Damage Identification and Health Monitoring of Structural and Mechanical Systems from Changes in Their Vibration Characteristics: A Literature Review

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Cover: These photos show structures that represent several of the current and future applications for the identification of structural damage using changes in measured vibration characteristics. The structures are (clockwise from top left): The Golden Gate Bridge; the Heidrun offshore oil platform (photo © Conoco, Inc., 1995, 1996 All Rights Reserved; the Elkraft IMW horizontal axis wind turbine located at Avedøre near Copenhagen, Denmark; the Space Shuttle Atlantis (photo courtesy of NASA); and the forward section of a DC9 in the Aging Aircraft Research Facility in Albuquerque, NM (photo courtesy of Sandia National Laboratories).

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Scott W. Doebling
Charles R. Farrar
Michael B. Prime
Daniel W. Shevitz

Los Alamos National Laboratory
Los Alamos, New Mexico 87545
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DAMAGE IDENTIFICATION AND HEALTH MONITORING OF STRUCTURAL AND MECHANICAL SYSTEMS FROM CHANGES IN THEIR VIBRATION CHARACTERISTICS: A LITERATURE REVIEW

Scott W. Doebling, Charles R. Farrar, Michael B. Prime, Daniel W. Shevitz

ABSTRACT

This report contains a review of the technical literature concerning the detection, location, and characterization of structural damage via techniques that examine changes in measured structural vibration response. The report first categorizes the methods according to required measured data and analysis technique. The analysis categories include changes in modal frequencies, changes in measured mode shapes (and their derivatives), and changes in measured flexibility coefficients. Methods that use property (stiffness, mass, damping) matrix updating, detection of nonlinear response, and damage detection via neural networks are also summarized. The applications of the various methods to different types of engineering problems are categorized by type of structure and are summarized. The types of structures include beams, trusses, plates, shells, bridges, offshore platforms, other large civil structures, aerospace structures, and composite structures. The report describes the development of the damage-identification methods and applications and summarizes the current state-of-the-art of the technology. The critical issues for future research in the area of damage identification are also discussed. This document, as well as links to other damage identification information sources, can be found at http://wvxvax7.esa.lanl.gov/damid/damidhome.html.

1 INTRODUCTION

The interest in the ability to monitor a structure and detect damage at the earliest possible stage is pervasive throughout the civil, mechanical and aerospace engineering communities. Current damage-detection methods are either visual or localized experimental methods such as acoustic or ultrasonic methods, magnet field methods, radiographs, eddy-current methods and thermal field methods. All of these experimental techniques require that the vicinity of the damage is known a priori and that the portion of the structure being inspected is readily accessible. Subjected to these limitations,
these experimental methods can detect damage on or near the surface of the structure. The need for additional global damage detection methods that can be applied to complex structures has led to the development of methods that examine changes in the vibration characteristics of the structure.

Damage or fault detection, as determined by changes in the dynamic properties or response of structures, is a subject that has received considerable attention in the literature. The basic idea is that modal parameters (notably frequencies, mode shapes, and modal damping) are functions of the physical properties of the structure (mass, damping, and stiffness). Therefore, changes in the physical properties will cause changes in the modal properties.

Ideally, a robust damage detection scheme will be able to identify that damage has occurred at a very early stage, locate the damage within the sensor resolution being used, provide some estimate of the severity of the damage, and predict the remaining useful life of the structure. (These four levels of damage identification are discussed in more detail in Section 1.B.) The method should also be well-suited to automation. To the greatest extent possible, the method should not rely on the engineering judgment of the user or an analytical model of the structure. A less ambitious, but more attainable, goal would be to develop a method that has the features listed above, but that uses an initial measurement of an undamaged structure as the baseline for future comparisons of measured response. Also, the methods should be able to take into account operational constraints. For example, a common assumption with most damage-identification methods reported in the technical literature to date is that the mass of the structure does not change appreciably as a result of the damage. However, there are certain types of structures such as offshore oil platforms where this assumption is not valid. Another important feature of damage-identification methods, and specifically those methods which use prior models, is their ability to discriminate between the model/data discrepancies caused by modeling errors and the discrepancies that are a result of structural damage.

The effects of damage on a structure can be classified as linear or nonlinear. A linear damage situation is defined as the case when the initially linear-elastic structure remains linear-elastic after damage. The changes in modal properties are a result of changes in the geometry and/or the material properties of the structure, but the structural response can still be modeled using a linear equation of motion. Nonlinear damage is defined as the case when the initially linear-elastic structure behaves in a nonlinear manner after the damage has been introduced. One example of nonlinear damage is the formation of a fatigue crack that subsequently opens and closes under the normal operating vibration environment. Other examples include loose connections that rattle and nonlinear material behavior. A robust damage-detection method will be applicable to both of these general types of damage. The majority of the papers summarized in this review address only the problem of linear damage detection.
1.A Previous Literature Surveys

A detailed survey of the technical literature and interviews of selected experts to determine the state of the art of the damage-detection field (using modal analysis procedures) as of 1979 was presented by Richardson (1980). The survey focused on structural integrity monitoring for nuclear power plants, large structures, rotating machinery, and offshore platforms, with by far the largest amount of literature associated with rotating machinery. The author stated that while monitoring of overall vibration levels for rotating machinery had become commonplace, attempts at relating structural damage to measured modal changes was still in its primitive stages.

Several doctoral dissertations that address damage detection and related issues have recently been published. Each dissertation contains a literature survey and a development of the theory relevant to its scope. These dissertations include Rytter (1993), Hemez (1993), Kaouk (1993), and Doebling (1995). Mottershead and Friswell (1993) present a survey of the literature related to dynamic finite element model (FEM) updating, which has been used extensively for structural damage detection. Their review contains a long list of references on the topic of model updating. Bishop (1994) reviews the literature in the field of neural networks. Because neural networks have been used extensively to solve inverse problems such as damage identification, the techniques reviewed by Bishop are directly applicable to the scope of this document. Neural-network-based damage-identification methods are reviewed in Section 2.G.

1.B Overview of Research Areas and Statement of Scope

As mentioned previously, the field of damage identification is very broad and encompasses both local and global methods. This survey will be limited to global methods that are used to infer damage from changes in vibration characteristics of the structure. Many different issues are critical to the success of using the mechanical vibration characteristics of a structure for damage identification and health monitoring. Among the important issues are excitation and measurement considerations, including the selection of the type and location of sensors, and the type and location of the excitations. Another important topic is signal processing, which includes such methods as Fourier analysis, time-frequency analysis and wavelet analysis.

In this review, these peripheral issues will not be directly addressed. The scope of this paper will be limited to the methods that use changes in modal properties (i.e. modal frequencies, modal damping ratios, and mode shapes) to infer changes in mechanical properties, and the application of these methods to engineering problems. The literature related to damage identification in rotating machinery has not been included.

A system of classification for damage-identification methods, as presented by Rytter (1993), defines four levels of damage identification, as follows:

- Level 1: Determination that damage is present in the structure
- Level 2: Determination of the geometric location of the damage
• Level 3: Quantification of the severity of the damage
• Level 4: Prediction of the remaining service life of the structure

The literature in this review can be classified mostly as Level 1, Level 2, or Level 3 methods because these levels are most often related directly to structural dynamics testing and modeling issues. Level 4 prediction is generally categorized with the fields of fracture mechanics, fatigue life analysis, or structural design assessment and, as such, is not addressed in the structural vibration or modal analysis literature.

1.C Organization of Report

This report is organized as follows: First, the literature relevant to the methods used to detect and locate damage are reviewed. Next, the applications of these techniques to specific structures and engineering problems are reviewed. Finally, the critical issues and recommended areas of future research are presented.
2 DAMAGE IDENTIFICATION METHODS

A summary of the literature pertaining to the various methods for damage identification and health monitoring of structures based on changes in their measured dynamic properties is presented in this section. The methods are categorized based on the type of measured data used, and/or the technique used to identify the damage from the measured data. Following the description of the methods is a table that presents a summary of the characteristics of each technique.

2.A FREQUENCY CHANGES

The amount of literature related to damage detection using shifts in natural frequencies is quite large. The observation that changes in structural properties cause changes in vibration frequencies was the impetus for using modal methods for damage identification and health monitoring. Because of the large amount of literature, we have not included all papers on this subject. An effort has been made to include the early work on the subject, some papers representative of the different types of work done in this area, and papers that are considered by the authors to be significant contributions in this area. There are a large number of papers that only duplicate previous work. These papers are largely excluded from this review, but are cited in a list of additional publications following the main reference list. Also, many papers are included only in Section 3 as they present only applications of these methods to different structures, rather than new theoretical work on the use of frequency shifts in damage detection.

It should be noted that frequency shifts have significant practical limitations for applications to the type of structures considered in this review, although ongoing and future work may help resolve these difficulties. The somewhat low sensitivity of frequency shifts to damage requires either very precise measurements or large levels of damage. For example, in offshore platforms damage-induced frequency shifts are difficult to distinguish from shifts resulting from increased mass from marine growth. Tests conducted on the I-40 bridge (Farrar, et al., 1994) also demonstrate this point. When the cross-sectional stiffness at the center of a main plate girder had been reduced 96.4%, reducing the bending stiffness of the overall bridge cross-section by 21%, no significant reductions in the modal frequencies were observed. Currently, using frequency shifts to detect damage appears to be more practical in applications where such shifts can be measured very precisely in a controlled environment, such as for quality control in manufacturing. As an example, a method known as “resonant ultrasound spectroscopy”, which uses homodyne detectors to make precise sine-sweep frequency measurements, has been used to determine out-of-roundness of ball bearings (Migliori, et al., 1993).

Also, because modal frequencies are a global property of the structure, it is not clear that shifts in this parameter can be used to identify more than the mere existence of damage. In other words, the frequencies generally cannot provide spatial information about structural changes. An exception to this limitation occurs at higher modal fre-
quencies, where the modes are associated with local responses. However, the practical limitations involved with the excitation and extraction of these local modes, caused in part by high modal density, can make them difficult to identify. Multiple frequency shifts can provide spatial information about structural damage because changes in the structure at different locations will cause different combinations of changes in the modal frequencies. However, as pointed out by several authors, there is often an insufficient number of frequencies with significant enough changes to determine the location of the damage uniquely.

2.A.1 The Forward Problem

The forward problem, which usually falls into the category of Level 1 damage identification, consists of calculating frequency shifts from a known type of damage. Typically, the damage is modeled mathematically, then the measured frequencies are compared to the predicted frequencies to determine the damage.

Vandiver (1975, 1977) examines the change in the frequencies associated with the first two bending modes and first torsional mode of an offshore light station tower to identify damage. Based on numerical simulations, the author concludes that changes in the effective mass of the tower resulting from sloshing of fluid in tanks mounted on the deck will produce only 1% change in the frequencies of the three modes being considered. By systematically removing members from a numerical model the authors demonstrated that failure of most members produces changes in resonant frequencies greater than 1%, and, thus, that damage in most of the members will be detectable. A numerical simulation of rust formation (reduction in wall thickness of the structural tubes by 1.27 mm (0.05 in.), actual wall thicknesses varied from 8.18 mm (0.322 in.) to 25.4 mm (1.00 in.) showed a 3.71% reduction in the bending mode frequencies.

Begg, et al. (1976) note that severance of members in a scale model offshore structure produced 5% to 30% change in resonant frequencies. When an extra bracing member was added to a North Sea platform, frequency increases of 10% were observed. When power spectra were examined for changes in resonant frequencies, it was noted that the lower-frequency peaks were associated with the spectral content of the input rather than resonances of the structure. Cracks were found to have little influence on the axial stiffness of members and, hence, did not reveal themselves when the global modes were examined. In these modes the members behaved primarily as truss elements. The authors suggest that monitoring local high-frequency bending modes of the individual members provides a better indication of cracking as both the crack and the associated water fill have more influence on these modes. Locating the damage from changes in frequencies alone was considered impractical.

Loland and Dodds (1976) use changes in the resonant frequencies, mode shapes, and response spectra to identify damage in offshore oil platforms. The mode shapes are necessary to ensure that the changes in modal frequencies are properly tracked. Changes in resonant frequencies of 3% were caused by changes to the mass on the decks and by changes in tide level. Frequency changes of 10% to 15% were observed.
when a structural modification was implemented that resembled a structural failure near the waterline. Thus, the authors conclude that changes in the response spectrum can be used to monitor structural integrity.

Wojnarowski, et al. (1977) examine the effects of eleven different parameters on the dynamic properties of an offshore lighthouse platform using finite element analysis. Foundation modeling assumptions, entrained water, marine growth, corrosion, variation in deck loads, and failed structural members are some of the parameters that were examined. The largest changes in frequencies were the result of changes in soil foundation properties.

Cawley and Adams (1979a) give a formulation to detect damage in composite materials from frequency shifts. They start with the ratio between frequency shifts for modes $i$ and $j$, $\delta \omega_i / \delta \omega_j$. A grid of possible damage points is considered, and an error term is constructed that relates the measured frequency shifts to those predicted by a model based on a local stiffness reduction. A number of mode pairs is considered for each potential damage location, and the pair giving the lowest error indicates the location of the damage. The formulation does not account for possible multiple-damage locations. Special consideration is given to the anisotropic behavior of the composite materials.

Coppolino and Rubin (1980) report the results of a numerical study on damage in an offshore platform. The authors benchmarked a FEM of a platform against measured modal data, and then the effect of severance of various members on the structural response was modeled numerically. Depending on the location of the damage, changes in resonant frequencies on the order of $1\%$ to $2\%$ were found to be indicative of damage. Other damage locations were not detected by changes in the resonant frequencies.

Duggan, et al. (1980) study the use of ambient, above-waterline vibration measurements on offshore platforms as a means of structural integrity monitoring. The program was aimed at determining the stability of vibration behavior under varying environmental and operating conditions, as well as the changes associated with structural modification. Three platforms in the Gulf of Mexico were monitored. On one platform, repairs and replacements of legs and braces took place during the study. The conclusion of this study was that changes in frequencies caused by removal of a bracing member could not be distinguished from shifts caused by normal operating changes. Because damage caused changes in the order of the modes, the authors state that it is essential to identify the mode shapes associated with the resonant frequencies to track the changes accurately.

Kenley and Dodds (1980) examine changes in resonant frequencies to detect damage in a decommissioned offshore platform. The authors find that only complete severance of a diagonal member can be detected by changes in the global modal frequencies. They state that damage has to produce a $5\%$ change in the overall stiffness before it can be detected. For global modes, the resonant frequency can be detected to within $1\%$, but for local modes the error increases to $2\%$ to $3\%$ because peaks in the power
spectrum are not as well defined. Flooding and half-severance of diagonal members were detected from local below-water measurements. The authors again point out that it is important to associate a resonant frequency with a mode shape when trying to track changes in frequency as an indicator of damage.

Crohas and Lepert (1982) describe a “vibro-detection device” that was mounted on structural members of an offshore oil platform. The device applies an input to the member and measures its response. Frequency response functions (FRFs) are then determined for the members by measuring the acceleration response that results from the excitation. Flooding of a test brace produced a 10% decrease in the resonant frequency, while the frequencies of neighboring braces were unaffected. A 30% through-crack located near the end of the test brace could also be detected.

Gudmundson (1982) proposes a technique for modeling cracks in a beam cross section using a static flexibility matrix. This flexibility matrix is determined using two different approaches. The first approach uses static stress-intensity factors, and the second approach uses a static FEM. The author compares numerical simulations of the beam modal frequencies using these two crack models for various crack lengths and crack positions. The results demonstrate that this method provides accurate predictions of the beam modal frequencies. The author also demonstrates that the modal frequencies decrease more slowly with a fatigue crack that opens and closes than with a crack that stays open.

Nataraja (1983) reports on a program designed to monitor North Sea platforms over a two-year period and to demonstrate the feasibility of such a system for structural damage detection. Results showed that only the lowest natural frequencies could be identified with certainty and that these frequencies were stable throughout the monitoring period. Changes in deck mass could be detected in the vibration signatures, and, hence, the author states that it is imperative to monitor the deck mass in order to distinguish changes in mass from structural damage. The author concludes that the measured accelerations can only be used to detect global changes in the structure and not localized damage.

Whittome and Dodds (1983) report results from a project where the response of British Petroleum’s Alpha Forties platform was monitored on a regular basis over 2.5 years. This study was undertaken to examine the feasibility of using changes in resonant frequencies to monitor the structural condition of offshore platforms. It was found that there was less than a 1.5% change in the resonant frequencies over the monitored time. Significant drops in frequencies were noticed when drilling operations were ongoing. These changes resulted from added mass on the deck. Damage was introduced in a numerical model of the structure that had been benchmarked against the measured response. It was concluded that changes in the resonant frequencies produced by damage or foundation deterioration were greater than the observed variations in resonant frequencies of the undamaged platform over time.

Tracy and Pardoen (1989) present an analytical solution for the vibrational frequencies and mode shapes for a composite (orthotropic) beam with a midplane delamination.
They divide the beam into four sections: above, below, and on either side of the delamination. The transverse bending and axial vibration differential equations are considered for each section. The model allows for independent extensional and bending stiffnesses. The transverse deflections of the sections above and below the delamination are constrained to vibrate together, which restricts the analysis to midplane delaminations. After all simplifications and boundary conditions are applied, 14 unknown constants are left. The characteristic equation is solved numerically to give frequencies and mode shapes.

Ismail, et al. (1990) demonstrate that the frequency drop caused by an opening and closing crack is less than that caused by an open crack. This property is a potentially large source of error that is considered by few of the researchers using frequency changes. It implies that the frequency drop is affected by factors such as preload and residual stress, not just crack size and location. It also serves to decrease the already low sensitivity of the method.

Osegueda, et al. (1992) report on a project that examines changes in the dynamic properties of a scale model offshore platform. Resonant frequencies were found to decrease with damage, and this decrease was an order of magnitude greater than the standard deviation of the measurement. These authors note that to track the changes in resonant frequencies properly, the mode shape associated with these frequencies must be identified.

Chowdhury and Ramirez (1992) report the results of modal tests performed on both reinforced and plain concrete beams. Qualitative observations of the changes in resonant frequencies and power spectra resulting from simulated damage, changes in strength, and changes in applied load were made. Although changes in the spectra were observed, no attempt to correlate these changes with the damage or other conditions was made.

Fox (1992) shows, using numerical and experimental data from a beam, that changes in the resonant frequencies are a poor indicator of damage in a beam with a saw cut. In the experimental data, resonant frequencies were actually observed to increase slightly for some modes after the damage had been introduced. These increases were attributed to inaccuracies in the methods used to estimate the resonant frequencies.

Srinivasan and Kot (1992) conducted an experimental study of the sensitivity of the measured modal parameters of a shell structure to damage in the form of a notch. The authors found that the resonant frequencies of the shell structure were insensitive to damage, with measured changes not exceeding the frequency resolution of the measurements.

Pape (1993) proposes a technique to identify damaged parts using statistical methods and measured natural frequencies. By looking for resonances outside of the mean standard deviations, he detected parts with gross defects. The ability to resolve smaller defects was not yet realized. It was hoped that this technique would replace the more expensive and time-consuming mechanical testing already done on these parts.
Penny, et al. (1993) present a method for locating the “most likely” damage case by simulating the frequency shifts that would occur for all damage cases under consideration. The measured frequencies are then fit to the simulated frequencies for each simulated damage case in a least-squares sense. The “true” damage case is indicated by the minimal error in this fit.

Slater and Shelley (1993) present a method for using frequency-shift measurements to detect damage in a smart structure. They describe the theory of modal filters used to track the frequency changes over time. They also describe how the system deals with sensor failures or sensor calibration drift over time.

Friswell, et al. (1994) present the results of an attempt to identify damage based on a known catalog of likely damage scenarios. The authors presume that the prior model of the structure is highly accurate. Using this model, they computed frequency shifts of the first \( n \) modes for both the undamaged structure and all the considered types of damage. Then ratios of all the frequency shifts were taken, resulting in \( n^2 \) numbers. For the candidate structure, the same \( n^2 \) numbers were computed, and a power-law relation was fit to these two sets of numbers. When the body of data is noise-free, and when the candidate structure lies in the class of assumed damages, the correct type of damage should produce a fit that is a line with unity slope. For all other types of damage the fit will be inexact. The likelihood of damage was keyed on the quality of the fit to each pattern of known damage. Two measures of fit were used: the first was related to the correlation coefficient; the second was a measure of how close the exponent and coefficient were to unity. Both measures were defined on a scale from 0 to 100. It was hypothesized that damage was present when both measures were near 100.

Meneghetti and Maggiore (1994) derive a sensitivity formulation for locating a crack in a beam from frequency shifts. Using an analytical result, the local stiffness change required to produce a given frequency shift was plotted as a function of crack position. Such a curve was plotted based on measured frequency shifts for several modes. The intersection of the curves was used as an indicator of the crack location.

Man, et al. (1994) present a detailed closed-form solution for the frequencies of a beam containing a slot. They investigate how the minimum detectable crack size can be determined from the frequency shifts predicted by the model of the slotted beam. The authors conclude that the minimum slot size that is detectable using the techniques of this paper is 10% of the beam depth.

Silva and Gomes (1994) present another method for solving the damage-detection problem. The technique requires an analytical model for the frequency shifts as a function of crack length and position. The program searches over combinations of crack location and length and selects the combination that minimizes the function...
At least two modes are required for this method, and better results are usually obtained by including more modes.

Brincker, et al. (1995b) use a statistical analysis method to detect damage in two concrete beams with different reinforcement ratios using changes in the measured vibration frequencies. The authors define a significance indicator for the $i^{th}$ modal frequency as

$$R = \sqrt{\frac{1}{m} \sum_{i=1}^{m} \left( \frac{(Q_i)_X - (Q_i)_A}{(Q_i)_X} \right)}$$

$$Q_i = \frac{\omega_i^d}{\omega_i^a}$$

Brincker, et al. (1995b) use a statistical analysis method to detect damage in two concrete beams with different reinforcement ratios using changes in the measured vibration frequencies. The authors define a significance indicator for the $i^{th}$ modal frequency as

$$(S_f)_i = \frac{f^d_i - f^d_j}{\sqrt{(\sigma^f_i)^2 + (\sigma^d_i)^2}}$$

where $(\sigma_f)_i$ is the experimentally-estimated standard deviation of the $i^{th}$ modal frequency. By scaling the observed change in modal frequency by the estimated standard deviation of the frequencies, measured frequencies with high confidence (low standard deviation) are weighted more heavily in the indicator function. A similar significance indicator is defined for the measured modal damping ratio. The authors define a "unified significance indicator" by summing the frequency and damping significance indicators over several measured modes. The significance indicator proved to be a sensitive indicator of structural damage, but it is not capable of providing an estimate of damage location. The authors state that knowledge of the input signal is not essential for the detection of damage using this technique.

Choy, et al. (1995) use a matrix transfer technique to relate the generalized forces and displacements at one end of a beam element to similar quantities at the other end of the beam element. The authors then extend this concept to a beam on an elastic foundation. The authors state that damage can be simulated by reductions in the Young's modulus of one or more of the beam elements. Assuming that all the original, undamaged Young's moduli of the system are known, an iterative procedure is developed to locate the elements with reduced moduli. It is assumed that the first beam element is degraded, and the modulus associated with this element is adjusted until the first natural frequency from the numerical model matches the first measured natural frequency. The process is repeated systematically assuming that the fault lies in each of the other elements. Again, the process is repeated for the second and third natural frequencies. The location of the damaged section is obtained from the intersection of flexural rigidity versus element location curves obtained from the iterative process using the different natural frequencies. A Newton-Raphson search procedure is employed to locate multiple faults. However, the number of damage locations must be known. Faults in the elastic foundation are located in a similar manner, but in this case adjust-
ments are systematically made to the stiffness of the springs that represent the elastic foundation. The method cannot distinguish damage at symmetrical locations in a symmetric structure.

2.A.2 The Inverse Problem

The inverse problem, which is typically Level 2 or Level 3 damage identification, consists of calculating the damage parameters, e.g., crack length and/or location, from the frequency shifts. In this section, summaries of some of the key works describing the inverse problem are presented.

Lifshitz and Rotem (1969) present what may be the first journal article to propose damage detection via vibration measurements. They look at the change in the dynamic moduli, which can be related to the frequency shift, as indicating damage in particle-filled elastomers. The dynamic moduli, which are the slopes of the extensional and rotational stress-strain curves under dynamic loading, are computed for the test articles from a curve-fit of the measured stress-strain relationships at various levels of filling.

Adams, et al. (1978) examine a method whereby damage in a structure that can be represented as one-dimensional can be identified from changes in the resonant frequencies associated with two modes. In particular, they looked at axial vibration modes. The method is based on the relationship between the receptance function on either side of the damage, \( \beta \) and \( \gamma \), respectively, and the stiffness of a spring representing the damage, \( k \). The relationship is defined by

\[
\beta + \gamma + \frac{1}{k} = 0 .
\]

They also point out a need to correct frequency measurements for changes in temperature, which is another possible source of error when frequency changes are used to locate damage.

Wang and Zhang (1987) estimate the sensitivity of modal frequencies and cross-spectral densities to changes in the structural stiffness parameters. The hypothesis is that modal characteristics themselves are not sensitive to damage, but that certain frequency ranges in the structural frequency response are sensitive to damage.

Stubbs, et al. (1990) and Stubbs and Osegueda (1990a, 1990b), discuss a method for damage identification that relates changes in the resonant frequencies to changes in member stiffnesses using a sensitivity relation. The relation between the normalized changes in squared frequencies \( \{z\} \), the fractional elemental stiffness reductions \( \{\alpha\} \), and the fractional elemental mass reductions \( \{\beta\} \) is given by

\[
\{z\} = [F]\{\alpha\} - [G]\{\beta\} .
\]
where $[F]$ and $[G]$ are the sensitivities of the frequency change to changes in elemental stiffness and mass magnitudes, respectively. The entries of $[F]$ and $[G]$ can be expressed for mode $i$ and member $p$ as

$$F_{ip} = \frac{\{\phi^d\}_i^T [K_E]_p \{\phi^d\}_i}{k_i}, \quad G_{ip} = \frac{\{\phi^d\}_i^T [M_E]_p \{\phi^d\}_i}{m_i},$$

(5)

where $[K_E]_p$ is the elemental stiffness matrix for member $p$, and $[M_E]_p$ is the elemental mass matrix for member $p$.

