Incoherent and Coherent Tune Shifts*

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Abstract

The longitudinal and transverse microwave instabilities are the two important limiting factors for the performance of an accelerator. Comparing with the fairly unified approach for the longitudinal microwave instabilities, different approaches have been used to define the transverse microwave instabilities. One reason of this is related to the role played by the space charge incoherent and coherent tune shifts. In this article, the transverse microwave instabilities will be discussed by defining separately the roles of the space charge incoherent and coherent tunes, which are represented by the space charge transverse impedances. Preliminary results for the AGS as proton driver are presented by using this approach.

1 Introduction

The longitudinal and transverse microwave instabilities are the two fundamental performance limiting factors for an accelerator. The longitudinal microwave instability threshold is usually obtained by using the Keil-Schnell and the Boussard criteria. The approach is fairly unified, and the obtained results are quite consistent. The approaches to the transverse microwave instability are usually not that unified, and the obtained results are somewhat diversified. One of the reasons is that the role played by the space charge incoherent and coherent tune shifts are not clearly defined, which can be shown by the conventionally defined transverse space charge impedance. This problem is probably not very important for higher energy machines, because of the relatively small space charge effects. It is, however, important for the machines operated at low or medium energy, such as the AGS. In this article, we separately define the incoherent and coherent space charge transverse impedances. The coherent space charge impedance, together with the broad band impedances, is responsible for the beam coherent motion, and the incoherent impedance, together with the chromatic and octupolar tune spread, etc., contributes to the transverse Landau damping. A preliminary study for the AGS as proton driver will be presented by using this approach.

2 Transverse Impedance

Derived from the Vlasov equation, taking $m = 0$ mode and also the first orthogonal polynomial in the expansion of radial modes, the transverse instability is determined by [1],

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Abstract

The longitudinal and transverse microwave instabilities are the two important limiting factors for the performance of an accelerator. Comparing with the fairly unified approach for the longitudinal microwave instabilities, different approaches have been used to define the transverse microwave instabilities. One reason of this is related to the role played by the space charge incoherent and coherent tune shifts. In this article, the transverse microwave instabilities will be discussed by defining separately the roles of the space charge incoherent and coherent tunes, which are represented by the space charge transverse impedances. Preliminary results for the AGS as proton driver are presented by using this approach.

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2 Transverse Impedance

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$$\omega - \omega_\beta = \frac{jeI_0}{2Rm_0\gamma\nu_0\omega_0} \sum_{p=-\infty}^{\infty} Z_T(p)(\Lambda_0^{(0)}(p))^2 \quad (1)$$

where ω_β and ω_0 are the betatron and revolution frequencies, respectively, I_0 is the average beam current, and m_0 is the rest mass of proton. Also R is the machine radius and ν_0 is the betatron tune. The transverse impedance $Z_T(p)$ is sampled at the harmonic number p , and $\Lambda_0^{(0)}(p)$ is the bunch spectrum of the first orthogonal polynomial for $m = 0$ mode.

For microwave instability, we consider only the space charge and broad band impedances. The transverse broad band impedance is obtained by using a rough estimate from the longitudinal counterpart $Z_{LBB}(p)/p$,

$$Z_{TBB}(p) = \frac{2c}{b^2\omega_0} \frac{Z_{LBB}(p)}{p} \quad (2)$$

where b is the radius of the vacuum chamber. At low frequency, $Z_{LBB}(p)/p$ is inductive, therefore, $Z_{TBB}(p)$ is inductive.

The conventional space charge impedance is defined as proportional to the difference between the coherent and incoherent tune shifts [2,3],

$$Z_{TSC} = j \frac{RZ_0}{\beta^2\gamma^2} \left(\frac{1}{b^2} - \frac{1}{a^2} \right) \quad (3)$$

where Z_0 is the impedance of free space, a is the radius of the beam. Since $a < b$, this impedance is capacitive.

Let us define the coherent and incoherent space charge transverse impedances as follows,

$$Z_{TSCcoh} = j \frac{RZ_0}{\beta^2\gamma^2 b^2} \quad (4)$$

and

$$Z_{TSCinc} = j \frac{RZ_0}{\beta^2\gamma^2 a^2} \quad (5)$$

where both impedances are inductive.

