q_D), q_D < 0.1$	-	-	0.56278	1.01046	0.77215	0.06918	-
$ft(M)$	-	-	0.00237	-0.29231	0.06269	-0.00440	-
$ft(x_{eD})$	0.00023	-	1.0	0.51495	-0.45280	0.05202	0.0
$ft(h_{wD})$	-	-	0.12106	-1.07741	4.23812	-2.28177	-
$ft(n_o)$	-0.07799	1.12590	1.0	1.09189	0.0	1.0	0.0
$ft(n_w)$	0.00833	1.0	1.0	0.99303	0.0	1.0	0.0
$aft(M)$	-	-	-0.02015	-0.37233	-0.05630	0.01269	-
$bft(M)$	-	-	-0.00173	-0.13537	-0.00481	0.00469	-
$aft(q_D)$	-	-	0.96112	-0.40901	-0.20875	-0.01565	-
$bft(q_D)$	-	-	0.03092	0.18297	0.09483	0.00875	-
$aft(n_o)$	-	-	1.0	0.14401	-0.39431	0.11374	-
$bft(n_o)$	-	-	0.0	0.05645	-0.16935	0.05053	-
$fa(q_D)$	-	-	2.44506	0.74312	-0.08108	-0.01379	-
$fa(h_{wD})$	-5.16519	-	0.95795	0.92981	-0.00023	0.34618	0.05673
$fa(n_o)$	0.43174	-	1.0	1.00177	0.0	1.0	0.0
$fa(n_w)$	1.0	-	1.0	0.33433	0.66566	-2.73073	0.0
$afa(M)$	-0.16534	-	1.0	0.96120	0.0	1.0	0.0
$bfa(M)$	-	-	0.18891	1.61377	0.0	0.0	-
$fm(q_D)$	-	-	-0.85501	-0.22928	-0.11837	-0.02331	-
$fm(M)$	-	-	0.99762	0.09478	-0.00288	-0.00150	0.0
$fm(x_{eD})$	-	-	1.05994	-0.22414	-0.14543	0.03404	0.0
$fm(h_{wD})$	-	-	0.22840	0.7787	0.0	0.0	0.0
$fm(n_o)$	-0.04130	1.0	1.0	1.03490	0.0	1.0	0.0
$afm(q_D), q_D \geq 0.02$	-	-	-0.35914	0.21539	0.17615	0.00654	-
$bfm(q_D), q_D \geq 0.02$	-	-	0.18917	0.13644	0.09088	0.02874	-
$afm(q_D), q_D < 0.02$	-	-	-0.07212	0.81646	0.50476	0.05771	-
$bfm(q_D), q_D < 0.02$	-	-	-0.20607	-0.67350	-0.33860	-0.03607	-

Table 9.8: Values of the constants for the correlations of t_{btD}^0, α and $m(q_{oD})$. A - means that the constant does not appear in the equation for that factor.

The value $m = 0.5$ can be used for a quick estimate of post-breakthrough behavior when the dimensionless variables are close to the base case, but as the dimensionless variables move away from the base case, m varies considerably.

A procedure similar to that used to obtain the correlations for the parameters of breakthrough time was used for the parameter m . Again the functions described by Equations 9.32 and 9.33 can be applied. Dimensionless rates are still matched using their original values and other parameters are added as correction factors. An extra correction factor to account for simultaneous variations of q_D and x_{eD} was also added.

Figure 9.5: Comparison of breakthrough time and q_{oD} correlations with simulation results for the base case.

The correlation can be written as:

$$m = fm(q_D)fm(M)fm(x_{eD})fm(h_{wD})fm(n_o)fm(q_D, x_{eD}) \quad (9.44)$$

where the functions and corrections factors are given by either Equation 9.32 or Equation 9.33.

The correction factor $fm(q_D, x_{eD})$ is given by:

$$fm(q_D, x_{eD}) = afm(q_D) + bf(q_D) \ln(x_{eD}) \quad (9.45)$$

The equations and functions to be used for each factor are given in Table 9.7. The values of each constant are given in Table 9.8.

Figure 9.5 shows a comparison of q_{oD} vs t_D calculated by the correlations and by simulations for the base case. The plot of WOR is in Figure 9.6. Except for a small deviation in the beginning of the water production, a good match is obtained.

9.4.3 Cumulative Oil Recovery

The dimensionless cumulative oil recovery is given by the area below the curve in Fig. 9.5. Numerically, it can be written for a given t_D as:

Figure 9.6: Comparison of breakthrough time and WOR correlations with simulation results for the base case.