Damage is defined as a reduction in the stiffness of one of the elements forming the structure, which can be represented as $\alpha_p < 0$. The stiffness reductions can be located by solving the general inverse problem

$$\{\alpha\} = [F]^+ \{\{z\} + [G]\{\beta\}\}$$

(6)

assuming that $\{z\}$ and $\{\beta\}$ can be measured or assumed. In this equation, $[F]^+$ represents the pseudoinverse of the stiffness sensitivity matrix. The use of the pseudoinverse operator will ensure that Eq. (6) holds when $[F]$ is not square, i.e., when the number of measured modes is not equal to the number of structural elements.

A similar analysis can be performed to include the effects of changes in damping on the observed changes in resonant frequencies, yielding

$$\{z\} = [F]\{\alpha\} - [G]\{\beta\} - [D]\{\gamma\},$$

(7)

where $[D]$ is the damping sensitivity matrix that relates changes in the resonant frequency to changes in element damping, and $\{\gamma\}$ is a vector of element damping changes. In this case, the changes in stiffness can be solved for as

$$\{\alpha\} = [F]^+ \{\{z\} + [G]\{\beta\} + [D]\{\gamma\}\}.$$

(8)

The authors have demonstrated that this sensitivity method has difficulty when the number of modes is much fewer than the number of damage parameters. This difficulty occurs because the system is significantly underdetermined, so there is not enough independent information to determine all of the stiffness reduction parameters. In this case, the pseudoinverse solution can become ill-conditioned.

Stubbs and Osegueda (1990a, 1990b) developed another damage detection method using the sensitivity of modal frequency changes that is based on work by Cawley and Adams (1979a). In this method, an error function for the $i^{th}$ mode and $p^{th}$ structural member is computed as
assuming that only one member is damaged, where \( z_i \) and \( F_{ik} \) are the same as defined for Eq. (4). The member that minimizes this error is determined to be the damaged member. This method is demonstrated to produce more accurate results than the previous method in the case where the number of members is much greater than the number of measured modes.

The authors point out that both of these frequency-change sensitivity methods rely on sensitivity matrices that are computed using a FEM. This requirement increases the computational burden of these methods and also increases the dependence on an accurate prior numerical model. To overcome this drawback, Stubbs, et al. (1992) developed a damage index method, which is presented in Section 2.C.

Hearn and Testa (1991) developed a damage detection method that examines the ratio of changes in natural frequency for various modes. Assuming that the mass doesn't change as a result of damage and neglecting second-order terms, the change in the \( i^{th} \) natural frequency that results from damage can be related to a change in global stiffness as

\[
\Delta \omega_i^2 = \frac{\{\phi_i\}^T[K]\{\Delta K\}\{\phi_i\}}{\{\phi_i\}^T[M]\{\phi_i\}}. \tag{10}
\]

When the global stiffness \([K]\) is decomposed into member stiffnesses \([k_n]\), and member deformations \(\{\epsilon_N(\phi_i)\}\) are computed from the mode shapes, the change in frequency for the \(i^{th}\) mode can be written as

\[
\Delta \omega_i^2 = \frac{\sum_N \{\epsilon_N(\phi_i)\}^T[\Delta k_n]\{\epsilon_N(\phi_i)\}}{\{\phi_i\}^T[M]\{\phi_i\}}. \tag{11}
\]

Noting that damage does not affect all components of the member stiffness the same, the change in member stiffness is expressed as

\[
[\Delta k_n] = [\alpha_n][k_n], \tag{12}
\]

where \([\alpha_n]\) is a matrix representing the fractional changes in the element stiffness matrix components.

If damage is limited to one component of the element stiffness, then the change in element stiffness can be represented with a scalar \(\alpha_n\) as
\[ [\Delta k_n] = \alpha_n[k_n]. \] (13)

The authors note that this representation is not meant to imply that the damage can be modeled as a uniform loss of stiffness within an element, but rather that the influence on the change in natural frequency is primarily a function of only one of the element stiffness parameters. Therefore, as long as the influential stiffness parameter is scaled properly, the change in natural frequency resulting from damage to a single member can be adequately represented as

\[
\Delta \omega_i^2 = \frac{\alpha_n[\epsilon_N(\phi_i)]^T[k_n][\epsilon_N(\phi_i)]}{\{\phi_i\}^T[M]\{\phi_i\}}. \tag{14}
\]

If the ratio of the changes in two natural frequencies \(\Delta \omega_i\) and \(\Delta \omega_j\) are considered, the dependence on the damage severity, \(\alpha_n\), can be eliminated, and the effects of damage are reduced to a function of the damage location only as

\[
\frac{\Delta \omega_i^2}{\Delta \omega_j^2} = \left(\frac{\{\epsilon_N(\phi_i)\}^T[k_n][\epsilon_N(\phi_i)]}{\{\phi_i\}^T[M]\{\phi_i\}}\right) \left(\frac{\{\epsilon_N(\phi_j)\}^T[k_n][\epsilon_N(\phi_j)]}{\{\phi_j\}^T[M]\{\phi_j\}}\right).
\tag{15}
\]

This equation shows the characteristic influence of each member on the natural frequency of the structure and that these influences can be calculated from pre-damaged modal properties. The authors then summarize a two-step procedure, both qualitative and quantitative, for correlating changes in the measured frequency ratios with the damage location.

Richardson and Mannan (1992) present a method that assumes that damage is limited to changes in stiffness. The method requires pre-damage mode shapes, pre-damage frequency measurements, and post-damage frequency measurements. Based on the orthogonality properties of the damaged and undamaged structure, a sensitivity equation for changes in stiffness can be obtained for each mode by subtracting the two orthogonality equations to obtain

\[
\{\phi_i + \delta \phi_i\}^T[\delta K]\{\phi_i + \delta \phi_i\} + 2\{\delta \phi_i\}^T[\delta K]\{\phi_i\} + \{\delta \phi_i\}^T[\delta K]\{\delta \phi_i\} = (\omega_i^d)^2 - (\omega_i^u)^2. \tag{16}
\]

If it is assumed that there is negligible change in the mode shapes, the sensitivity equation reduces to

\[
\{\phi\}^T[\delta K]\{\phi\} = (\omega_i^d)^2 - (\omega_i^u)^2. \tag{17}
\]
By similar analysis, sensitivity equations for changes in mass and changes in damping can also be developed. An inherent assumption in this method is that the largest stiffness changes occur between test points near the fault. A difficulty with this method is that it typically leads to a set of underdetermined equations by which the changes in stiffness are evaluated. To circumvent this problem, the authors have implemented a pseudo-inverse search routine, similar to that discussed by Stubbs and Osegueda (1990a), to estimate the changes in stiffness.

Sanders, et al. (1992) use the frequency sensitivity method of Stubbs and Osegueda (1990a) combined with an internal-state-variable theory to detect damage in composites. The damage theory includes parameters which indicate two possible types of damage: extensional stiffness changes caused by matrix microcracking and changes in bending stiffness caused by transverse cracks in the 90-degree plies. The technique is applicable in general to any internal variable theory that can predict stiffness changes resulting from changes in the measured parameters.

Narkis (1994) developed a closed-form solution for the inverse problem of determining crack location (but not length) from measuring either bending or axial frequencies. The author derives simple closed-form solutions for the crack position, $e$, as a function of the frequency shifts of two modes, $\Delta f_i$ and $\Delta f_j$. For a simply supported beam in longitudinal bending, the function is

$$ e = \frac{1}{\pi} \cos \left(1 - \frac{R_{ji}}{2} \right) $$

$$ R_{ji} = \frac{\Delta f_j / f_j}{\Delta f_i / f_i} $$

Similarly, for a simply supported beam in axial vibrations (axial displacement constrained at one end with the other end free of load), the function is

$$ e = \frac{2}{\pi} \sin \left( \pm \sqrt{R_{ji}} \frac{2}{2} \right) $$

and for a free-free beam using the first two axial frequencies the function is

$$ e = \frac{1}{\pi} \cos \left(1 - \frac{R_{ji}}{2} \right) $$

Similar equations can be written for any combination of two modes. It should be noted that this result is independent of crack size, crack shape, and crack configuration.

Brincker, et al. (1995a) apply an auto-regressive moving average (ARMA) model to measured acceleration-time histories to estimate modal parameters (resonant frequencies and modal damping) of a reinforced concrete offshore oil platform. The gen-
eral form of the \([n, m]\) order ARMA model that expresses the response as a linear combination of \(n\) past responses as well as the present and \(m\) past inputs is

\[
y(t) = \sum_{i=1}^{n} c(i)y(t-i) - \sum_{i=1}^{m} d(i)e(t-i) + e(t),
\]

(21)

where \(y(t)\) is the response at time \(t\), \(y(t-i)\) are the responses at previous times, \(c(i)\) is the auto-regressive parameter, \(d(i)\) is the moving average parameter, and \(e(t)\) and \(e(t-i)\) are the present and past inputs, respectively, which are assumed to be white noise. The AR and MA parameters are obtained by minimizing an error function based on the measured response and the response predicted by Eq. (21). Roots of the characteristic polynomial containing the AR parameters can be related to the modal frequencies and damping of the system, and the ARMA model can be used to examine the time variation of these parameters. The authors then develop a probability-based damage indicator that examines the changes in resonant frequencies to determine a quantifiable estimate of structural change.

Balis Crema, et al. (1995) use the modal parameter sensitivity equations presented by Stubbs and Osegueda (1990a) to locate damage. The authors examine the effects of the location of the damage on successful damage detection, as well as the relationship between the modes used in the analysis and the position of the damage.

Skjaerbaek, et al. (1996) developed a procedure to locate and quantify damage in a multi-story reinforced concrete frame structure from a single response measurement made at the top of the structure. Damage in a substructure is defined as the average relative reduction of the stiffness matrix of the substructures that reproduces the two lowest eigenvalues of the overall structure. First, the time-varying nature of the resonant frequency in a damaged concrete structure is approximated by a smoothed fit to the measured frequency-time history. The frequency-time history is estimated using ARMA models. A maximum softening damage index, \(\delta_m\), is then defined as

\[
\delta_m = 1 - \frac{T_0}{T_m},
\]

(22)

where \(T_0\) is the undamaged period and \(T_m\) is the maximum period calculated during the softening portion of the response. An iterative procedure for dividing the structure into substructures is then developed. First, the stiffness matrix is partitioned in two matrices representing the first two substructures

\[
[K_0] = [K^{(1)}_{1,0}] + [K^{(1)}_{2,0}],
\]

(23)

where the subscripts 1 and 2 refer to substructures 1 and 2, the subscript 0 refers to the initial state of the structure, and the superscript refers to the first iteration of substructuring.
An equivalent linear stiffness matrix, \([K^e_e(t)]\), for the damaged structure is defined as

\[
[K^e_e(t)] = (1 - \delta_1^{(1)}(t))^2[K^{(1)}_{1,0}] + (1 - \delta_2^{(1)}(t))^2[K^{(1)}_{2,0}],
\]

where \(\delta_1^{(1)}(t)\) and \(\delta_2^{(1)}(t)\) can be interpreted as the average stiffness loss in each of the substructures. The authors summarize an iterative method to estimate these parameters based on the smoothed resonant frequencies and mode shapes of the structure in which linear equations for \((1 - \delta_1^{(1)}(t))^2\) and \((1 - \delta_2^{(1)}(t))^2\) are obtained from the Rayleigh fraction.

\[
(\omega_j(t))^2 = \frac{[\Phi_{j,0}]^T[K_e(\delta_1^{(1)}(t), \delta_2^{(1)}(t), t)][\Phi_{j,0}]}{[\Phi_{j,0}]^T[M][\Phi_{j,0}]},
\]

where \((\omega_j(t))^2\) is the value of the jth natural frequency obtained from the smoothed frequency-time function. Next, substructure 2 is further divided into two substructures, and the equivalent linear stiffness matrix is defined as

\[
[K^e_e(t)] = (1 - \delta_1^{(1)}(t))^2[K^{(1)}_{1,0}] + (1 - \delta_2^{(2)}(t))^2[K^{(2)}_{1,0}] + (1 - \delta_2^{(2)}(t))^2[K^{(2)}_{2,0}],
\]

where

\[
[K^{(1)}_{2,0}] = [K^{(2)}_{1,0}] + [K^{(2)}_{2,0}].
\]

This process can be repeated further to the desired level of refinement. The values of \(\delta_2^{(1)}\) are used as measures of the damage in each substructure. When this method was applied to numerical simulations of the nonlinear response of reinforced concrete framed structures subjected to earthquake excitations, the method correctly located the damage in the structure at higher levels of excitation.

2.B  
**MODE SHAPE CHANGES**

West (1984) presents what is possibly the first systematic use of mode shape information for the location of structural damage without the use of a prior FEM. The author uses the modal assurance criteria (MAC) to determine the level of correlation between modes from the test of an undamaged Space Shuttle Orbiter body flap and the modes from the test of the flap after it has been exposed to acoustic loading. The mode shapes are partitioned using various schemes, and the change in MAC across the different partitioning techniques is used to localize the structural damage.

Yuen (1985) examined changes in the mode shape and mode-shape-slope parameters by computing...
The changes in these parameters were simulated for a reduction in stiffness in each structural element, then the predicted changes were compared to the measured changes to determine the damage location. The author identified the need for some orthonormalization process in order to look at higher mode shapes.

Rizos, et al. (1990) developed an analytical model for vibration of a beam with an open crack. The sections on either side of the crack are considered to be standard slender beams in transverse vibration. The compatibility condition between the two sections is derived based on the crack-strain-energy function. The result is a system of equations for the frequencies and mode shapes in terms of crack length and position. To determine the crack length and location, the beam is excited at a natural frequency, and vibration amplitudes at only two locations are measured. The Newton-Raphson method is used to solve the system of equations for the crack parameters.

Osegueda, et al. (1992) report on a project that examines changes in the dynamic properties of a scale model of an offshore platform subjected to damage. Mode shape changes could not be correlated with damage in this study.

Fox (1992) shows that single-number measures of mode shape changes such as the MAC are relatively insensitive to damage in a beam with a saw cut. “Node line MAC,” a MAC based on measurement points close to a node point for a particular mode, was found to be a more sensitive indicator of changes in the mode shape caused by damage. Graphical comparisons of relative changes in mode shapes proved to be the best way of detecting the damage location when only resonant frequencies and mode shapes were examined. A simple method of correlating node points—in modes that show relatively little change in resonant frequencies—with the corresponding peak amplitude points—in modes that show large changes in resonant frequencies—was shown to locate the damage. The author also presents a method of scaling the relative changes in mode shape to better identify the location of the damage.

Kam and Lee (1992) present an analytical formulation for locating a crack and quantifying its size from changes in the vibration frequency and mode shape. The crack is located by discretizing the structure and looking at the reduced stiffness in each element. The formulation is based on a first-order Taylor expansion of the modal parameters in terms of the elemental parameters. Once located, the crack length is determined by a formulation based on considering the change in strain energy resulting from the presence of a crack. The Newton-Raphson method is used to solve the resulting equations for the crack parameters.
Kim, et al. (1992) investigate the use of MAC and its variations in the location of structural damage. They use the Partial MAC (PMAC) to compare the MAC values of coordinate subsets of the modal vectors. By using the Coordinate MAC (COMAC) and the PMAC in conjunction, they are able to isolate the damaged area of the structure.

Mayes (1992) presents a method for model error localization based on mode shape changes known as structural translational and rotational error checking (STRECH). By taking ratios of relative modal displacements, STRECH assess the accuracy of the structural stiffness between two different structural degrees of freedom (DOF). STRECH can be applied to compare the results of a test with an original FEM or to compare the results of two tests.

Srinivasan and Kot (1992) found that changes in mode shapes were a more sensitive indicator of damage than changes in resonant frequencies for a shell structure. These changes are quantified by changes in the MAC values comparing the damaged and undamaged mode shapes.

Ko, et al. (1994) present a method that uses a combination of MAC, COMAC and sensitivity analysis to detect damage in steel framed structures. The sensitivities of the analytically derived mode shapes to particular damage conditions are computed to determine which DOF are most relevant. The authors then analyze the MAC between the measured modes from the undamaged structure and the measured modes from the damaged structure to select which mode pairs to use in the analysis. Using the modes and DOF selected with the above criteria, the COMAC is computed and used as an indicator of damage. The results demonstrate that particular mode pairs can indicate damage, but when all mode pairs are used, the indication of damage is masked by modes that are not sensitive to the damage.

Salawu and Williams (1994) compare the results of using mode shape relative change and mode shape curvature change to detect damage. They demonstrate that the relative difference measure does not typically give a good indication of damage using experimental data. They point out that the most important factor is the selection of the modes used in the analysis. Salawu and Williams (1995) show that the MAC values can be used to indicate which modes are being affected most by the damage.

Lam, et al. (1995) define a mode shape normalized by the change in natural frequency of another mode as a “damage signature.” The damage signature is a function of crack location but not of crack length. They analytically computed a set of possible signatures by considering all possible damage states. The measured signatures were matched to a damage state by selecting which of the analytical signatures gave the best match to the measurements using the MAC.

Salawu (1995) proposes a global damage integrity index that is based on a weighted ratio of the damaged natural frequency to the undamaged natural frequency. The weights are used to reflect the relative sensitivity of each mode to the damage event. When damage is indicated, local integrity indices are calculated to locate the defective areas. The local integrity index is calculated from the global integrity index by further
weighting the global index by the square of the ratio of damaged mode amplitude to the undamaged mode amplitude for a particular measurement point.

2.C MODE SHAPE CURVATURE/STRAIN MODE SHAPE CHANGES

An alternative to using mode shapes to obtain spatial information about vibration changes is using mode shape derivatives, such as curvature. It is first noted that for beams, curvature and bending strain are directly related as

\[ \varepsilon = \frac{y}{R} = \kappa y, \tag{29} \]

where \( \varepsilon \) is strain, \( R \) is radius of curvature, and \( \kappa \) is curvature or \( 1/R \). The practical issues of measuring strain directly or computing it from displacements or accelerations are discussed by some researchers.

Pandey, et al. (1991) demonstrate that absolute changes in mode shape curvature can be a good indicator of damage for the FEM beam structures they consider. The curvature values are computed from the displacement mode shape using the central difference approximation for mode \( i \) and DOF \( q \)

\[ \phi_{q,i}'' = \frac{\phi_{q-1,i} + 2\phi_{q,i} + \phi_{q+1,i}}{h^2}, \tag{30} \]

where \( h \) is the length of each of the two elements between the DOF \( (q-1) \) and \( (q+1) \).

Stubbs, et al. (1992) present a method based on the decrease in modal strain energy between two structural DOF, as defined by the curvature of the measured mode shapes. For a linearly elastic beam structure, the damage index for the \( p^{th} \) element, \( \beta_p \), can be written as

\[ \beta_p = \left( \sum_{i=1}^{m} \mu_{i,p}^{d} \right) / \left( \sum_{i=1}^{m} \mu_{i,p}^{u} \right), \tag{31} \]

where the \( \mu_{i,p} \) terms are measures of the experimentally determined fractional strain energy for mode \( i \) between the endpoints of element \( p \), denoted by \( a \) and \( b \). For a Bernoulli-Euler beam, these fractional strain energies are expressed by Stubbs, et al. (1995) as
The values of $p$, with the largest magnitudes indicate members which are probably damaged.

At the probable damage locations, the severity of damage estimate for the $p^{th}$ element, $p$, is computed as

$$\alpha_p = \left( \sum_{i=1}^{m} g_{ip}^p \right) / \left( \sum_{i=1}^{m} g_{ip}^d - 1 \right),$$

(33)

where

$$g_{ip}^u = \left( \int_a^b [\phi^u(x)]_i'' dx \right) / \left( \int_0^L [\phi^u(x)]_i'' dx \right),$$

$$g_{ip}^d = \left( \int_a^b [\phi^d(x)]_i'' dx \right) / \left( \int_0^L [\phi^d(x)]_i'' dx \right).$$

(34)

The parameter $\alpha_p$ represents the fractional change in bending stiffness. Thus, larger magnitudes of $\alpha_p$ represent locations where the damage is more severe. In contrast to other inverse modeling methods, this technique does not require computation of sensitivity matrices from a prior model. It does, however, require differentiation of the measured mode shapes, and it also requires the structure to be well-suited for representation by beam elements. This method is applied to damage detection data by Stubbs, et al. (1995).

Chance, et al. (1994) found that numerically calculating curvature from mode shapes resulted in unacceptable errors. They used measured strains instead to measure curvature directly, which dramatically improved results. Chen and Swamidas (1994) found that strain mode shapes facilitated the location of a crack in a cantilever plate using FEM simulation.

Dong, et al. (1994) study a parameter based on the change in strain mode shape and the change in frequency. It is formulated as

$$\{ \Delta \phi^* \}_i = \{ \phi^d \}_i \left( \frac{\omega_i^d}{\omega_i^u} \right)^2 - \{ \phi^u \}_i,$$

(35)
where \( \{ \phi \}_i \) is the \( i \)th strain mode shape. The authors demonstrate that this parameter is more sensitive to structural damage than the equivalent parameter computed using displacements.

Kondo and Hamamoto (1994) determine modal properties from ambient vibration data using a multivariate ARMA model similar to that described by Eq. (21). Damage in the system is then located by examining changes in curvature of mode shapes for an equivalent lumped mass model of the structure. Changes in curvature are estimated from the central difference operator given in Eq. (30).

Salawu and Williams (1994) use a mode shape curvature measure computed using a central difference approximation as defined in Eq. (30). They compare the performance of this relative difference method to a mode shape relative difference method. They demonstrate that the curvature change does not typically give a good indication of damage using experimental data. They point out that the most important factor is the selection of which modes are used in the analysis.

Nwosu, et al. (1995) evaluated strain changes with the introduction of a crack in a tubular T-joint. They found these changes to be much greater than any frequency shifts and to be measurable even at a relatively large distance from the crack.

2.D METHODS BASED ON DYNAMICALLY MEASURED FLEXIBILITY

Another class of damage identification methods uses the dynamically measured flexibility matrix to estimate changes in the static behavior of the structure. Because the flexibility matrix is defined as the inverse of the static stiffness matrix, the flexibility matrix relates the applied static force and resulting structural displacement as

\[
\{ u \} = [G] \{ F \}.
\]  

(36)

Thus, each column of the flexibility matrix represents the displacement pattern of the structure associated with a unit force applied at the associated DOF.

The measured flexibility matrix is estimated from the mass-normalized measured mode shapes and frequencies as

\[
\]  

(37)

The formulation of the flexibility matrix in Eq. (37) is approximate due to the fact that only the first few modes of the structure (typically the lowest-frequency modes) are measured. The synthesis of the complete static flexibility matrix would require the measurement of all of the mode shapes and frequencies.

Typically, damage is detected using flexibility matrices by comparing the flexibility matrix synthesized using the modes of the damaged structure to the flexibility matrix synthesized using the modes of the undamaged structure or the flexibility matrix from a
FEM. Because of the inverse relationship to the square of the modal frequencies, the measured flexibility matrix is most sensitive to changes in the lower-frequency modes of the structure.

2.D.1 Comparison of Flexibility Changes

Aktan, et al. (1994) propose the use of measured flexibility as a “condition index” to indicate the relative integrity of a bridge. They apply this technique to 2 bridges and compare the measured flexibility to the static deflections induced by a set of truck-load tests.

Pandey and Biswas (1994) present a damage-detection and -location method based on changes in the measured flexibility of the structure. This method is applied to several numerical examples and to an actual spliced beam where the damage is linear in nature. Results of the numerical and experimental examples showed that estimates of the damage condition and the location of the damage could be obtained from just the first two measured modes of the structure.

Toksoy and Aktan (1994) compute the measured flexibility of a bridge and examine the cross-sectional deflection profiles with and without a baseline data set. They observe that anomalies in the deflection profile can indicate damage even without a baseline data set.

Mayes (1995) uses measured flexibility to locate damage from the results of a modal test on a bridge. He also proposes a method for using measured flexibility as the input for a damage-detection method (STRECH) which evaluates changes in the load-deflection behavior of a spring-mass model of the structure.

Peterson, et al. (1995) propose a method for decomposing the measured flexibility matrix into elemental stiffness parameters for an assumed structural connectivity. This decomposition is accomplished by projecting the flexibility matrix onto an assemblage of the element-level static structural eigenvectors.

Zhang and Aktan (1995) suggest that changes in curvatures of the uniform load surface (deformed shape of the structure when subjected to a uniform load), calculated using the uniform load flexibilities, are a sensitive indicator of local damage. The authors state that changes in the uniform load surface are appropriate to identify uniform deterioration. A uniform load flexibility matrix is constructed by summing the columns of the measured flexibility matrix. The curvature is then calculated from the uniform load flexibilities using a central difference operator as in Eq. (30).

2.D.2 Unity Check Method

The unity check method is based on the pseudoinverse relationship between the dynamically measured flexibility matrix and the structural stiffness matrix. An error matrix is defined as
which measures the degree to which this pseudoinverse relationship is satisfied. The relationship uses a pseudoinverse rather than an inverse since the dynamically measured flexibility matrix is typically rank-deficient.

Lin (1990) proposes the unity check method for locating modeling errors and uses the location of the entry with maximum magnitude in each column to determine the error location. He applies the method to FEM examples and also investigates the sensitivity of the method to non-orthogonality in the measured modes.

Lin (1994) extends the unity check method to the problem of damage detection. He defines a least-squares problem for the elemental stiffness changes—that are consistent with the unity check error—in potentially damaged members.

2.D.3 Stiffness Error Matrix Method

The stiffness error matrix method is based on the computation of an error matrix that is a function of the flexibility change in the structure and the undamaged stiffness matrix. The stiffness error matrix is defined as

\[ [E] = [G^d][K^u] - [I], \]

which measures the degree to which this pseudoinverse relationship is satisfied. The relationship uses a pseudoinverse rather than an inverse since the dynamically measured flexibility matrix is typically rank-deficient.

He and Ewins (1986) present the stiffness error matrix as an indicator of errors between measured parameters and analytical stiffness and mass matrices. For damage identification, the stiffness matrix generally provides more information than the mass matrix, so it is more widely used in the error matrix method.

Gysin (1986) demonstrates the dependency of this method on the type of matrix reduction used and on the number of modes used to form the flexibility matrices. The author compared the reduction techniques of elimination, Guyan-reduction, and indirect reduction, and found that the latter two techniques gave acceptable results, while the first technique did not.