In the following, we show that substituting the incoherent and coherent impedances (4,5) into the beam dynamic equation (1) yields the same incoherent and coherent tune shifts obtained by using the conventional Laslett approach.

For circular chamber, the incoherent and coherent tune shifts can be calculated by [4,5],

$$\Delta\nu_{coh} = \frac{-NRr_0}{\pi\nu_0 B_f \beta^2 \gamma^3} \left(\frac{1}{b^2} \right) \quad (6)$$

and

$$\Delta\nu_{inc} = \frac{-NRr_0}{\pi\nu_0 B_f \beta^2 \gamma^3} \left(\frac{1}{a^2} \right) \quad (7)$$

where N is the number of particles, r_0 is the classical radius of proton, and B_f is the bunching factor.

For space charge and broad band impedances, $Z_T(p)$ can be taken out of the summation, therefore for a Gaussian distribution we can use

$$\sum_{p=-\infty}^{\infty} (\Lambda_0^{(0)}(p))^2 \approx \frac{1}{\pi^{3/2} B_f} \quad (8)$$

Substituting, say, Z_{TSCcoh} of (4) into (1), using (8) and the following relations,

$$I_0 = \frac{Ne\omega_0}{2\pi} \quad (9)$$

and

$$Z_0 = \frac{1}{\epsilon_0 c} \quad (10)$$

where ϵ_0 is the permittivity in free space, and also

$$r_0 = \frac{1}{4\pi\epsilon_0} \frac{e^2}{m_0 c^2} \quad (11)$$

we get the space charge coherent tune shift, which differs from the one obtained using (6) by a factor of 1.13. Similar result can be obtained for the incoherent tune shift. This shows that applying the space charge incoherent and coherent transverse impedances (4,5) into (1) generates consistent results with other approaches. Therefore, the transverse coherent frequency shifts and the Landau damping can be studied in a more unified formulation based on the equation (1).

Substituting Z_{TSC} of (3) into (1), and writing on the left side by following convention ,

$$\omega_\beta = \omega_{\beta 0} + \Delta\nu_{inc}\omega_0 \quad (12)$$

and also replacing $\omega_\beta \approx \omega_{\beta 0}$ on the right side, we get

$$\omega - \omega_{\beta 0} - \Delta\nu_{inc}\omega_0 = \frac{jeI_0}{2Rm_0\gamma\omega_{\beta 0}} \frac{1}{\pi^{3/2} B_f} (Z_{TSCcoh} - Z_{TSCinc}) = \Delta\nu_{coh}\omega_0 - \Delta\nu_{inc}\omega_0 \quad (13)$$

where we observe that the incoherent tune shift is cancelled from both sides. This shows that ideally the incoherent tune shift plays no role in the transverse coherent motion. One reminds that in the longitudinal case, the space charge induced incoherent frequency shift directly affects the longitudinal focusing and therefore it has to be included in the longitudinal coherent motion. In addition, in the longitudinal case, the broad band impedance also gives rise to the incoherent frequency shift, while in the transverse case, the broad band impedance only contributes to the coherent tune shift. Therefore, the scenarios are quite different for the longitudinal and the transverse motions.

Note that the exact physical implication of applying the transverse impedance defined in (3) in a beam dynamic equation is not clear. A more straightforward approach is to use the impedances separately defined in (4,5). In this way, the space charge coherent impedance, together with the broad band impedances, is responsible for the beam coherent tune shift, and the space charge incoherent impedance, together with the chromatic and octupolar effects, etc, is responsible for the tune spread.

We note also that sometimes the transverse space charge impedance defined in (3) is used for the beam coherent tune shift, which is not only quantitatively incorrect, but also gives rise to a tune shift that is in the opposite polarity. For example, using the equation (6), or applying (4) into (1), we get the same space charge coherent tune, which is decreased. Applying (3) into (1), the obtained coherent tune is increased.