Figure 9.7: Comparison of N_{pD} vs t_D between the correlations and the simulation results for the base case.

$$N_{pD} = q_D t_{btD} + \int_{t_{btD}}^{t_D} q_{oD}(\tau) d\tau \quad (9.46)$$

where the first term indicates N_{ptbtD} , that is, N_{pD} up to breakthrough time and the second indicates N_{pD} for post-breakthrough behavior. Using Equation 9.41 and integrating, N_{pD} can be written as:

$$N_{pD} = \frac{1-2m}{2-2m} N_{ptbtD} + \frac{1}{2-2m} [q_{oD}(t_D)(2t_D - t_{btD})] \quad (9.47)$$

The plot of N_{pD} vs t_D for the base case for both simulation and correlation is shown in Figure 9.7, where a good match is still obtained.

9.4.4 Optimum Grid

The optimum grid is defined here as the gridblock size that gives the minimum number of blocks for the simulation with a small variation in breakthrough time, N_{pD} and WOR behavior when compared with a refined grid. A refined grid is the gridblock size that gives practically no variation in those parameters when its dimension is reduced.

Here we just need to find the variation of the vertical block dimension Δz_D , since the horizontal dimension Δx_D can be specified using the pattern described earlier.

Since t_{btD}^0 is the limit for breakthrough time when the number of blocks in the vertical direction tends to infinity, this value can be used as a reference to calculate a refined or an optimum grid. Equation 9.31 can be used to find a value of Δz_D for a specified ratio t_{btD}^0/t_{btD} .

At first, any practical criterion can be used to specify a refined vertical gridblock size. From what is reported in the literature and the experience with several runs, it is suggested that a vertical gridblock size that gives a difference less or equal to 1% in breakthrough time when compared with the parameter t_{btD}^0 be used. From Equation 9.31, using $t_{btD} = 0.99 t_{btD}^0$, a refined grid can be found as:

$$\Delta z_{Dref} = 0.01\alpha^{-1} \quad (9.48)$$

Similarly, it can be suggested that the optimum grid is the vertical gridblock size that obeys the following criteria:

1 - maximum variation of 5% in breakthrough time when compared with a refined vertical gridblock size result.

2 - Maximum variation of 3% in N_{pD} at $t_D = 3 t_{btD}$ when compared with a refined vertical gridblock size result.

3 - Visual inspection for WOR variation, that normally loses its smooth behavior for coarse grids.

Using Equation 9.31, the criterion for breakthrough time (variation of 5% for a refined grid) can be written as:

$$\Delta z_{Dopt} = 0.06\alpha^{-1} \quad (9.49)$$

The runs showed that this criterion is normally enough to calculate the optimum grid, as the other two criteria defined above are also almost satisfied.

Table 9.9 shows the variation of breakthrough time and N_{pD} for the base case for several Δz_D . Some of them are plotted in Fig. 9.8. Equation 9.48 gives $\Delta z_{Dref} = 0.0062$ for

Δz_D	t_{btD}	Var(%)	N_{pD}	Var(%)
0.005	4.3547	-	0.9617	-
0.01	4.3144	0.9	0.9555	0.6
0.02	4.2542	2.3	0.9472	1.5
0.03	4.2191	3.1	0.9419	2.1
0.04	4.1111	5.5	0.9257	3.7
0.05	4.0522	6.9	0.9162	4.7
0.1	3.8794	8.6	0.8872	7.7
0.2	3.5770	17.8	0.8292	13.8

Table 9.9: Variations in breakthrough time and N_{pD} with vertical gridblock size for the base case.

this situation, which can be roughly rounded to 0.005. Using Equation 9.49, the optimum grid for the base case is $\Delta z_D = 0.038$, which can be rounded to 0.04, the value used here.

9.5 Applications

Besides the immediate calculation of breakthrough time, q_{oD} , f_w , WOR, N_p , RF and optimum grid, the derived correlations can be used for some additional applications.

9.5.1 Maximum Rate

The derived correlations allow the calculation of any dimensionless variable if the others and the breakthrough time are known. Then, for a given t_{btD}^0 , a maximum rate to be imposed on the production well can be found. This means that any rate below this q_{max} gives a breakthrough time longer than the desired one. This maximum rate is not the critical rate in the classical meaning, since the boundary conditions used to develop these correlations are no-flow boundaries. Some works in the literature, however, refer to this rate as the critical rate [16] [17].

The value of q_{Dmax} can not be calculated explicitly from Equation 9.34, so a numerical procedure must be performed to obtain it. Newton's method was chosen here for such numerical calculations. The equation to be solved is:

$$F(q_D) = Aft(q_D)ft(q_D, M)ft(q_D, x_{eD})ft(q_D, n_o) - t_{btD}^0 = 0 \quad (9.50)$$

where A is the product of the other factors in Equation 9.34. Once q_{Dmax} is known, q_{max} can be calculated from Equation 9.12.

9.6 A field produced by several wells

For a field with evenly spaced wells as shown in Figure 9.9 and equal production rate for each well, the total production can be calculated using the derived correlations for any number of wells, located at the same reservoir height.