Park, et al. (1988) present a weighted error matrix, where the entries in \([E]\) are divided by the variance in natural frequency resulting from damage in each member. The authors apply their formulation to both beam models and plate models.
2.D.4 Effects of Residual Flexibility

The residual flexibility matrix, \([G_r]\), represents the contribution to the flexibility matrix from modes outside the measured bandwidth so that the exact flexibility matrix can be related to the measured modes and the residual flexibility as

\[
\begin{align*}
\end{align*}
\] (41)

Doebling, et al. (1995, 1996) and Doebling (1995) present a technique to estimate the unmeasured partition of the residual flexibility matrix because only one column of the FRF matrix can be measured for each modal excitation DOF. This technique does not add any new information into the residual flexibility, but it does complete the reciprocity of the residual flexibility matrix so that it can be used in the computation of measured flexibility. The authors demonstrate that the inclusion of the measured residual flexibility in the computation of the measured flexibility matrix yields a more accurate estimate of the static flexibility matrix.

2.D.5 Changes in Measured Stiffness Matrix

A variation on the use of the dynamically measured flexibility matrix is the use of the dynamically measured stiffness matrix, defined as the pseudoinverse of the dynamically measured flexibility matrix. Similarly, the dynamically measured mass and damping matrices can be computed. Salawu and Williams (1993) use direct comparison of these measured parameter matrices to estimate the location of damage.

Peterson, et al. (1993) propose a method to use the measured stiffness and mass matrices to locate damage by solving an “inverse connectivity” problem, which evaluates the change in impedance between two structural DOF to estimate the level of damage in the connecting members.

2.E Matrix Update Methods

Another class of damage identification methods is based on the modification of structural model matrices such as mass, stiffness, and damping to reproduce as closely as possible the measured static or dynamic response from the data. These methods solve for the updated matrices (or perturbations to the nominal model that produce the updated matrices) by forming a constrained optimization problem based on the structural equations of motion, the nominal model, and the measured data. Comparisons of the updated matrices to the original correlated matrices provide an indication of damage and can be used to quantify the location and extent of damage. The methods use a common basic set of equations, and the differences in the various algorithms can be classified as follows:

1. Objective function to be minimized
2. Constraints placed on the problem

3. Numerical scheme used to implement the optimization

**2.E.1 Objective Functions and Constraints**

There are several different physically based equations that are used as either objective functions or constraints for the matrix update problem, depending upon the update algorithm. The structural equations of motion are the basis for the “modal force error equation.” An \( n \)-DOF FEM is assumed and given as

\[
[M^u]\{\dot{x}\} + [C^u]\{x\} + [K^u]\{x\} = \{f(t)\}.
\]

(42)

The eigenvalue equation for the above equation is given as

\[
((\lambda_i^u)^2[M^u] + (\lambda_i^u)[C^u] + [K^u])\{\phi^u\}_i = \{0\},
\]

(43)

where \( \lambda_i^u \) and \( \{\phi^u\}_i \) are the measured \( i \)th eigenvalue and eigenvector of the undamaged structure. It is assumed that this equation is satisfied for all measured modes.

Now consider the eigenvalues and eigenvectors corresponding to the damaged state, \( \lambda_i^d \) and \( \{\phi^d\}_i \). Substituting these quantities into the above eigenvalue equation yields

\[
((\lambda_i^d)^2[M^u] + (\lambda_i^d)[C^u] + [K^u])\{\phi^d\}_i = \{E\}_i,
\]

(44)

where \( \{E\}_i \) is defined as the “modal force error,” or “residual force,” for the \( i \)th mode of the damaged structure. As described by Ojalvo and Pilon (1988), this vector represents the harmonic force excitation that would have to be applied to the undamaged structure represented by \( [M^u], [C^u], [K^u] \) at the frequency of \( \lambda_i^d \) so that the structure would respond with mode shape \( \{\phi^d\}_i \).

There are several methods that have been used to compute the analytical model matrices of the damaged structure \( [M^d], [C^d], [K^d] \) such that the resulting equation of motion (EOM) is balanced,

\[
((\lambda_i^d)^2[M^d] + (\lambda_i^d)[C^d] + [K^d])\{\phi^d\}_i = \{0\},
\]

(45)

where the damaged model matrices are defined as the model matrices of the undamaged structure minus a perturbation,

\[
[M^d] = [M^u] - [\Delta M] \\
[C^d] = [C^u] - [\Delta C] \\
[K^d] = [K^u] - [\Delta K]
\]

(46)
Substituting Eq. (46) into Eq. (45) yields

$$((\lambda_i^d)^2[M^u - \Delta M] + (\lambda_i^d)[C^u - \Delta C] + [K^u - \Delta K])\{\phi_i^d\} = \{0\}. \quad (47)$$

Moving the perturbation terms to the right side of the equation then yields

$$((\lambda_i^d)^2[M^u] + (\lambda_i^d)[C^u] + [K^u])\{\phi_i^d\} = ((\lambda_i^d)^2[\Delta M] + (\lambda_i^d)[\Delta C] + [\Delta K])\{\phi_i^d\}. \quad (48)$$

The left side of this equation consists of known quantities and has previously been defined as the modal force error, so the equation to be solved for the matrix perturbations can be written as

$$((\lambda_i^d)^2[\Delta M] + (\lambda_i^d)[\Delta C] + [\Delta K])\{\phi_i^d\} = \{E_i\}. \quad (49)$$

The modal force error is used as both an objective function and a constraint in the various methods described below.

Preservation of the property matrix symmetry is used as a constraint. This constraint can be written for each property matrix as

$$[\Delta M] = [\Delta M]^T$$
$$[\Delta C] = [\Delta C]^T$$
$$[\Delta K] = [\Delta K]^T$$

Preservation of the property matrix sparsity (the zero/nonzero pattern of the matrix) is also used as a constraint. This constraint can be written for each property matrix as

sparse([M^u]) = sparse([M^d])
sparse([C^u]) = sparse([C^d])
sparse([K^u]) = sparse([K^d]) \quad (51)$$

The preservation of sparsity is one way to preserve the allowable load paths of the structure in the updated model.

Preservation of the property matrix positivity is also used as a constraint. This constraint can be written for each property matrix as

$$\{x\}^T[\Delta M]\{x\} \geq 0$$
$$\{x\}^T[\Delta C]\{x\} \geq 0,$$
$$\{x\}^T[\Delta K]\{x\} \geq 0 \quad (52)$$
where \( \{x\} \) is any arbitrary vector.

2.E.2 Optimal Matrix Update Methods

Methods that use a closed-form, direct solution to compute the damaged model matrices or the perturbation matrices are commonly referred to as optimal matrix update methods. Reviews of these methods have been published by Smith and Beattie (1991a), Zimmerman and Smith (1992), Hemez (1993), and Kaouk (1993). The problem is generally formulated as a Lagrange multiplier or penalty-based optimization, which can be written as

\[
\min_{\Delta M, \Delta C, \Delta K} \left\{ J(\Delta M, \Delta C, \Delta K) + \lambda R(\Delta M, \Delta C, \Delta K) \right\},
\]

where \( J \) is the objective function, \( R \) is the constraint function, and \( \lambda \) is the Lagrange multiplier or penalty constant.

Baruch and Bar Itzhack (1978), Kabe (1985), and Berman and Nagy (1983) have a common formulation of the optimal update problem that is essentially minimization of the Frobenius norm of global parameter matrix perturbations using zero modal force error and property matrix symmetry as constraints.

Chen and Garba (1988a, 1988b) present a method for minimizing the norm of the property perturbations with a zero modal force error constraint. They also enforce a connectivity constraint to impose a known set of load paths onto the allowable perturbations. The updates are thus obtained at the element parameter level, rather than at the matrix level. This method is demonstrated on a truss FEM.

Another approach to this problem used by Kammer (1988) and Brock (1968) can be formulated as minimization of modal force error with a property matrix symmetry constraint. The symmetry constraint preserves the reciprocity condition in the updated structural model.

McGowan, et al. (1990) report ongoing research that examines stiffness matrix adjustment algorithms for application to damage identification. Based on measured mode shape information from sensor locations that are typically fewer than the DOF in an analytical model, mode shape expansion algorithms are employed to extrapolate the measured mode shapes such that they can be compared with analytical model results. These results are used to update the stiffness matrix while maintaining the connectivity and sparsity of the original matrix.

Smith and Beattie (1991a) extend the formulation of Kabe to include a sparsity preservation constraint and also formulate the problem as the minimization of both the perturbation matrix norm and the modal force error norm subject to the symmetry and sparsity constraints.
Smith (1992) presents an iterative approach to the optimal update problem that enforces the sparsity of the matrix at each iteration. The sparsity is enforced by multiplying each entry in the stiffness update by either one or zero, depending upon the correct sparsity pattern.

Kim and Bartkowicz (1993) investigate damage detection capabilities with respect to various matrix update methods, model reduction methods, mode shape expansion methods, numbers of damaged elements, numbers of sensors, numbers of modes, and levels of noise. The authors develop a hybrid model reduction/eigenvector expansion approach to match the order of the undamaged analytical model and the damaged test mode shapes in the matrix update. They also introduce a more realistic noise level into frequencies and mode shapes for numerical simulation. From both numerical and experimental studies, the authors showed that the number of sensors is the most critical parameter for damage detection, followed by the number of measured modes.

Lindner, et al. (1993) present an optimal update technique that formulates an over-determined system for a set of damage parameters representing reductions in the extensional stiffness values for each member. For the $p^{th}$ member, they write the extensional stiffness as

\[
(1 - d_p)(k_a)_p,
\]

where $(k_a)_p$ is the undamaged axial stiffness of the member. The value of $d_p$ represents the amount of stiffness reduction in that member. Lindner and Kirby (1994) extend the technique to account for changes in elemental mass properties.

Liu (1995) presents an optimal update technique for computing the elemental stiffness and mass parameters for a truss structure from measured modal frequencies and mode shapes. The method minimizes the norm of the modal force error, as defined in Eq. (49). The author demonstrates that if sufficient modal data are available, the elemental properties can be directly computed using the measured modal frequencies, measured mode shapes, and two matrices which represent the elemental orientations in space and the global connectivity of the truss. In this case, the solution for the elemental properties is shown to be unique and globally minimal. The method is used to locate a damaged member in a FEM of a truss using the first four measured modes in sets of three at a time.

Another type of approach to the optimal matrix update problem involves the minimization of the rank of the perturbation matrix, rather than the norm of the perturbation matrix. This approach is motivated by the observation that damage will tend to be concentrated in a few structural members, rather than distributed throughout a large number of structural members. Thus, the perturbation matrices will tend to be of small rank. This approach has been published extensively by Zimmerman and Kaouk (see Refs. below). The solution for the perturbation matrices is based on the theory that the unique minimum rank solution for matrix of the underdetermined system

30
\[ [A] \{X\} = \{Y\} \quad \text{with} \quad [A]^T = [A] \quad (55) \]

is given by

\[ [A] = \{Y\} [H] \{Y\}^T \quad \text{where} \quad [H] = ((\{Y\}^T \{X\})^{-1}) . \quad (56) \]

Zimmerman and Kaouk (1994) present the basic minimum rank perturbation theory (MRPT) algorithm. This basic algorithm defines the modal force error of the \(i\)th mode as the damage vector \(\{d\}_i\), so that the perturbation error equation can be written as

\[ \{d\}_i = [Z^d]_i \{\phi^d\}_i \quad (57) \]

where

\[ [Z^d]_i = (\lambda_i^d)^2 [\Delta M] + (\lambda_i^d)[\Delta C] + [\Delta K] \quad (58) \]

By observing that the \(j\)th element of \(\{d\}_i\) will be zero when the \(j\)th rows of the perturbation matrices are zero, a nonzero entry in \(\{d\}_i\) is interpreted as an indication of the location of damage. However, changes in the perturbation matrices are not the only possible source of nonzero entries in \(\{d\}_i\), as can be seen by rewriting the above equation at the \(q\)th structural DOF as

\[ d_{iq} = z_{iq}^d \phi_{iq}^d = \|z_{iq}^d\| \|\phi_{iq}^d\| \cos(\theta_{iq}) . \quad (59) \]

The deviation of the angle \(\theta_{iq}\) from 90° is shown to be a better indicator of damage location than the nonzero entries of \(\{d\}_i\), particularly when the row norms of \([Z^d]_i\) have different orders of magnitude.

In the case of a single nonzero perturbation matrix (for example \([\Delta K]\)), the perturbation error equation can be solved using the MRPT equations as

\[ [\Delta K] \{\phi^d\}_i = \{d\}_i \quad \text{with} \quad [\Delta K] = [\Delta K]^T . \quad (60) \]

The solution to this problem is

\[ [\Delta K] = \{d\}_i [H] \{d\}_i^T \quad \text{where} \quad [H] = ((\{d\}_i^T \{\phi^d\}_i)^{-1}) . \quad (61) \]

The resulting perturbation has the same rank as the number of modes used to compute the modal force error. It is demonstrated that the MRPT algorithm preserves the rigid body modes of the structure, and the effects of measurement and expansion errors in the mode shapes are demonstrated and discussed.
Kaouk and Zimmerman (1994a) further develop this algorithm and demonstrate how perturbations to two of the property matrices can be estimated simultaneously by using complex conjugates of the modal force error equation. The method is demonstrated numerically for a truss with assumed proportional damping. Also, the technique is used experimentally to locate a lumped mass attached to a cantilevered beam.

Kaouk and Zimmerman (1994b) extend the MRPT algorithm to estimate mass, stiffness, and proportional damping perturbation matrices simultaneously. The computation of these individual perturbation matrices is accomplished by exploiting the cross-orthogonality conditions of the measured mode shapes with respect to the damaged property matrices. The authors examine the results by computing a cumulative damage vector.

Kaouk and Zimmerman (1994c) present a technique that can be used to implement the MRPT algorithm with no original FEM. The technique involves using a baseline data set to correlate an assumed mass and stiffness matrix, so that the resulting updates can be used as the undamaged property matrices.

Zimmerman and Simmermacher (1994, 1995) compute the stiffness perturbation resulting from multiple static load and vibration tests. This technique is proposed partially as a method for circumventing the mismatch in the number of modes between test and FEM. They apply this technique to a FEM of a structure similar to a NASA test article. They also present two techniques for overcoming the rank deficiency that exists in the residual vectors when the results of one static or modal test are linear combinations of the results of previous tests.

Kaouk and Zimmerman (1995a) introduce a partitioning scheme into the MRPT algorithm by writing the parameter matrix perturbations as sums of elemental or substructural perturbations. The partitioning procedure reduces the rank of the unknown perturbation matrices and thus reduces the number of modes required to successfully locate the damage. The technique is demonstrated on data from the NASA 8-bay Dynamic Scale-Model Truss (DSMT) testbed. In a related paper, Kaouk and Zimmerman (1995b) further examine the reduction of the number of modes required for model updating using a two-level matrix partitioning technique.

Zimmerman, et al. (1995a) extend the theory to determine matrix perturbations directly from measured FRFs. This method is implemented by solving for the perturbation in the dynamic impedance matrix, \([\Delta Z]\), from the generalized off-resonance dynamic force residual equation

\[
[Z(\omega) + \Delta Z] \{X(\omega)\} = \{F(\omega)\} .
\]  

(62)

They discuss the benefits of this formulation, including the elimination of the need to match modes between FEM and test, reduction in the amount of frequencies required in the test (and thus test time), and the elimination of the need to perform modal parameter identification.
Zimmerman, et al. (1995b) investigate the role of engineering insight and judgment in the implementation of the MRPT techniques to damage detection. Specifically, the issues of evaluation of the damage location, selection of how many measured modes to use, filtering of the eigenvectors and the damage vector, and decomposition of the damage vector into contributions from individual property matrices are addressed. This paper also contains a list of publications related to the theory and application of MRPT.

Doebling (1996) presents a method to compute a minimum-rank update for the elemental parameter vector, rather than for global or elemental stiffness matrices. The method uses the same basic formulation as the MRPT, but constrains the global stiffness matrix perturbation \([\Delta K]\) to be a function of the diagonal elemental stiffness parameter perturbation matrix \([\Delta P]\) as

\[
[\Delta K] = [A][\Delta P][A]^T, \tag{63}
\]

where \([A]\) is a static sensitivity matrix relating the elemental stiffness parameters to the entries in the global stiffness matrix. By substituting Eq. (63) into Eq. (60), the unknown in the problem becomes \([\Delta P]\), and the minimum-rank stiffness perturbation that minimizes the modal force error \(\{E\}_i\) in Eq. (49) is explicitly constrained to preserve the FEM connectivity. The author shows that this method performs better than a minimum-norm parameter update technique on the data from a truss damage detection experiment. A limitation of this method, as with all minimum-rank procedures, is that the rank of the perturbation is always equal to the number of modes used in the computation of the modal force error.

### 2.E.3 Sensitivity-Based Update Methods

Another class of matrix update methods is based on the solution of a first-order Taylor series that minimizes an error function of the matrix perturbations. Such techniques are known as sensitivity-based update methods. An exhaustive list and classification of various sensitivity-based update techniques is given in Hemez (1993). The basic theory is the determination of a modified parameter vector

\[
\{p\}^{(n+1)} = \{p\}^{(n)} + \{\delta p\}^{(n+1)}, \tag{64}
\]

where the parameter perturbation vector \(\{\delta p\}^{(n+1)}\) is computed from the Newton-Raphson iteration problem for minimizing an error function

\[
J(\{p\}^{(n)} + \{\delta p\}^{(n+1)}) = J(\{p\}^{(n)}) + \left[\frac{\partial J}{\partial \{p\}}(\{p\}^{(n)})\right] \otimes \{\delta p\}^{(n+1)} = 0, \tag{65}
\]

where \(J(\{p\})\) is the error function to be minimized. Typically the error function is selected to be the modal force error, as defined by Eq. (44).
A main difference between the various sensitivity-based update schemes is the method used to estimate the sensitivity matrix. Basically, either the experimental or the analytical quantities can be used in the differentiation. For experimental sensitivity, the orthogonality relations

\[
[\Phi]^T [M] [\Phi] = [I]
\]
\[
\]

(66)
can be used to compute the modal parameter derivatives \( \partial \omega^2 / \partial p \) and \( \partial \Phi / \partial p \). Such an approach has been proposed by Norris and Meirovitch (1989), Haug and Choi (1984) and Chen and Garba (1980).

Analytical sensitivity methods usually require the evaluation of the derivatives \( [\partial M / \partial p] \) and \( [\partial K / \partial p] \), which are less sensitive than experimental sensitivity matrices to noise in the data and to large perturbations of the parameters. Hemez (1993) presents a method for computing the global analytical sensitivity matrices based on assembly of the element-level analytical sensitivities.

Ricles (1991) presents a methodology for sensitivity-based matrix update, which takes into account variations in system mass and stiffness, center of mass locations, changes in natural frequency and mode shapes, and statistical confidence factors for the structural parameters and experimental instrumentation. The method uses a hybrid analytical/experimental sensitivity matrix, formed as

\[
[T] = \begin{bmatrix}
\partial \omega^2 & \partial \omega^2 \\
\partial \Phi & \partial \Phi \\
\partial K & \partial K \\
\partial M & \partial M
\end{bmatrix}
\begin{bmatrix}
\partial K \\
\partial M \\
\partial p \\
\partial p
\end{bmatrix},
\]

(67)

where the modal parameter sensitivities are computed from the experimental data, and the matrix sensitivities are computed from the analytical model. This method is further developed and applied to more numerical examples by Ricles and Kosmatka (1992).

Sanayei and Onipede (1991) present a technique for updating the stiffness characteristic of a FEM using the results of a static load-displacement test. A sensitivity-based, element-level parameter update scheme is used to minimize the error between the applied forces and forces produced by applying the measured displacements to the model stiffness matrix. The sensitivity matrix is computed analytically. The structural DOF are partitioned such that the locations of the applied loads and the locations of the measured displacements are completely independent. The technique is demonstrated on two example FEMs.

In a related paper, Sanayei, et al. (1992) examine the sensitivity of the previous algorithm to noisy measurements. The influence of the selected measurement DOF set on
the errors in the identified parameters is studied. A heuristic method is proposed that recursively eliminates the measurement DOF that the elemental stiffness parameters are the most sensitive to. In this manner, the full FEM DOF set is reduced to a manageable size while preserving the ability to identify the structural stiffness parameters. In later work, Sanayei and Saletnik (1995a, 1995b) extend the algorithm and the error analysis to use static strain, rather than displacement, measurements.

Hemez (1993) presents a sensitivity-based matrix update procedure which formulates the sensitivities at the element level. This has the advantage of being computationally more efficient than forming the sensitivities at the global matrix level. It also allows the analysis to “focus” on damage in specific members. A modified version of this algorithm, developed by Alvin (1996), improves the convergence, utilizes a more realistic error indicator, and allows the incorporation of statistical confidence measures for both the initial model parameters and the measured data.

2.E.4 Eigenstructure Assignment Method

Another matrix update method, known as “eigenstructure assignment,” is based on the design of a fictitious controller which would minimize the modal force error. The controller gains are then interpreted as parameter matrix perturbations to the undamaged structural model.

Lim (1994, 1995) provides a clear overview of the eigenstructure assignment technique. Consider the basic structural EOM with a controller

\[ [M^u]\{\dot{x}\} + [C^u]\{\dot{x}\} + [K^u]\{x\} = [F]\{u\} \]
\[ \{u\} = -[G][F]^T\{x\} \] (68)

Suppose that the control gains are selected such that the modal force error between the nominal structural model and the measured modal parameters from the damaged structure is zero

\[ ((\lambda^d_i)^2[M^u] + (\lambda^d_i)[C^u] + [K^u + FGF^T])\{\phi^d_i\} = \{0\} \] (69)

making the definition

\[ L_{ij} = ((\lambda^d_i)^2[M^u] + (\lambda^d_i)[C^u] + K^u)^{-1}[F]_j[F]^T \] (70)

Then the “best achievable eigenvectors” \( \{\phi^d_i\}_i \) (which lie in the subspace spanned by the columns of \( L_{ij} \)) can be written in terms of the measured eigenvectors as

\[ \{\phi^d_a\}_i = L_{ij}^{-1}\{\phi^d\}_j \] (71)
The relationship between the best achievable eigenvectors and the measured eigenvectors is then used as a measure of damage location. Specifically, if damage is in member \( j \), then the measured and best achievable eigenvectors are identical. Thus, the angle between the vectors gives an indicator of how much a particular member contributes to the change in a particular mode. This information can be used to hypothesize the location of the structural damage. The magnitude of the damage is then computed using the eigenstructure assignment technique such that the best achievable eigenvectors, undamaged model matrices, and controller satisfy the modal force error equation. Lim and Kashangaki (1994) introduce the use of the best achievable eigenvectors for the location of structural damage and apply the technique to the detection of damage in an 8-bay cantilevered truss.

Zimmerman and Kaouk (1992) implement such an eigenstructure assignment technique for damage detection. They include algorithms to improve the assignability of the mode shapes and preserve sparsity in the updated model. They apply their technique to the identification of the elastic modulus of a cantilevered beam.

Lindner and Goff (1993) define damage coefficients for each structural member so that the updated elastic modulus is expressed as in Eq. (54). They then use an eigenstructure assignment technique to solve for the damage coefficient for each member. They apply this technique to detect simulated damage in a 10-bay truss FEM.

Lim (1994, 1995) applies a constrained eigenstructure technique experimentally to a twenty-bay planar truss. His approach identifies element-level damage directly, rather than finding perturbations to the stiffness matrix. The computation of element-level perturbations is accomplished by diagonalizing the control gains, then interpreting the diagonal entries as changes to the elemental stiffness properties. The technique is shown to work well even with limited instrumentation.

Schulz, et al. (1996) present a technique similar to eigenstructure assignment known as "frequency response function assignment." The authors formulate the problem as a linear solution for element-level stiffness and mass perturbation factors. They point out that using FRF measurements directly to solve the problem is more straightforward than extracting mode shapes. They use measured mobility functions (FRFs from velocity measurements) to obtain higher numerical accuracy, since the velocity response is flatter over the entire spectrum than either the displacement or acceleration response. The technique is applied to an FEM of a bridge structure.

### 2.E.5 Hybrid Matrix Update Methods and Other Considerations

Baruh and Ratan (1993) use the residual modal force as an indicator of damage location. They separate the residual modal force into the effects of identification error in the measurements, modeling error in the original structural model, and modal force error resulting from structural damage. They examine the sensitivity of the damage location solution to errors in the original structural model and to inaccuracies in the modal identification procedure.
Kim and Bartkowicz (1993, 1994) and Kim, et al. (1995a) present a two-step damage-detection procedure for large structures with limited instrumentation. The first step uses optimal matrix update to identify the region of the structure where damage has occurred. The second step is a sensitivity-based method, which locates the specific structural element where damage has occurred. The first advantage of this approach lies in the computational efficiency of the optimal update method in locating which structural parameters are potentially erroneous. The second advantage lies in the small number of parameters updated by the sensitivity-based technique.

Li and Smith (1994, 1995) present a hybrid model update technique for damage identification which uses a combination of the sensitivity and optimal-update approaches. This method constrains the stiffness matrix perturbation to preserve the connectivity of the FEM, and the solution minimizes the magnitude of the vector of perturbations to the elemental stiffness parameters. The hybrid technique is shown to be more computationally efficient than the iterative sparsity-preserving algorithm presented by Smith (1992).

Dos Santos and Zimmerman (1996a) examine the effects of model reduction via component mode synthesis (specifically using the Craig-Bampton technique) on the accuracy of damage identification results obtained using the MRPT force residual and angle residual vectors. Numerical examples were conducted on an FEM of a clamped-clamped beam divided into five substructures of 3 to 4 elements each. Damage was simulated on one of the elements within one of the substructures by reducing the cross-sectional moment of inertia by 25%. The results indicated that the MRPT force residual vector was unable to accurately locate the damaged substructure. The results of applying the angle residual vector indicated that the damaged substructure could be identified using a highly truncated component mode set, and the damaged element could be identified using a more rich component mode set.

