Reducing space charge tune shift with a barrier cavity¹

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Implementation of a barrier cavity rf system appears to be a straightforward and relatively inexpensive way to increase the output current of second stage synchrotrons. This note serves as a general introduction to the relevant beam dynamics and addresses the problem of driving a cavity in the short burst mode.

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INTRODUCTION

A barrier cavity rf system (1,2) is proposed as a tool in the continuing effort to obtain higher beam currents. Unlike a single frequency rf system a barrier cavity rf system employs a single turn voltage waveform similar to that shown in Figure 1. Each of the pulses making up the waveform tends to repel particles resulting in two bunches separated by narrow gaps. By varying the amplitude and phase of the pulses some novel rf manipulations become possible.

SINGLE PARTICLE BEAM DYNAMICS

Before proceeding it will be useful to define some relevant coordinates. The azimuth θ refers to longitudinal position with respect to the physical machine and increases by 2π each turn. I assume the RF field is limited to a narrow range around $\theta = 0$, several gaps will be similar as long as the time delays are correct. The RF system creates an electric field whose component along the beam velocity is given by $V(t)\delta_p(\theta)/R$ where $V(t)$ is the gap voltage as a function of time, R is the machine radius and $\delta_p(\theta)$ is the periodic delta function. I assume a constant magnetic field and an ideal energy E_0 with corresponding angular frequency ω_0 . A comoving coordinate is defined by the relative phase $\phi = \theta - \omega_0 t$. Let ϕ_n and E_n correspond to the relative phase and the total energy a particle has just after traversing the cavity for the n th time. To leading order and neglecting collective effects,

$$\phi_{n+1} - \phi_n = -\frac{2\pi\eta(E_n - E_0)}{\beta^2 E_0} \quad (1)$$

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FIG. 1. Voltage vs. time for one revolution period. The phases and amplitudes of the two components may vary slowly from one turn to the next.

$$E_{n+1} - E_n = qV(nT_0 - \phi_{n+1}/\omega_0) \quad (2)$$

where q is the charge of the particle, η is the frequency slip factor, $\beta = v/c$ and T_0 is the revolution period. The difference equations can be approximated by differential equations. With n as the smooth time-like variable the differential equations for coordinate ϕ with conjugate momentum E can be obtained from the Hamiltonian,

$$H(\phi, E) = -\frac{\pi\eta(E - E_0)^2}{\beta^2 E_0} + U(\phi). \quad (3)$$

The potential is given by

$$U(\phi) = -q \int_0^\phi V(x/\omega_0) dx. \quad (4)$$

Slow changes in the confining potential can be included in the usual way to produce $U(t, \phi)$. As long as $U(t, \phi)$ varies slowly enough in t , the adiabatic invariant given by $J = \oint E d\phi$, where the integral is over one period of the motion, will be nearly conserved. Figure 2 shows simulation results for a possible scenario using the AGS and AGS Booster which have a 4 : 1 circumference ratio. To make the connection with adiabatic invariants visible $\sqrt{H}$ and $\sqrt{U}$ are plotted. Figure 2a shows four Booster cycles worth of particles

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outside the barrier bucket and another Booster cycle has just been injected. In Figure 2b the injected particles have been squeezed yielding a similar energy distribution for particles inside and outside the bucket. In Figure 2c the second voltage component has collapsed. In 2d this component has returned in phase with the first. In Figures 2e and f the voltage components separate and grow making room for the next Booster cycle. In plot units, the initial peak momentum is 4 while the final peak momentum is 5 and the number of particles outside the barrier increases in the same ratio. There is very little emittance dilution during the process.

MULTIPARTICLE BEAM DYNAMICS

From a multiparticle standpoint a barrier cavity rf system has several features which distinguish it from a single frequency rf system. By optimizing the injection process it is possible to reduce the peak current without significantly diluting the longitudinal emittance. Additionally, the peak current in a barrier cavity system is time dependent, growing with each transfer. Reducing the peak current results in a smaller space charge tune shift which should reduce stopband losses. As an example consider a single bunch in the Booster having a parabolic line density of full length equal to half the Booster circumference. The gap in the AGS (*cf.* Figure 2) is taken to be the same length. Figure 3 shows the peak current in the AGS in units of Booster peak current assuming that the initial filamentation of the parabolic bunch in the gap is essentially instantaneous. The initial filamentation causes a fractional emittance increase of $4/\pi$ and reduces the peak current to $2/3$ of the Booster value. Notice that the peak current for the stored bunch does not reach the initial injected intensity until the sixth Booster transfer and that the peak current for eight transfers always remains below the peak current in the Booster. After the injection process is completed the beam could be captured in a conventional rf system and quickly accelerated. The important point is that the time average space charge tune depression is greatly reduced with the barrier cavity system.