Figure 9.8: Effect of vertical gridblock size on WOR as a function of time.

The rate of each well is given by:

$$q = \frac{q_T}{n_{wel}} \quad (9.51)$$

The dimensionless rate q_D can be calculated by Equation 9.12. The drainage distance x_{eD} for each well is given by:

$$x_{eD} = \frac{X_D}{2n_{wel}} \quad (9.52)$$

where X_D is the dimensionless total extent of the reservoir, given by:

$$X_D = \frac{X}{h_o} \sqrt{\left(\frac{k_v}{k_h}\right)} \quad (9.53)$$

N_{pD} for the entire field can be calculated by Equation 9.47 using the total dimensionless rate q_{DT} . Figure 9.10 shows the cumulative oil recovery for the entire field with four wells using the rate of the base case as the rate q_T . Again, the match with simulation when all wells are considered is good.

9.7 Estimation of breakthrough time

One relevant application of the procedure used to correlate t_{btD} with Δz_D is the calculation of an accurate breakthrough time from coarse grid simulations. For the simulation of any field, the pattern described in Section 9.3 can be used to find the value of t_{btD}^0 using just two coarse grid simulations. This is shown in Figure 9.11. The two points on the straight line represent the coarse grid simulations for $\Delta z_D = 0.05$ and $\Delta z_D = 0.04$. Matching these

Figure 9.9: Evenly spaced wells in a field with a specified total rate

two points with a straight line, the extrapolation of the line to $\Delta z_D = 0$ gives the value of t_{btD}^0 , which represents the breakthrough time for an infinite number of blocks in the vertical direction. This value of breakthrough time can then be used to predict the performance of the reservoir.

9.8 Pseudofunctions

The application described in the last section can be extended for the development of pseudofunctions to match the breakthrough time of coarse gridblock size simulations with breakthrough time from refined ones. The main idea is to change one of the dimensionless variables related with the shape of the relative permeability curves, that is, the exponents n_o or n_w or one of their end-points (which ultimately changes the mobility ratio M), so that the velocity of the flow is also changed and this change compensates for the numerical diffusion created with the use of coarse gridblock sizes.

Section 9.4 described how the families of straight lines of t_{btD} vs Δz_D were obtained by linear regression. Figure 9.12 shows straight lines from the family of n_o . The idea is to use a coarse grid simulation ($\Delta z_D = 0.05$ in Figure 9.12) and obtain the same breakthrough time if a refined grid simulation ($\Delta z_D = 0.005$ Figure 9.12) was used. In other words, the objective is to find a value of n_{op} (pseudo n_o) that matches the breakthrough time from a coarse grid simulation that uses n_o and $\Delta z_D(\Delta z_D(n_o))$, with that from a smaller $\Delta z_{Dp}(\Delta z_D(n_{op}))$. The value $t_{btDp}(t_{btD}(n_{op}))$ for the refined Δz_{Dp} can be obtained throughout the straight line of n_o , that is, from the point A to point B shown in the Figure. The value of n_{op} is then obtained by extending a horizontal straight line from this value (point B) toward right to the vertical line that passes Δz_D (point C). The value of n_o from the family of straight lines that crosses this latter point is n_{op} . Normally, an interpolation is needed for this graphical procedure.

Figure 9.10: N_{pD} vs t_D curves for a field with four wells

Numerically, this means that we need to use Equation 9.31 to calculate n_{op} , once t_{btDp} is known. As in the case of the calculation of maximum rate, a numerical procedure is needed to obtain n_{op} . Again, Newton's method is applied. Using the values of n_{op} and t_{btDp} in Equation 9.31 and rearranging, the equation to be solved is:

$$F(n_{op}) = \ln[Aft(n_{op})ft(q_D, n_{op})] - Bfa(n_{op}) = 0 \quad (9.54)$$

Where A is the product of the other factors in Equation 9.34 divided by t_{btDp} and B is the product of the other factors in Equation 9.35 times $\Delta z_D(n_o)$.

Figure 9.13 shows an application for the base case for n_o pseudofunction. The original value of n_o is 2 and the procedure described above leads to a value of n_{op} equal to 1.08. The match of breakthrough time is practically exact, with some difference in WOR at the beginning but, after that, all curves merge into one.

It should be noted that the new value of n_{op} is always less than the original value n_o , which means that n_{op} can be less than 1, a situation where the relative permeability curve does not have a physical meaning, but still can be used to match the breakthrough time. The procedure to obtain pseudofunctions based on n_w (n_{wp}) and M (M_p) is similar to that for n_{op} .