Dos Santos and Zimmerman (1996b) propose a method for damage identification that uses MRPT in conjunction with ordinary least-squares estimation to preserve the connectivity of the FEM during the update procedure. The method produces estimates of the damage extent in the form of element-level stiffness parameter perturbations. The procedure is conducted in two steps: First, the damaged global stiffness matrix perturbation is estimated using the MRPT algorithm, as described in Section 2.E.2. Next, a set of parameters representing the loss of stiffness in each element is estimated by minimizing the error between the MRPT matrix perturbation and the global stiffness matrix perturbation computed using the elemental stiffness matrices and the stiffness reduction parameters. The unique estimation of the parameters requires that the number of measurement be greater than or equal to the number of parameters being estimated.

Gafka and Zimmerman (1996) evaluate the performance of a mode shape expansion algorithm known as Least-Squares Dynamic Residual Force Minimization with Quadratic Measurement Error Inequality Constraint (LSQIC). The method is used to estimate the component of the measured mode shapes at the unmeasured FEM DOF. The method minimizes the error in the residual modal force vector that results from
substituting the expanded measured mode shape into the FEM eigenequation. The magnitude of the difference between the expanded and measured mode shape at the measurement DOF is constrained to be less than a certain fraction of the magnitude of the measured mode shape. The method is compared to two standard techniques—Guyan (or static) expansion and dynamic expansion—for application to both FEM model correlation and damage identification. The results demonstrate that the expansion method allows for accurate FEM correlation in the general case where the errors are distributed somewhat evenly in the structure. However, in the case of damage identification, where the discrepancies between the test data and the model are isolated at a few DOF, a smearing effect resulting from the use of a singular value decomposition in the solution procedure can impede the accurate identification of the damage.

2. F Nonlinear Methods

Actis and Dimarogonas (1989) develop a FEM for a beam with an opening and closing crack. They determine if the beam is open or closed by looking at the sign of the bending moment at the crack location. They make the approximation that the mode shape does not change as a result of the crack. They also present results showing the harmonics generated from the opening and closing of the crack.

Lin and Ewins (1990) propose a process for locating a structural nonlinearity using modal testing. The basis for the process is a model update technique with modal data, measured at different response levels, used to localize the nonlinearity. Writing the EOM for the two different response levels, \( a \) and \( b \), yields

\[
((\lambda^a_i)^2[M + \Delta M] + [K + \Delta K])\{\phi^a\}_i = \{0\}
\]

\[
((\lambda^b_i)^2[M + \Delta M] + [K + \Delta K + \Delta K_n])\{\phi^b\}_i = \{0\},
\]

where \( \lambda^a_i, \{\phi^a\}_i, \lambda^b_i, \{\phi^b\}_i \) are the eigenparameters for the \( i \)th mode measured at response levels \( a \) and \( b \), respectively. The matrix perturbations \( [\Delta M] \) and \( [\Delta K] \) represent the errors in the FEM, and \( [\Delta K_n] \) represents the stiffness nonlinearities. Post-multiplying the first equation by \( \{\phi^b\}_iT \) and the second by \( \{\phi^a\}_iT \), subtracting and then rearranging gives

\[
[\Delta M](\lambda^b_i)^2[\phi^b]_i[\phi^a]_iT - (\lambda^a_i)^2[\phi^a]_i[\phi^b]_iT
+ [\Delta K](\{\phi^b\}_iT\{\phi^a\}_iT - \{\phi^a\}_i\{\phi^b\}_iT + [\Delta K_n](\{\phi^b\}_i\{\phi^b\}_iT)
= ((\lambda^a_i)^2[M] + [K])\{\phi^a\}_iT - ((\lambda^b_i)^2[M] + [K])\{\phi^b\}_i\{\phi^a\}_iT_t
\]

Define the right hand side of this equation as \([A]\), which is a known matrix from the eigenparameters measured at two response levels and the original (uncorrected) mass and stiffness matrices. Consider \( \{\phi^b\}_i \) to be a perturbation of \( \{\phi^a\}_i \), and assume that the modeling errors \( (\lambda^a_i)^2[\Delta M] \) and \( [\Delta K] \) are of the same order as the nonlinearity \( [\Delta K_n] \). These assumptions allow the simplifications...
\( \{ \phi^b \} \llbracket \{ \phi^a \} \rrbracket^T - \{ \phi^a \} \llbracket \{ \phi^b \} \rrbracket^T \approx 0 \)

\( (\lambda^b)^2 \{ \phi^b \} \llbracket \{ \phi^a \} \rrbracket^T - (\lambda^a)^2 \{ \phi^a \} \llbracket \{ \phi^b \} \rrbracket^T = 0 \)

which leaves the equation

\[ [\Delta K_n] \{ \phi^b \} \llbracket \{ \phi^a \} \rrbracket^T = [A] . \]

Thus, the nonlinear term \([\Delta K_n]\) can be calculated from the measured \([A]\) matrix. The nonlinear modeling errors are not included in this term. From this point on the procedure follows that of other model update techniques, as described in Section 2.E, and the authors address issues such as interpolation of unmeasured coordinates and selection of sensitive modes.

Shen and Chu (1992) and Chu and Shen (1992) develop a closed-form solution for the vibration of a beam with an opening and closing crack subjected to low-frequency harmonic forcing. They model the beam as a bilinear oscillator. The crack is determined to be opened or closed in one paper based on the sign of the axial strain at the crack location and in the other paper based, apparently, on the sign of the transverse displacement at the crack location. They present results that indicate the possibility of identifying damage based on the frequency and magnitude of harmonic vibrations.

Krawczuk and Ostachowicz (1992) develop a model for transverse vibration of a Bernoulli-Euler beam with an opening and closing crack. They model the beam as two sections on either side of the crack connected by a torsional spring. The model is described in more detail in Section 2.A.2, in the discussion of the work of Narkis (1994). To handle the opening and closing of the crack, the authors consider the crack length to vary sinusoidally as a function of time during the closing half of the vibration cycle. This method contrasts with previous work, where the crack is typically considered to be either fully open or fully closed. The authors solve a numerical equation of motion (including damping) based on this formulation. The results include the effects of crack parameters on the first resonance and regions of vibrational instability. (Presumably these vibrations occur under forced harmonic loading, but it is not clear from the paper).

Huang and Gu (1993) use higher-order cumulants to detect the presence of nonlinearities in a structure. Cumulants are expectation values of polynomials of a random variable. Cumulants above the second order are designed to have zero value for Gaussian processes. The authors use this fact to make a detector for nonlinearities in a beam.

Manson, et al. (1993) present a method for simulating nonlinear systems using only linear techniques. The motivation for this research is strong because nonlinear elements such as cracks are notoriously difficult to model using finite element analysis. The authors develop a technique based on the Volterra series. This technique yields a perturbation series for nonlinear responses based on generalizations of FRFs. The
proposed method works by adding auxiliary inputs to model the effects of the nonlinearities. The superposition of the linear response resulting from the actual and auxiliary inputs simulates the true nonlinear response. The strengths of the auxiliary inputs are determined by the form of the nonlinearity and the true input. One problem with this method is that the effective inputs are functions of the entire time histories and must be recomputed if the input changes so the method cannot be used in real time. A second problem is that a different series must be computed for each location where a measured response is desired. A final problem is that to exactly simulate the nonlinear response, one must compute an infinite number of terms.

Klein, et al. (1994) experimentally observe a different form of frequency change. They measured FRFs on a fatigue cracked cantilever beam. They found that the measured natural frequency in the cracked beam increased noticeably as the accelerometer location approached the crack. They also found that the natural frequencies of the beam increased during the fatigue process up to and beyond the point of crack initiation.

Surace and Ruotolo (1994) use the complex Morlet wavelet transformation to study damage in finite element simulations of a 300mm (11.8 in.) x 20mm (0.8 in.) x 20mm (0.8 in.) cantilever beam. The author found that when the damage was sufficiently large, corresponding to a crack somewhere between 20% to 45% of the beam thickness, the amplitude of the wavelet transform showed modulations consistent with the crack opening and closing.

Feldman and Braun (1995) use a quantity known as the “analytic signal” to perform modal parameter estimation. The analytic signal has the actual signal as its real part and the Hilbert transform as its imaginary part. The authors transform the differential equation for a spring-mass system into the analytic signal domain. They then solve an algebraic equation for an instantaneous estimate of the damping and frequency. The signal is further conditioned by low-pass filtering. The authors apply the method to three types of nonlinearities: the backlash spring model, the preloaded spring model, and the dry friction model. The results show reasonable estimates for the modal parameters.

Crespo, et al. (1996) present an analytical method for describing the nonlinear vibration of a cracked cantilever beam using higher order FRFs. A Volterra series is used to generate the analytical formulation of the higher-order FRFs. To use a Volterra series analytically, the bilinear stiffness of the cracked beam is approximated using a 4th-order power series for force as a function of displacement. Applying this technique to a simulated cracked beam indicates that the higher-order FRFs are more sensitive than conventional FRFs to both crack size and location.

Prime and Shevitz (1996) present results from an experimental study of a cantilever beam containing an opening and closing crack. They present results showing that “harmonic mode shapes” are more sensitive to crack depth and locate cracks better than conventional mode shapes. Harmonic mode shapes are computed using the magnitude of harmonic peaks in the cross-power spectra. They use the analytic signal (see review of Feldman and Braun (1995) above) to calculate an instantaneous fre-
frequency, which displays the bilinear behavior of the opening and closing crack. They also use the analytic signal to perform a Wigner-Ville time-frequency transform on the data. The Wigner-Ville transform appeared to be more sensitive to the nonlinearity than Fourier transform techniques. They conclude that the nonlinear aspect of the crack provides increased ability to locate the crack compared to just using the associated stiffness change.

2.G  **Neural Network-Based Methods**

In recent years there has been increasing interest in using neural networks to estimate and predict the extent and location of damage in complex structures. Neural networks have been promoted as universal function approximators for functions of arbitrary complexity. A general overview of neural networks can be found in Bishop (1994). The most common neural network in use is the multilayer perceptron (MLP) trained by backpropagation. In this section, terminology will be defined consistent with common usage by calling a MLP trained by backpropagation a “backprop neural network.” The backprop neural network is a system of cascaded sigmoid functions where the outputs of one layer, multiplied by weights, summed, then shifted by a bias are used as the inputs to the next layer. Once an architecture for the network is chosen, the actual function represented by the neural network is encoded by the weights and biases. The backpropagation learning algorithm is a way of adjusting the weights and biases by minimizing the error between the predicted and measured outputs. In the following studies there were typically more adjustable weights than experiments, and the body of data was repeatedly run through the training algorithm until a criterion for the error between the data and the neural network was satisfied. Each error-generating run is called an epoch. The terms neuron and node are used interchangeably in the following discussion.

Kudva, et al. (1991) used a backprop neural network to identify damage in a plate stiffened with a 4 x 4 array of bays. Damage was modeled by cutting holes of various diameters in the plate at the centers of the bays. The bays were sized 305 mm (12 in.) x 203 mm (8 in.) and the holes were from 12.7 mm (0.5 in.) to 63.5 mm (2.5 in.). A static uniaxial load was applied to the structure, and strain gage readings were taken from elements in the bays. The neural network was used to identify the map from the strain gage data to the location and size of the hole. In different experiments 8, 20, and 40 strain gages were used as input. The structure of the network was chosen to be two hidden layers, each with the same number of hidden nodes as the number of inputs. The network was trained with 3, 12, or 32 patterns, depending on which experiment was being tested. The authors claimed the networks converged in less than 10 minutes on a 386 PC, depending on the example. It should be noted that in one example the neural network failed to converge, and the authors were forced to modify their procedure to a two-step algorithm, which first predicted the hole quadrant, then the correct bay within the quadrant. The authors found that the neural network was able to predict the location of the damaged bay without an error but that predicting hole size was more difficult with sometimes erratic results. In the cases where the neural network successfully identified the hole size, typical errors were on the order of 50%.
Wu, et al. (1992) used a backprop neural network to identify damage in a three-story building modeled by a two-dimensional “shear building” driven by earthquake excitation. The damage was modeled by reducing member stiffness by 50% to 75%. The neural network was used to identify the map from the Fourier transform of acceleration data to the level of damage in each of the members. The first 200 points in the fast Fourier transform (FFT) (0 Hz to 20 Hz) were used as network inputs. A network architecture with one hidden layer and 10 hidden nodes was selected, and 42 training cases were used. No information was given on how long it took for the neural net to converge. The first attempt relied on using only acceleration data from the top floor. On test data, the neural network was only able to identify third-floor data with any accuracy. A second network was implemented that used acceleration data from the second two floors as inputs. This network was able to diagnose damage on the first and third floors to within approximately 25% but was still unable to predict damage to the second floor with any accuracy. The latter method relied on a complete knowledge of the time histories of two of the three DOF.

Elkordy, et al. (1993) used backpropagation neural networks to identify damage in five-story buildings. Damage was modeled by reducing member stiffness in the bottom two stories from 10% to 70%. The neural network was used to identify the map from the mode shapes to the percent change in member stiffness. The authors chose a network with a single hidden layer of fourteen nodes. The network was trained on two mathematical models and verified with experimental data. The models were two-dimensional, finite element representations of increasing complexity. The first model gave eleven training patterns, the second model gave nine. The authors do not specify how long it took the neural net to converge. The model was reasonably successful at predicting damage to the first and second stories and predicting the extent of the damage. In general, the neural network trained on the first model of lower complexity made poorer predictions, and it incorrectly diagnosed damage in two of the eleven experiments. The neural network trained on the second model never made an incorrect diagnosis, but it was indeterminate in one example. If the correct diagnosis was made, the predictions of damage were generally correct to within 10%. Elkordy, et al. (1994) is a slightly modified version of this paper.

Leath and Zimmerman (1993) used an MLP neural network based on a training method they developed to identify damage in a four-element cantilevered beam. Most of the paper was spent developing the training algorithm, which was subsequently used in the damage identification. The training algorithm was designed to fix the architecture of the network. The idea involved the creation of a network that exactly fit the data with a minimal number of hidden nodes. This criterion fixed the number of hidden nodes to be one less than the number of data points (the bias to the output neuron was the final adjustable parameter). There are two issues which raise questions about the intended structure of the neural network. The first issue is that the number of training data points is essentially arbitrary, and one should not need to change architecture to incorporate more data. The second and much more important issue is that exactly fitting the data can cause problems in general. If there is any noise in the data, then an exact fit will model the noise as well. Additional measurements give no noise rejection
and instead serve only to confuse the algorithm. This problem occurs, for example, when repeated measurements are made at the same input. The training algorithm to compute the weights for the hidden nodes was based on the principles of computational geometry and as such could only be applied with input dimensions of one or two. The training was intended to put the thresholds for the hidden layer sigmoids for maximum discrimination between the training data. The damage in the beam was modeled by reducing Young's modulus up to 95%. The neural network was used to identify the map from the first two bending frequencies to the level of damage in each member. It should be noted that only the first two frequencies are used because no higher dimension can be handled by the training algorithm. The algorithm was able to identify damage to within 35%.

In Spillman, et al. (1993) the authors used a feedforward neural network to identify damage in a steel bridge element. The element was roughly 4.5 m (14.75 ft) long. Damage was introduced by cutting the element and bolting a plate reinforcement over top of the cut. With the plate attached, the element was considered to be undamaged. With the bolts loosened, the element was considered to be partially damaged; with the plate removed, the element was considered to be fully damaged. There were three sensors mounted to the element: two accelerometers and a fiber optic modal sensor. The beam was struck in four different locations with a calibrated impact. A total of eleven tests was performed. The time-history signal from each sensor was Fourier transformed, and the height and frequency of the first two modal peaks were used as inputs to the neural network. The impact intensity and location were also provided as inputs. A network configuration was selected with the 14 inputs already mentioned, a hidden layer with 20 neurons, and 3 outputs, one for each of the possible damage states. The body of training data was cycled through the training algorithm until the self-prediction error converged to a minimum. Generally, convergence took less than 100 epochs. Other network configurations were also tried that used fewer of the inputs to see if, for example, the fiber optic sensor was providing any useful information. The results were moderately successful. Using all three sensors, the authors found the proportion of correct diagnoses to be 58%. The authors credited this number by citing the small size of the training data set.

Worden, et al. (1993) used a backpropagation neural network to identify damage in a twenty-member framework structure. Damage was modeled by removing one of the structural members completely. The neural network was used to identify the map from static strain data to a subjective measure of the damage. The strain in eight members was used for input, and there was one output for each of the eight members where damage was to be identified. The subjective scale was between 0 and 1. A three-hidden-layer design with 12, 12, and 8 hidden nodes, respectively, was chosen. The network was trained on data generated by a FEM and tested on an experimental model of the same geometry. There were 192 training patterns, and the neural network took approximately 50,000 epochs to converge. When applied to experimental data, the system was mostly able to identify the location of the damage. However, there were frequent misclassifications owing partially to the large size of the test set.
Yen and Kwak (1993) used a cerebellar model articulation controller (CMAC) network to identify sensor failures in damage detection. The CMAC network is a nearest-neighbor-type network. The function is approximated by interpolating the nearest data points. This paper was not really a damage-identification paper because triply redundant sensors, which were perfect indicators of damage, were used. The network was used to determine if there were any sensor failures in the triply redundant sensing system. The basic idea of this network is that if one sensor's signal is sufficiently different from the other two, it has failed. If all three sensors produce different signals, then two sensors have failed. The inputs to the neural network are the sensor readings, and the output is a binary value for each of five damage states: no damage, damage in one of the three sensors, or multiple damage. In the case of multiple damage, no attempt was made to determine which sensors had failed. The network was able to discriminate single failures but had difficulties with multiple failures.

Kirkegaard and Rytter (1994) used a backprop neural network to identify damage in a 20-m steel lattice mast subject to wind excitation. Damage was modeled by replacing lower diagonals with bolted joints of diminished thickness. The neural network was used to identify the map from the first five fundamental frequencies to the percent damage in member stiffness. One output was used for each element of interest. The authors chose a network architecture with two hidden layers of five nodes each. There were four outputs corresponding to four of the diagonals. The network was trained with 21 examples generated from a FEM. The training set used data with 0% to 100% reduction in diagonal cross sectional area. The network was able to reproduce the training data, but it had less success on the test data. At 100% damage the neural network was able to locate and quantify the damage. At 50% the network was able to predict the existence of damage, but not the magnitude. Damage less than 50% was not identified. This result was consistent for test data from both the FEM and the actual experiment.

Manning (1994) used backprop neural networks to identify damage in a 10-bar truss structure and a 25-bar transmission tower with active members. Damage was modeled as changes in member cross-sectional area. Pole and zero locations were extracted from the FRFs between the member actuator and the two piezoceramic sensors on the same member. A measure of the member stiffness was also extracted from each FRF. The imaginary part of the pole and zero locations and the stiffness information were the data given to the neural network. The neural network identified the map from pole-zero location and member stiffness to member cross-sectional area. For the 10-bar truss a network architecture with a single hidden node with 9 hidden neurons was used. For the 25-bar transmission tower, an architecture with 40 inputs, two hidden layers with 7 and 5 nodes, respectively, and 4 outputs was used. The author does not discuss how large the training sets were but claims that convergence occurred within 8000 epochs for the transmission tower. The networks were tested on three examples not in the training sample and predicted member area within 10% for most of the members.
Povich and Lim (1994) used a backpropagation neural network to identify damage in a 20-bay planar truss composed of 60 struts. Damage was modeled by removing struts from the structure. The structure was excited by a shaker in the 0 Hz to 50 Hz frequency range. Two accelerometers were placed on the structure to provide input data. The frequency range studied contained the first four bending modes. The neural network was used to identify the map from the Fourier transform of the acceleration history to damage in each desired member. The network had 394 inputs corresponding to the acceleration FFTs at the frequencies of interest for two points and 60 outputs, one for each strut in the structure. The authors chose a two-hidden-layer network with 125 and 40 nodes in the respective layers. There were 61 training examples consisting of the removal of each strut and an undamaged case. The network took approximately 1000 epochs to converge. The neural network was able to correctly identify the missing strut in 21 cases and was able localize the damage to two adjacent struts in 38 cases. No cross validation or testing of the network was done; the reported results are simply a measure of how well a neural network could fit the data. The network’s generalization capabilities were not tested.

Rhim and Lee (1994) used a backpropagation neural network to identify damage in a composite cantilevered beam. The damage was modeled as delamination in a FEM of the beam. The simulations were dynamic with both the force input and the measured output located at the beam tip. Before the neural network was applied, a preprocessing step was done on the data. An auto-regressive system identification was performed on the transfer function of the beam from the force input to the displacement output. The denominator of the transfer function or characteristic polynomial (equivalent to the poles) was then used in subsequent damage identification. The advantage of doing the system identification first was that it reduced the body of data to a smaller number of physically meaningful parameters. The neural network was used to identify the map from the characteristic polynomial to an empirical damage scale. Each of the four outputs represented a different level of damage, where zero indicated no damage at that level, and one indicated total damage at that level. Ideally, there would be at most one nonzero output at a time. The authors chose a network architecture with 13 inputs, corresponding to a fixed 12th-order characteristic polynomial (the maximum number of resonances seen), one hidden layer with 30 nodes, and 4 outputs. The network was trained with 10 training patterns, and the network took 330,000 iterations to converge. The network generalization capability was tested on three examples and correctly identified the damage in those cases.

Stephens and VanLuchene (1994) used a backpropagation network to identify damage in multistory buildings. The body of data was accumulated on a one-tenth scale reinforced concrete structure. The damage was modeled by introducing actual cracks into a concrete model of the structure. The network was used to identify the map from three empirical damage indices to a qualitative scale of damage in the structure. The three indices were measures of maximum displacement, cumulative energy dissipated in the building, and stiffness degradation. The output was a number between zero and one with the former being no damage, the latter total collapse. The neural network was trained on 60 data points and tested on a training set of 32 data points. The authors
experimented with different network topologies and settled on a single hidden layer with seven hidden neurons. The number of training epochs depended on the network architecture but was always less than 10,000. The neural network correctly identified the damage index for about 25 of the 32 test data points. Interestingly, the researches applied the neural network to the Imperial County Services Building, the one building to suffer a major earthquake while adequate sensors were in place to measure the damage indices. The network correctly identified the building as being lightly damaged. The neural network result was shown to be superior to a linear regression model of the data by about 25%.

Szewczyk and Hajela (1994) used a counter-propagation neural network to identify damage in truss structures. The counter-propagation network builds what is essentially an adaptive look-up table from the data. The look-ups were keyed by the position of the input vector. During training, nodes were moved closer to adjacent input nodes. If no node was sufficiently near the input pattern, then a new node was added. Predictions were made by interpolating adjacent nodes. The advantage of the counter-propagation network is that the body of data does not have to be cycled through more than once, as there is no error criterion to limit the convergence. Another advantage of the network is that the architecture is selected by the data, not user-specified. The disadvantage of this network is that it may take a very large number of training points to adequately sample the desired function. In the paper, the authors show that the size of the training set does not seem to be a problem in their case. Damage was modeled by reducing Young’s modulus in the truss members up to 100% (complete removal). The neural network was used to identify the map from static deformation under load to the Young’s modulus of the members. The analysis is completely static, and no modal or frequency analysis is required. The damage identification algorithm was run on three structures of increasing complexity: a 2-dimensional, 6-DOF system; a 2-dimensional, 18-DOF system; and a 3-dimensional, 12-DOF system. The network was trained with 200, 3600, and 3000 examples respectively. The neural network was then verified separately on test data and found to have a maximum error of approximately 30%.

Tsou and Shen (1994) used backpropagation neural networks to identify damage in two spring-mass systems: a 3-DOF system and the 8-DOF Kabe system, which has widely spaced eigenvalues. Damage was modeled by changing spring constants from 10% to 80%. For the 3-DOF system the neural network was used to identify the map from the change in modal frequencies to the percent change in the spring stiffnesses. In the 8-DOF problem, the neural network was used to identify the map from the residual modal force, as defined in Eq. (44), to the percent change in each spring constant. In the 8-DOF problem, the damage map was first broken up into a binary determination of whether damage was present in each spring. This body of data was then used to train another neural network (via a look-up table) to estimate the extent of damage. This procedure led to a combinatorial explosion in the number of training patterns which had to be stored. The architecture of the neural networks was a single hidden layer with 40 hidden nodes for the 3-DOF problem. For the 8-DOF problem, different architectures were tried including 100, 60, and 40 hidden nodes. The networks for the two systems were trained with 27 and 105 patterns respectively. The 3-DOF problem
took 80,000 epochs to converge. The authors do not state how many epochs were required for the neural network to converge in the 8-DOF case, but the computation was performed on a Cray supercomputer. The authors were able to identify the extent of damage to within 5% accuracy, provided the body of data was in an interpolation of existing data. However, data extrapolation produced errors up to 30%.

Barai and Pandey (1995) used backpropagation neural networks to identify damage in a truss structure simulating a bridge. The authors used the neural network to identify the map from various nodal time histories to changes in stiffness. To train the neural network, the authors used a finite element simulation of the truss with a moving point force to simulate a vehicle being driven at constant velocity. The time histories at small time intervals for 1, 3, and 5 nodes were used as inputs to the neural network. Depending on the run, approximately 70 inputs were chosen. The authors do not say how many training examples were given nor the extent of damage, hence it is hard to comment on the sensitivity of the method. The authors claimed to be able to predict stiffness changes to 4% accuracy. The authors also do not discuss whether this accuracy was on independent test data or the training set. The authors found that the time history from a single, carefully selected node produced the best predictions.

In Ceravolo and De Stefano (1995), the authors use a backpropagation neural network to identify damage in a truss structure simulated by finite element methods. The authors use the neural network to identify the map from modal frequencies to the \((x, y)\) coordinates corresponding to the location of damage. Damage was modeled by removing elements of the truss. The network architecture was chosen to be 10 input nodes corresponding to 10 modal frequencies, a hidden layer with 10 nodes, and two output nodes corresponding to the \(x\) and \(y\) positions. Only single-damage scenarios were considered. The network was trained on 18 sample runs and cross-validated on 5 sample runs. The neural network located the damage well. The authors do not discuss how noisy measurements or multiple damage would effect the results.

Kirkegaard, et al. (1995) used recurrent neural networks to predict solutions for general nonlinear ordinary differential equations (ODEs). This work is not a damage-identification paper as such because no structures were studied. The implication of the paper is that structural damage will change the equations of motion, which will lead to different time histories. The time history of the ODE was predicted using parameters called “delay coordinates” and a technique known as the “innovations approach.” The neural network was used to identify the map from past sample data values to the next sample data value. The authors demonstrated both approaches to modeling a hysteretic oscillator system. The number of hidden nodes was chosen to be nine in one hidden layer. The training took approximately 2000 and 650 data points, respectively, with a single epoch. Recurrent neural networks were determined to be able to adequately predict nonlinear ODEs. The authors do not comment on how the network training sample size or structure would change with the dimension of the ODE.