Coherent instabilities with a barrier cavity rf system are different from the sort found with a single frequency rf system. When the barrier half width is small compared to the bunch length (ϕ_b), the synchrotron tune (ν_s) is proportional to the fractional energy deviation,

$$\nu_s \approx \frac{\pi}{\phi_b \beta^2} \left| \eta \frac{E - E_0}{E_0} \right|. \quad (5)$$

The large spread in synchrotron tune implies a lot of Landau damping and in a first approximation the coasting beam stability criteria with the same peak current should give reasonable limits.

FIG. 2. Simulation of an injection scenario using a barrier cavity rf system.

FIG. 3. (Peak current in AGS)/(Peak current in Booster) for eight Booster transfers.

RF CONSIDERATIONS

Generating an rf waveform like that shown in Figure 1 can be done in at least two ways. The first way is to approximate the waveform by a truncated Fourier series and to create the net voltage per turn using several single frequency rf systems operating at several revolution frequency harmonics. This appears to be a fairly complicated solution and will not be considered further.

Another way to generate the voltage waveform in Figure 1 is to assume that only a single type of rf cavity is available and to tailor the current pulse driving the cavity to create the desired voltage. For simplicity assume that the cavity impedance can be characterized by a single resonator

$$Z(\omega) = \frac{R_s}{1 + iQ(\omega_r/\omega - \omega/\omega_r)}, \quad (6)$$

where R_s is the shunt impedance, Q is the quality factor, and ω_r is the resonant frequency. For a current pulse $I(t)$ the voltage across the cavity is given by

$$V(t) = \int_0^\infty I(t - \tau)W(\tau)d\tau, \quad (7)$$

$$W(t) = \frac{\omega_r R_s}{\tilde{\omega} Q} \frac{d}{dt} \sin(\tilde{\omega} t) e^{-\omega_r t/2Q}, \quad (8)$$

where $W(t)$ is the wake potential and $\tilde{\omega} = \omega_r \sqrt{1 - 1/4Q^2}$. The cavity need not ring after the current pulse is turned off. Let $I_0(t)$ be some easily created current pulse of finite duration. Driving the cavity with a current $I(t) = I_0(t) + \exp[-\pi/\sqrt{(4Q^2 - 1)}]I_0(t - \pi/\tilde{\omega})$ results in no cavity voltage after the current $I(t)$ stops flowing. Another way to create a voltage pulse of finite duration is to define the voltage pulse required and then, via Fourier analysis, calculate the necessary current pulse. A particularly simple result is (4)

$$I(t) = \begin{cases} I_0 \left(1 + \frac{\sin(\omega_r t)}{Q} \right), & 0 < t < 2\pi/\omega_r \\ 0, & \text{otherwise} \end{cases} \quad (9)$$

$$V(t) = \begin{cases} \frac{I_0 R_s}{Q} \sin(\omega_r t), & 0 < t < 2\pi/\omega_r \\ 0, & \text{otherwise.} \end{cases} \quad (10)$$

Note that the discontinuity of the current in equation 9 is not a fundamental problem. Convolution of the current pulse with a Bartlett window (isosceles triangle function) yields a current pulse that is continuous with continuous derivative and smooths out the voltage without creating any ringing.

The results of a hardware test are shown in Figure 4 (3). The persistent high frequency ringing after the main voltage pulse is due to a parasitic resonance and would be removed by active feedback in an operational mode.

CONCLUSIONS

A barrier cavity rf system appears feasible on both theoretical and practical grounds. The main effect of such a system is to reduce the time average of the peak current. The benefits involve a reduction in space charge tune shift and an increase in synchrotron frequency spread. Longitudinal phase space dilution is small. Since the rf system generally contributes a small fraction to the total cost of a synchrotron the installation of barrier buckets is relatively inexpensive as well.

REFERENCES

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FIG. 4. Experimental confirmation of pulsed operation using existing hardware at the AGS. The time increment is $1\mu\text{s}$ per box. The top trace is the gap voltage (1160 volts per box), while the bottom trace is the tube current (40 amps per box).

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