The procedures described above can be used in any simulation regardless of correlations. For this, two coarse grid simulations using the suggested pattern must be performed for each value of n_o , n_w or M (k_{rwro}). These simulations generate a family of straight lines that can be used to calculate the new value n_{op} , n_{wp} or M_p by a procedure similar to that described above. Fig. 9.14 shows an example of such a procedure. Each straight line corresponds to a value of a dimensionless variable n_o , n_w or M . Using the graphical procedure described to obtain the pseudofunctions, the value of the dimensionless variable that matches breakthrough time can be found by interpolation from the straight lines. This value can be

Figure 9.11: Extrapolation of two coarse grid simulations to find t_{btD}^0 .

Figure 9.12: Procedure to obtain n_{op} for a given n_o and Δz_D .

Figure 9.13: Application of pseudo n_o for the base case data.

Figure 9.14: Variation of n_o , n_w or M to generate pseudofunctions for any simulation.

used to match breakthrough time from one of the coarse grid simulations with the value of t_{btD}^0 or a value of t_{btD} of a refined grid block.

9.8.1 Gas Cresting Behavior

The correlations were derived for water-oil flow, but since only dimensionless variables were used and the flow is governed mainly by the differences in densities and mobilities, they can be used exactly in the same way for gas-oil flow. The parameters and subscripts used for water should be now used for gas instead.

$$GOR = \frac{B_o}{B_g} \left(\frac{q_D}{q_{oD}} - 1 \right) + R_s \quad (9.55)$$

Figure 9.15 shows one example of breakthrough time and GOR performance for a gas-oil case application using the same reservoir parameters as the base case, but with $\Delta\rho = 400 \text{ kg/m}^3$, $B_g = 0.012$ and $M = 10$. The cumulative oil recovery is shown in Fig. 9.16. Both matches show that the correlations derived are robust and they can also be used for gas-oil flow with good accuracy.

9.9 A Field Example

A program was developed for the correlations and applications shown in this work and applied for a real field. The data for the field were obtained from an input data for the ECLIPSE simulator [14] supplied by the field operator and are shown in Table 9.10. The relative permeability data are given in Tables 9.11 and 9.12.

The field has both an aquifer and a gas cap and is produced by a horizontal well placed in the middle of the oil column. Since the correlations were derived for two-phase flow, they were applied for each phase separately assuming the well penetrated the entire width of the reservoir, made equal to the well length. For this situation, only half of the oil column must be used to apply the correlations. This means that the well will be at the top of the bottom half of reservoir for water-oil flow calculations and at the bottom of the top half of the reservoir for gas-oil flow calculations. It is assumed only half of the total rate to be used for each situation. This assumption is only valid until breakthrough time and if the flow resistances above and below the well are the same, that is, the contrast in densities and mobilities are the same for water-oil and gas-oil flows. This is not the case for this real field, but this assumption can be used as a first approach to check the accuracy of the correlations when applied using field data.

9.9.1 Water-oil flow

The water data generate the dimensionless variables shown in Table 9.13. Also shown in the same Table are the values of t_{btD} and m calculated for these data using the derived correlations.

The results given by the correlations were compared with two simulations using the ECLIPSE simulator [14]: (1) the two-phase flow using the half of the oil column and half of the total rate with the well at the top of the reservoir and, (2) a three-phase flow using the original data for the field, with the well centered in the reservoir.

Figure 9.15: Comparison of breakthrough time and GOR from correlations with simulation results for gas-oil flow

Figure 9.16: Comparison of N_{pD} calculated from correlations to simulation results for gas-oil flow

Oil column, h	40 m
Reservoir width, W	1300 m
Reservoir extension, X	900 m
Horizontal permeability, k_h	3 d
Vertical permeability, k_v	1 d
Porosity, ϕ	0.2
Oil formation volume factor, B_o	1.965 rm^3/sm^3
Water formation volume factor, B_w	1.03 rm^3/sm^3
Gas formation volume factor, B_g	0.004 rm^3/sm^3
Oil density, ρ_o (standard)	784.7 kg/m^3
Water density, ρ_w (standard)	1065 kg/m^3
Gas density, ρ_g (standard)	0.995 kg/m^3
Gas-oil solubility ratio, Rs	312 sm^3/sm^3
Oil viscosity, μ_o	0.22 cp
Water viscosity, μ_w	0.5 cp
Gas viscosity, μ_g	0.02 cp
Well length, L	500 m
Well height, h_w	20 m
Production rate, q	1000.00 m^3/day

Table 9.10: Field data set used to apply the developed correlations.

S_w	k_{rw}	k_{row}
0.260	0.0	0.890
0.314	0.0116	0.623
0.368	0.0232	0.427
0.422	0.0348	0.276
0.476	0.0580	0.169
0.530	0.0928	0.107
0.584	0.1421	0.0445
0.638	0.2175	0.0267
0.692	0.3103	0.0178
0.746	0.4353	0.0045
0.800	0.580	0.0

Table 9.11: Water-oil relative permeability curves for field data.