Klenke and Paez (1996) used two probabilistic techniques to detect damage in an aerospace housing component. The first technique used a probabilistic neural network (PNN). The second technique used a probabilistic pattern classifier (PPC) of the au-
thors' design. Both methods attempted to ascertain the existence of damage, but neither attempted to quantify the extent or location of the damage. The PNN applies a Bayesian decision criterion to determine set membership. In the problem studied, the two sets were either "damaged" or "undamaged." Data from each class were presented to the PNN, and probability density functions for each class were estimated. The idea was that for a piece of new data, an estimated likelihood was computed for membership into both classes, damaged and undamaged. The greater of these two likelihoods was taken as the guess for class membership. In order to apply the method, a probability density function needed to be estimated from the data. The probability density function was approximated by a Gaussian kernel-density estimator.

The PPC method attempted to determine membership in a single class. Given class data such as instances of undamaged structures, the Gaussian kernel-density estimator was computed. The space of measurements was then mathematically transformed into normal and independent random variables. A new data instance could then be transformed into this same space, and a statistical chi-square test could be made to predict if the new instance lay in the same class. In both techniques, the necessary computational effort went up dramatically with the dimensionality of both the input and output spaces. It was therefore necessary to dimensionally reduce the input data to a manageable number of DOF, as well as to ensure that these DOF contained predictive information. For the aerospace housing component the number of measurements was reduced to five static flexibilities estimated from experimental FRFs. The raw data were vibrational spectra. Multiple data sets were created by taking small sequential subsets of the data. Damage was modeled by five progressively worse cuts made in the housing. Both methods were perfect detectors of damage because in all cases damage was clearly identified.

Schwarz, et al. (1996) used a backpropagation neural network to identify linear damage in spring-mass systems. The authors used a commercial package to implement the neural network and were unspecific about the details of the network except to say that it was a three-layer backpropagation network. The neural network was used to identify the changes in spring constants as a result of changes in modal frequencies. The function-mapping frequency shifts to changes in stiffness are multi-valued in the sense that more than one stiffness change can cause the same frequency shift. The authors circumvented this problem by eliminating inconstant data sets; only the smallest stiffness change that produces the desired frequency shift was retained.

The neural network was applied to a trial system consisting of two springs and two masses. One output was assigned to the stiffness change of each spring. The neural network was trained with 1000 data points corresponding to changes in stiffness of up to approximately 100%. The authors found they could identify changes in stiffness to within 10%. One problem with this technique is that as the complexity increases to more realistic levels, the degeneracy becomes more of an issue. In addition, noise was not simulated, which would serve to further confound the algorithm. It is also known that regardless of the modeling technique, neural networks included, modal frequency shifts are rather insensitive damage indicators.
As shown in the reviews contained in this section, the identification of damage using neural networks is still in its infancy. Most researchers use backpropagation neural networks, although no single paper compared the performance of two different neural network types. Most of the papers contained attempts to identify damage from information related to modal frequencies. All damage was modeled by linear processes; specifically, most papers used changing member shapes and/or cross-sectional areas. None of these produced a nonlinear dynamic system, which is what may be expected in real structures. Most of the papers assume detailed knowledge of the mechanical structure including mass and stiffness matrices. A few performed the identification of system parameters based on measured data so that no detailed knowledge of the structure was assumed. Generalizations of these non-model-based methods would seem to be more useful for practical applications.

2.H Other Methods

Yang, et al. (1980, 1984) apply the random decrement technique to identify damage in a scale-model offshore platform. The response of a linear system depends on the initial conditions and the applied forcing function. This response can be decomposed into three parts: that caused by the initial displacement, that caused by the initial velocity, and that caused by the forcing function. A total of $N$ averages is taken, each of time length $T$. The starting time for each average is taken such that the initial displacement is a constant, and the initial velocity is alternating from positive to negative value. This process is represented mathematically as

$$\delta(t) = \frac{1}{N} \sum_{i=1}^{N} x_{i}(t_{i} + \tau) \quad 0 \leq \tau \leq T ,$$

(76)

where

$$x_{i}(t_{i}) = x_{s} \quad i = 1, 2, 3, ...$$

$$\dot{x}_{i}(t_{i}) \geq 0 \quad i = 1, 3, 5, ...$$

$$\ddot{x}_{i}(t_{i}) \leq 0 \quad i = 2, 4, 6, ...$$

(77)

If the input is random, then taking numerous averages will cause the response to the initial velocity to average out because it alternates from positive to negative. Response to the excitation will also diminish with numerous averages because the excitation is random. Therefore, only the response to the initial displacements remains, and this vibration decay curve can be used to identify the resonant frequency of the structure and its damping. The technique requires 400 to 500 averages to produce a repeatable signature. Changes in the resonant frequencies are then used to indicate damage. The process does not locate the damage or give an estimate of its severity.

Afolabi (1987) proposed the use of changes in anti-resonances rather than resonances to detect and locate damage. He developed an analytical expression that demon-
strated that while resonance shifts are independent of measurement location, anti-
resonances are strongly dependent on measurement location.

Lee, et al. (1987) use the damping loss factor to look at damage in composites. The
damping loss factor $\eta$ is defined as

$$\eta = \frac{(\omega_a/\omega_b)^2 - 1}{(\omega_a/\omega_b)^2 + 1}$$

(78)

where $\omega_a$ and $\omega_b$ are the frequencies above and below resonance where the real part
of compliance or inertance reaches peaks of opposite sign in the FRF. Shifts in this
factor are used to identify damage in the same manner as frequency shifts.

Cawley (1990) reviews several techniques for detection of disbonds and delamina-
tions in composites. The techniques are vibration-based, but in general they require
special tools. The tests are the coin-tap test, the mechanical impedance method, the
membrane resonance method, and the velocimetric method. A comparison is made of
the strengths and weaknesses of each method.

Law, et al. (1992) develop a sensitivity formulation based on the change in the FRF at
any point, not just the resonances. In practice, many points of the FRF around the res-
onances are taken, and a least squares fit is used to determine the changes in the
physical parameters. This method requires both a before- and an after-damage FRF
and a physical model relating the damage parameter to a physical parameter such as
stiffness.

Ju (1993) presents what is primarily a review of the author's work in health monitoring
of structures. The author first reviews modal frequency theory, where he shows that
modal frequency shifts can be used to determine crack location and extent, and he re-
iterates that the cracks can be suitably modeled by a "fracture hinge." The author then
notes that the analysis becomes more complicated when more than a single crack is
present, motivating the need for a probabilistic theory of crack presence. Next is a re-
view of transmissibility theory, which is simply a measurement of two-point signal cor-
relation. The author argues that transmissibility is locally influenced by cracks. The
idea is to cover the structure with sensors to acquire local information. Damage mon-
itoring with response records is reviewed next. The idea here is to take structures that
are subject to strong signals of primarily known frequency content. The goal of this
technique is the assignment of probabilities of multiple cracks based on a Poisson dis-
tribution. Finally, the author discusses reliability assessment, which is based on the
notion of total accumulated damage. The idea is that a distribution for reliability can be
derived to be conditioned on the total accumulated damage.

In Mioduchowski (1993), the author presents analytic results for the propagation of
shear waves in N-story buildings. The paper contains more of a description of the phe-
nomenon than a damage-identification algorithm. The author assumes that microc-
racks can be equivalently modeled by a change in the shear modulus. By assuming
different shear moduli in different columns in the structure, changes in the shear wave
velocities, reflection coefficients, and transmission coefficients can be observed. The
author computes an analytic expression for these changes. He applies the results to
a simulated two-story structure with base isolation subjected to a seismic loading.

Springer and Reznicek (1993) use a “stress intensity correction factor” to locate the
damage in an L-section beam containing a crack. The crack is modeled as a spring-
mass system. The authors derive a cracked beam element, which is suitable for use
in finite element modeling.

Casas (1994) presents a method for identifying the cracked portions of concrete
beams by using a nonlinear least-squares method to identify the equivalent moments
of inertia for beam elements in a FEM. The method is based on minimizing the error
between measured resonant frequencies and modal amplitudes and those calculated
by FEM. The method is also used to evaluate the actual boundary conditions for the
beam.

Juneja, et al. (1994) present a method known as “contrast maximization.” This method
is based on the selection of a force vector that maximizes the strain energy ratio of the
damaged and undamaged structures. The force vector that maximizes this ratio is
found by simulating the damage in each member of the structure. The force vector that
maximizes this ratio for the experimental data is then matched to each of the force vec-
tors for the simulated damage cases. The one that matches most closely corresponds
to the true damage case.

Ma and Zheng (1994) applied the Wigner-Ville distribution to the response of a
cracked tuning fork. The nonlinearity of the crack introduced responses and nondriving
frequencies. The Wigner-Ville distribution changed in time and was deemed to be su-
perior to the spectrogram, which was shown to be stationary.

Saravanos, et al. (1994) examine the detection of delamination in composites using
embedded piezoelectric sensors. A theory is developed to predict the dynamic re-
sponse of a composite laminate containing delaminations and embedded sensors. The theory also predicts the output voltage in the sensors. The composites are excited
at a frequency below the first resonance, and the magnitudes of the responses are ob-
served.

Schuetze, et al. (1994) use an expert system to diagnose whether a building satisfies
an ambient vibration regulation. The expert system helped the engineer select sensors
given knowledge of the structure, suggested where the sensors should be located,
and then organized the data after the sensors were installed. The system, as tested,
checked compliance with a German Standard (DIN 4150) for the amplitude of ambient
vibrations. The standard was relatively easy to implement; it required only that the vi-
bartion velocity of the floor be less than 15 mm/s (0.6 in./s). The expert system coor-
dinated the results effectively, but beyond checking the velocity, it presented no new
signal processing or damage-identification methods.
Balís Crema and Mastroddi (1995) present a technique based on changes in the overall FRF. They apply the technique to FEM results and show that it successfully locates damage but is insensitive to averaging in the presence of noise.

Fritzen, et al. (1995) describe a vibrating system via FEM and reformulate it as a state-space model reduced by a modal transformation. Kalman filters are used to compare the time history response of the system to FEM predictions for the undamaged case and various damaged cases. The output is a distribution of probabilities indicating which of the test cases most likely corresponds to the actual measurements. Generally the filters are applied once to locate a damaged element or section and then a second time to further refine the location and quantify the damage. Because the work is all done in the time domain, the technique applies directly to nonlinear damage. This method requires FEM models of possible damage states.

Koh, et al. (1995) present a condensation method for local damage detection of a multi-story frame building. Damage is quantified in terms of the reduction in story stiffness. The number of DOF in the model is first reduced (all rotational DOF are removed) using a static condensation procedure. A remedial stiffness matrix is then derived to make the condensed model a more accurate representation of the actual structure. An extended Kalman filtering process is applied to the measured time-histories and is used to identify the stiffness parameters of the remedial matrix. The process is repeated until the stiffness parameters converge. The remedial stiffness parameters are used to define stiffness reduction factors associated with each story. The method is developed to be used with low-level excitation and assumes non-time-varying parameters.

Liang, et al. (1995) propose the use of the modal “energy transfer ratio” (ETR) as an indicator of structural damage. A flow chart for the procedure to calculate the ETR is given in Huang, et al. (1996). The energy transfer ratio is a measure of how much energy is transferred to a particular part of the structure during one cycle of a particular mode. The ETR is defined using the equation

$$ \frac{\{\phi\}^T[C]\{\psi\}_{i}}{\{\phi\}^T\{\psi\}_{i}} = 2\omega_{i}(\zeta_{i} + j(ETR_{i})) . $$

The authors demonstrate that the ETR is much more sensitive to changes in the system dynamics than modal frequencies are. For example, for $ETR_{i} = 0.001$, a 0.1% change in modal frequency corresponds to a 100% change in ETR. The authors use this method to identify the changes in the physical parameters of a bridge resulting from repair work, a scale model bridge with one support roller removed (Kong, et al. (1996)), and a steel angle (Huang, et al. (1996)).

Sibbald, et al. (1995) have used plots of the dynamic stiffness, $E’$, to identify damage in arch bridges. The dynamic stiffness is defined as
\[ E_{ij} = \frac{F_i(\omega)}{X_j(\omega)} \]  

where \( X_j(\omega) \) is the Fourier transform of the response measurement at location \( j \), and \( F_i(\omega) \) is the Fourier transform of the force input at location \( i \). This function is mode- and location-dependent. The dynamic stiffness measures the dynamic force applied at node \( i \) that is needed to produce a unit displacement at node \( j \). The authors were able to detect spandrel wall separation by examining changes in the plots of the dynamic stiffness.

Carlin and Garcia (1996) present a study of the application of a genetic algorithm to the problem of damage identification. The genetic algorithm uses a "survival of the fittest" analogy to test each "generation" of possible solutions for satisfaction of the objective function. An objective function that incorporates the magnitudes of the errors in both the measured frequencies and the measured mode shapes is used as the measure of fitness. The selection of values for the following parameters was studied: 1) the number of bits in the gene length, which determines the numerical resolution of the solution; 2) the rate of genetic crossover, which determines how often the chromosomes are cut and recombined randomly; 3) the rate of mutation, which controls the random change of a gene; and 4) the population size, which is the number of candidate solutions to choose from. Optimal ranges for these parameters were selected, and the resulting genetic algorithm was applied to three different numerical examples from the literature. The authors also introduced a new genetic algorithm structure known as an "intron" to offset the negative effects of mutation and crossover.

Choudhury and He (1996) present a method for locating damage that uses the mass and stiffness matrices of the undamaged FEM and the pre- and post-damage FRF data. They form a "Damage Location Vector" (DLV) by multiplying the change in measured receptance (FRF between displacement and force) at a particular frequency with the "dynamic stiffness" of the undamaged FEM. The DLV at a particular frequency, \( \{d(\omega)\} \) is defined as

\[ \{d(\omega)\} = [Z(\omega)] \{\Delta \alpha(\omega)\} \]  

where \([Z(\omega)]\) is the dynamic stiffness matrix for the undamaged FEM at frequency \( \omega \), defined as

\[ [Z(\omega)] = [K] - \omega^2[M] , \]  

and \( \{\Delta \alpha(\omega)\} \) is the change in the measured receptance between the undamaged and damaged data sets. Nonzero values of the DLV at particular DOF at particular frequencies can be used as indicators of damage when the undamaged FEM is assumed to be accurate. The authors use Eq. (81) to generate a three-dimensional "Damage Location Plot" (DLP), which shows the value of the DLV for each combination of frequency and DOF. The DLP is used to evaluate several algorithms for estimating the value.
In a study to develop a near-real-time damage monitoring system for civil structures subject to extreme events, Straser and Kiremidjian (1996) note that it is not sufficient to monitor the elastic response before and after the event. Rather, because most large civil engineering structures are designed such that they will exhibit nonlinear response during extreme events, nonlinear analysis of the response during the actual extreme event such as an earthquake must be performed. In this study analysis of measured responses is carried out by ordinary least squares, extended Kalman filtering (EKF) and a substructure approach. Hysteretic models are used to represent the nonlinear behavior of the structure during an extreme event. The EKF is employed to estimate the parameters of the nonlinear hysteretic model from measured input and response data. The authors discuss two shortcoming of the EKF approach: the sensitivity to initial estimates of the system parameters and the difficulty identifying multiple parameters.

2.1 COMPARATIVE STUDIES

Jauregui and Farrar (1996a, 1996b) and Farrar and Jauregui (1996) have performed a comparative study of five damage-identification procedures by applying these methods to a common set of numerical and experimental modal data obtained from a bridge. All the methods that were studied require mode shape data. Some of the methods require mass-normalized mode shapes, and some methods require resonant frequencies. These methods include ones that examine changes in modal strain energy (Stubbs, et al. (1992)), changes in mode shape curvature (Pandey, et al. (1991)), changes in flexibility (Pandey and Biswas (1994)), changes in the uniform flexibility shape curvature (Zhang and Aktan (1995)), and changes in stiffness (Zimmerman and Kaouk (1994)).

Ambient and forced vibration modal data were acquired from the bridge when it was in both a damaged and undamaged state (Farrar, et al. (1994)). Numerical simulations of the bridge consisted of time-history finite element analyses using models that had been benchmarked against measured modal properties (Farrar, et al. (1996)). These analyses and the subsequent post-processing of results replicated actual experiments in terms of excitation, measured response, and data reduction methods. With the numerical models additional damage scenarios were examined. These additional cases include one multiple-damage case and one case where two different sets of data from the undamaged structure were compared to test for false-positive readings. Parameters that were varied in these studies include the number of modes used in the damage detection process, the number and location of the sensors, and the use of mass-normalized modes as opposed to modes normalized assuming a unit diagonal mass matrix.

Results of this study showed that when damage was severe, all methods could accurately locate the damage. For the less severe damage cases, the modal strain energy
method was most successful at locating the damage. However, this method performed worse on the noise-free numerical data than on the actual experimental data when the lowest levels of damage were examined. A disturbing result of this study was that all methods except the method that examines changes in the modal strain energy gave false-positive indications of damage when the two sets of numerical data from the undamaged structure were compared. The authors state that this problem could possibly be eliminated by taking more averages of the data or by developing a decision-making algorithm such as the one used in conjunction with the modal strain energy method.

2.J **Classification Table for Damage Identification Methods**

The following table is a compilation that chronologically categorizes the various damage-identification and model update methods that have been reviewed in Section 2. The table is similar in format and content to that presented by Hemez (1993). The abbreviations used in the table are listed in a footnote at the end of the table.

**Table 1. Classification of Damage Identification Methods (Sheet 1 of 6)**

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Table 1. Classification of Damage Identification Methods (Sheet 4 of 6)

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Table 1. Classification of Damage Identification Methods (Sheet 6 of 6)

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<tr>
<th>Author(s)</th>
<th>Data/Correlation(a)</th>
<th>Scheme(b)</th>
<th>Criterion/Constraint(c)</th>
<th>Result/Property(d)</th>
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<td>TD / TA</td>
<td>N / I</td>
<td>E</td>
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</table>

a. F = Frequencies, M = Mode Shapes, MD = Mode Shape Derivatives (including dynamic strain data), C = Damping, G = Flexibility or Reduced-Rank Stiffness, GD = Flexibility Derivatives, S = Static Data, R = Response Spectra or Frequency Response Function, TD = Time Domain Response / TA = Modal Test/Analysis Correlation (uses analytical model), UD = Undamaged Test/Damaged Test Correlation (uses no analytical model)

b. F = Forward, N = Inverse, T = Neural Network / D = Direct, I = Iterative
c. (This column is only relevant for inverse or iterative modeling techniques which use some sort of optimization procedure.)

- F = Frequency Change
- E = Equation of Motion
- P = Parameter Change
- R = Parameter Vector (or Matrix) Rank
- RF = Response Spectra or Frequency Response Function Change
- C = Connectivity
- S = Symmetry
- D = Positive Definiteness
- Z = Sparsity (It should be noted that constraining connectivity inherently constrains sparsity.)

- I = Indication Only (no spatial information)
- L = Location of Damage only (no property information)
- GM = Global Matrix Change
- EM = Element Matrix Change
- P = Element or Localized Parameter Change
- M = Mass
- C = Damping
- K = Stiffness
- H = Hybrid Property (e.g., dynamic impedance)
- G = Crack Geometry or Location (including composite delamination)
- V = Environmental or Operational Conditions
- N = Response Nonlinearity
In this section, the literature that focuses on application issues is reviewed. The literature is categorized according to the type of structures analyzed.

3.A  Beams

3.A.1 Metal and Miscellaneous Beams

In Gudmundson (1983), the author discusses the dynamic behavior of cracked beams. The idea is that the loss of stiffness resulting from the crack can be represented by an equivalent flexibility matrix. The flexibility matrix is determined by an energy balance equation where the elastic energy of the flexibility matrix is balanced with the strain energy near the crack. The method is used to compute frequency shifts for a cantilevered beam with edge cracks. The results agree well with experimental data.

Yuen (1985) performed FEM modeling on cantilevered beams where damage was modeled using an element with reduced stiffness. He found his mode-shape-based eigenparameters to be slightly more sensitive to the damage than frequency shifts.

Ju and Mimovich (1986) model damage in cantilevered beams by a “fracture hinge,” a concept developed by the authors. The beams were aluminum with dimensions 9.5 mm (0.375 in.) x 63.5 mm (2.5 in.) x 457 mm (18 in.). The authors made twenty beams, introducing damage at five positions along the length by milling slots at two depths and two widths. The slots were wide enough to never close on the compressive part of the bending cycle. The frequencies of the first four bending modes were measured before and after the cracks were machined. The authors used the frequency shifts of the first four bending modes to estimate the location and extent of damage in the beams. Damage was modeled by introducing a fracture hinge at the location of the crack. The fracture hinge is a torsional spring whose stiffness depends on the crack geometry. The strength of the fracture hinge could be measured experimentally, and when incorporated into the subsequent analysis, could be used to predict frequency shifts resulting from the cracks to within 0.3%. Using the frequency shifts, the authors were able to locate the crack damage to within 3% on average, provided the crack occurred at a position in the beam with high bending moment. The authors were much less successful at estimating crack intensity.

Sanders, et al. (1989) present a method, also based on the measurement of modal parameters, to detect the location and extent of damage in structures. The work was based on the use of modal sensitivity equations, and is applied to fiber-reinforced composite beams.

Ismail, et al. (1990) investigated the effect of fatigue crack closure on the frequency changes of cracked cantilever beams. Based upon a combined experimental-numerical program, the authors conclude that the drops in resonant frequencies alone, espe-
cially for the higher modes, are insufficient measures of crack severity. The reliability of the vibration testing method for detecting the presence and nature of the crack was, however, demonstrated.

Lin and Ewins (1990) applied their nonlinearity localization technique to a specially designed nonlinear structure. A rectangular frame, suspended by springs, was driven by a shaker with a feedback loop such that the force applied was proportional to the cube of the displacement. The measured matrix describing the nonlinearity, based on measured modes at two response levels, indicated the region of the nonlinearity but also showed false indications in other regions.

Rizos, et al. (1990) performed tests on 300mm (11.8 in.) x 20mm (0.8 in.) x 20mm (0.8 in.) steel cantilever beams. Saw cuts of 2mm (0.078 in.) to 14mm (0.55 in.) depth were introduced at locations throughout the length of the beam. The beam was excited at a resonance, and the amplitude of vibration was measured at several points along the length. From these amplitude measurements, the crack location and length were calculated. The values given were within 8% of the actual values in all cases. Kam and Lee (1992) applied their crack-detection and location algorithm to the same data set and also reported good results.

Silva and Gomes (1990) present detailed experimental results on natural frequencies in slotted free-free beams. They give tables of all the frequencies they measured so that other investigators can use the results. The beam considered was steel, 0.72m (2.36 ft) long, with a 32 mm (1.26 in.) x 16 mm (0.63 in.) cross section. A slot was introduced into the beams with a 0.5-mm wide milling cutter. Slots ranged in depth from 1/8 to 1/2 of the beam thickness, and were introduced in both dimensions (x and y) of the cross section. For each of the 32 beams, the first four frequencies were measured both before and after damage.

Stubbs and Osegueda (1990a) apply their sensitivity method of damage identification to numerical examples of a 10 m (32.8 ft) long by 200 mm (7.8 in.) wide by 600 mm (23.6 in.) deep, simply-supported concrete beam with various damage scenarios. Stiffness reductions for bending modes are modeled by reducing the moment of inertia over finite lengths at one or more locations along the beam. For axial modes, the stiffness reductions are modeled by reducing the area of the beam over finite lengths. For single damage locations, the sensitivity method accurately predicted the extent and location of the damage, while occasionally identifying additional low-level damage at locations adjacent to the actual damage location. Results for multiple damage locations are similar to those obtained for the single damage location, but with one failure to locate the second damage location. The authors conclude by posing four questions that must be asked of this damage-identification method:

1. What is the smallest defect that can be detected?

2. Can small frequency changes associated with damage scenarios studied actually be measured accurately?
3. How will nonlinear behavior affect the accuracy of the method?

4. How accurately can an analyst develop the sensitivity matrices for an existing structure?

Their response is that general answers to these questions are not available, and they must be studied with regard to the particular application being investigated.

Subsequently, Stubbs and Osegueda (1990b) apply the same techniques to a series of 1 m (3.3 ft) long by 25 mm (1 in.) wide by 12.7 mm (0.5 in.) deep aluminum cantilevered beams. Damage was simulated by reducing the cross-section area of the beam over finite length intervals. Impact excitation was used to excite the structure. Resonant frequencies were computed by fitting a parabola to peaks in the Fourier spectrum of the frequency response signal. Practical aspects to the testing, such as removing and replacing the beam in its support fixture, are discussed. Results of the damage identification procedure are similar to those obtained from numerical examples reported in Stubbs and Osegueda (1990a) and show that this method can accurately locate the damage and accurately predict the extent of the damage.

Fox (1992) performed impact modal tests on a 1 m (3.3 ft) long by 12 mm (0.47 in.) deep beam with saw cuts 1 mm (0.04 in.), 3 mm (0.12 in.) and 6 mm (0.24 in.) deep located 200 mm (8 in.) from one end. Soft springs were use to simulate free boundary conditions during the tests. Modal analyses of the test specimens were also performed numerically with finite element analysis. Although the resonant frequencies and MAC values showed little change, damage could be located by examining relative changes in the mode shapes whose frequencies were found to shift as a result of damage.

Huang and Gu (1993) studied a finite element simulation of a cantilever beam 400 mm long with a cross-section of 6 mm (0.24 in.) x 35 mm (1.38 in.). The authors used a single displacement node and calculated an ARMA model to fit the response. The residual was the quantity whose cumulant was estimated. The authors showed that in numerical studies both with and without noise the cumulant was very near zero for the uncracked beam, but nonzero for the cracked beam because a nonlinearity was present.

Kim and Bartkowicz (1993) conducted a numerical trade study using a 40-DOF simply supported aluminum beam. The authors showed how the number of sensors and modes affects the ability to locate structural damage.

