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ABSTRACT

We demonstrate how the formulation of a nonlinear scale-space filter can be used for edge detection and junction analysis. By casting edge-preserving filtering in terms of maximizing information content subject to an average cost function, the computed cost at each pixel location becomes a local measure of edgeness. This computation depends on a single scale parameter and the given image data. Unlike previous approaches which require careful tuning of the filter kernels for various types of edges, our scheme is general enough to be able to handle different edges, such as lines, step edges, corners and junctions. Anisotropy in the data is handled automatically by the nonlinear dynamics.

Keywords: edge detection, corner detection, junction analysis, energy filters, maximum entropy, clustering, edge-preserving filtering, nonlinear filter, anisotropic, scale-space

2. INTRODUCTION

Edge detection is a basic operation for many image analysis and machine vision systems. It is not surprising that numerous schemes have been invented for various purposes. The earlier schemes such as Roberts and Sobel operators are gradient-based [1]. Over the years, ideas from many different fields have been applied to this problem. For example, one approach is based on surface fitting [2, 3, 4]. Fitting functions resembling edge profiles to the data, one can extract the edges based on the residuals or gradient strengths. Mathematical morphology has also been used to detect edges [5]. The most popular approaches, however, are based on convolving the images with Gaussian-like kernels. Typically, peaks or ridges in the filtered output are identified as the edges, with some proper thresholding. For example, zero crossings of the Laplacian of Gaussians were considered to be edges in [6]. In his seminal work [7], Canny derived Gaussian-like filters that maximize the SNR and minimize the localization error. Recently, energy filters have been proposed [8] in which the squared responses of a pair of quadrature filters are computed and the local maxima were defined to be the edges [8]. To maximally tune the responses of the filters to the edges, it is advantageous to use a set of oriented filters. This idea was implemented in the Binford-Horn linefinder [9].

Corners and junctions are very important for object description and recognition. Since their descriptions are even more complicated than edges, their extraction are harder. There are two types of approaches for corner and junction detection: 1) extracting the edges and then look for points with maxima curvature [10, 11]; and 2) working directly on the grey-scale images [12, 13, 14, 15, 16].

Since edges can be present in multiple orientations and scales, one major drawback of these linear-filter-based techniques is that careful choice of the shapes, orientations and scales of the kernels [17, 18] is required to extract meaningful edges. This becomes much more complicated if accurate detection of corners and junctions is needed.

Recently, a clustering filter [19, 20] that can remove noise, preserve edges and smooth data was derived, all conditioned upon a scale parameter. In this work, we discuss another aspect of the filter. It is shown that the filter contains a mechanism suitable for edge detection and junction analysis.

For completeness, let us review the essential ideas of the filter.

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3. CLUSTERING FILTER

Let $\mathbf{x}$ be the coordinate of a pixel in an image¹ and y its gray level, or really any real-valued attribute. It is well-known that the pixels are highly redundant due to spatial correlation. Thus, given a scale, we can estimate the new pixel value y at position $\mathbf{x}$ by its neighboring pixels. Common sense tells us that the data points near $(\mathbf{x}, y)$ ² should give more information while those far away should give less. This can be implemented by having each data point contributing to the estimate y pay a cost. The cost function should be small for nearby data points but larger for those further away. To make this estimate robust, the information should be spread among the neighboring data. If we treat the contributions to the determination of y from the neighboring data as a probability distribution, then this probability distribution should be chosen such that its entropy is maximized subject to linear cost constraints [21].

The above reasoning implies that we can estimate each pixel independently. Say we are given a neighborhood of data points $S = \{(\mathbf{x}_i, y_i) : i = 1, \dots, N\}$. Let P_i denote the contribution of $(\mathbf{x}_i, y_i)$ to $(\mathbf{x}, y)$, or equivalently, the probability that $(\mathbf{x}, y)$ is influenced by $(\mathbf{x}_i, y_i)$. To specify the cost function, we note that the input and output domains should be treated differently because the former is fixed and known but the latter is really environment-dependent. Thus, let the cost function have two components $e_x(\mathbf{x}_i)$ and $e_y(y_i)$. Our criterion seeks to maximize the entropy $S = -\sum_i P_i \log P_i$, subject to the linear constraints

$$\sum_i P_i e_x(\mathbf{x}_i) = C(x), \quad \sum_i P_i e_y(y_i) = E(x) \quad (1)$$

which are obtained by averaging the costs in the neighborhood.

Using Lagrangian multipliers, the contribution of i^{th} pixel to the determination of the filter output y at pixel location x was found to be

$$P_i = e^{-\alpha e_x(\mathbf{x}_i) - \beta e_y(y_i)} / Z \quad (2)$$

where $Z = \sum_i e^{-\alpha e_x(\mathbf{x}_i) - \beta e_y(y_i)}$. Using an analogy with statistical physics, we can define a free energy $F = \frac{-1}{\beta} \log Z$.

If one uses squared distance for the cost functions e_x and e_y , one further obtained that, by minimizing F , the output y at x is given by

$$y = \frac{\sum_i y_i w_i e^{-\beta(y_i - y)^2}}{\sum_j w_j e^{-\beta(y_j - y)^2}}, \quad (3)$$

the weighted mean of the data and $w_i = e^{-\alpha \|\mathbf{x}_i - \mathbf{x}\|_2^2}$.

Let us now explain what α means. Clearly, a large α implies that only the pixels very close to $\mathbf{x}$ have significantly non-zero w_i 's. Thus, only a few data points can influence the output. Conversely, a small α implies that more neighbors of $\mathbf{x}$ can contribute. Hence, α is a measure of scale in the input space.