S_g	k_{rg}	k_{rog}
0.0	0.0	0.890
0.074	0.0275	0.3218
0.148	0.0688	0.209
0.222	0.1100	0.1348
0.296	0.1650	0.0825
0.370	0.2283	0.04125
0.444	0.2943	0.02475
0.518	0.3768	0.0110
0.592	0.4593	0.0055
0.740	0.5500	0.0

Table 9.12: Gas-oil relative permeability curves for field data.

q_D	0.0052
M	0.287
x_{eD}	13.0
h_{wD}	1.0
n_o	3.25
n_w	2.70
t_{btD}	477.0
m	1.84

Table 9.13: Dimensionless variables for water-oil flow for the field data.

q_D	0.0085
M	6.798
x_{eD}	13.0
h_{wD}	1.0
n_o	7.35
n_w	1.23
t_{btD}	211.0
m	1.58

Table 9.14: Dimensionless variables for gas-oil flow for real field data.

Figure 9.17 shows the comparison between these runs with the results of the correlations for *WOR* vs $t(days)$. The value of breakthrough time calculated from the correlations is $t = 618 \text{ days}$, which compared with the value $t = 638 \text{ days}$ calculated by the simulator for two-phase flow gives a difference of 3%, which is within the expected range of errors of the correlations. The *WOR* behavior also shows that the correlations give good results for the field situation.

The results of the simulation for three-phase flow shows larger deviations from the correlations. The difference in breakthrough time, although still small (3%), is caused by the differences in the properties of water and gas. As explained, the use of half of the rate for each two-phase flow is only valid if the contrasts in densities and mobilities are the same for water-oil and gas-oil flow. The larger differences in *WOR* are due to another factor besides the contrast in densities and mobilities between water and gas: the well calculations performed by the simulator after breakthrough time are made using three-phase relative permeabilities [14], which changes both *WOR* and *GOR* behaviors. However, the general behavior of simulation and correlation results is very similar.

9.9.2 Gas-oil flow

The gas data generate the dimensionless variables shown in Table 9.14. Also shown in the same table are the values of t_{btD} and m calculated for these data using the derived correlations.

The results of the correlations were again compared with those from the ECLIPSE simulator using two approaches: (1) two-phase flow using the half of the oil column with the well at the bottom of the reservoir and, (2) three-phase flow using the original data for the field, with the well centered in the reservoir, that is, the same simulation used to obtain the results for water-oil flow discussed in Section 9.9.1.

Figure 9.18 shows the comparison between these runs and the results of the correlations for *GOR* vs $t(days)$. The value of t_{bt} calculated by the correlations is $t = 580 \text{ days}$, which compared with the value $t = 565 \text{ days}$ from two-phase simulation gives again a difference of about 3%. The *GOR* behavior shows larger differences, which can be expected since the dimensionless variables have extreme values in this case, far from the base case studied. However, the correlations still gives a good estimate of the post-breakthrough behavior.

The results of the three-phase simulation when compared with those obtained from

Figure 9.17: Comparison of breakthrough time and WOR correlations with simulation results for a field application.

Figure 9.18: Comparison of breakthrough time and GOR correlations with simulation results for a field application.

the correlations again show larger differences. As in the water-oil case, these differences are due to the differences in properties of the water and gas and the three-phase relative permeability calculations performed by the simulator after breakthrough time. However, the general behavior is not far from that found by the correlations.

9.10 Conclusions

Water and gas cresting behavior are important reservoir phenomena that occur in reservoirs with aquifers and/or gas caps. A good understanding of this phenomenon along with calculations based on simple correlations forms a good basis to start a reservoir simulation study. The correlations and procedures developed in this work help to improve this understanding.

The conclusions of this work can be summarized as follows:

1. A set of dimensionless variables has been defined, leading to clear identification of important parameters for a reservoir study.
2. A grid pattern to start a reservoir simulation study has been suggested. The proposed pattern leads to minor loss of accuracy in the results compared with very refined grid results and is useful to generate procedures to calculate breakthrough time, develop correlations and generate pseudofunctions.
3. A new set of correlations for breakthrough time, post breakthrough behavior and optimum grid has been developed using just dimensionless variables derived from the flow equations. These correlations take into account the main reservoir parameters that affect the flow, including the effects of relative permeability end-points and shapes. Numerical diffusion effects are taken into account by the inclusion of grid block sizes in the correlations. The correlations can be used for either water-oil or gas-oil flow. Such correlations can be used at the beginning of a reservoir study to estimate breakthrough time, post-breakthrough behavior and give an initial estimate of for the grid block sizes to use in the simulation runs.
4. A procedure to calculate an accurate breakthrough time using just coarse grid simulations and the suggested pattern has been presented.
5. Pseudofunctions to match breakthrough time using coarse grid runs have been generated. These pseudofunctions can be used together with the developed correlations or even independently, to obtain the right breakthrough time even with a coarse grid.
6. Applicability of the correlations to an entire field with multiple wells has been demonstrated.
7. A real field example has shown that the correlations are reliable and can be used at the beginning of a reservoir simulation study to give an estimate of breakthrough time and post-breakthrough behavior.