Salawu and Williams (1993) apply the error matrix method to a beam FEM and compare the results to some other criteria, such as changes in mass-normalized mode shapes and changes in the measured stiffness matrix. They also introduce the matrix cursor method, which uses vector space theory to identify the nonzero rows and columns of a dynamic error matrix. They use measured parameter matrices in all of these methods.
Budipriyanto and Swamidas (1994) measured the modal parameters of a notched cantilevered beam in air and both partially and fully submerged in water. The beam was notched symmetrically on both top and bottom surfaces. Measurements were taken with accelerometers and strain gages. Results presented include natural frequencies, damping ratios, and peak response magnitudes for all combinations of sensor type and degree of submergence.

Chance, et al. (1994) tested a beam containing a "bilinear crack device." A groove was machined into the beam, and two pieces were inserted into the groove. The two pieces were designed to open and close like a fatigue crack. FRFs from stepped-sine excitation clearly showed nonlinear behavior. Mode shapes from accelerometers were not able to locate the crack. Curvature shapes from measured strains clearly located the damage, provided that the strain gages were located close enough to the crack.

Dong, et al. (1994) measured their strain-mode-shape-based parameter on a beam containing a fatigue-induced crack. They were easily able to locate the crack, and the results agreed with FEM predictions. They gave no details on the beam material or dimensions or how deep the crack was.

Kaouk and Zimmerman (1994a) apply the MRPT matrix update technique to a cantilevered beam with a discrete lumped mass. The beam was tested with the lumped mass in place, and then the mass was removed and the beam retested. The technique was able to locate this damage case uniquely using the first four measured modes.

Klein, et al. (1994) measured modal parameters on a steel cantilever beam with a fatigue-induced crack. Their observations are discussed in Section 2.A.

KO, et al. (1994) apply a MAC/COMAC-based damage identification technique to data acquired from a 2 m (6.6 ft)-square steel portal frame. The frame was instrumented at eleven locations. An impact hammer test was performed to estimate mode shapes and frequencies, then a shaker test was performed to obtain accurate identification of the mode shapes. Tests were conducted with the joints in both rigid and pinned conditions, and then damage was simulated by removing bolts at 2 locations—one at a column-beam connection, and one at a column-base connection.

Meneghetti and Maggiore (1994) apply a sensitivity-based damage-detection method to experimentally measured results on a steel beam 600 mm (23.6 in.) long with a 15 mm (0.6 in.) square cross section. Slots 0.3 mm (0.012 in.) wide were milled into several beams. They were able to locate a slot only 2 mm (0.08 in.) deep by measuring the pre- and post-damage frequencies.

Perchard and Swamidas (1994) obtained measurements with accelerometers and strain gages on a cantilever beam with a machined notch. Frequency shifts did not match FEM predictions. Changes in damping and residues did not correlate well with the damage, but changes in off-peak amplitudes in FRFs did correlate well with the damage. Strain-based FRFs were also very sensitive to damage, provided that the strain gages were near enough to the damage location.
Silva and Gomes (1994) applied a frequency shift method to data collected experimentally from fatigue-cracked steel beams. They used a cantilever beam 600 mm (23.6 in.) long with a 18 mm (0.71 in.) x 32 mm (1.26 in.) cross section. Fatigue cracks 8 mm (0.31 in.) to 16 mm (0.63 in.) long were introduced into the beam at locations ranging from the fixed end to 75 mm (3 in.) from the free end. The authors do not specify the directions of the cracks. In all cases they were able to determine the crack location to within 12 mm (0.5 in.) and the length to within 1 mm (0.04 in.). The best results were obtained when the first four frequencies were used.

Chen, et al. (1995) tested two C 3 x 4.1 channel sections that were 3.6 m (12 ft) long. Damage was introduced by successively cutting away portions of the flange. The step-relaxation method (quick release of a suspended weight) was used to excite the beams. Acceleration response measurements were made along the length of the beam. The authors found that relatively severe damage (damage that under the original design load produced the onset of a plastic hinge) caused less than 5% changes in the resonant frequencies of the beams. Mode shapes identified from the free vibration decay of the structure were not found to be useful in locating the damage.

Fritzen, et al. (1995) applied a Kalman filtering technique to FEM-generated time histories of a pinned-pinned beam with an opening and closing crack. They were able to precisely detect, locate, and quantify the crack. They also applied the technique experimentally to a T-frame consisting of two solid aluminum beams welded together. A non-closing notch with depth of 30% of the beam thickness was machined using a procedure known as wire electric discharge machining. The technique successfully located and quantified the notch.

Prime and Shevitz (1996) present experimental results from the vibration of a cantilever polycarbonate beam containing an opening and closing crack. The beam is 61 cm (24 in.) long, 5.1 cm (2 in.) wide and 1.21 cm (0.48 in.) thick with a crack penetrating to half of the thickness. The crack was made by bonding together 3 pieces of polycarbonate. The excitation was step relaxation, i.e., pull and release. A variety of techniques was used to examine the nonlinear response.

Zimmerman, et al. (1996) describe the development of an integrated structural damage detection system. The system includes data acquisition hardware and software, damage-identification software, and a damage-identification demonstration test article consisting of a cantilevered beam with various damage scenarios. The authors explain the details of the test article design, the operation of the software, and particular difficulties that were encountered during the development. The primary difficulty encountered was the systematic extraction of the modal parameters from the measured frequency response data.

3.A.2 Concrete Beams

Chowdhury and Ramirez (1992) performed impact modal tests on 27 reinforced and unreinforced concrete beams 762 mm (30 in.) long by 89 mm (3.5 in.) wide by 152 mm
(6 in.) deep. Some of the beams had defects cast into them to simulate delaminations and cracks. Free boundary conditions were simulated by suspending the beam with plastic belts during these tests. The authors also examined changes in frequency and power spectra caused by changes in strength, applied loads, and incremental loads.

Slastan and Pietrzko (1993) measured frequencies and mode shapes on reinforced concrete beams before and after damage. The beams were 6 m (19.7 ft) long, had a T-shaped cross-section, and were tested with both simply supported and cantilevered boundary conditions. Damage was introduced incrementally by static loading to three levels. Results were considered for both hammer and shaker excitation. They found the frequency shifts to be measurable but small. These authors found that the mode shapes and damping values contained little useful information.

Allbright, et al. (1994) measured mode shapes on a deteriorated, prestressed concrete beam. The beam, taken from a bridge, was a 23.3 m (76.5 ft) long box section, 910 mm (36 in.) wide, 840 mm (33 in.) high with 127 mm (5 in.) thick walls. The beam had eighteen 13 mm (0.5 in.)-diameter prestressing steel tendons. Three tendons in one corner of the beam had corroded along the length of the beam except for a 1 m (3.3 ft) length at each end. An impact hammer and accelerometers were used to measure a FRF for the beam. The damage in the beam manifested itself as torsional coupling in the bending modes of the beam. The flexibility matrix measured from the modal tests agreed well with the flexibility matrix obtained using static tests.

Casas (1994) and Casas and Aparicio (1994) report the testing of four pairs of beams. Each pair consisted of an undamaged beam and a beam with cracks introduced by the form work at various locations along the length. Impact excitation was applied to the top of the beam without a driving point response measurement. Two acceleration responses were measured at the center point and quarter point. Based on the measured resonant frequencies and the two measured modal amplitudes, a nonlinear least-squares algorithm was employed to determine equivalent moments of inertia for beam elements of a FEM in the damaged region. Static testing was then performed to assess the accuracy of the identified damage model. The damage conditions as well as the end boundary conditions were successfully identified by this method.

3.B Trusses

Smith and McGowan (1989) and McGowan, et al. (1990) present the results of experimental and analytical modal analyses on a 10-bay aluminum truss structure, where each bay was 0.5 m (1.64 ft) square. The truss was cantilevered from one end, and plates weighing 39.1 kg (86.25 lbs) (60% of total weight) were attached to the free end. Burst random excitation was provided by two shakers, and acceleration response was measured at all truss nodal locations. Tests were repeated with various truss elements removed to simulate damage. In general, damage was more apparent in the resonant frequency changes than in changes associated with mode shapes as quantified by MAC values. The ability to locate damage was shown to be a function of the modes.
that are examined and the sensor locations that are selected. Analytical simulations showed results similar to those obtained from the experiments.

Stubbs, et al. (1990) applied a sensitivity-based method of damage identification to a numerical simulation of a cantilevered, 20-bay plane truss, where each bay was 1-m square. A continuum approximation of the truss consists of a 10-element Bernoulli-Euler beam for bending behavior and a uniform rod for axial behavior. A total of 14 damage scenarios, including multiple damage locations, was considered. Based on resonant frequencies calculated for the undamaged and damaged structures and mode shapes calculated based on closed form solutions for cantilever beams, sensitivity matrices were constructed and used to identify the location and the extent of the damage. The location and extent of the damage was accurately predicted in most cases. However, the authors point out that false predictions may be made near the support or free end of the structure. For the multiple damage cases, the magnitude of the damage was predicted accurately and, in most cases, the damage locations were also accurately predicted.

Kashangaki (1991) proposes the use of the DSMT testbed at NASA-Langley Research Center for damage detection research. One component of the DSMT, an 8-bay truss, was tested in a cantilevered configuration. A series of tests was performed, with different structural members removed between each set of tests. A total of 15 damage cases were implemented in this manner. This body of data from the DSMT test has been widely disseminated and used for many different damage-detection studies.

Kashangaki, et al. (1992) use the DSMT data in their examination of some issues inherent in the use of modal data for detecting damage in truss structures. They conclude that damage detection is feasible for members that contribute significantly to the strain energy of the measured modes. They also demonstrate how the modes that are most effective for detecting damage in certain critical members can be identified and targeted. A relationship is drawn between the accuracy of the measured modes and frequencies and the feasibility of detecting damage in the truss members.

Doebling, et al. (1993a, 1993b) apply a sensitivity-based FEM update scheme to an 8-bay suspended truss with asymmetrically placed lumped masses, the so-called "MUDDE" truss. The choice of suspended boundary conditions and the large amount of non-structural mass resulted in a structure with little global modal behavior. Thus, particular modes were only sensitive to damage in specific regions of the structure. This localized sensitivity made the choice of modes used in the update critical to the successful location of damage. The results demonstrated that modes whose selection is based on highest overall strain energy content in the damaged structural configuration provide the most information about the damage and thus are the best modes to use in the model update.

Kaouk and Zimmerman (1993) apply the MRPT matrix update technique to the DSMT 8-bay truss data. The damaged member was successfully located in all but two cases. In one of the two cases, the member was an x-batten, and the damage was narrowed down to this member and one other member. In the other case, the member was a di-
agonal located along one of the cross-sectional planes of the cantilevered truss. In both of these cases, the member in question was along a batten plane of the truss and thus carried very little of the modal strain energy. Thus, changes to these elements caused very little change to the measured modes. This result provides a good example of the difficulty of locating damage when the modes do not store a high level of strain energy in the members which undergo damage.

Peterson, et al. (1993) use an inverse-connectivity technique to locate the damage in the MUDDE truss from the measured mass and stiffness matrices. The results demonstrate that the ability of the measurements to distinguish rotational DOF is crucial when structural changes are manifested primarily in bending behavior.

Slater and Shelley (1993) apply an adaptive modal filtering scheme to a 9-bay, 4.5 m (14.75 ft) suspended truss. They demonstrated the ability of the system to detect frequency shifts over time by adding and removing constraints. They also showed the ability of the system to handle faulty sensors by disconnecting sensors during the test.

Kaouk and Zimmerman (1994c) use the MRPT matrix update technique to perform damage detection when a baseline data set is available in lieu of an analytical model. The baseline data set is used with the MRPT to update a hypothesized mass and stiffness model to form an estimate of the prior analytical model. This estimated model is then used as the baseline mass and stiffness model for the damage-identification algorithm. The results were shown to be successful overall, although the algorithm still performed better with a correlated baseline FEM.

Kondo and Hamamoto (1994) apply an ARMA model system identification method to a 2-D numerical model of a truss. Damage was simulated by reducing the stiffness of one element by 75%. Changes in the mode shape curvature of a lumped mass model of the truss were used to locate the damage. Within the damaged region, the damaged member was located, and the extent of damage was quantified using an incremental modal perturbation method.

Lim (1994, 1995) applies an eigenstructure assignment damage-detection technique to a 20-bay planar truss structure. He first locates the areas of damage using the concept of best achievable eigenvectors, then he uses constrained eigenstructure assignment to find the magnitude of the damage. He shows that the method works well even with a limited number of instrumented DOF.

Zimmerman and Kaouk (1994) use the results of the DSMT test to demonstrate the effectiveness of the MRPT matrix update technique on experimental data. They examine the effects of eigenvector measurement errors and demonstrate the use of multiple modes to enhance the results in the presence of such errors.

Hemez (1995) presents some of the key issues surrounding the practical application of damage-identification techniques to complex structures such as trusses. He discusses the problems of overdetermined realizations, including a phenomenon known as “modal splitting,” whereby the system realization procedure determines that two or
more modes are present at a spectral location where only a single mode exists. He also analyzes the effects of out-of-bandwidth modes in terms of their residual effect on the flexibility matrix. He presents a damage indicator based on the structural flexibility matrix and examines the sensitivity of model-updating methods to the quality of the identified modes and to numerical ill-conditioning during the update process.

Hemez and Farhat (1995) apply their element-by-element sensitivity update method to the DSMT 10-bay truss data. They examine some specific issues surrounding the location of damage in this structure, including the selection of the type and number of finite elements, the modeling of the cantilever boundary condition, the selection of the modes used in the update, and the limitations of the sensitivity-based technique.

Hinkle, et al. (1995) examine the effects of gravity preloading on the joints of a precision deployable truss structure. The structure is tested with a series of off-load masses to determine the sensitivity of the flexibility shapes to the level of gravitational preloading. Directional dependencies consistent with the orientation of the deployment joints were discovered in the measured flexibility.

Kim and Bartkowicz (1995) present case studies of the McDonnell Douglas Aerospace HexTruss. The HexTruss is a ten-bay (5 full-hexagon cross-section, 5 half-hexagon cross-section) 5.1 m (16.7 ft) long truss structure that simulates a segment of the current International Space Station Alpha truss design. The HexTruss was tested in an undamaged condition and 20 different damage conditions with 96 accelerometer measurements. The authors present a two-step damage identification technique to quantify the location and extent of damage in the HexTruss.

Zimmerman, et al. (1995b) discuss and evaluate several of the issues surrounding the use of engineering judgment in the application of the MRPT matrix update technique, and examine these issues in terms of their effects on successful damage location in the 8-bay DSMT truss. The topics discussed include: 1) Using the angle damage vector rather than the nonzero entries in the damage vector to locate the damaged structural member. 2) The selection of the number of modes used in the update. It is shown that using a larger number of modes may degrade the results of the algorithm when the actual matrix perturbations have smaller rank. 3) The use of eigenvector filtering with the modified damage vector. The damage vector is first modified by setting certain entries to zero which correspond to known undamaged DOF, then filtered eigenvectors are computed using the modified damage vector. 4) The decomposition of the damage vector into components corresponding to each property matrix. 5) Techniques for estimating the proper rank of the perturbation matrices. The study concludes that application of the MRPT in conjunction with engineering insight and judgment provides a strong tool for determining damage in structures.

Doebling (1996) applies a parameter-level minimum-rank update procedure to the data from the 8-bay DSMT truss. The method successfully located the damage in the two damage cases considered—one single-element damage and one double-element damage. The author showed that this method produced results superior to those from a parameter-level minimum-norm update technique. Results of a simple mode-selec-
tion procedure confirmed the conclusions of other authors regarding a deficiency in minimum-rank update techniques that constrain the rank of the solution to be equal to the number of modes used in the matrix update.

Dos Santos and Zimmerman (1996b) apply a hybrid stiffness matrix update algorithm to the data from the 8-bay DSMT truss. The hybrid method is compared to the basic MRPT algorithm in its ability to locate the damage precisely in the truss. Numerical results show that there is little difference between the two algorithms except in the case of high levels of measurement noise. The results from the application of the hybrid technique to the 8-bay DSMT truss are consistent with the numerical results. The hybrid method is shown to reduce the amount of spurious damage indications in the MRPT result by producing a perturbed stiffness matrix estimate that is consistent with the connectivity of the original FEM. The authors also point out that the quality of the results from the hybrid method are strongly dependent on the accuracy of the estimated stiffness matrix perturbation from the MRPT algorithm.

3.C PLATES

Wolff and Richardson (1989) examine the changes in modal parameters after several damage cases are simulated on an aluminum plate with a centerline rib stiffener. The first damage case is the removal of a bolt at the center of the plate. The second damage case is the removal of a bolt at the end of the plate. The first damage case was clearly observed in frequency changes of the first several modes, while the second was not. The local effects of the second case vs. the global effects of the first case are cited as the explanation for these results.

Richardson and Mannan (1992) applied a stiffness sensitivity method to a 500 mm (19.7 in.) x 190mm (7.48 in.) x 8mm (0.31 in.) aluminum plate. A 25 mm (1 in.) saw cut was made in the edge of the plate to simulate damage. Mode shapes and resonant frequencies of the plate were measured before damage using an impact excitation method. Boundary conditions for the test are not described. After damage, the sensitivity method required only the measurement of resonant frequencies. Damage was successfully located using a pseudoinverse search technique to locate the largest negative changes in stiffness.

Chen and Swamidas (1994) and Swamidas and Chen (1995) present FEM results for a cantilever plate containing a crack. Their best data for locating damage were determined to be strain mode shapes. Chance, et al. (1994) present FEM results for a cantilevered plate containing a hole. Mode shape curvatures are shown to be better indicators of damage location than mode shape displacements.

Chen and Swamidas (1996) performed swept-sine modal tests of a T-plate joint. Modal parameters that were examined include resonant frequencies, acceleration FRFs, and strain FRFs. Resonant frequencies showed less than a 4% change for a crack halfway through the thickness of the plate. Changes in magnitude of the strain FRFs obtained relatively close to the cracked region gave better indications of the damage.
Saitoh and Takei (1996) have applied the modal sensitivity method developed by Richardson and Mannan (1992) to a plate with a crack in it. The results showed small decreases in frequencies associated with the damaged plate. Changes in damping were inconsistent, and MAC values showed no change when the damaged plate was compared to the undamaged plate.

3.3 SHELLS AND FRAMES

Srinivasan and Kot (1992) tested a 305 mm (12 in.) diameter by 686 mm (27 in.) long cylindrical structure with 2.5 mm (0.1 in.) wall thickness, which had a circumferential notch machined in it at mid-height. Impact excitation was applied to the cylinder, which was suspended by relatively soft springs to simulate free boundary conditions. No specific damage identification method was employed. Instead, the authors examined changes in the resonant frequencies and changes in the mode shapes. Resonant frequencies were found to be insensitive to the damage with changes not exceeding the frequency resolution of the measurements. Mode shapes were found to be a more sensitive indicator of damage as quantified by the MAC values between the damaged and undamaged mode shapes.

Friswell, et al. (1994) applied their method to two structures: a frame structure and a cantilevered plate. Both were made of steel, and the length of both was approximately 0.5 m (1.64 ft). Damage was simulated by saw cuts in the structures. The algorithm was applied and found to be reasonably successful. The algorithm was able to correctly find the damage but gave some false positive responses when the actual structure was intact.

Nwosu, et al. (1995) performed FEM analyses on a tubular T-joint using shell elements for the tubes and line spring elements for a crack. They modeled one cracked and one uncracked configuration. Their results showed large changes in bending moment with the introduction of the crack. These could presumably be measured with strain gages. Significantly, these changes were quite noticeable even in regions relatively distant from the crack.

Choudhury and He (1996) apply their method for detecting damage using FRF changes and FEM mass and stiffness matrices to data from a planar frame structure with dimensions of approximately 900 mm (35.4 in.) by 420 mm (16.5 in.). The damage consisted of two cuts, each 5 mm (0.20 in.) wide, made on opposite sides of one of the beam members at the same cross section. They were able to locate the damage successfully using their method in conjunction with a dynamic expansion procedure (used to estimate the values for the FRFs at unmeasured FEM DOF).

Saitoh and Takei (1996) have applied the modal sensitivity method developed by Richardson and Mannan (1992) to forty automobile car doors with over 100 spot welds. When applied to the car doors, the data obtained from damaged structures appears to be in the scatter of the data from the undamaged structures.
3.E BRIDGES

A recent extensive survey of bridge failures in the United States since 1950 is presented by Shirole and Holt (1991). These authors point out that recent responses of engineers to bridge failures have been reactive. Bridge design modifications and inspection program changes are often made only in response to catastrophic failures. The collapse of the Tacoma Narrows Bridge a half century ago is a classic example of this reactive attitude because it led to the inspection and modification of other suspension bridges. The widespread introduction of systematic bridge inspection programs was directly attributed by Shirole and Holt to the catastrophic bridge collapse at Point Pleasant, WV, in 1967. Design modifications for seismic response of bridges have been made as a direct consequence of the 1971 San Fernando Earthquake (Gates, 1976).

At present, bridges are generally rated and monitored during biennial inspections, largely with the use of visual inspection techniques. There is the possibility that damage could go undetected at inspection or that cracks in load-carrying members could grow to critical levels between inspection intervals (see Gorlov, 1984). Sudden damage leading to bridge collapse also occurs as a result of collision. For example, the AMTRAK railroad bridge collapse in the Southeastern US in 1993 involved the collision of a barge with the bridge. (According to statistics presented by Shirole and Holt, more than 13% of identified failures of US bridges since 1950 are attributed to collisions.)

Based on the above information, a quantitative, possibly continuous, mechanism of bridge damage detection may be appropriate for certain types of bridges; specifically, those bridges with non-redundant structural members. Additionally, the use of an active damage-detection system may be appropriate in some cases. For example, such a system could detect sudden significant damage to the bridge structure resulting from collision and trigger a system to close the bridge to traffic.

Since 1979, numerous studies involving the development and application of damage detection techniques for bridge structures have been reported. Salane, et al. (1981) use changes in the dynamic properties of a three-span highway bridge during a fatigue test as a possible means of detecting structural deterioration resulting from fatigue cracks in the bridge girders. The authors found that changes in bridge stiffness and vibration signatures (mechanical impedance plots) can be used as indicators of structural deterioration resulting from fatigue. Stiffness coefficients were calculated from experimentally-determined mode shapes. Excitation was provided by an electrohydraulic actuator.

Kato and Shimada (1986) perform vibration measurements on an existing prestressed concrete bridge during a failure test. A reduction in natural frequencies could be detected as the statically applied load approached the ultimate load; however, damping

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1. Details of current bridge inspection techniques are given by White, et al. (1992).
values were largely unaffected. The ambient vibration method of system identification was used.

Turner and Pretlove (1988) perform a numerical vibration analysis on a simple beam representation of a bridge subjected to random traffic loading. The authors state that the measured response of a bridge to traffic appears to provide a method of determining resonant frequencies. These frequencies could then be monitored, and a 5% change would indicate significant damage. The motivation for the work was the development of a structural condition monitoring system that did not require a measured excitation force.

Biswas, et al. (1990) discuss the state of degradation of bridges in the US, emphasizing that the current 24-month inspection interval for highway bridges has two major drawbacks. First, bridge failure could occur between inspection intervals. Second, incipient failures may go unnoticed during inspection. They performed modal tests on a 2-span continuous composite bridge in both undamaged and damaged conditions. The damage consisted of a large fatigue crack simulated by unfastening a set of bolts at a steel girder splice connection. Changes in FRFs obtained using a shaker were found to be detectable and quantifiable. Modal frequencies showed small but consistent drops as a result of the presence of the simulated crack. In related work by the same authors (Samman, et al., 1991), a scale model of a typical highway bridge was used to investigate the change in FRF signals caused by the development of girder cracks. The authors used a procedure from the field of pattern recognition to accentuate the differences in the FRFs between cracked and uncracked bridges. The method also provided some crack location information.

Spyrakos, et al. (1990) performed a series of experiments on a set of beams designed to have dynamic responses similar to actual bridges. Each beam was given different damage scenarios (type, location, degree), and low-level free vibration tests were performed. The authors found a definite correlation between the level of damage and the dynamic characteristics of the structure. It was found that frequency change may be an insufficient indicator of structural safety (less than 5% change in frequency was associated with critical damage). However, the study suggests that the method may be applicable to more severely damaged structures to provide an indication of remaining serviceability.

Mazurek and DeWolf (1990) present strong arguments in favor of a continuous automated vibration monitoring system for highway bridges, citing several unexpected collapses and near collapses of bridges. (The collapse of one Rhode Island bridge was prevented when a passerby observed severe cracking of a primary girder at midspan.) In their experimental study of a bridge-monitoring technique, the authors performed laboratory modal tests on a 2-span aluminum plate-girder bridge, with vibrations induced by vehicular excitation. The authors found that major structural degradation can cause significant changes to both resonant frequencies and mode shapes. The greatest changes in mode shapes occur in the vicinity of the structural defect. Therefore, once it is determined that a structural defect is present, mode shapes can be used for detection of the defect location.
Jain (1991), also using modal methods, investigated the performance characteristics of a continuously deteriorating railway bridge using as excitation a locomotive run at constant speed. Jain concluded that modal parameters, specifically frequencies and mode shapes, can furnish only general information on the damage state of the structure. Deviation of these parameters indicates that damage has occurred, but not its location, extent, or underlying cause.

Tang and Leu (1991) performed experiments on a defective prestressed concrete girder bridge. They found that mode shape changes may be a more effective indicator of damage in bridges than frequency shifts (for damage detection, they state a frequency shift on the order of 0.01 Hz must be detectable). The step relaxation method was used to excite the bridge.

Law, et al. (1992) performed vibration tests on a one-fifth scale model of a reinforced concrete beam-slab bridge deck. The model had five precast main beams connected transversely by five diaphragms. The model was statically loaded incrementally to failure. An FRF from simulated ground motion was measured after each increment of loading. Using their sensitivity method, the authors were able to identify the regions of reduced stiffness resulting from damage. The results based on the FRF or its magnitude appear to be superior to those based on the phase of the FRF.

Raghavendrachar and Aktan (1992) performed impact tests on a three-span reinforced concrete bridge with a goal of detecting local or obscure damage, as opposed to severe, global damage. The authors concluded that modal parameters may not be reliable as damage indicators if only the first few modes are measured. For this type of damage, modal information for higher modes would be required.