3 Transverse Landau Damping

Following the rule of thumb, i.e. the coherent tune shift must be smaller than the tune spread for the effective Landau damping, the transverse microwave instability can be determined by,

$$\Delta\nu > \frac{eI_{peak}}{4\pi Rm_0\gamma\omega_\beta\omega_0} |Z_T(p)| \quad (14)$$

where $\Delta\nu$ is the tune spread, and

$$I_{peak} = \frac{2\sqrt{2}}{\sqrt{\pi}B_f} I_0 \quad (15)$$

is the peak current for a Gaussian distribution.

Now for the coherent tune shift we use the space charge coherent impedance and the broad band impedance. For incoherent tune shift, we consider the space charge incoherent impedance, and also others, such as the chromatic and the octupolar tune spreads, and also the synchrotron oscillation frequency. The chromatic tune spread is not very effective for the weak instabilities with the growth rate comparable to the synchrotron oscillation period, it is effective, however, for the fast instabilities. The synchrotron tune is effective in stabilizing the weak instabilities.

Below transition at the lower energy, the space charge induced tune shift is large compared with contributions of the chromatic, the octupolar tune spread and the synchrotron frequency. The space charge incoherent tune spread therefore provides Landau damping. Since this tune spread is quite large, therefore the transverse microwave instability is not of serious concern at the low energy.

Above transition at the higher energy, both the incoherent and coherent space charge effects have been substantially reduced, the broad band impedance contributed coherent tune shift, therefore, becomes dominant. The stabilizing force has to come from other than the space charge incoherent tune spread. This implies that the situation above transition is much more serious than the one below transition.

Strictly speaking, only the chromatic and the octupolar tune spreads, and also the synchrotron oscillation frequency, which are not dependent on the beam current, can contribute to the microwave instability threshold. On the other hand, the space charge incoherent tune spread depends on the beam current, therefore it affects the growth rate. This situation is similar to the longitudinal case, where the incoherent frequency shifts affect the growth rate and the RF nonlinearity induced synchrotron oscillation frequency spread gives rise to the instability threshold.

The estimate of the microwave instability, however, is not exactly the same as the calculation of the instability threshold. In an estimate of the beam instabilities using the criterion (14), both the instability threshold and the growth rate have been accounted for. After all, the influence of the beam current on the both sides in (14) is already included in the calculation. This problem can also be looked in other ways. For example, given a beam current, the transverse beam size, which directly affects the space charge incoherent tune spread, can be seen as the variable. Thus, applying the equation (14) gives rise to an instability threshold in term of the beam size, rather than the current.

4 AGS as Proton Driver

We only present the longitudinal and the transverse microwave instabilities based on some incomplete upgrade of the AGS, using the usual Keil-Schnell criterion for the longitudinal case and the approach developed in this article for the transverse case. Under these upgrade, the repetition rate of the AGS becomes 2.5 Hz, and the RF peak voltage is increased to about 800 KV. The longitudinal broad band impedance is assumed to be a modest 30 Ω , and the longitudinal emittance is taken as 4 eVs. The 10^{14} protons are in 8 bunches, and the normalized transverse emittance including 95% particles is assumed to be 50 $\pi mm-mr$. For longitudinal microwave instabilities, the momentum spread and the required momentum spread are shown in Fig.1, where we used the 'momentum spread threshold' to indicate the required momentum spread. The valley of the momentum spread threshold at about 40 ms from the beginning of the cycle is due to the cancellation between the broad band and the space charge impedances. The peak of the momentum spread threshold indicates the transition. For transverse microwave instabilities, the required tune spread, and the space charge incoherent tune spread are shown in Fig.2, where also the chromatic tune spread is shown with $\xi = -0.2$.

The stability margins are barely met, yet the requirement for the beam for the 250 on 250 GeV muon collider proton driver is not satisfied. To meet the requirement, a bunch compression is needed. In order to do this, a new vacuum chamber, a more powerful RF system and other damping systems are required.

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Fig.1. *Longitudinal Microwave Instabilities*

Fig.2. *Transverse Microwave Instabilities*