Nomenclature

B	formation volume factor, m^3/m^3
c	compressibility, Pa^{-1}
D	depth, distance from the surface, m
f_w	fractional water flow (water-cut)
FVF	formation volume factor, m^3/m^3
g	gravitational acceleration = $9.81m/s^2$
GOC	gas-oil contact
GOR	gas-oil ratio
h	height
h_w	well height
k	permeability, m^2
l	phase
L	well length, m
M	mobility ratio
m	exponent for q_{0D}
N	cumulative oil recovery, m
n_l	exponent for kr_l
n_b	number of blocks
p	pressure, Pa
q	rate at reservoir conditions, m^3/s
Q	rate at standard conditions, m^3/s
Rs	solubility ratio, m^3/m^3
RF	Recovery Factor
r	radial direction, m
s	length in the streamline direction, m
S	saturation
t	time, s
u	velocity, m/s^2
V	volume, m^3
x	main horizontal direction, with the origin in the well, m
W	width (y direction), m^3
WOC	water-oil contact
WOR	water-oil ratio
y	horizontal y direction, m
z	vertical direction

Greek

α	slope of the straight lines for t_{btD}
γ	hydrostatic gradient = ρg , Pa/m
λ	phase mobility $Pa.s^{-1}$
μ	viscosity $Pa.s$
ρ	density, kg/m^3

ϕ	porosity
ψ	ratio between the density differences
Φ	potential, Pa
θ	angle
ζ	mass fraction of each component

Subscripts

b	block
bp	bubble point
bt	breakthrough
c	critical
cp	component
D	dimensionless
e	external (lateral edge)
ef	effective
g	gas
h	horizontal direction
i	grid block discretization index
l	phase
o	oil
opt	optimum
or	oil residual
p	pseudo function
r	relative
rc	rock
ref	refined
res	reservoir
std	standard conditions
v	vertical direction
T	total
x	horizontal x direction
w	water ; well
wc	connate water
z	vertical z direction

Superscripts

$-$	average
$\sim$	per unit volume
n	time discretization level
0	related with $\Delta z_D = 0$

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10. Use of Streamline Grids in Reservoir Simulation

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Abstract

Commonly used Cartesian grids should be aligned with reservoir heterogeneities to represent their effective properties. This is often difficult and sometimes impossible due to the complexity of some reservoir heterogeneity. A new numerical scheme for flexible grids has been proposed and developed by Santosh Verma [5,6]) in an Object Oriented reservoir simulator called FLEX. Different types of grids have been considered such as Voronoi, Control Volume Finite Element (CVFE), and Hexagonal grids. Streamline-based grids have also been used in FLEX but only in limited cases. Our work will consist of developing within FLEX (in C++) a new grid generator based on streamlines and also on isopotential lines, in order to improve the accuracy of reservoir flow simulation in heterogeneous media.

10.1 FLEX Simulator

FLEX is an Object Oriented, flexible grid, black-oil reservoir simulator. The Object Oriented approach is particularly useful because of the complexity of grid generation and of flexible grid geometries (such as Voronoi, CVFE, Hexagonal grids, etc.). Further developments will therefore be made following this approach using the C++ language.

10.1.1 Why Flexible Grids?

Flexible grids are made of polygons whose shape and size vary from place to place in the reservoir. Such grids have been introduced because of the limitations of the commonly used Cartesian grids and their associated numerical schemes. The types of grid and the corresponding numerical model implemented in FLEX overcome most of these limitations. Indeed, FLEX is able to align grids with :

- Reservoir boundaries
- Beds with varying permeability tensors (FLEX accepts full and asymmetric tensors)
- Faults
- Horizontal/Inclined wells

FLEX can generate 2D grids based on modules of given grid-block geometry as proposed originally by Palagi [3]. Several modules can be assembled together to generate the desired grid. These modules can be scaled, rotated and then placed at any particular location (in the global coordinate system) in the reservoir using simple coordinate transformations.

In 2D, FLEX can use any one of the following modules as a base module (the modules are shown in Figure 10.1) :

- Cartesian

- Hexagonal
- Parallel Cartesian
- Tilted hexagonal
- Radial

Cartesian

Hexagonal

Parallel Cartesian

Tilted Hexagonal

Radial

Figure 10.1: Modules Used for Grid Generation in 2D (After Ref [1], Chapter 4)

Figure 10.2 shows a Voronoi grid whose grid nodes were generated using a hexagonal base grid. Radial and Cartesian modules were also placed in different regions, close to wells. Figures 10.3 and 10.4 give other examples of Voronoi and CVFE grids.