An extensive survey and analysis of structural damage detection has been completed by Kim and Stubbs (1993). The authors assessed the relative impact of model uncertainty on the accuracy of nondestructive damage detection in structures. The authors applied their approach to a plate-girder bridge and a 3-dimensional truss-type bridge.

Aktan, et al. (1994) assess the reliability of modal flexibility as an indicator of bridge condition by comparing the measured flexibility to the flexibility obtained using a static-load truck test. They estimated that the error in measured flexibility resulting from modal truncation was about 2% after 18 modes had been included. They used the measured modes to calibrate their FEM, which they then used for condition assessment when no baseline data set was available.

In Biswas, et al. (1994) the authors study crack detection in a scale model of a bridge structure. The structure consists of three 2.5 m (8.2 ft) flanged steel beams supporting a concrete deck 1.5 m (5 ft) wide. Damage was introduced by changing the properties of bolted steel splice plates attached to the flanged beams. Specifically, cuts were made in the splice plates between 6 mm (0.25 in.) and 25 mm (1 in.). The beams were excited on their flanges with an impact hammer. The sensor was an accelerometer mounted on the same flange opposite the hammer. This body of data was used to compute a transfer function of the response at the flange resulting from the impact
loading. This transfer function provided the raw data for the damage-identification algorithm. The authors used a modified chain code to further analyze the data. The modified chain code is simply the slope and curvature of the transfer function of the damaged response subtracted from the known undamaged response. Four damage indicators were used: both the maximum and total integrated signal for both the slope difference and curvature difference. Damage was indicated when these parameters exceeded a certain threshold. In each of the four cases, the indicator was a monotonic function of crack length. The method was also applied to data with noise added and still found to be sensitive to crack lengths as small as 13 mm.

Farrar, et al. (1994) present the results of a damage-detection experiment performed on the I-40 bridge over the Rio Grande river in Albuquerque, NM. This bridge was designed so that the two main plate girders carry all the loads of the bridge. Such a design is called “fracture critical,” since failure of either of the main girders is assumed to produce catastrophic failure of the bridge. The bridge was tested in its undamaged state, using both ambient excitation (automobile traffic on the adjacent bridge) and standard forced modal excitation (a proof-mass actuator). Damage was applied to the bridge by cutting through one of the main girders with a cutting torch incrementally. The bridge was then tested in each damage state. This report contains a full description of the bridge test setup and procedure, the resulting modal parameters in each damage state, and the results of several damage detection algorithms that were applied to the data. In general, the results indicate that modal frequency is not a sensitive indicator of damage, as it took a large reduction in the bridge bending stiffness to see any changes in the measured modal frequencies. However, the mode shapes were shown to be more sensitive indicators of damage.

Samman and Biswas (1994) tested a scale model bridge consisting of a concrete slab bonded to three steel girders. Channel sections provided cross bracing at the ends and at intermediate locations. The bridge was excited with an impact hammer. Damage was introduced by making cuts in the splice plates. Vibration tests were performed on the damaged structure as well as the undamaged structure and the repaired structure. Waveform recognition techniques were then applied to the FRFs. The various waveform recognition techniques were ranked according to their ability to identify the damage with and without artificial noise. All methods were successful in detecting the cracks, but readings taken directly on the girders as opposed to the decks were found to be more sensitive to the cracks.

Toksoy and Aktan (1994) use the measured flexibility matrix to assign a condition index to a bridge. By comparing cross-sectional deflection profiles from the flexibility matrices, they are able to detect structural damage and anomalies. Results are presented both with and without original data. When an original data set is not available, the measured flexibility is compared to FEM flexibility, with anomalies in the deflection profile of the measured flexibility used to locate damage. When a baseline data set is available, the deflection profiles are compared directly.

Alampalli, et al. (1995) repeatedly tested a single span, steel girder bridge with an integral concrete deck in an undamaged condition to examine the variability in resonant
frequencies, modal damping, and mode shape data caused by random test variations and environmental effects. MAC and COMAC values were used to quantify the changes in mode shape data from a baseline measurement. Standard deviations and covariance statistics were developed for each of these modal parameters. For these tests the bridge was excited by impact with an instrumented hammer. Saw cuts were then made at various locations in the steel girders to simulate damage. The cuts were introduced in incremental levels of severity. Modal tests were then repeated, and statistical analyses of the variations in the modal parameters for the undamaged and damaged case were conducted. The authors concluded that changes in resonant frequencies can be used to indicate damage as these changes were beyond the statistical variations caused by random test variations and environmental effects, but that changes in mode shapes as measured by MAC or COMAC values are not sensitive enough to locate damage.

Farrar and Cone (1995) present further analysis of the I-40 bridge damage detection data set described by Farrar, et al. (1994). They identify the modal properties from the ambient test, when the bridge was undamaged, and from the forced-excitation tests for each of the damage cases. There are two primary results presented: First, the ambient vibration data provided adequate estimates of the modal frequencies and modal damping ratios. The accuracy of these parameters supports the use of ambient vibration data for bridge health monitoring. Second, significant changes in the measured frequencies and mode shapes (obtained via forced vibration) were observed only in the final damage state. This implies that modal frequencies, modal damping ratios, and mode shapes may not be sensitive enough indicators to detect damage at an early enough stage to be practical.

James, et al. (1995) present the results of two damage-location techniques applied to the body of data from the I-40 bridge damage detection test described by Farrar, et al. (1994). The STRECH technique is used to locate differences in stiffness between the measurements and the FEM on a mode-by-mode basis. The matrix completion (MAXCON) technique is a method for completing the rank of the measured mass matrix such that the mass-matrix sparsity is enforced. The measured stiffness matrix is then computed from this rank-enriched measured mass matrix. The results of this test indicate that the STRECH technique provides a better global indication of damage, but that the MAXCON technique appears to be more robust to measurement errors and more applicable to measured dynamic data. These techniques are also applied to a horizontal axis wind turbine (see Section 3.G).

Liang, et al. (1995) apply their ETR damage-identification method to data obtained on the steel Peace Bridge over the Niagara River near Buffalo, NY. Since no damage could be added to the bridge, the authors used the test as an opportunity to study the repeatability of the necessary parameters and to observe the changes in the structure resulting from construction repair work. Accelerometer measurements were used in the testing, and the excitation was provided using both impact hammer and ambient signals from automobile traffic. It is demonstrated that the ETR has the highest repeat-
able signal-to-noise ratio of any of the damage measures considered. The impact hammer tests yielded better overall results than the ambient input tests.

Salawu (1995) and Salawu and Williams (1995) apply the global damage integrity index and local integrity index method to a voided-slab reinforced concrete bridge that was being repaired. The structure was excited by a hydraulic actuator. Frequencies of the bridge were found to decrease slightly after the repair. The local integrity index identified the repaired zones, but this index also indicated that locations near the south support were areas of significant change. MAC and COMAC values were also used to identify the repair zones. The MAC matrix gave some indication of the modes that were affected by the damage, and the COMAC values indicated the location of the damage, but these parameters also indicated changes at locations that did not correspond to damage.

Sibbald, et al. (1995) used plots of the dynamic stiffness to identify damage in scale-model masonry arch bridges. The authors were able to detect spandrel wall separation and different levels of fill by examining changes in the plots of the dynamic stiffness.

Simmermacher, et al. (1995) examine the effects of FEM mesh density on successful application of the MRPT matrix update algorithm. This study is motivated by the fact that large models are necessary to reduce discretization error in the FEM. Matrix updating procedures inherently require model reduction and/or mode shape expansion, which destroy the load paths and therefore decrease the ability of such algorithms to locate damage at the element level. This research examines the trade-offs between large and small FEMs for application of the MRPT to the I-40 bridge data.

Stubbs, et al. (1995) present the application of a previously developed damage identification technique to the I-40 bridge damage-detection data. The method involves the computation of local structural strain energy using the curvature of the measured mode shapes. The procedure is shown to locate reduced-stiffness elements successfully using the modal parameters from a FEM of a beam. The procedure is also applied to the I-40 bridge data, and the results show indication of damage at the expected location. It is noted that an advantage of this algorithm is the ability to locate damage without knowledge of the structure’s material properties.

Zhang and Aktan (1995) use changes in curvatures of the uniform load surface (the deformed shape of the structure when subjected to a uniform load), calculated using the uniform load flexibilities, to identify damage in a numerical simulation of the Cross County highway bridge near Cincinnati, OH. The bridge is a three-span concrete deck supported by steel stringers. Results from impact and forced vibration modal tests were used to benchmark a numerical model of the bridge. Damage was introduced in the numerical model by changing the stiffness of an element in the model. The change in curvature was shown to be a sensitive indicator of this local damage. Changes in other modal parameters (resonant frequencies and mode shapes) were shown to be insensitive to the damage.
Kong, et al. (1996) performed ambient vibration studies on a scale-model steel-girder bridge in both undamaged and damaged conditions. Damage was imposed by removing a roller support under the girder. Resonant frequencies, damping ratios, and modal ETRs were measured before and after the damage. The ETR was observed to be the most sensitive indicator of damage. The resonant frequencies and damping were found to be inconsistent indicators of damage.

Villemure, et al. (1996) report the tests on reinforced concrete bridge piers after damage had been introduced by quasi-static lateral loads. Impact and ambient vibration tests were conducted after various levels of damage had been introduced. Frequency and damping changes as a function of the ductility were reported. Both viscous and hysteretic damping models were studied. Resonant frequencies were observed to decrease with increasing levels of damage. No consistent trends were observed for the viscous damping, but the hysteretic damping was shown to increase with increasing damage levels.

To summarize, it appears that over the past 15 years there has been repeated application of modal properties of bridges to the fields of damage detection and structural monitoring. The work has been motivated to a great extent by several catastrophic bridge failures. Earlier work utilized primarily modal frequency changes to detect damage, but more recent work has shown that frequency changes are insufficient. Changes in mode shapes are more sensitive indicators and might be more useful for detection of the defect location. Damping changes have not generally been found to be useful for damage detection in bridges. Finally, other more sensitive methods of computing damage from modal properties are being developed (e.g., using pattern recognition to accentuate changes in FRFs measured on cracked and uncracked bridges).

3.F Offshore Platforms

Vandiver (1975, 1977) examined damage (caused by the impact of a vessel) to a steel pile-supported offshore light station tower. Visual inspection revealed no damage above the water line. Measurements of the two fundamental bending mode frequencies and the first torsional mode frequency had been made before collision. Based on computer simulations, where each structural element was systematically removed and the change in resonant frequencies calculated, it was concluded that measured changes in the resonant frequencies obtained with just three accelerometers mounted on the deck were not significant enough to indicate damage. The primary source of error in the calculations was associated with fluid in tanks on the platform. However, the effects of this uncertainty on the calculated changes in resonant frequency were shown to be less than the effects of a failed member in most cases. The conclusion that the platform was undamaged was subsequently verified by ultrasonic inspections performed by divers.

Begg, et al. (1976) discuss results from tests of 4.8 m (16 ft) scale model of a four-leg, K-braced North Sea platform with sine sweep and random excitations applied by an
electrodynamic shaker. They examined the change in the first three resonant frequencies of the structure caused by failure of a series of members. These failures produced changes in the resonant frequencies ranging from 5% to 30%. In addition, they report tests on a 16-leg North Sea platform where an additional bracing member was added to the structure resulting in a 10% change in the resonant frequencies. Changes in frequency were measured down to 0.1% of the frequencies of interest (usually in the 0.4 Hz to 10 Hz range) indicating that damage levels observed above could be accurately detected. Cross-power spectra were used to evaluate the mode shapes of the structure. Piles were found to introduce nonlinear effects into the measured power spectra. These nonlinear effects are attributed to inadequate grout between the piles that run through the legs of the platform and the legs themselves.

Loland and Dodds (1976) discuss practical experiences learned by monitoring three North Sea platforms for six to nine months. This paper begins by summarizing the inspection requirements that came into effect in 1974 regarding the structural integrity of UK offshore oil platforms. Next, five requirements for the monitoring system are given as follows:

1. Ambient (sea and wind) excitation must be used to extract the resonant frequencies.
2. Vibration spectra must remain stable over long periods of time.
3. Instruments must withstand environmental challenges.
4. Mode shapes must be identified from above-water measurements.
5. The system must offer financial advantages over the use of divers.

Discussions of platform geometry, instrumentation, environmental conditions during measurements, and system cost are presented. In actual application, spectra were stable to within 3%. Variations were attributed to change of the mass on the decks and to changes in the tides (water level will change the effective mass of the structure). Minor structural modifications caused changes of 10% to 15% in the measured resonant frequencies. Tracking of these frequencies required the corresponding mode shapes to be identified.

Wojnarowski (1977) examined the effects of eleven different parameters on the dynamic properties of an offshore light house platform using finite element analysis. Foundation modeling assumptions, entrained water, marine growth, corrosion, variation in deck loads, and failed structural members are some of the parameters that were examined. The largest changes in frequencies were the result of changes in soil foundation properties. These numerical models were correlated with accelerometer measurements made on the Ambrose lighthouse tower. Resonant frequencies for the translational modes were found to be within 12% of the analytical model prediction and torsional frequencies were found to be within 7% of the analytical model prediction.
Only a change in the foundation model or the effective length of the legs could produce a difference in the dynamic properties that would account for these discrepancies.

Coppolino and Rubin (1980) use the measured modal response from ambient excitation of the Shell Platform SP-62C in the Gulf of Mexico to benchmark a FEM of the platform. The platform is an eight-leg, diagonally-braced jacket structure in a water depth of 100 m (327 ft). Numerous damage cases in the form of member severance were introduced into the numerical model. Depending on the location of the damage, changes in resonant frequencies on the order of 1% to 2% were found to be indicative of damage. Damage at other locations was not detected by changes in the resonant frequencies.

Duggan, et al. (1980) study the use of ambient vibration measurements taken during a seven-month period on three offshore platforms in the Gulf of Mexico (Conoco Main Pass 296A, Gulf South Pass 62B, and Ship Shoal 274A) as a means of structural integrity monitoring. The program was aimed at determining if vibration behavior was stable under varying environmental and operating conditions, but changed as a result of structural modification. On the Ship Shoal platform, repairs and replacements of legs and braces took place during the study. Seven different above-water accelerometer locations were monitored. The conclusion of this study was that changes in frequencies caused by removal of a bracing member could not be distinguished from shifts caused by normal operating changes. Because damage caused changes in the order of the modes, the authors state that it is essential to identify the mode as well as the resonant frequency to accurately track its changes.

Kenley and Dodds (1980) examine changes in resonant frequencies to detect damage in a decommissioned offshore platform. The West Sole WE platform is approximately 51.82 m (170 ft) tall and in a water depth of 23.77 m (78 ft). Eight accelerometers placed above water line were used to monitor the ambient vibration response caused by wave action. These eight accelerometers were also placed under water to measure local modes of individual members. Damage consisted of a small hole cut to allow a diagonal member to flood, half-severance of a member, and full severance of a member. The authors found that only complete severance of a diagonal member could be detected by changes in the global modal frequencies. The authors state that damage has to produce a 5% change in the overall stiffness before it can be detected. For global modes, the resonant frequency can be detected to within 1%, but for local modes the error increases to 2% to 3% because peaks in the power spectrum are not as well defined. Flooding and half-severance of diagonal members were detected from local below-water measurements. The authors again point out that it is important to associate a resonant frequency with a mode shape when trying to track changes in frequency as an indicator of damage.

Yang, et al. (1980) apply the random decrement method to a scale model offshore platform that was tested on a shake table with simulated seismic inputs. Response time histories were band-passed filtered between 4kHz and 8kHz preceding the random decrement analysis. Saw cuts, initially 3 mm (0.125 in.) deep, were added to member at its welded joint with increasing depth at 1.5 mm (0.0625 in.) increments un-
til the member was severed. Data corresponding to initial cuts could not be distinguished from the undamaged data. When the cuts exceeded 6 mm (0.25 in.), the change in the random decrement signature was easily distinguishable.

Crohas and Lepert (1982) describe the application of a “vibro-detection device” that was mounted on a test brace of the Total-ABK living quarters platform in the Arabian gulf. The structure is located in 28 m (92 ft) of water. The vibro-detection device applies a 44 kN (5 ton) input over a frequency range of 0.5 to 45 Hz to the member and measures its response. FRFs are then determined for the members by measuring the below-water acceleration response that results from the excitation with submersible triaxial accelerometers. Flooding of a test brace showed that it produced a 10% decrease in the resonant frequency while the frequencies of neighboring braces were unaffected. A 30% through-crack located near the end of the test brace could also be detected.

Nataraja (1983) reports on a program designed to monitor three jacketed north sea platforms (BP Forties Alpha, Amoco Montrose Alpha, and Occidental Claymore) over a two-year period and to demonstrate the feasibility of such a system for structural damage detection. The platforms are located in water depths ranging from 91 m (300 ft) to 122 m (400 ft). Up to 55 accelerometers were used to monitor the ambient vibration response. Some below-water measurements were made. Results showed that only the lowest natural frequencies could be identified with certainty, and that these frequencies were stable throughout out the monitoring period. The author states that changes in deck mass could be detected in the vibration signatures. Thus, it is imperative to monitor the deck mass in order to distinguish changes in mass from structural damage. The author concludes that monitoring the surface accelerations can only detect global changes in the structure.

Whittome and Dodds (1983) report results from a project where the response of British Petroleum’s Forties Alpha platform was monitored on a regular basis over 2.5 years. Eight accelerometers were used to make above-water measurements at two levels. This body of data was used to examine the changes in the measured resonant frequencies over time. It was found that there was less than a 1.5% change in the resonant frequencies over the monitored time period. Significant drops in frequencies were noticed when drilling operations were ongoing. These changes result from added mass on the deck. To reduce scatter in the data, a “batch mean” process was developed. In this procedure, results from consecutive sets of data are averaged. It was shown that for a particular parameter, such as resonant frequency, the standard deviation of the average of \( n \) measurements about its mean is much less than the standard deviation of the individual measurement about the mean. Damage was introduced in a numerical model of the structure that had been benchmarked against the measured response. It was concluded that changes in the resonant frequencies produced by damage or foundation deterioration were greater than the observed variations in resonant frequencies over time in the undamaged platform.

Yang, et al. (1984) apply the random decrement technique to blind test data from a 1:13.8 scale model of a tubular-steel offshore oil platform. Horizontal random excita-
ations at three different locations were applied to the structure, and acceleration responses in the horizontal direction at four locations corresponding to above-water locations were measured. An initial undamaged data set was measured first. Potential damage cases were cracks, completely severed members, changed foundation conditions, changed deck masses, simulated marine growth, and no damage. The authors present their findings but do not give the correlation with the actual damage scenarios. Some of the features of the method that are discussed include its inability to locate damage or identify the type of damage. Because higher-frequency portions of the spectrum are analyzed, the local effects of damage can be better identified.

Osegueda, et al. (1992) report on a project that examines changes in the dynamic properties of a 90" scale model of a jacket-type offshore platform tested in a water tank. Three damage scenarios were examined by cutting the structural tubing at various locations. One damage scenario was repeated five times to examine statistical variations in the measurements. Sine and random excitations were applied to the top of the model with an electrodynamic shaker. The response of the structure was monitored by 24 accelerometers. Tests were performed with and without water in the tank. Resonant frequencies and mode shapes were calculated from the test data. The resonant frequencies were found to decrease with damage, and this decrease was an order of magnitude greater than the standard deviation of the measurement. These authors note that to properly track the changes in resonant frequencies, the mode shape associated with these frequencies must be identified.

Swamidas and Chen (1992) present a technique for identifying damage in a scale-model tripod tower platform. The tripod is 8.6 m (28 ft) tall and is a 1:50 scale idealization of an actual offshore oil structure. The authors observe changes in mode shapes, frequencies, and damping ratios from FRFs measured with LVDTs and strain gages. The location of damage is inferred from the location of the sensor detecting the largest change.

Brincker, et al. (1995a) apply an ARMA model to measured acceleration time histories to estimate the changes in resonant frequencies and modal damping levels of an offshore oil platform. The platform, built in 1993, is a 58 m (190 ft) x 20 m (65 ft), multiple-pile, reinforced-concrete structure supporting a steel superstructure. The platform is located in 30 m (98 ft) of water. Two sets of measurements, made in a 12-month interval, showed definite changes in the resonant frequencies of the first two modes of the structure. The authors are able to determine contributions to these changes from damage, changes in the foundation conditions, and increased mass resulting from marine growth.

3.G OTHER LARGE CIVIL ENGINEERING STRUCTURES

Petroski and Glazik (1980) present a simple model for a cracked cylindrical shell, which is used to model cracks in nuclear reactor components such as vessels and piping. This paper is not damage-identification research in a strict sense, but it does address modeling issues that are significant for understanding the effects of damage on
structural integrity. The authors model the crack as a pair of moments that are a function of crack depth, loading and shell geometry. They examine the behavior of the cracked cylinders under static pressurization, as well as the dynamic responses to both uniform and nonuniform pressure loadings. The results indicate that cracks can have a large influence on both the bending deformations and the stress intensity factors in the cylinder walls.

Hearn and Testa (1991) formed fatigue cracks in a welded steel frame. The cracks formed at connections of the steel angles to the connection plates. The authors applied their method of damage assessment that examines the initial mode shapes of the structure and changes in the ratios of resonant frequencies. Using this method they were able to locate the damage. They note that the method cannot distinguish the location between damage in symmetric portions of the structure, but that construction tolerances should be sufficient to eliminate any true symmetry in a structure. These authors also tested wire rope under tension. The changes in the natural frequency of the wire rope were relatively insensitive to damage and could be observed only at the higher tension levels.

Salawu (1994) gives an overview on applying vibration testing to the nondestructive evaluation of civil engineering structures. This summary discusses many of the practicalities associated with this type of testing including the influence of environmental factors and the need for an accurate baseline measurement.

James, et al. (1995) apply two damage identification techniques to a horizontal axis wind turbine (HAWT) blade. The blade was subjected to an extended fatigue test at the National Renewable Energy Laboratory. Modal tests and acoustic emissions tests were performed periodically during the course of the fatigue test. (The two methods are described in Section 3.E.) As with the I-40 bridge data, the results of this test indicate that the STRECH technique provides a better global indication of damage, but that the MAXCON technique appears to be more robust to measurement errors and more applicable to measured dynamic data. A field test capability that uses non-contact sensing apparatus such as a laser doppler vibrometer is proposed to monitor the health of HAWTs.

Koh, et al. (1995) apply a condensation method for local damage detection of multi-story frame buildings to a numerical simulation of a 12-story plane-frame structure and a 6-story steel frame structure. Damage, ranging from 10% to 45.6% reductions in the story stiffness, were successfully identified without false indications in the undamaged floors. The method was found to be insensitive to reasonable assumed damping values for the structures.

Lam, et al. (1995) applied a mode-shape-based detection routine to a steel frame. The frame consisted of two 100 mm (4 in.)-deep steel I-beams, 2.82 m (9.25 ft) tall, connected laterally by two 150 mm (6 in.)-deep steel I-beams at the top and midway point of the structure. The columns were welded to a steel plate at the ground, and the beam-column connections were four bolted angles: one above the top flange, one below the lower flange, and one on either side of the web. Damage was simulated by un-
bolting the top and bottom angles at a connection. The authors' method was able to select the correct damage state from six possible states, which were defined to be loose connections at the four beam-column joints and the two column-ground joints.

Prion, et al. (1996) performed impact and ambient vibration tests on a four-story steel frame with steel shear walls. Quasi-static cyclic loading was applied to the top story to simulate earthquake excitation. Resonant frequencies, modal damping, and mode shapes were calculated for the structure in both the undamaged and damaged state. Resonant frequencies were observed to decrease when the structure was damaged. Changes in viscous damping did not show a consistent trend, and it was concluded that this parameter is a poor damage indicator.

Skjaerbaek, et al. (1996) developed a procedure to locate and quantify damage in a multi-story reinforced concrete frame structure from a single response measurement made at the top of the structure. Damage in a substructure is defined as the average relative reduction of the stiffness matrix of the substructure that reproduces the two lowest eigenvalues of the overall structure. The authors apply this method to numerical simulations of a degrading reinforced concrete frame subjected to various seismic inputs. The method correctly located the damage in the structures at higher levels of excitation but identified undamaged areas as damaged when very low levels of excitation were studied.

Straser and Kiremidjian (1996) apply their method of damage identification using EKF and nonlinear hysteretic models to a scale-model six-story structure that was constructed with steel plates for the floors and threaded steel rods to simulate the supporting columns. A lumped mass was placed at the top of the structure to produce larger responses. The structure was subjected to simulated seismic inputs on a shake table. Numerical integration of the measured acceleration-time histories were used to calculate the velocities and displacements. The single-DOF EKF fit of the nonlinear model to the measured response produced calculated time-history responses that accurately predicted the measured response; however, initial estimates of the system parameters had to be within 10% of the final values for the method to converge. For the nonlinear six-DOF case the EKF did not converge. Subsequently, the structure was divided into two 3-DOF substructures, but the nonlinear model still had trouble converging. The authors discuss two shortcoming of the EKF approach: the sensitivity to initial estimates of the system parameters and the difficulty of identifying multiple parameters.

3.H Aerospace Structures

West (1982) applies modal test techniques to nondestructive inspection of the Space Shuttle Orbiter structure. Specifically, testing was performed on the orbiter body flap, which is used to shield the main engines from heat and to provide pitch control during atmospheric re-entry. Single-point random excitation was used to acquire approximately 370 FRFs from the flap, which was equipped in two different test configurations with steel dummy actuators and actual flight-type actuators. Between modal tests, the flap was exposed to an acoustic environment similar to operating conditions. It was ob-
served that the frequencies of the first three modes decreased following the acoustic exposure, with both the dummy and flight-type actuators. Upon disassembly and inspection of the test article, indication of galling in the spherical bearings at the actuator-rib interfaces was discovered and was determined to be responsible for the reduction in frequency of the first mode. Additionally, shear clips in the interface between the trailing edge wedge and the flap ribs were found to contain significant cracking. This damage was found to be responsible for the observed reduction in the frequency of the third mode. It is also noted that the conventional visual, X-ray, and ultrasonic inspection techniques had failed to locate the damage. Also, the conventional techniques require the removal of at least some orbiter thermal protective system tiles, whereas the modal inspection technique does not.