Once the scale α in the input space is selected, it is clear that the particular estimate one obtains depends on β and initial y . A simple procedure was used in [20]. Let us compute the mean $\bar{y} = \sum_i y_i w_i / \sum_i w_i$ and variance $\sigma_y^2 = \sum_i (y_i - \bar{y})^2 w_i / \sum_i w_i$. One then sets $\beta = (2\sigma_y^2)^{-1}$. To compute the filter output, one simply iterates (3), with initial $y = \bar{y}$, until it converges.

It has been observed that this filter can accomplish three tasks [19, 20]:

- removing impulsive noise;
- improved smoothing of nonimpulsive noise and
- preserving edges.

¹ $\mathbf{x} = (i, j)$ for an image with rectangular grids. We choose this notation for simplicity.

² We use $(\mathbf{x}, y)$ to denote the joint image plane and intensity space.

4. APPROACH FOR EDGE AND JUNCTION ANALYSIS

The clustering filter uses a new mechanism, namely, saddle-node dynamics, for edge-preserving filtering. Can we use this new mechanism for edge detection? A key observation here is that the energy (cost) function

$$E(x) \doteq \sum_i (y - y_i)^2 P_i \quad (4)$$

is a measure of edgyness at x in the image data. The energy is small for smooth areas and large for areas containing "edges." The reasoning is as follows: Over a smooth area, the pixels in a neighboring area are highly correlated. Thus, the cost of having a smooth estimate is low. At an edge, the usual notion of spatial redundancy breaks down and strong nonlinear action is needed to preserve an edge. Thus, the cost is high.

Furthermore, the energy is invariant to the orientation of an edge. Imagine that one rotates an edge profile within a circular region. The contributions P_i 's are rotated too. Thus, both $E(x)$ and y will remain unchanged. Our formulation gives a response which is orientation invariant. It is also unnecessary to tune the shape of the neighborhood used in the nonlinear filtering.

We now illustrate this observation with some synthetic and real images. For visualization purposes, the square root of the energy image E is shown, unless other specified. When the outputs are generated for different α s, no scaling is performed. This allows one to compare the magnitudes of the outputs. White means small and dark means large in the figures.

4.1 Energy at Lines

Figure 1a is an image with a horizontal line and a pair of crossed lines. Figures 1b and 1c are the energy images for $\alpha = 1/2, 1/8$ respectively. One can make three observations:

- The energies are highly localized along the lines and their terminals.
- At the corners and junctions, the energies become local maxima. This can serve as a scheme to corner and junction detection.
- The ridges in the energy image are the edges.

4.2 Energy at Step Edges and Corners

Figure 2a shows an image with step edges and corners. Figures 2b and 2c show the energy images for $\alpha = 1/2, 1/8$ respectively. Again, one can see that the energies are concentrated at the boundaries which can clearly be identified by the ridges. At the corners, the energies are distinctly local maxima. At a larger scale, two local maxima are generated at the corners. They can be merged by noting that there cannot be two adjacent corners at a large scale.

4.3 Energy at Junctions

Junctions pose great difficulty for Canny-like edge detectors. The behavior of the edges obtained by linear filters is very complicated at junctions because an ideal junction can be characterized by four parameters. To demonstrate that the energy image can be useful for detecting such junctions, Figure 3a shows a corner and Figure 3b is the filtered image. Figures 3c and 3d show the energies at $\alpha = 1/2$ and $1/8$ respectively. Indeed, the ridges are the edges and they meet at the junction, the energy of which is a local maxima.

4.4 Results using real data

Edges in real data differ from the synthetic ones considerably due to sampling. A step edge has nonzero components at all frequencies. For two-dimensional images, depending on the sampling periods in the x and y directions and the edge's orientation, sampled edges can appear jagged. Thus, care must be taken in using our formulation because the filter is nonlinear. Edges are detected as follows: For each pixel, it is determined if the energy is a local

maximum in any of the four directions. If the energy is a local maximum in one or more directions, the maximum gradient among these directions is computed. Otherwise the gradient is set to zero. One then thresholds this gradient image to get the edges. Figure 4a shows a girl image. Figure 4b shows the edges extracted by our method. Figure 5a shows a MR image of a human hand. Figure 5b shows the edges extracted.

5. SUMMARY

We have shown how the formulation of a nonlinear scale-space filter can be utilized for edge and junction analysis. Using the nonlinear filter, an energy value can be calculated at each pixel. Energy is large at an edge and small over smooth areas. Moreover, it is invariant to the orientation of an edge. It was observed that the ridges correspond to edges and the local maxima correspond to the corners and junctions. Our main contribution is that the use of saddle-node dynamics allows us to perform tasks quite effortlessly which would require careful tuning of the shapes and orientation of the filter kernels in conventional methods. Future work would include more detailed experiments and integration of edges over scales to generate a complete and robust edge detection scheme.

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Figure 1a. Line image.

Figure 1b. Energy image, $a=1/2$.

Figure 1c. Energy image, $a=1/8$.

Figure 2a. Step edges and Corners.

Figure 2b. Energy image, $a=1/2$.

Figure 2c. Energy image, $a=1/8$.

Figure 4a. Girl image.

Figure 4b. Edges from Fig. 4a.

Figure 5a. MR Hand image.

Figure 5b. Edges from Fig. 5a.

Figure 3a. Junction image.

Figure 3b. Filtered image, $a=1/2$.

Figure 3c. Energy image, $a=1/2$.

Figure 3d. Energy image, $a=1/8$.