Figure 10.2: A 2D Voronoi Grid Using Modules (After Ref [1], Chapter 4)

(a) X-Y Cross Section of a Point Distributed Cartesian Grid

(b) 2D Hexagonal Grid

Figure 10.3: 2D Voronoi Grids with Radial Modules around Wells

Figure 10.4: 2D CVFE Grid with Radial Modules around Wells

10.1.2 Numerical Model

Grid types and assumptions about reservoir properties affect numerical properties of the solution. The mathematical formulation used to solve for the unknowns is strongly related to these assumptions.

The numerical scheme implemented in FLEX assumes that every control volume has uniform porosity and permeability. This model is based on the fact that flow at any block boundary is constrained by the velocity normal to that boundary.

Figure 10.5: Example of a Control Volume and its Connections to the other Grid Blocks

In the example of Figure 10.5, the constraint at the interface a-d can be expressed as follows:

$$\vec{v}_0 \cdot \vec{n}_{ad} = \vec{v}_1 \cdot \vec{n}_{ad}$$

where $\vec{v}_i$ is the velocity in region i ($i = 0, 1$), $\vec{n}_{ad}$ is the outward normal on the interface a-d.

The potential at the nodes are obtained by an interpolation between potentials at the interfaces. The flow term corresponding to the nodes considered is :

$$q_{0a} = - \int_a^d \vec{v}_0 \cdot \vec{n}_{ad} dL = -A_{ad} \vec{v}_0 \cdot \vec{n}_{ad}$$

where A_{ad} is the length of the interface a-d and this becomes a surface in 3D.

Substituting for the velocity using Darcy's law: $\vec{v}_0 = -[\bar{k}_0 \cdot \vec{\nabla} \Phi_0]$ and solving for the face potentials in terms of the node potentials, the constraint equation can be written in the following form :

$$\begin{bmatrix} \Phi_a \\ \Phi_b \\ \Phi_c \end{bmatrix} = \begin{bmatrix} R_{00} & R_{01} & R_{02} \\ R_{10} & R_{11} & R_{12} \\ R_{20} & R_{21} & R_{22} \end{bmatrix} \begin{bmatrix} \Phi_0 \\ \Phi_1 \\ \Phi_2 \end{bmatrix}$$

The flow terms can thus be expressed as follows:

$$\begin{bmatrix} q_a \\ q_b \\ q_c \end{bmatrix} = \underbrace{\begin{bmatrix} T_{a0} & T_{a1} & T_{a2} \\ T_{b0} & T_{b1} & T_{b2} \\ T_{c0} & T_{c1} & T_{c2} \end{bmatrix}}_T \begin{bmatrix} \Phi_0 \\ \Phi_1 \\ \Phi_2 \end{bmatrix}$$

T is the transmissibility matrix for node 0 which represents the contribution of the triangle Δ_{012} .

All the flow terms for a given node i for each triangle (tetrahedron in 3D) are then collected and the conservation equation for the node is put in the following form :

$$\sum_{j=1}^{n_c} [\sum_{k=1}^{n_{sc}} (\sum_{l=1}^{n_{sc,l}} T_{kl} \Phi_l)]_i = \underbrace{\frac{V_i}{\Delta t} [(\frac{\Phi S_p}{B_p})^{n+1} - (\frac{\Phi S_p}{B_p})^n]}_{\text{Accumulation Term}}_i$$

where n_c is the number of connections for a grid node, n_{sc} is the number of subconnections of a connection and $n_{sc,l}$ is the number of nodes which affect flow for a subconnection.

10.2 Use of Streamline Grids in Simulation

Streamlines are being used more and more extensively in reservoir flow simulation to characterize flow patterns (Thiele et al. 1996 [4]). Indeed, this is a very efficient way to deal with displacement processes governed by reservoir heterogeneities.

S.Verma and M.Thiele proposed a method to generate a grid based on streamlines. This has been implemented in FLEX but with the assumption of approximating the isopotential lines by straight lines. Figure 10.6 shows the streamlines for an example in 2D (XY) where the wells are located at both ends of the reservoir. The injector is at the left boundary and the producer at the right boundary.

The streamlines are arbitrarily selected at equal intervals and grid-blocks are constrained only to the selected streamlines. A coarser streamline-based grid is then generated (See Figure 10.7).

Figure 10.6: Streamline Grid Generated after Selecting Some Streamlines and Upscaling Permeabilities

Figure 10.7: Streamlines generated by Injecting at the Most Left Boundary and Producing at the Most Right Boundary

It was observed that such a grid (aligned to streamlines) significantly improves the water-cut response when compared with a Cartesian grid with the same number of grid nodes.