West (1984) uses the MAC to analyze the results of another acoustic environment test for locating damage in space shuttle control surfaces. The author proposes a procedure for partitioning the measured mode shapes before the computation of the MAC. A comparison is made between several such partitioning schemes. The inconsistencies between mode shape partitions before and after damage are used to show how mode shapes can be used to localize damage.

Hunt, et al. (1990) describe the development and implementation of the Shuttle Modal Inspection System (SMIS). Performance of modal tests on shuttle orbiter specimens had been used to locate damage on the wing leading edge, the body flap, and the forward, middle, and aft fuselage panels. This paper discusses the results of these specimen tests, as well as tests on a full orbiter structure. The use of a Cessna airframe for further tests is also described. The layout, implementation, and acceptance tests for the SMIS are outlined.

Mayes (1992) examined error localization between a FEM and modal test results for a two-link robot arm. A technique known as STRECH was used to localize the areas that required stiffness adjustments in the model. An optimization technique based on frequency sensitivity was then used to adjust the stiffnesses.

Grygier (1994) provides a summary of the implementation of modal test and analysis techniques for the nondestructive evaluation of Space Shuttle structures. The key testing projects are summarized, including the orbiter acoustic fatigue certification tests, the Enterprise modal tests, Cessna airframe modal tests, and the Columbia and Discovery body flap tests. The SMIS is described, and the current implementation procedure for orbiter inspections is outlined, including operational constraints, system performance, and future developments.

Grygier, et al. (1994) report the results of a SMIS test on the control surfaces and wings of Orbiter Vehicle 102 (OV-102) following flight STS-65. The tests were conducted in Bay One of the Orbiter Processing Facility. The test results showed a decrease of 2% in the modal frequency of the first two spanwise bending modes of the orbiter body flap. Also, testing of the vertical tail structure demonstrated less than 0.5% frequency change in over half of the modes, and no more than 1.5% frequency change on any of them. The first two symmetric bending modes of the wings showed a fre-
quency decrease of 1.6%, and the first bending mode of the elevons showed a frequency decrease of 3%. The results of this test will be used as a baseline for future tests, and the measured changes in frequencies provide information about specific areas of the structure which should be monitored and inspected more closely.

Kim, et al. (1994) present a free-decay modal identification method for health monitoring of aerospace structures using time-domain zooming and random data processing techniques. The authors also determined the frequency changes resulting from propellant mass loss using flight data from a Delta II launch vehicle.

Kim, et al. (1995b) present the integrated damage-detection procedure and software, which allows spacecraft operations personnel to detect the structural damage in real time without an extensive background in structural damage detection. The authors demonstrated the procedure with an overlay of the damage on a structural diagram using data from the NASA LaRC 10-bay truss damage-detection experiment.

Robinson, et al. (1996) and Doebling (1995) analyzed modal test results to locate damage in a stringer of a DC-9 aircraft skin support structure. The test structure is the forward section of a DC-9 fuselage in the Aging Aircraft Non-Destructive Testing Center in Albuquerque, N.M., which is a joint project between the Federal Aviation Administration and Sandia National Laboratories. The stringer had cross-sectional cuts applied to it incrementally for a total of four damage cases. The undamaged and damaged data sets were analyzed using flexibility analysis, and the changes in point flexibilities clearly indicated the damage location for the cases when the stringer cross-sectional stiffness was reduced significantly. For the smaller damage cases, the damage indication was less exact. Comparisons were made between two damage identification methods: a method using direct assessment of the point flexibility change, and a method utilizing disassembly of the measured flexibility matrix under a presumed elemental connectivity. The direct comparison of point flexibility changes provided more accurate damage identification. It is assumed by the authors that the errors in the flexibility disassembly technique are due to both the inadequacy of the simple assumed elemental connectivity and the fact that measured flexibility matrix was not statically complete.

3.1 COMPOSITES

Lifshitz and Rotem (1969) measured resonant frequencies and damping on quartz-particle-filled epoxy and polyester. Damage was induced via static loading, and measurements were taken continuously during the loading process. The authors found damping to be more sensitive to damage than changes in the dynamic moduli.

Schultz and Warwick (1971) measured the forced vibration response of glass-fiber-reinforced epoxy beams at various intervals during fatigue loading. They found changes in the resonant frequencies to be relatively insensitive to damage. They also found, however, that the damping ratio and response magnitude were very sensitive to damage.
Adams, et al. (1975) tested unidirectional carbon-fiber- and glass-reinforced plates to attempt to detect damage after both static and fatigue torsional loading. The main mode of failure was matrix shear cracking. They found damping changes to be more sensitive than frequency shifts for detecting the onset of damage. They also noted that some changes in dynamic properties in the early stages of damage could be recovered after a rest period.

Cawley and Adams (1979b) apply a frequency-shift-based damage detection routine to several damage cases in composite materials. The experimentally implemented damage included holes, saw cuts, crushing with a ball bearing, local heating with a flame, and impact. Specimens included unidirectional and cross-ply carbon-fiber-reinforced plastic (CFRP) plates and honeycomb panels with CFRP faces. They were able to locate low levels of damage accurately. This successful location did require, however, controlled temperatures and testing of the undamaged and damaged specimens in a short amount of time to prevent long-term frequency shift.

Reddy, et al. (1984) experimentally measured natural frequencies in composite plates containing delaminations. The plates were graphite fiber/epoxy resin panels with a $[\pm 45,0,90]_{2s}$ layup. Known delaminations comprising 10% of the total panel area were implanted by inserting fiberglass fabric sandwiched by Teflon. They found the resonant frequencies to be insensitive to the delaminations, even looking at the higher modes.

Lee, et al. (1987) looked at the damping loss factor as a possible indicator for detecting and locating four types of damage in composites. In all cases, 150 mm (6 in.) x 25 mm (1 in.) cantilever beams were excited with a force hammer. A non-contact motion transducer was used to measure displacement at the free end of the beam. The first damage case involved milling two notches, which were symmetric about the neutral axis of the cross section, into beams of SMC R50 chopped-fiber composite, unidirectional Kevlar/epoxy, and a hybrid composite made of both randomly oriented chopped fibers and continuous glass fibers. In all cases the change in the damping loss factor was more sensitive to damage than the frequency change was. Notches that removed less than 5% of the cross-sectional area were difficult to detect. Since the damping change depended on notch location and mode number, it was postulated that locating the crack from damping measurements is possible. The second damage case involved matrix cracking in the 90 degree plies of a $[90_2/0_{19}/90_2]$ glass/epoxy composite. The cracking was introduced by bending from 30% to 90% of the ultimate bending strength. No measurable changes in damping were detected as a result of the matrix cracks. The third damage case involved creating delaminations by gluing together two pieces of $[0_3]$ glass/epoxy and leaving particular regions unglued. The damping loss factor was found to be sensitive to this type of damage, with the sensitivity apparently increasing for higher modes. The fourth damage case involved manufacturing an inclusion by including pieces of mylar in a $[90_5/0_4/90_3]$ glass/epoxy composite. The damping change in this case was very small.

Tracy and Pardoen (1989) experimentally examined frequency shifts in graphite-epoxy laminates with midplane delaminations. Delaminations were manufactured by in-
cluding various lengths of 0.013 mm (511 x 10^{-6} in.) fluorinated ethylene propylene tape at the midplane of the 16-ply 2.4 mm (0.1 in.) laminate with a [(90±45/0)_{2}]_{sym} stacking sequence. Beams 287 mm (11.3 in.) long by 25 mm (1 in.) wide were tested in a pinned-pinned configuration. The resulting frequency shifts corresponded well with the analytical predictions.

Paolozzi and Peroni (1990) calculated frequency shifts resulting from debonding in a composite structure using finite elements. The structure was a panel with a honeycomb core and CFRP faces. Several different FEMs were considered. A general conclusion was that the maximum frequency shifts occurred in the modes where the wavelength was approximately the same as the size of the debonding area.

Engblom and Havelka (1991) quantified, both numerically and experimentally, the effects of fiber breakage, matrix cracking, local buckling, and delamination based on variations in stiffness and damping characteristics. The authors also developed an interactive procedure and software for easier damage detection in composite structures.

Alvelid and Gustavsson (1992) use FRFs, cross-correlation functions, and local impedance measurements to characterize the effects of damage on the dynamic response of a glass-fiber-reinforced plastic plate. Of the methods implemented, they recommend the use of FRFs to locate damage. The authors state that the advantages lie in the global nature of the response contained in the FRF, and in the fact that the size of the damage can be determined from the wavelength of the mode affected the most by the damage.

Sanders, et al. (1992) measured modal parameters on damaged graphite/epoxy [0/90]_{s} beams of approximate dimensions 304 mm (12 in.) x 26 mm (1 in.) x 1.1mm (0.043 in.). Damage was induced by tensile loading the beams to 60%, 75%, and 85% of the ultimate tensile strength. Damage was predicted using a sensitivity method and the measured frequencies. Because the measured mode shapes were of poor resolution, they were not used in the prediction. Results agreed well with independently obtained results based on static stiffness measurements and crack densities from edge replication. Because this damage was approximately uniform throughout the beam, the ability of the method to localize damage was not demonstrated.

Nokes and Cloud (1993) used laser Doppler vibrometry and electronic speckle pattern interferometry to measure modal parameters on a composite beam at high frequencies (up to 10 kHz) with high spatial resolution. The beams were glass/epoxy cross-ply beams with dimensions of 2.5 mm (0.1 in.) x 25mm (1 in.) x 135 mm (5.3 in.). They found the damping loss factor to be a sensitive indicator of global material damage. They also found the higher torsion modes to be especially sensitive to local damage.

Saravanos, et al. (1994) used their theory to numerically predict the output of embedded piezoelectric sensors in [p/0/90/0/90]_{s} composite laminates with delaminations. The results indicated the ability to locate delaminations of various sizes and in various locations, including on and off the midplane of the laminate.
CRITICAL ISSUES FOR FUTURE RESEARCH IN DAMAGE IDENTIFICATION AND HEALTH MONITORING

This section contains a summary of the critical issues, as perceived by the authors, in the field of structural damage identification and health monitoring. The importance of many of these issues is suggested in the reviewed publications or was discussed by the participants of the LANL Damage Identification workshop. The purpose behind this section is to focus on the issues that must be addressed by future research to make the identification of damage using vibration measurements a viable, practical, and commonly implemented technology.

One issue of primary importance is the dependence on prior analytical models and/or prior test data for the detection and location of damage. Many algorithms presume access to a detailed FEM of the structure, while others presume that a data set from the undamaged structure is available. Often, the lack of availability of this type of data can make a method impractical for certain applications. While it is doubtful that all dependence on prior models and data can be eliminated, certainly steps can and should be taken to minimize the dependence on such information.

Almost all of the damage-identification methods reviewed in this report rely on linear structural models. Further development of methods that have an enhanced ability to account for the effects of nonlinear structural response has the potential to enhance this technology significantly. An example of such a response would be the opening and closing of a fatigue crack during cyclic loading, in either an operational situation or in the case of a forced-vibration test. Many methods are inherently limited to linear model forms and, therefore, cannot account for the nonlinear effects of such a damage scenario. Another advantage of methods that detect nonlinear structural response is that they can often be implemented without detailed prior models.

The number and location of measurement sensors is another important issue that has not been addressed to any significant extent in the current literature. Many techniques that appear to work well in example cases actually perform poorly when subjected to the measurement constraints imposed by actual testing. Techniques that are to be seriously considered for implementation in the field should demonstrate that they can perform well under the limitations of a small number of measurement locations, and under the constraint that these locations be selected a priori.

An issue that is a point of controversy among many researchers is the general level of sensitivity that modal parameters have to small flaws in a structure. Much of the evidence on both sides of this disagreement is anecdotal because it is only demonstrated for specific structures or systems and not proven in a fundamental sense. This issue is important for the development of health-monitoring techniques because the user of such methods needs to have confidence that the damage will be recognized while the structure still has sufficient integrity to allow repair. A related issue is the discernment of changes in the modal properties resulting from damage from those resulting from statistical variations in the measurements: a high level of uncertainty in the measurements will prevent the detection of small levels of damage.
With regards to long-term health monitoring of structures such as bridges and offshore platforms, the need to reduce the dependence upon measurable excitation forces is noted by many researchers. The ability to use vibrations induced by ambient environmental or operating loads for the assessment of structural integrity is an area that merits further investigation.

The literature also has scarce instances of studies where different health-monitoring procedures are compared directly by application to a common data set. Some data sets, such as the NASA 8-Bay DSMT data set and the I-40 Bridge data set, have been analyzed by many different authors using different methods, but the relative merits of these methods and their success in locating the damage have not been directly compared in a sufficiently objective manner.

Overall, it is the opinion of the authors that sufficient evidence exists to promote the use of measured vibration data for the detection of damage in structures, using both forced-response testing and long-term monitoring of ambient signals. It is clear, though, that the literature in general needs to be more focused on the specific applications and industries that would benefit from this technology, such as health monitoring of bridges, offshore oil platforms, airframes, and other structures with long design life. Additionally, research should be focused more on testing of real structures in their operating environment, rather than laboratory tests of representative structures. Because of the magnitude of such projects, more cooperation will be required between academia, industry, and government organizations. If specific techniques can be developed to quantify and extend the life of structures, the investment made in this technology will clearly be worthwhile.
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A. Emin Aktan, University of Cincinnati
David Allen, Texas A&M University
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Thomas Baca, Sandia National Laboratories
R. G. Bell, Sandia National Laboratories
Jim Bryant, TRACOR
Robert Carlin, Vanderbilt University
Thomas Carne, Sandia National Laboratories
Lowell Cogburn, Association of American Railroads
Phil Cornwell, LANL
Thomas Duffey, Consulting Engineer
David Ewins, Imperial College of Science
Carlos Ferregut, The University of Texas at El Paso
Michael Grygier, NASA Johnson Space Center
David Hayden, LANL
Gwanghee Heo, University of New Mexico
Norm Hunter, LANL
George James, Sandia National Laboratories
Frederick Ju, University of New Mexico
Hyoung-Man Kim, McDonnell Douglas Aerospace
Anne Kiremidjian, Stanford University
Scott Klenke, Sandia National Laboratories
John Kosmatka, University of California - San Diego
Takashi Kuroda, Shimizu Corporation
Brett Lewis, APTEK, Inc.
Dave Martinez, Sandia National Laboratories
Sami Masri, University of Southern California
Randy Mayes, Sandia National Laboratories
Gerald Mok, Lawrence Livermore National Laboratory
Denby Morrison, Shell E&P Technology Company
Haruyuki Namba, Shimizu Corporation
Roberto Osegueda, The University of Texas at El Paso
Loukas Papadopoulos, Vanderbilt University
K. C. Park, University of Colorado
Lee Peterson, University of Colorado
Mark Richardson, Vibrant Technology, Inc.
James Ricles, Lehigh University
Masoud Sanayei, Tufts University
Craig Schoof, EQE International, Inc.
James Sirkis, University of Maryland
Suzanne Smith, University of Kentucky
Erik Straser, Stanford University
Norris Stubbs, Texas A&M University
Arisi Swamidian Swamidas, Memorial University of Newfoundland
Ming Wang, University of New Mexico
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NOMENCLATURE AND CONVENTIONS

\{
\}\quad \text{Vector}

\[
\]
\quad \text{Matrix}

\{ \}^T, [\]^T \quad \text{Transposed vector or matrix}

\omega_i, f_i \quad i^{th} \text{Modal frequency (rad/s, Hz)}

\zeta_i \quad i^{th} \text{Modal damping ratio (viscous)}

m \quad \text{Number of included/measured modes}

p \quad \text{Number of current structural member}

i \quad \text{Number of current structural vibration mode}

q \quad \text{Number of current structural DOF}

\{ \phi \}_i \quad i^{th} \text{Mass-normalized mode shape vector}

[\Phi] \quad \text{Mass-normalized mode shape matrix}

\lambda_i \quad \text{Complex eigenvalue for } i^{th} \text{ mode}

\{ \psi \}_i \quad i^{th} \text{ Arbitrarily normalized mode shape vector}

[\Psi] \quad \text{Arbitrarily normalized mode shape matrix}

[M] \quad \text{Global mass matrix}

[C] \quad \text{Global viscous damping matrix}

[K] \quad \text{Global stiffness matrix}

[\Delta M] \quad \text{Global mass matrix perturbation}

[\Delta C] \quad \text{Global viscous damping matrix perturbation}

[\Delta K] \quad \text{Global stiffness matrix perturbation}

[k]_p \quad p^{th} \text{Elemental stiffness matrix}

[m]_p \quad p^{th} \text{Elemental mass matrix}

k_i \quad i^{th} \text{Modal stiffness}

m_i \quad i^{th} \text{Modal mass}

\{ f(t) \} \quad \text{Applied force vector}

\{ E_i \} \quad \text{Modal force error (or residual force) for } i^{th} \text{ mode}

[G] \quad \text{Structural flexibility matrix } ([K]^{-1})

[L] \quad \text{Best achievable eigenvector matrix}
Modal Assurance Criterion (MAC)

\[
MAC(\{\phi\}_p, \{\phi\}_j) = \frac{[\{\phi\}_p^T \{\phi\}_j]^2}{(\{\phi\}_p^T \{\phi\}_j)(\{\phi\}_p^T \{\phi\}_j)}
\]

Coordinate MAC (COMAC)

\[
COMAC(\{\phi^u\}_L, \{\phi^d\}_L, q) = \frac{\left(\sum_{L=1}^{L_{\text{max}}} |(\phi^u_L)(\phi^d_L)|^2\right)^2}{\left(\sum_{L=1}^{L_{\text{max}}} (\phi^u_L)^2\right)\left(\sum_{L=1}^{L_{\text{max}}} (\phi^d_L)^2\right)}
\]

Superscripts
- \(u\): Undamaged properties, or data from undamaged structure
- \(d\): Damaged properties, or data from damaged structure
- \((n)\): Value resulting from iteration \(n\)

Subscripts
- \(A\): Analytically or theoretically predicted values
- \(E\): Elemental (member) parameters
- \(X\): Experimentally measured values
- \(L\): Index of correlated mode pairs
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ADDITIONAL SOURCES


R. Cantieni  
Uberlandstrasse 129  
Dubendorf, CH-8600  
Switzerland

Steve Chase  
Federal Highway Administration  
6300 Georgetown Pike  
McLean, VA 22101-2296

Franklin Cheng  
University of Missouri Rolla  
Dept. of Civil Eng.  
Rolla, MO 65401

Ken Chong  
National Science Foundation  
4201 Wilson Blvd., Rm. 545  
Arlington, VA 22230

Lowell Cogburn  
Association of American Railroads  
P.O. Box 11130  
Pueblo, CO 81001

Roy R. Craig, Jr.  
University of Texas at Austin  
Aerospace Engineering and Engineering Mechanics Dept., Mail Code C0600  
Austin, TX 78712–1085.

Tim Darling  
Los Alamos National Laboratory  
MST-10, MS K764  
P.O. Box 1663  
Los Alamos, NM 87545

Barry Davidson  
Compusoft Engineering Ltd.  
PO Box 9493  
Newmarket, Auckland  
New Zealand

Moshe M. Domb  
deHavilland Inc.  
Structures Research and Development  
123 Garratt Blvd., MS N18-06  
Downsview, Ontario  
Canada, M3K 1Y5

Thomas Duffey  
P.O. Box 1239  
Tijeras, NM 87059

Dave Ewins  
Imperial College  
Mechanical Engineering Dept.  
Exhibition Road  
London SW7 2BX, UK

Richard Fale  
W. S. Atkins  
160 Aztec West  
Park Avenue  
Almondsburg  
Bristol U.K. BS124TG

M. Ferner  
Anlauf Ingenieur - Consulting GMBH  
Posttach 101259  
D-69002 Heidelberg, Germany

Gongkang Fu  
New York State Dept. of Transportation  
1220 Washington Ave.  
Albany, NY 12232

Chris Gannon  
Penguin Engineering Ltd  
PO Box 33 093  
Petone, New Zealand

F. Gantenbein  
C.E.A.-CE/ Saclay - DMT/SEMT/EMS  
91191 - GIF - SUR - YVETTE Cedex  
France

Ephrahim Garcia  
Vanderbilt University  
Dept. of Mechanical Engineering  
Box 1592, Station B  
Nashville, TN 37235

Michael Grygier  
NASA Johnson Space Center  
ES43  
Houston, TX 77058
Ozden O. Ochoa  
Offshore Technology Research Center  
1200 Mariner Dr.  
Texas A&M University  
College Station, TX 77845

Roberto Osegueda  
The University of Texas at El Paso  
Department of Civil Engineering  
El Paso, TX 79912

Gerard Pardoen  
University of California-Irvine  
101 ICEF-Civil Eng.  
Irvine, CA 92717

K. C. Park  
Center for Aerospace Structures  
University of Colorado, Boulder  
Campus Box 429  
Boulder, CO 80309-0429

Bruno Piombo  
Departmento di Meccanica  
Politecnico di Torino  
Corso Duca degli Abruzzo, 24  
1-10129 Torino, Italy

Lee Peterson  
Center for Aerospace Structures  
University of Colorado, Boulder  
Campus Box 429  
Boulder, CO 80309-0429

Darryll J. Pines  
Dept. of Aerospace Engineering  
Room 3154 Engineering Classroom Bldg  
University of Maryland  
College Park, MD 20742

Ron Polivka  
EQE, Inc.  
44 Montgomery St., Suite 3200  
San Francisco, CA 94104

Don Rabern  
Los Alamos National Laboratory  
ESA-EA, MS P946  
P.O. Box 1663  
Los Alamos, NM 87545

John Reed  
Jack Benjamin & Associates  
Mountain Bay Plaza  
444 Castro St. Suite 501  
Mountain View, CA 94041

Mark Richardson  
Vibrant Technology, Inc.  
18141 Main Street  
Jamestown, CA 95327

Jim Ricles  
Lehigh University  
Department of Civil Engineering  
117 AT LSS Drive, H Building  
Bethlehem, PA 18015-4729

David Robert  
Litton Laser Systems / CLS Operations  
5 Jeffrey Drive  
South Windsor, CT 06074

S. Rubin  
Aerospace Corporation  
M.S. M4-899  
Los Angeles, CA 90009-2957

John Ruminer  
Los Alamos National Laboratory  
DDESA, MS P945  
P.O. Box 1663  
Los Alamos, NM 87545

Romualdo Ruotolo  
Dip. Ingegneria Aeronautica e Spaziale  
Politecnico di Torino  
10100 Torino  
Italy

Anders Rytter  
RAMBOLL  
Kjaerulfsgade 2  
DK-9400 Norresundby  
Denmark

Erdal Safak  
U.S. Geological Survey  
DFC, Box 25046, MS.966  
Denver, CO 80225
M. Saiidi
College of Engineering
Department of Civil Eng./258
Reno, NV 89557-0152

Mike Salmon
EQE Engineering, Inc.
Lakeshore Towers
18101 Von Karman Ave., Suite 400
Irvine, CA 92715

Masoud Sanayei
Tufts University
Dept. of Civil and Env. Engineering
Lexington, MA 02173

Jose Maria Campos dos Santos
UNICAMP
Caixa Postal 6122
13083-970 Campinas, SP
Brazil

Jim Sirkis
University of Maryland
SMART Materials and Structures Res. Center
College Park, MD 20742

Paul Smith
Los Alamos National Laboratory
ESA-MT, MS C931
P.O. Box 1663
Los Alamos, NM 87545

Christopher Smith
FAA Technical Center
Atlantic City Int'l Airport, NJ 08405

Suzanne Smith
University of Kentucky
Department of Engineering Mechanics
467 Anderson Hall
Lexington, KY 40506-0046

Cecily Sobey
Earthquake Engineering Research Center Library
Gift & Exchange Dept.
University of California/RFS 453
1306 South 46th Street
Richmond, CA 94804-4698

Mete Sozen
2113 Newmark Civil Engineering Lab.
208 North Romine Street
Urbana, IL 61801

John Stevenson
Stevenson & Associates
9217 Midwest Avenue
Cleveland, OH 44125

Norris Stubbs
Texas A&M University
Department of Civil Engineering
Mechanics & Materials Center
College Station, TX 77843-3136

Arisi Swamidas
Memorial University of Newfoundland
St. Johns Facility of Engineering
Newfoundland, Canada, A1B 3X5

Yvette Tang
National Research Council
NASA Langley Research Center
Mail Stop 463
Hampton, VA 23681-0001

Fred Tasker
Dept. of Mechanical Engineering
University of Maryland Baltimore County
Baltimore, MD 21228-5398

Geof. Tomlinson
The University of Sheffield
Department of Mechanical and Process Engineering
PO Box 600
Mappin St
Sheffield S1 4DU

126
Ward Turner  
Exxon Production Research Company  
P.O. Box 2189  
Houston, TX 77252

C. E. Ventura  
The University of British Columbia  
Dept. of Civil Engineering  
2324 Main Mall  
Vancouver, B.C.  
Canada, V6T 1Z4

Sara Wadia-Fascetti  
Northeastern University  
Dept. of Civil Engineering  
443 Snell Engineering Center  
Boston, MA 02115

Ming Wang  
University of New Mexico  
Department of Civil Engineering  
209 Tapy Hall  
Albuquerque, NM 87131

Lloyd Welker, Jr.  
Ohio Department of Transportation  
25 South Front Street  
Columbus, OH 43216-0899

Robert West, Jr.  
Virginia Polytechnic Institute of State Univ.  
Structural Imaging and Modal Analysis Lab.  
Mechanical Engineering Department  
Blacksburg, VA 24061-0238

Ed White  
McDonnell Douglas Aerospace  
Mailcode 1021310  
P.O. Box 516  
St. Louis, MO 63166

Ken White  
Dept. of Civil, Agricultural, and Geological Eng.  
Box 30001/Dept. 3CE  
Las Cruces, NM 88003-0001

P. Winney  
P&P Engineering  
Consultant Engineers  
P.O.Box 36  
Billingshurst, West Sussex RH14 OYG

Felix S. Wong  
Weidlinger Associates  
4410 El Camino Real, Suite 110  
Los Altos, CA 94022

J. T. P. Yao  
Dept. of Civil Engineering  
Texas A&M University  
College Station, TX 77843-3136

Bill Young  
W. P. Young Construction, Inc.  
426 Lincoln Centre Drive  
Foster City, CA 94404-1127

Dave Zimmerman  
University of Houston  
Department of Mechanical Engineering  
Houston, TX 77204-4792