10.2.1 New Work on streamline grids

A new grid generation algorithm is being implemented in FLEX in an object oriented framework (in C++). The new grid will be defined by streamlines and equipotentials. These grids are of interest because they reflect variations in the pressure field caused by permeability heterogeneities in the reservoir and can cluster grid lines in high flow regions. This should increase local accuracy of the finite-volume discretization.

First, the streamlines are generated (using for example a streamline-based simulator). The isopotential lines are then calculated using an interpolation algorithm on the pressure values along the streamlines. The intersections between streamlines and isopotential lines are calculated. These intersections correspond to the vertices of the control volumes. Therefore, every control volume is a quadrilateral. The grid nodes are placed in the center of each control volume. This center corresponds to the intersection of the segments joining the mid-points of opposite faces (See Figure 8).

Figure 10.8: Construction of Grid Blocks Using Streamlines and Equipotentials

Once the nodes and the control volumes are generated, we can create the connections between nodes and then calculate the transmissibilities for all the connections.

Flux continuity is imposed following Edwards and Rogers 1994 [2], using the same approach as the one used for the triangles (Verma and Aziz 1996 [6]). The numerical scheme is the same but instead of triangular connections, we have a quadrilateral. The potential and velocity continuity points are located as shown in Figure 9:

Using the same approach as in Section 2.2 on triangles, we can express the flow continuity for the node 0 by the following constraint :

$$\vec{v}_0 \cdot \vec{n}_a = \vec{v}_1 \cdot \vec{n}_a$$

Then, we obtain the potential at the interfaces as a linear function of the potentials at the nodes and put it in the following matrix form :

Figure 10.9: Locations of Pressure Points for Interpolation and Transmissibility Calculations

$$\begin{bmatrix} \Phi_a \\ \Phi_b \\ \Phi_c \\ \Phi_d \end{bmatrix} = \begin{bmatrix} R_{00} & R_{01} & R_{02} & R_{03} \\ R_{10} & R_{11} & R_{12} & R_{13} \\ R_{20} & R_{21} & R_{22} & R_{23} \\ R_{30} & R_{31} & R_{32} & R_{33} \end{bmatrix} \begin{bmatrix} \Phi_0 \\ \Phi_1 \\ \Phi_2 \\ \Phi_3 \end{bmatrix}$$

This method allows the points of continuity P_a , P_b , P_c and P_d to be chosen along their corresponding segments, using a factor α .

$$\alpha \in]0; 1[\text{ and } \alpha = 0.5 \text{ gives the mid-point.}$$

Figure 10.10 shows an example of a grid generated by this process:

10.2.2 Results

Grids generated with streamlines and isopotential lines are now being tested in order to test for improvements in the accuracy of such an approach, compared to classical Cartesian grids. Some results are shown in this section. The first example considered here corresponds to the permeability field shown on the top of the Figure 10.11. This fine grid (256x128) has been upscaled using a flow-based upscaling method (Figure 10.12). As shown by Figure 10.13, the grid is not very far from a Cartesian grid because there are no big distortions on the distribution of permeabilities. The water-cut response given in figure 10.14 shows that cartesian grid and streamline grids give very close results. Nevertheless, we observe that the more the grid is aligned along heterogeneities, higher the water-cut at early time. This is the effect of taking into account the flow-paths to build the grid.

The second example is based on the permeability field shown in Figure 10.15 (CASE 2). We can see important differences between the cartesian grid and the one built with streamlines and isopotential lines (Figure 10.16). The impact of this grid on the water-cut performance is shown in Figure 10.17

Figure 10.10: Streamline/Isopotential Line Grid

Figure 10.11: Permeability Field 1

Figure 10.12: Permeability Field 1

Figure 10.13: Three Different Grids Corresponding to the Permeability Field 1

Figure 10.14: Water-Cut Response for the Three Grids Used in CASE 1

Figure 10.15: Permeability Field 1

Figure 10.16: Three Different Grids Corresponding to the CASE 2

Figure 10.17: Water-Cut Response for the Three Grids Used in CASE 2

10.2.3 Future Work

Future work will involve the extension of the streamline generator (in 2-D first and then in 3-D) to arbitrary well configurations and application to:

- Compressible flow
- Flow with diffusive/dispersive components added to the total flow, like tranverse diffusion and tranverse capillary pressure
- Gravity dominated flow (this problem with mild gravity has just recently been dealt with by operator splitting in a streamline simulator)

A longer term goal is to couple a streamline simulator (such as 3DSL [1,4]) with FLEX. This would yield essential information on flow paths and help to generate realistic grids to be used by FLEX which can handle complex grid geometries in 2D and 3D. Periodic updates of the grid may also be considered. Currently, effects of upscaling and grid level are being investigated compared to time grid simulations.

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